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Gravitational Radiation Produced by High Energy Accelerators and High Power Lasers.

GIORDANO DIAMBRINI PALAZZI
University of Rome 'La Sapienza' and INFN (Sezione di Roma), Italy.

1. INTRODUCTION

Astrophysical impulsive events like galactic supernovae have been considered up to now as the only sources capable of producing fluxes of gravitational waves (GWs) strong enough to be detectable on the earth. Binary systems and pulsars produce continuous gravitational waves but their intensity is hard to estimate.

On one side the rarity of the supernova events and the consequent difficulty of detecting their GW signal, and on the other side the great physical interest of such a detection independently from the kind of sources involved, pushed us to study the possibility of realizing man made sources of *detectable* gravitational waves.

In fact a systematic experimental study of the GWs would allow to check the predictions of the Einstein general relativity and perhaps, in a more distant future, the quantum properties of such waves.

For example,by measuring the polarization features of the GWs, one could verify if they are transverse and traceless, which are the classical consequences of spin-2 gravitons.

In the following we will show as the today technology of the particle accelerators and the perspectives of the development of very high power Laser and Maser make the construction of sources of detectable GWs possible in a foreseeable future. One important aspect of this program would be the development of a detector capable to peak up a GW flux which is expected to be quite tiny, but also continuous, monochromatic and controllable.

Once this ambitious scientific goal was reached, a great leap forward would be achieved, greater than that realized in elementary particle physics when particle accelerators where considered instead of cosmic rays as sources of particles.

2- PRODUCTION OF GRAVITATIONAL WAVES BY PARTICLE ACCELERATORS AND BY ELECTROMAGNETIC RADIATION IN STATIC MAGNETIC FIELD.

In this paper we will make use of the results obtained first by Gertsenshtein and Pustovoit, and subsequently by others [1-6],in order to estimate the fluxes of GWs produced by circulating particles in high energy accelerators and by electomagnetic radiation interacting with a static magnetic field.

According to the results derivable from the Einstein equations in the approximation of the weak gravitational field ,the total GW power emitted by a single particle traveling in a circular orbit of radius R, due to the presence of a magnetic field H, *extended outside of the orbit area*, is given by:

$$P_{gw}=39/8\ (G/c)\, m^2\, \omega^2\, \gamma^4 + 2/3\ (G/c)\, m^2\, \omega^2\, \gamma^6\ (L/R)^2; \qquad (1)$$

L being the path length of the *electro-magnetic* (e.m.) synchrotron radiation inside the region where the magnetic field H is present.

In the following two sub-sections we will consider GW sources based on both the mass radiation and the resonant radiation , respectively related to the first and second term on the right side of eq. (1).

2a- -The Mass Radiation Term.

The first term of the eq(1) represents the direct emission of the GWs due to the mass acceleration.

This term, because of a γ^4 dependence, seems to be negligible at high energies, compared to the second one,which has a γ^6 dependence . But if the GW length is larger than the particle bunch dimensions, then the direct emission is coherent and the mass term must be multiplied by N^2 instead than by N , where N is the number of particles inside the bunch. In principle this allows to recover a factor 10^{12}-10^{14}.The angular distribution is peaked in the forward direction (so it is tangent to the orbit) inside an angle θ of the order of $\approx \gamma^{-1}$ The spectral density of GW radiation (mass term) as a function of the harmonic number n= ω/ω_0 for a particle in a circular orbit of angular speed ω_0, in the low frequency region ,where $\omega << \omega_0 \gamma^3$, is given by [4,6] :

$$P_n \approx 39/8\,(G/c)\, m^2 \omega_0^2 \gamma^2\, 1/2\{1+(-1)^n\}\, n^{-1/3} \times$$

$$\times \{1+n^{2/3}\gamma^{-2} + n^{4/3}\gamma^{-4}\} \qquad (3)$$

while for $\omega > \omega_0 \gamma^3$ the function P_n vanishes exponentially and can be neglected. Because we are interested in the energy radiated in the low frequency range of the spectrum,we sum (3) over the harmonic number from n=1 to n, and we get [6] :

$$I_n \approx 39/16(\,G/c)\, m^2 \omega_0^2 \gamma^2\, 1/2\;\; F(n,\gamma) \qquad (3)$$

where:

$$F(n,\gamma) = \{\,(3/2)\, n^{2/3} + 3n^{4/3}\,(4\gamma^2)^{-1} + n^2\,(2\gamma^4)^{-1}\,\}$$

Let us consider the case of the Large Electron Positron Storage Ring, second phase (LEP-II); assume a special circulating bunch of 10^{14} electrons , 1 cm long and with transverse dimension less than 1 mm, then by taking an upper limit for the angular speed $\omega < 10^{13}$ rad/s, we obtain for the coherent GW radiated power: $I_n \approx 10^{-18}$ erg/s $\approx 10^{-6}$ eV/s or $\approx 10^2$ gravitons per day.

For the Large Hadron Collider (LHC) which is proposed to build in the LEP tunnel,with the same cut and proton number one obtains:

$I_n \approx 5*10^{-15}$ erg/s or $\approx 3*10^{-3}$ eV/s , with more than $\approx 10^6$ gravitons per day.

It is worth noting that eq (3) is depending on the square of the particle energy E^2, through to the factor $m^2\gamma^2$, whereas, for a fixed energy, the factor $F(n,\gamma)$ is almost independent of the value of γ for $\gamma^3 >> \approx n$, while it behaves as $\approx 1/\gamma^4$ for $\gamma^3 < \approx n$. Therefore in the former case, when the coherence is experimentally feasible, the GW power is essentially proportional to the product $(NE\,\omega_0)^2$.

The 20 TeV Super Conducting Super Collider (SSC) to be built in the USA, has, a larger energy E but a smaller ω_0 than the LHC owing to its larger radius. As a consequence the GW power radiated by the SSC proton bunch , turns out to be the same as that from LHC, assuming to have in both machines one circulating bunch with the same features as stated above .

The results are summarized in Table 1 , where an estimation of the GW power emitted by a bunch of particles circulating in the SPS (Super Proton Synchrotron) of CERN it has been added

These values show that such acccelerators represent the highest continuous and periodic GW sources available on the earth.

In fact let us compare the numbers of the Table1 with the two following typical examples of other possible terrestrial sources.

Firstly, let us suppose there is a massive steel rod with radius of 1 m, length 20 m, mass 490 tons rotating around its axis at an angular speed ω = 28 rad/s , i.e. near the breakdown value.Then the radiated GW power is $2*10^{-22}$ erg/s, four order of magnitude less than from LEP-II, and 7 order of magnitude less than from LHC.

As a second example, let us consider the explosion of an atomic weapon of 17 kiloton and 10% efficiency [8]. Then the GW radiated energy is $2.5*10^{-12}$ erg.Therefore it can be seen that the integrated GW power from LHC, is equivalent to the energy delivered in GWs, from *the explosion of such an atomic bomb every 8 minutes.*

Nevertheless the absolute value of the emitted power is quite tiny; moreover the GW radiation is spread out along the accelerator circunference so that the useful detectable power is further reduced and hardly could match the available detector sensitivity .The situation can be much improved if one considers to build an undulator or a free electron Laser, two devices capable to concentrate the whole produced GW power in the very forward direction, and at a preselected frequency .This important issue will be the subject of forthcoming papers

2b)-- Resonant Gravitational Radiation-.

Let's go back to the second term of Eq. (1), i.e.

$$P_{res} = 2/3 \ (G/c) \, m^2 \, \omega^2 \, \gamma^6 \ (L/R)^2 \ ; \qquad (4)$$

To understand the physics meaning of this expression when applied to the GW radiation emitted by a charged particle orbiting along a circle of radius R in an homogeneous magnetic field, we remember that the expression $R = \gamma \, mc^2/eH$ holds:, and by substitution in Eq(4) we get:

$$P_{gw} = (G/c^4) \, L^2 H^2 P_{em} \qquad (5)$$

where we introduced the electromagnetic synchrotron radiation power emitted by the electric charge of the particle, which is given by:

$$P_{em} = 2/3\,(e^2/c)\,\omega^2\,\gamma^4\ . \qquad (6)$$

Therefore Eq.(5) represents the GW power produced by the conversion into gravitational radiation of the e.m. radiation (emitted by the rotating particle), while it is crossing the constant magnetic field along the path of length L. The gravitational waves will have the same frequency and angular distribution of the electromagnetic waves.In an accelerator like LEP the $(L/R)^2$ ratio is of the order 10^{-6} so that only a tiny fraction of P_{em} spread out along the LEP orbit is converted in GW radiation.

In order to design an effective GW source based on this method, we need a strong source of monochromatic electromagnetic radiation, properly focused, which is able to cross a vacuum pipe on which is acting an high and constant magnetic field.

As an example of e.m. source let us consider a very high power laser. The laser beam should be sent in a vacuum pipe on which the highest possible magnetic field H is provided (perpendicular to the beam direction) along the pathlength L. At the end of the pipe the e.m. beam will be traveling with a GW of *the same frequency and angular distribution* transporting a power given by eq. (6), that we rewrite for convenience of numerical evaluation as:

$$P_{gw\,(erg/s)} = 8.23*10^{-50}\ L^2{}_{(cm)}\,H^2{}_{(Gauss)}\,P_{em(erg/s)}\ ; \qquad (7)$$

In order to have an upper limit on P_{gw} let assume to have H=15 Tesla and L=30 km, i. e. the highest values that could be reached in a foreseeable future. But in order to put a constraint on the size of the volume where such a high constant field H must be present,we have to put an upper limit to the cross section radius r of the pipe. Taking for the laser beam divergence the approximate expression $a_d \approx \lambda/d$, where λ, d are the beam wavelength and laser output diameter, respectively, and assuming we want an upper limit r=d with d=10 cm we need the following condition to be respected:

$$\lambda \lesssim d^2/L \approx 3300\ \text{Å}. \qquad (8)$$

Recently a study group of the American Physical Society pubblished a review of the results and perspectives concerning the developement of "Directed Energy Weapons". The Authors describe (see Ref. [7], p.S58) the experimental performances of a Free Electron Laser (FEL) delivering a power of $\approx$100 MW at a wavelength of λ = 8.6 mm.Moreover they consider that the FEL is "promising" also for ultrahigh power applications, like 100 MW power at λ=1μm (corresponding to 1.24 eV).

Now let us assume using as an e.m. source a FEL of the latter features. Then we would obtain for the GW power radiated at the end of the pipe:

$P_{rgw} = 1.7\text{x}10^{-11}$ erg/s = 17 eV/s ,

corresponding to an intensity : $I_0 = 10^{-13}$ erg/(s.cm^2)

In order to increase the radiated P_{gw} let us install to the line extremities a system of 2 highly reflecting mirrors,having a reflectivity $C_r = 0.9999$, able to reflect the RF waves forwards and backwards along the line. In fact, according to some opinions (see K.S.Thorne, Ref. [8], p. 425) the present technology of mirror coating limits the reflectivity at $C_r = 0.9999$, while in a few years from now perhaps this limit could be increased to $C_r = 0.999\,99$.

Let assume that the path length between the mirrors and the Laser frequency are so adjusted as to make an optical resonant cavity of the Fabry Perot type. Then the power output would be given by :

$$dW/dt\,ds = I_0 \approx 10^{-13}(1 - C_r)^{-2} = 10^{-5}\ \text{erg/(s.cm}^2) =$$
$$\approx 10^7\ \text{eV/(s. cm}^2) = 10^7\ \text{gravitons/ (s.cm}^2)\,, \qquad (9)$$

where $(1 - C_r)^{-2}$ = represents the number of reflections squared.

This power is of the same order as that which could arrive on the earth, inside the band width of a bar detector of the Weber type, if a galactic supernova was exploding every few hours .Of course formidable engineering problems have to be faced to design such an experimental project . For example, obtaining the maximum possible magnetic field on a very long distance, dissipation of power in the mirrors, phase shift of the e.m. radiation crossing the magnetic field, etc.

The 3000 Å limit to the wavelength has the consequence that it is much harder to find a suitable detector resonating on the corresponding frequency of 10^{15} Hz. RF cavities or laser interferometer gravitational detectors proposed up to now could work with wavelength much larger than ≈ 1 mm.

We need to find a way either to produce larger wavelength without loosing GW power, or to find a new detector with high sensitivity to GWs in the optical frequency range .

In order to evaluate a numerical example concerning the former case, let's assume to use a source of the type which has known to be produced [7] in the USA. It has a peak power of one gigawatt at a wavelength of 8 mm. Assuming $\lambda = 1$cm, and this time accepting r=d = 100 cm, we have for the maximum allowed L :

$L \underline{<} \approx d^2/\lambda \approx 100$ meters.

Assume to have a field H=15 Tesla , and $P_{em}=10^{16}$ erg/s, we obtain $P_{rgw}=1.85*10^{-15}$ corresponding to an intensity $I_0 \approx 10^{-18}$ erg/s·cm^2. Then by installing the two mirror cavities of the Fabry Perot type as before, the produced power would be:

$$d^2W/dt\,ds \approx 10^{-18}\,(1-C_r)^{-2} \approx 10^{--10}\ \text{erg/s.cm}^2\,,$$

where a mirror reflectivity of C_r=0.9999 was also assumed.

Or, with a reflectivity $1-C_r= 10^{-5}$, $d^2W/dt\,ds = 10^{-8}$

This simple estimate gives results far from being optimized. By a special design aimed to have a better focusing of the radiated GW energy, probably it would be possible to obtain an intensity of $10^{-7}\cdot\cdot\,10^{-5}$ (erg/s. cm^2), now at a frequency which makes the detection in principle possible with proposed detectors, as we will see in the next section.

3)-- A DETECTOR FOR HIGH FREQUENCY AND MONOCHROMATIC GWs.-

The peculiar features of the GW such as those emitted by the sources considered in the previous sections, are the frequency, say $\geq 10^8$ Hz, which is much higher than that expected from the astrophysical sources, the high degree of monochromaticity, and the continuous emission.The expected density flux of energy is at the best 10^{-5}-10^{-10} erg/s.cm^2.

Unfortunately the features stated above seem to rule out the use of the GW bar detector of the Weber type, the only one that has been succesfully developed and put in operation for a long time(8); in fact their normal sensitivity is in the frequency range 10^3-10^4 Hz.

Therefore we will consider in the following a kind of GW detector which promise to have the best sensitivity to match our aim. It was proposed by Bernard,Pegoraro,Picasso and Radicati (1978) [10,11] and by Caves(1978)(12).

The configuration of the detector consists in a two resonant and coupled RF cavities at right angle, say along y and x axes. The cavities have two modes of oscillations, with frequency ω_s (simmetric mode), and ω_a (antisymmetric mode) which satisfy the resonant condition, $\omega_a - \omega_s = \Omega$.

The incoming GW propagating along z will induce a transition between the two levels, provided: a)- the two resonating cavities are put at right angle and in the plane perpendicular to the z axis, and b)- the frequency time 2π of the incoming GW, Ω_{gw}, is equal to $\Omega = \omega_a - \omega_s$. It is also assumed that initially the upper level ω_a is empty. The interacting GW, after a time t will induce a rate of transition of the EM energy to the level ω_a given by:

$$P_a = h^2 \omega_a t \; \Omega_{gw} E_s, \qquad (10)$$

where E_s is the energy stored at the level ω_s.

Now this detector can be operated as a frequency converter if instead of measuring the total absorbed power, only the power transmitted to the upper level is measured.

Let us take now the following relationship involving the adimensional GW amplitude h :

$$W_{gw} = A\,\Omega^2_{gw}\, h^2 \qquad (11)$$

where $A= c^3/(32\pi G) = 4*10^{36}$ erg.s/cm^2 and W(erg/s.cm^2) is the power for surface unit transported by the GW

then by substitution by (11) in (10), we found for the minimum detectable GW's energy flux:

$$(W_{gw})_{min} = A\,(\Omega_{gw}/\omega_a)^2\,(P_a)_{min}/[\,E_s\,t\,] \quad . \qquad (12)$$

From the previous equation it can be seen that the detector sensitivity ($\propto 1/(W_{gw})_{min}$), is higher as $(P_a)_{min}$, the minimum detectable power on the ω_a level, and the ratio $(\Omega_{gw}/\omega_a)^2$, are smaller, and as the stored energy at the level ω_s, E_s, is higher.

It is clear that in order to maximize the detector sensitivity, it is essential to utilize high Q superconducting cavities . This implies an upper limit to the possible energy stored in the coupled cavity resonance. For example in case of a resonating frequency of 10 GHz, it could not exceed .5 joule, otherwise the RF magnetic field would destroy the superconductivity regime. In order to increase the E_s, one should consider the possibility to arrange a large number N_c of such small cavities in parallel , and therefore to make the substitution $N_c E_s \rightarrow E_s$ in the second member of the eq.(12).

For example in case of the GW radiated power by high power Laser we obtain a value of 10^{-5} erg/s. From eq.(12) it can be seen that such a value for $(W_{gw})_{min}$ can be reached by our detector, by assuming:

$N_c E_s$= 5x 10^5 joule ; $(P_a)_{min} = 10^{-23}$ Watt ; $(\Omega_{gw} / \omega_a) = 10^{-3}$; $t = 10^7$ sec .

It would need an important R & D in order to realize that the needed value for $N_c E_s$ is experimentally feasible .

In Ref. [6] has been also considered the possibility to utilize Laser interferometers as detector of artificial sources of GW's.

CONCLUSIONS .

We have shown that it is today conceivable to build sources on the earth capable to produce fluxes of continuous and monochromatic gravity waves of the order of 10^{-10}-10^{-5} erg / s.cm^2 with frequencies in the range $10^{15} \div 10^8$ Hz .These fluxes if integrated over a time of several hours would correspond to the GW energy delivered by the explosions of a galactic supernova inside the bandwidth of a bar detector of the Weber type and are of the same order or higher than those expected from periodic astrophysics sources like the Crab pulsar.

Finally we have shown that for a certain range of frequencies a detector exist whose sensitivity could become, in principle, adequate to pick up the GW signal.

Despite the many open problems,these results are encouraging. Of course further feasibility studies and important R & D should be undertaken in order to optimize a kind of source and the detector.

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Table I

Coherent emission of GW power by high energy particle accelerators.

	LEP	SPS	LHC	SSC
Particles	e	p	p	p
Energy (GeV)	100	450	8000	20000
Particles/bunch	10^{14}	10^{14}	10^{14}	10^{14}
GW Power ; erg/s (eV/s)	10^{-18} (10^{-6})	10^{-16} (10^{-4})	$4*10^{-15}$($2*10^{-3}$)	$5*10^{-15}$ ($3*10^{-3}$)
Min. Grav. Wave- length (cm)	0.1	0.1	0.1	0.1
Gravitons/sec	0.002	0.25	12	12

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