Calculation of the magnetic rigidity cutoff and the asymptotic cone of acceptance for the site of the Pierre Auger Observatory in Malargüe, Argentina

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Abstract: The calculation of asymptotic directions of approach of cosmic ray particles is an important tool in the determination of the rigidity cutoff and the cone of acceptance for a given geographical site. We present our results of these computations for the site of the Pierre Auger Observatory in Argentina, based on the method of M.A. Shea and D.F. Smart. These results are most useful in the study of low and mid energy phenomena observed by this detector.

Keywords: Cutoff rigidity, directions of approach, cone of acceptance

1 The equation of motion of a charged particle

The equation of motion of a charged particle inside a magnetic field is

$$m\frac{d^2\vec{R}}{dt^2}=\frac{e}{c}\left(\frac{d\vec{R}}{dt}\times\vec{B}\right),$$

where m is the particle's *inertial* mass, e is its electric charge in esu, $\vec{R}$ the position vector of the particle, c the speed of light in vacuum, $\vec{E}$ is the electric field intensity vector and $\vec{B}$ is the magnetic field intensity vector. This equation is equivalent to the following system of first order linear differential equations [2]:

$$\left.\begin{aligned}
\frac{dv_r}{dt}&=\frac{e}{mc}\left(v_\theta B_\varphi-v_\varphi B_\theta\right)+\frac{v_\theta^2}{r}+\frac{v_\varphi^2}{r},\\
\frac{dv_\theta}{dt}&=\frac{e}{mc}\left(v_\varphi B_r-v_r B_\varphi\right)-\frac{v_r v_\theta}{r}+\frac{v_\varphi^2}{r\tan\theta},\\
\frac{dv_\varphi}{dt}&=\frac{e}{mc}\left(v_r B_\theta-v_\theta B_r\right)-\frac{v_r v_\varphi}{r}-\frac{v_\theta v_\varphi}{r},\\
\frac{dr}{dt}&=v_r,\\
\frac{d\theta}{dt}&=\frac{v_\theta}{r},\\
\frac{d\varphi}{dt}&=\frac{v_\varphi}{r\operatorname{sen}\theta},
\end{aligned}\right\} \qquad (1)$$

where r is the particle's radial distance from the center of the earth; θ is the geographic colatitude; φ is the geographic longitude measured eastward; v_r, v_θ, v_φ are the velocity vector's components along the directions r, θ, φ; c is the speed of light; and B_r, B_θ, B_φ are the components of the magnetic field along the directions r, θ, φ expressed in Gauss. This system can be solved numerically if the quantities B_r, B_θ, B_φ are represented explicitly as functions of the coordinates r, θ, φ. If we assume that the earth's magnetic field is irrotational, we can define a magnetic potential U, which can be expanded into a (convergent) series of spherical harmonics:

$$U(r,\theta,\varphi)=\\ a\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left(g_n^m\cos m\varphi+h_n^m\operatorname{sen}m\varphi\right)P_n^m(\cos\theta)\left(\frac{a}{r}\right)^{n+1},$$

where g_n^m y h_n^m are the *Gauss coefficients* of the magnetic field, $P_n^m(\cos\theta)$ are partially normalized Legendre functions and a is the earth's mean radius. So, the magnetic field intensity at the point (r, θ, φ) is given by

$$\begin{aligned}
B_r&=-\frac{\partial U(r,\theta,\varphi)}{\partial r},\\
B_\theta&=-\frac{1}{r}\frac{\partial U(r,\theta,\varphi)}{\partial\theta},\\
B_\varphi&=-\frac{1}{r\operatorname{sen}\theta}\frac{\partial U(r,\theta,\varphi)}{\partial\varphi}.
\end{aligned}$$

Thus, B is known at every point and, therefore, it is possible to determine any cosmic ray trajectory solving system (1). Usually, this is done applying Runge –Kutta's second order method [2, 3, 4, 5].

2 Directions of approach

To calculate directions of approach, as is customary since the work of Sandoval-Vallarta and Lemaitre, in the equation of motion we reverse the electric charge and the sense of the motion, the equation remains unaltered; in other words, the trajectory of a departing negative particle is the same as that of an approaching positive particle. Thus, to find the direction of approach corresponding to a location P on the surface of the earth of a positive particle with given momentum and incidence direction, we have to calculate the trajectory of a negative particle with the same rigidity going away from P in an antiparallel direction. The position of P is chosen 20 km above the surface, which is the average height where air showers begin.

3 Calculation of the cutoff rigidity

To study the behavior of cosmic ray trajectories inside the earth's magnetic field, we consider particles with different energies departing from the same point. A high energy particle will reach interplanetary space without much deviation; but, as we diminish the starting energy, it will get deflected before it can escape. Below certain threshold, the particle will not have enough momentum to escape the magnetic field and it will end hitting the surface. This kind of trajectories are the *forbidden trajectories*; all other are *allowed trajectories*. The algorithm used to determine geomagnetic cutoff rigidities is as follows: we start calculating the trajectory of a particle of a given rigidity and proceed decreasing this value by a given step. We determine if the trajectory was allowed or forbidden in each case. Usually, we will get a sequence of allowed trajectories, until we get the first forbidden trajectory; this indicates a discontinuity in the directions of approach and constitutes the beginning of an interval with alternate forbidden an allowed trajectories, called the *penumbra region*. The incidence of particles whose rigidities fall into this region tends to be chaotic because there arises an instability in the system of equations (1). The most important features of the penumbra region are (a) the *upper cutoff* R_u, that is the rigidity of the last allowed trajectory before the first forbidden trajectory; (b) the *lower cutoff* R_l, that is the rigidity value from which all trajectories are forbidden, in decreasing order; and (c) the *effective cutoff* R_c, which is a weighted average between R_u and R_l that takes into account the transparency of the penumbra [1]. It has been established that the most adequate integration step for the numerical solution of system (1) is 0.01 GV, though it has been shown that finer structures in the penumbra, which need a higher resolution, can be found [5]. In this work, we chose a step of 0.001 GV. Trajectory calculations were performed using the FORTRAN program TJ2000 kindly provided to us by M.A. Shea and D.F. Smart, available in the site http://ccmc.gsfc.nasa.gov/modelweb/sun/cutoff.html. This program makes use of the epoch 2010 coefficients of the International Geomagnetic Reference Field (IGRF, http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html).

4 Results

A representation of the vertical incidence penumbral zone for the site of the Pierre Auger Observatory in Malargüe, Argentina (Lat. 35.25°S, Long. 290.75°E) is shown in figure 1. Dark strips indicate forbidden trajectories, while grey strips indicate allowed trajectories. We found a value of 9.5 GV for the effective cutoff rigidity of Malargüe. Figure 2 shows the dependence of the cutoff rigidity on zenithal distance and azimuth, first for particles approaching the site with an inclination of 16º and then with an inclination of 32º. The east-west asymmetry is evident. Dashed lines represent the width of the corresponding penumbral zones. The set of allowed trajectories accessible by charged particles at a given site on the surface of the earth is called the *asymptotic cone of acceptance*. This cone could be understood as the direction of sight of the detector. We present a plot of the cone of acceptance for Malargüe in figure 4; this plot represents the asymptotic directions of approach of vertically incident particles and also those of the particles arriving at a zenithal distance of 16°. A complementary plot (figure 5) shows the cone of acceptance for particles hitting the detector at 32º of zenithal distance. We also plotted in figure 6 the cone of acceptance for particles with rigidities inside the penumbral zone; it is interesting to note that such particles can get through the geomagnetic field almost from any direction above the earth's equator.

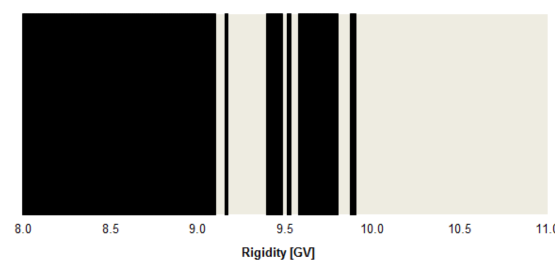

Figure 1. Vertical penumbral zone for the site of Malargüe. Black strips represent forbidden trajectories and white strips indicate allowed trajectories.

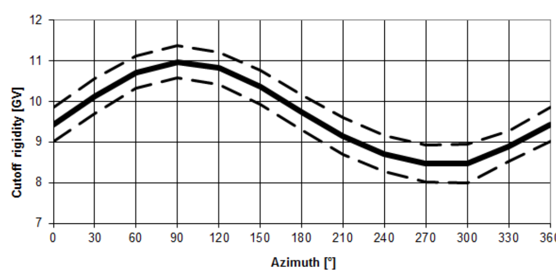

Figure 2. Dependence of the cutoff rigidity on azimuth for particles approaching the surface at an inclination angle of 16º. Dashed lines represent the width of the penumbral zone.

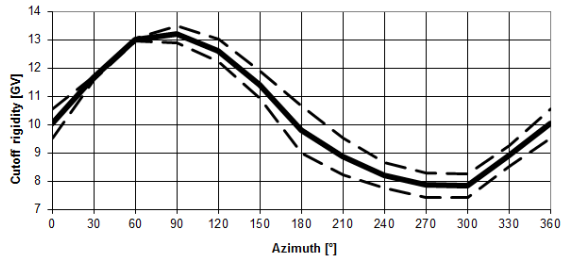

Figure 3. Dependence of the cutoff rigidity on azimuth for particles approaching the surface at an inclination angle of 32º. Dashed lines represent the width of the penumbral zone.

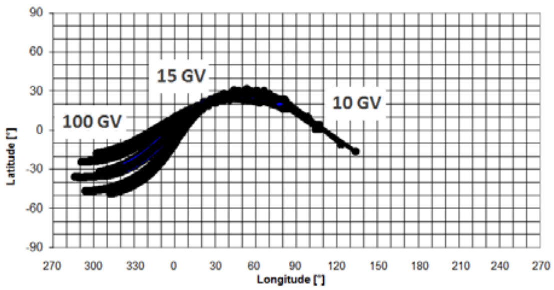

Figure 4. Asymptotic cone of acceptance for the site of Malargüe. Asymptotic directions of approach of vertically and 16º oblique incident particles are plotted. Rigidity thresholds of 10, 15 and 100 GV are indicated.

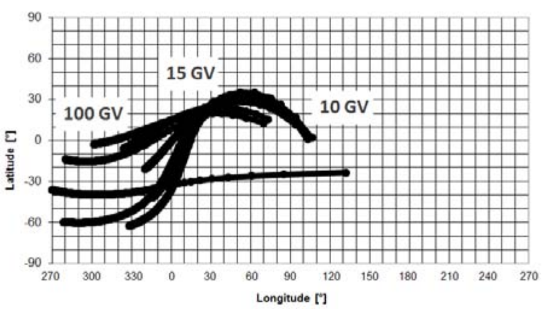

Figure 5. Asymptotic cone of acceptance for the site of Malargüe. Asymptotic directions of approach of vertically and 16º oblique incident particles are plotted. Rigidity thresholds of 10, 15 and 100 GV are indicated.

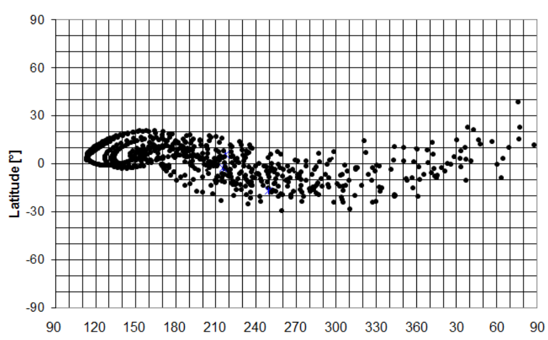

Figure 6. Asymptotic directions of approach of particles with rigidities inside the penumbral zone for the site of Malargüe.

5 References

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