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Suppression of Bunch Transverse Instabilities by the Chamber Asymmetry

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Suppression of Bunch Transverse Instabilities by the Chamber Asymmetry

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Axial asymmetry of a vacuum chamber gives rise to wake forces producing betatron tune shifts for tail particles. In the result, the bunch transverse instabilities could be suppressed or even eliminated.

I. INTRODUCTION

An ultrarelativistic charged particle leave electromagnetic fields behind it in the vacuum chamber. The net effect of these fields on a following charge is determined by the forces integrated over the vacuum chamber periodicity L. The integrated transverse force $\vec{F}$ caused by a slight offset $\vec{r}_0$ of the leading particle from the axis of a round chamber is conventionally expressed in terms of the wake function [1]:

$$\int_L \vec{F} ds = -q^2 \vec{r}_0 W(z), \tag{1}$$

where q is the particles charge and z is the distance between head and tail particles. Due to the axial symmetry, the force (1) does not depend on the offset of the tail particle however high it is. In the result, the betatron frequencies of the trailed particles are not shifted by the coherent fields of the leading ones; all the bunch particles are in the resonance with each other.

This property is not valid for axially asymmetrical structures generally discussed in [2]. So the wake fields of the head particles not only drive the oscillations of the tail ones but also detune them from the resonance with the driving force. An importance of such a detune was shown in [3] where an influence of the betatron tune spreading along the bunch was analyzed. It was demonstrated that this spreading introduced by means of an RF quadrupole has a stabilizing role for the bunch transverse oscillations. It is naturally to suppose that the tune spread produced by the bunch coherent fields is a stabilizing factor as well. In this case, the detuning part of the wake might increase or even eliminate the thresholds of bunch transverse instabilities.

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II. DRIVING AND DETUNING WAKES

The transverse wake forces are regular functions of the transverse offsets of the leading and trailing particles, $\vec{r}_0$ and $\vec{r}$ and can be expanded over them [2]. Assuming for simplicity a mirror symmetry for at least one of the transverse axis and neglecting the nonlinear terms, the forces can be presented as follows:

$$\begin{aligned} \int_L F_x ds &= -q^2 x_0 W_x(z) + q^2 x D(z) \\ \int_L F_y ds &= -q^2 y_0 W_y(z) - q^2 y D(z) \end{aligned} \tag{2}$$

where insignificant constant terms are omitted. The first terms in the right hand sides describes the forces caused by the offsets of the leading particle; the functions $W(z)$ can be referred to as the driving wake functions. The second terms are responsible for the tunes shift of the tail particle; the function $D(z)$ can be called as the detuning wake function. The detuning terms for x and y axis are described in Eqs.(2) by the single function $D(z)$. This is a consequence of the Maxwell equations which can be proved by means of Panofsky-Wenzel theorem [4]

$$\frac{\partial}{\partial z} \int_L \vec{F}_\perp ds = \vec{\nabla}_\perp \int_L \vec{F}_\parallel ds \tag{3}$$

for the harmonic function $\int_L \vec{F}_\parallel ds$.

To give examples, the wake functions caused by the wall resistivity are presented below for various simplified cases: a source charge inside a round chamber, near an infinite plane and outside a small cylinder; the three cases are sketched at Figs. 1a, 1b and 1c.

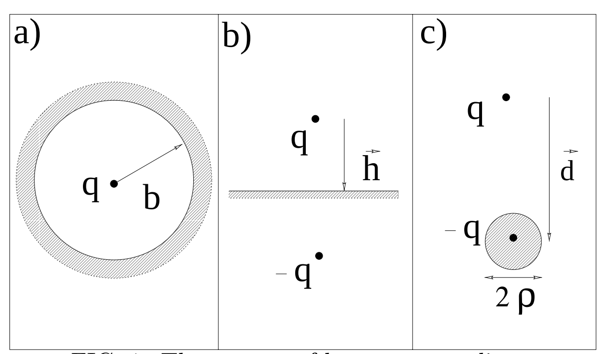

FIG. 1. Three types of beam surroundings.

The conventional procedure for these problems suggests the Fourier transformation over the longitudinal coordinate and to treat the wall resistivity as a perturbation. So, for relevant frequencies $\omega = ck$, the fields for the perfect conductivity can be taken as an initial approximation. The fields caused by the resistivity are harmonic functions in the transverse plane xy; the tangential electric field satisfies the Leontovich boundary condition at the metal surface [5]:

$$\vec{E}_t = \zeta \vec{H}_t \times \vec{n}, \tag{4}$$

where $\vec{H}_t$ is the magnetic field calculated for perfect conductivity, $\vec{n}$ is a unit normal vector pointed inside the wall. The so-called surface impedance

$$\zeta = \sqrt{\frac{\omega}{8\pi\sigma}}(1-i) \tag{5}$$

is determined by the finite metal conductivity σ at the given field frequency ω; this factor has to be small, $|\zeta| \ll 1$.

For the first case of the round vacuum chamber the wake functions can be found in e. g. [1]:

$$W_x(z) = W_y(z) = -\frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma z}}L, \quad D(z) = 0 \tag{6}$$

where c is a velocity of light and b is the vacuum chamber radius.

For the case of the resistive plane, the boundary condition (4) can be presented in the following form:

$$E_{\parallel} = 2\zeta E'_n \tag{7}$$

where $E_{\parallel}$ is the longitudinal electric field and E'_n is the normal component of the image charge electric field. Taking into account that the field E'_n satisfies the Laplas equation, it can be concluded that Eq. (7) describes the resistive wall field $E_{\parallel}$ not only on the metal surface but everywhere in the volume. After the longitudinal field is found, the transverse wake forces are found from the Panofsky-Wenzel theorem (3), which gives:

$$\vec{F}_{\perp}(z) = \frac{2q^2}{\pi}\sqrt{\frac{c}{\sigma z}}\vec{\nabla}\frac{\vec{n}(\vec{r}-\vec{r}'_0)}{(\vec{r}-\vec{r}'_0)^2} \tag{8}$$

with $\vec{r}'_0 = (x_0, -y_0)$ standing for the image charge position (the axis y is assumed here to go normally to the plane).

To find the wakes (2), the work of the force (8) has to be expanded over the leading and trailing particle offsets. In the result, it comes out:

$$W_x(z) = W_y(z) = D(z) = -\frac{L}{2\pi h^3}\sqrt{\frac{c}{\sigma z}} \tag{9}$$

where $h = |\vec{h}|$ is the distance from the beam to the plane.

This geometry demonstrates the possibility for the detuning wake to be the same value as the driving ones. The next and the last example treats the case of the beam passing along a small resistive cylinder, Fig. (1c); the detuning wake is shown to dominate here. The image charge is located at the position $\vec{r}' = \vec{r}\rho^2/r^2$, with the cylinder axis taken as as $z-$axis. For a simplicity sake, the radius of cylinder ρ is taken to be much smaller than the distance between the beam and the cylinder: $\rho \ll r_0 = |\vec{d}| = d$. Solution of the Laplas equation for the longitudinal electric field with the boundary condition (4)

$$E_s = -\frac{2\zeta q}{\rho}\left(1 + \frac{\rho^2 \vec{r}\vec{r}_0}{r^2 r_0^2}\right) + Cln(r/\rho), \tag{10}$$

includes an arbitrary constant C. The small dipole term in the brackets reflects a weak dependence of the fields on the source position $\vec{r}_0$. To find the constant C, a boundary condition at infinity is needed. It can be assumed that this system is bounded by a conducting cylinder with the radius $R \gg r_0$. Then the constant C can be found by equating the expression for the monopole part of E_s to zero at this remote surface, which gives

$$C = \frac{2\zeta q}{\rho \ln(R/\rho)}.$$

Using (3) one can obtain the integrated transverse force and finally the wakes:

$$D(z) = -\frac{L}{\pi d^2 \rho \ln(R/\rho)}\sqrt{\frac{c}{\sigma z}} \tag{11}$$

$$W_x(z) = W_y(z) = -\frac{L\rho}{\pi d^4}\sqrt{\frac{c}{\sigma z}} \tag{12}$$

Introducing the detuning factors $\kappa_x = D(z)/W_x(z)$, $\kappa_y = D(z)/W_y(z)$, the results for the various geometry are expressed as

$$\kappa_x = \begin{cases} 0, & \text{axial symmetry} \\ 1, & \text{plane wall } \vec{n}_x \perp \vec{h} \\ -1, & \text{plane wall } \vec{n}_x \parallel \vec{h} \\ d^2/(\rho^2 \ln(R/\rho)), & \text{small cylinder } \vec{n}_x \perp \vec{d} \\ -d^2/(\rho^2 \ln(R/\rho)), & \text{small cylinder } \vec{n}_x \parallel \vec{d} \end{cases} \tag{13}$$

Here $\vec{n}_x$ is the unit vector in the x direction, vectors $\vec{h}, \vec{d}$ are defined in Fig. 1.

The driving wake function $W(z)$ for the small cylinder (12) are in a factor $\propto \rho/d \ll 1$ smaller than the wake functions of the round chamber (6) or parallel plates (9) with the same aperture. This result gives an idea how the transverse instability can be suppressed due to the decrease of the driving wake function. The detuning wakes, which arise here, work in the same direction: they damp the instability even more.

III. COHERENT STABILIZATION BY THE DETUNING WAKE

The detuning wake modulates the betatron frequencies along the bunch. Such a modulation introduced by means of an RF quadrupole was studied in [3]. It was shown there that the transverse instabilities can be strongly damped in this case because the particles stay away from the resonance with each other. Following this paper, the numerical results for an influence of the detuning wake on the transverse mode coupling instability are presented below.

Assuming the bunch to consist of particles with the same synchrotron amplitude a, the transverse equation of motion is written:

$$\frac{d^2x(\phi)}{dt^2} + \omega_b^2 x(\phi) = F_x(\phi)$$

$$F_x(\phi) = -\frac{Nq^2}{2\pi\gamma m L}\int_{-|\phi|}^{|\phi|}(W(z)x(\phi') - D(z)x(\phi))d\phi'$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \omega_s\frac{\partial}{\partial\phi}, \quad z = a\cos\phi - a\cos\phi' \qquad . \tag{14}$$

Here ϕ is the synchrotron phase, ω_b and ω_s are the betatron and the synchrotron frequencies, N is the number of particles in the bunch. An expansion of the deviation $x(\phi)$ over the synchrotron harmonics

$$x(\phi) = e^{-i\omega_b t}\sum_{n=-\infty}^{+\infty} x_n e^{-i\alpha\omega_s t + in\phi}, \tag{15}$$

amounts Eq. (14) to solving a set of algebraic equations for the eigenvector components x_n and the eigenvalues α :

$$x_n(\alpha - n) = K\sum_{m=-\infty}^{+\infty} x_m K_{nm}, \quad K = \frac{Nq^2}{2\pi^2\gamma m\omega_b\omega_s L}$$

$$K_{nm} = \int_0^\pi cos(n\phi)d\phi\int_0^\phi W(z)cos(m\phi')d\phi' - \int_0^\pi cos((n-m)\phi)d\phi\int_0^\phi D(z)d\phi' \tag{16}$$

where the influence of the coherent interaction was supposed to be small in a comparison with the transverse focusing, $\alpha\omega_s \ll \omega_b$. To resolve such equations, the sum has to be truncated to a finite number of lower modes. In the numerical calculations, five modes were initially taken; then, the results were compared with the nine-mode truncation. All the resistive wall wake functions has the following form:

$$W(z) = -Q/\sqrt{z}, \quad D(z) = -\kappa Q/\sqrt{z},$$

where Q is the geometry factor, the examples for the detuning factor κ are given by (13).

Figure 2 presents plots for dimensionless eigenvalues α as functions of the dimensionless intensity parameter

$$\mathcal{I} = KQ/\sqrt{a} \tag{17}$$

at various detuning factors κ. The dependence of the modes behavior on this factor is seen to be significant.

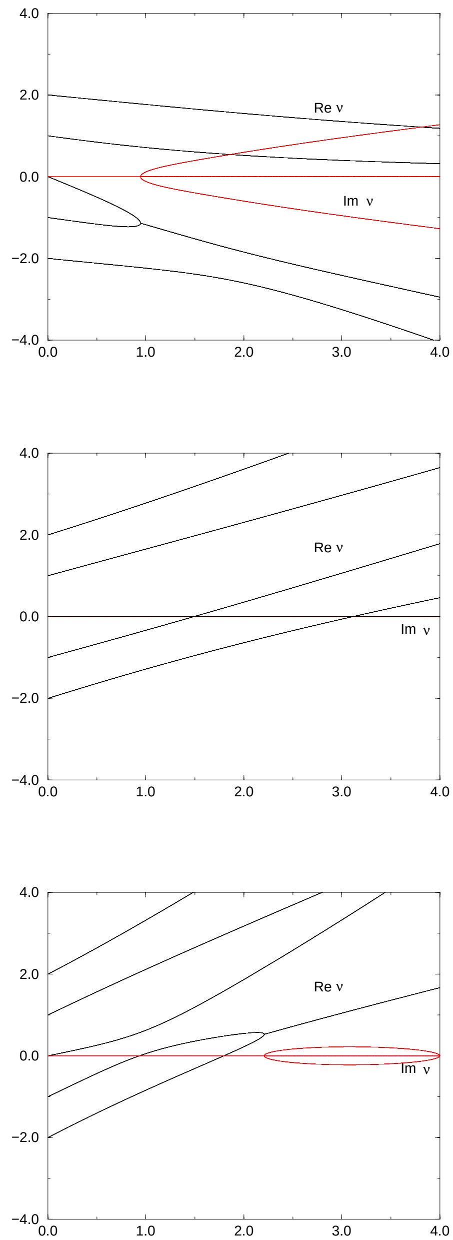

FIG. 2. Eigenvalues α versus the intensity parameter $\mathcal{I}$ for various detuning factors κ. From top to bottom: $\kappa = 0, 1$ and 1.5.

The mode coupling instability threshold is minimal for the symmetrical case with $\kappa = 0$. At $\kappa = 1$ coupling and decoupling thresholds merge (degenerate case) and the beam is stable for any current. This result is valid for the 9-mode calculations as well, so it looks as a rigorous statement.

The instability threshold appears again at higher κ, the bottom plot at Fig. 2 shows the coupled modes to decouple for a higher current.

The Figure (3) shows the threshold's behavior versus coefficient of asymmetry κ for both the 5-modes and 9-modes calculation. One can see that the threshold disappears for $\kappa \geq 2.5$ and the situation becomes stable for all currents. Actually, this ultimate threshold for the detuning factor was found to be dependent on the number of the accounted modes. With the increase of κ, intensity thresholds go up and disappear for certain modes at certain κ. However, the higher order modes still have there own thresholds at higher intensity parameters. So, the instability threshold increase with the absolute value of the detuning factor, but it is hard to say if it ever disappears.

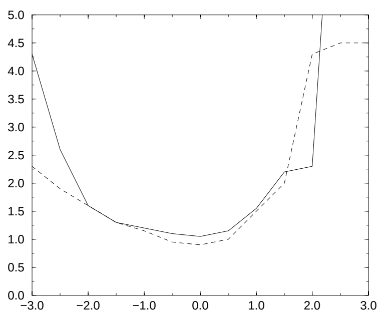

FIG. 3. Intensity threshold of the transverse mode coupling instability $\mathcal{I}$ versus the detuning factor κ for the 5-mode (the solid line) and 9-mode (the dash line) truncations.

IV. HOW IT COULD BE USED

Axial asymmetry of the chamber changes the conventional (driving) wake function and introduces the detuning wake function. With a proper chamber configuration, the thresholds of bunch transverse instabilities could be significantly increased. This might be done due to the following circumstances.

- Asymmetry of the vacuum chamber can reduce the driving wake function.
- The asymmetry causes the stabilizing detuning wake function.

An example of the vacuum chamber with these properties is sketched in Fig. 4.

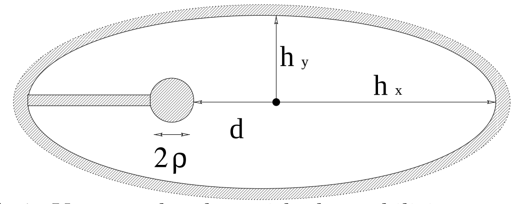

FIG. 4. Vacuum chamber with the stabilizing asymmetry insertion.

A particular possibility is realized for the detuning factor $\kappa = 1$, where all the instability thresholds disappear, see Fig. 2, center. Variation of signs of the detuning factors κ_x, κ_y in the correlation with the variations of the $\beta-$functions could effectively provide this beneficial condition for both transverse degrees of freedom.

The suggested method could be used for the damping of transverse instabilities driven by any kind of wake forces, not only by the resistive one taken here only as an example. Moreover, the method might have wider field of application than one discussed above. First, the depression of the direct wake function looks promisible for damping of the couple-bunch instabilities. Second, the detuning wakes could be the way for the BNS damping [6] in linear accelerators where the asymmetry might be introduced as in CLIC [7]. The chamber like one shown in Fig. 4 could be used for the suppression of the mode-coupling instability predicted for the VLHC project [8,9].

V. ACKNOLEDGEMENTS

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