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Bianchi types I and V bulk viscous fluid cosmological models in $f(R, T)$ gravity theory

Research Article

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Abstract: In this paper we present non-singular Bianchi types I and V cosmological models, in the presence of bulk viscous fluid and within the framework of f(R,T) gravity theory. Exact solutions to the field equations are obtained by choosing a particular form of the function f(R,T) and a special value for the average scale factor of the model, which corresponds to a time- dependent deceleration parameter. The cosmological models initially accelerate for a certain period of time and thereafter decelerate. The physical and kinematical properties of the models of the universe are discussed.

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1. Introduction

The simplest representation of the expanding universe is given by Friedmann-Robertson-Walker models which are spatially homogeneous and isotropic. These models are in some sense a good global approximation to the present day universeIt is however unreasonable to assume that the early stages of evolution of the universe may be suitably described by the regular expansion predicted by these models . The aim of modern cosmology is to study the past history, the present state and future evolution of the universe. Recent observational data indicate that our universe is accelerating (Riess et al. [1], Perlmutter et al. [2]). Also observations such as cosmic microwave background radiation (Spergel et al. [3]) and large scale structure (Tegmark et al. [4]) provide indirect evidence for the late time accelerated expansion of the universe.This acceleration is explained in terms of so-called dark energy.

In view of the late-time acceleration of the universe and the existence of the dark energy and dark matter, several modified theories of gravitation have been proposed as alternatives to Einstein's general theory of relativity. Noteworthy among them is the cosmologically important f(R) gravity theory. It has been shown that f(R) gravity theory is indeed a realistic alternative to general relativity, being consistent in dark epoch. It has been suggested that cosmic acceleration could be achieved by replacing the Einstein's Hilbert action of general relativity with a general function of Ricci scalar R. Nojiri and Odintsov [5] developed a general program for unification of the matter -dominated era with the accelerated epoch for scalar

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-tensor theory or dark fluid. Nojiri and Odintsov [6] presented an extensive review of modified gravity theories which is considered to be a gravitational alternative for dark energy. Bertolami et al. [7] have proposed a generalization of f(R) gravity theory by including in the theory an explicit coupling of an arbitrary function of Ricci scalar R to the matter Lagrangian density. Shamir [8] has also proposed a physically viable f(R) gravity model, with the unification of early time inflation and late time acceleration, Shamir and Jhangeer[9] investigated static plane symmetric vacuum solutions in f(R) gravity for $(n+1)$ dimensional space time. Recently, Adhav [10] studied a Bianchi type-III cosmological model in f(R) theory of gravity in the presence of cosmic strings.

The role played by viscosity and the consequent dissipative mechanism in cosmology have been discussed by several authors (Misner [11, 12], Murphy [13], Belinsky and Khalatnikov [14]). The heat represented by the large entropy per baryon in the microwave background provides a useful clue as to the nature of the early universe. A possible explanation for this huge entropy per baryon is that it was generated by physical dissipative processes acting at the beginning of evolution. These dissipative processes may indeed be responsible for the smoothing out of initial anisotropics (Weinberg [15]). Misner [11, 12] suggested that neutrino viscosity acting in the early era might have considerably reduced the present anisotropy of the black-body during the process of evolution. Belinskyand and Khalatnikov [14] presented some general characteristics of anisotropic cosmological models in the presence of viscosity. Bulk viscosity is the only dissipative phenomenon occurring in FRW models and is significant in causing the accelerated expansion of the universe known as inflationary phase, as discussed by Setare and Sheyki [16]. Murphy [13] has obtained a zero curvature FRW type model in the presence of bulk viscosity alone, which exhibits an interesting property that the big-bang singularity appears in the infinite past. Roy and Tiwari [17] presented plane symmetric solutions to Einstein's field equations representing inhomogeneous cosmological models with viscous fluid and constant bulk viscosity. Szyddowski and Heller [18] constructed models of the universe filled with interacting matter and radiation, including dissipation due to bulk viscosity . Mohanty and Pradhan [19] obtained a class of exact non-static solution in closed elliptic Robertson-walker space-time filled with viscose fluid in the presence of an attractive scalar field. Banerjee et al. [20] obtained some Bianchi type-I solutions in the case of stiff matter under the assumption that the shear viscosity coefficient is a power -law function of energy density. Goener and Kowallski [21] developed a method for obtaining irrotational anisotropic viscous fluid solutions of a Bianchi type-I model with a barotropic equation of state. Banerjee and Sanyal [22] presented an irrorational Bianchi type V model under the influence of both shear and bulk viscosity together with heat flow. Coley [23], Coley and Hoogan [24], while generalizing the work of Coley and Tupper [25], studied diagonal Bianchi type-V imperfect fluid models with both viscosity and heat condition with and without the cosmological term. Bali and Meena [26] have investigated tilted cosmological models filled with disordered radiation for perfect fluid and heat flow. A tilted Bianchi type I cosmological model for perfect fluid distribution in the presence of a magnetic field is investigated by Bali and Sharma [27]. Also, Bali and Anjali [28] presented Bianchi type-I bulk viscous fluid string dust magnetized cosmological models in general relativity. Adhav et al. [29] studied Bianchi type-III anisotropic cosmological models with varying Λ. Baghel and Singh [30] considered spatially homogeneous and anisotropic Bianchi type-V space-time with a bulk viscous fluid source, and time varying gravitational constant G and cosmological term Λ. Several authors have discussed the role of bulk viscosity in the early evolution of the universe in different physical contexts.

Harko et al. [31] developed another modification of Einstein's gravity theory, known as $f(R,T)$ gravity theory, wherein the gravitational Lagrangian is an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor T_{ij}. It is to be noted that the dependence from T may be induced by exotic imperfect fluid or quantum effects. They have derived the field equations of f(R,T) gravity by varying the action of the gravitational field equations with respect to the metric tensor and have presented a physically realistic model with a certain choice of the function f(R,T). Subsequently, several authors viz. Myrzabulov [32], Adhav [33], Reddy et al. [34], Chaubey and Shukla [35], Ram et al. [36], Chandel and Ram [37] etc. presented spatially homogeneous Bianchi type cosmological models in the presence of a perfect fluid in f(R,T) gravity theory. Samanta [38] studied the Kantowski -Sachs space time cosmological model filled with perfect fluid matter in f(R,T) gravity. Further, Reddy et al. [39], and Ram and Priyanka [40], have investigated five dimensional Kaluza-Klein cosmological models filled with perfect fluid in f(R,T) gravity theory. Naidu et al. [41] investigated a Bianchi type -V bulk viscous string cosmological model in f(R,T) gravity theory. Reddy et al. [42] considered LRS Bianchi type II space-time and obtained the solutions of field equations with cosmic string and bulk viscous fluid within the framework f(R,T) theory of gravity. Recently, Ahmed and Pradhan [43] investigated a cosmological model in f(R,T) gravity of Bianchi type-V by assuming $f(R,T) = f_1(R) + f_2(\mathrm{T})$. Chakraborty et

al. [44] formulated an alternative f(R, T) gravity theory and the dark energy problem. Recently, Sharif and Zubair [45] studied Bianchi type-I anisotropic models in f(R,T) gravity theory. Sahoo et al. [46] considered an axially symmetric space -time in the presence of a perfect fluid source within the framework of f(R,T) gravity theory. Mishra and Sahoo [47] investigated Bianchi type VI cosmological models filled with perfect fluid within the framework of f(R,T) gravity theory. The spatially homogeneous and totally anisotropic Bianchi type-II cosmological solutions of massive strings in the presence of a magnetic field in the f(R,T) theory of gravity have been studied by Sharma and Singh [48]. Singh and Singh [49] presented the cosmological viability of reconstructing an alternative gravitational theory, namely the modified f(R,T) gravity theory.
Motivated by the above studies, we investigate new classes of spatially homogeneous Bianchi type I and V bulk viscous fluid cosmological models in the f(R, T) theory of gravity. We also discuss certain physical and kinematical features of the cosmological models.

2. Field equations

We assume that the cosmic matter may be represented by the energy-momentum tensor of an imperfect bulk viscous fluid

$$T_{ij} = (\rho + \bar{p})\, u_i u_j - \bar{p} g_{ij}, \tag{1}$$

where $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \zeta u^i_{;i}, \tag{2}$$

satisfying a linear equation of state

$$p = \epsilon\rho, \quad 0 \leqslant \epsilon \leqslant 1. \tag{3}$$

Here p is the equilibrium pressure, ρ is the energy density of matter, ζ is the coefficient of bulk viscosity and u^i is the flow vector of the fluid satisfying $u_i u^i = 1$. The semicolon stands for covariant differentiation. On thermodynamic grounds bulk viscosity coefficient ζ is positive, assuring that the viscosity pushes the dissipative pressure $\bar{p}$ towards negative values. However, the correction applied to the thermodynamical pressure p due to bulk viscous pressure is very small. Therefore, the dynamics of cosmic evolution are not fundamentally influenced by the inclusion of the viscous term in the energy-momentum tensor.
The field equations in f(R, T) gravity theory with the particular choice of the function f(R, T), given by

$$f(R,T) = R + 2f(T) \tag{4}$$

When the matter source is a bulk viscous fluid, these are given by (Reddy et al. [30]):

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} + \left[2\bar{p} f'(T) + f(T)\right] g_{ij} \tag{5}$$

We further choose that f(T)=λ T, where λ is a constant. This f(R,T) gravity model is equivalent to a cosmological model with an effective cosmological constant $\Lambda \propto H^2$, where H is the Hubble function [50]. It is also interesting to note that generally for this choice of f(R,T) the gravitational coupling becomes an effective and time dependent coupling, of the form $G_{eff} = G \pm 2f'(T)$. Thus the term 2f(T) in the gravitational action modifies the gravitational interaction between matter and curvature, replacing G by a running gravitational coupling parameter.

3. Bianchi type-I model

We consider a spatially homogeneous Bianchi type-I metric given as:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2 \tag{6}$$

where A, B and C are cosmic scale functions.
To discuss the kinematical properties of the models, we introduce the expansion scalar (θ), shear scalar (σ), the average Hubble parameter (H) and the anisotropy parameter A_m for the metric (6) as follows:

$$V = a^3 = ABC, \tag{7}$$

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{8}$$

$$\theta = 3H = \frac{\dot{V}}{V} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{9}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \tag{10}$$

$$A_m = \frac{1}{3}\Sigma_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{11}$$

where $\Delta H_i = H_i - H, (i = 1,2,3)$ and $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$, and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.
An important observational quantity is the deceleration parameter q (DP) which is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{12}$$

Here a dot denotes derivatives with respect to time t. The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to a standard decelerating model whereas the negative sign indicates inflation.
For the metric (6), the field equations (1), (4) and (5) in comoving coordinates lead to the following set equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=\lambda\rho-(8\pi+3\lambda)\bar{p}, \tag{13}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}=\lambda\rho-(8\pi+3\lambda)\bar{p}, \tag{14}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=\lambda\rho-(8\pi+3\lambda)\bar{p}. \tag{15}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=(8\pi+3\lambda)\rho-\lambda\bar{p}, \tag{16}$$

These are four highly non-linear equations in six unknowns A,B, C, ρ, $\bar{p}$ and ζ. Therefore to find a consistent solution to these equations, valid assumptions will need to be made to simplify the physics or the mathematics.
Subtracting Eq. (14) from Eq. (13), Eq. (15) from (14), Eq.(15) from Eq.(13) and integrating the resulting equations, we obtain

$$\frac{B}{A}=d_1\exp\left(c_1\int\frac{dt}{a^3}\right), \tag{17}$$

$$\frac{C}{B}=d_2\exp\left(c_2\int\frac{dt}{a^3}\right), \tag{18}$$

$$\frac{A}{C}=d_3\exp\left(c_3\int\frac{dt}{a^3}\right) \tag{19}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are integration constants which satisfy the relations

$$c_1+c_2+c_3=0, \qquad d_1d_2d_3=1. \tag{20}$$

From Eqs. (17)-(19), we can obtain the scale factors A, B and C metric functions explicitly as

$$A=ap_1\exp\left(q_1\int\frac{dt}{a^3}\right), \tag{21}$$

$$B=ap_2\exp\left(q_2\int\frac{dt}{a^3}\right), \tag{22}$$

$$C=ap_3\exp\left(q_3\int\frac{dt}{a^3}\right) \tag{23}$$

where

$$p_1=(d_1^{-2}d_2^{-1})^{\frac{1}{3}},\quad p_2=(d_1d_2^{-1})^{\frac{1}{3}},\quad p_3=(d_1d_2^2)^{\frac{1}{3}} \tag{24}$$

and

$$q_1=-\frac{2c_1+c_2}{3},\qquad q_2=\frac{c_1-c_2}{3},\qquad q_3=\frac{c_1+2c_2}{3}. \tag{25}$$

The constants p_1, p_2, p_3 and q_1, q_2, q_3 satisfy the relations

$$p_1p_2p_3=1, \qquad q_1+q_2+q_3=0. \tag{26}$$

It is obvious that we determine the scale factors A, B, C from Eqs. (21)-(23) the average scale factor a(t) is known. For constructing physically relevant cosmological models, the Hubble parameter and deceleration parameter (DP) play important roles. It has been common practice to use a constant DP. Berman [51], Berman and Gomide [52] proposed a law of variation of Hubble parameter in the FRW model that yields a constant value of DP, which subsequently leads to power-law and exponential forms of the average scale factor. The recent observations of SNe Ia (Riess et al. [1], Perlmutter et al. [2]) indicate that the universe is presently accelerating while there was decelerated expansion in the past, and the universe undergoes transition from decelerated expansion to accelerated expansion and vice-versa at present. Therefore, in general DP is expected to be not a constant but rather a function of time. Some authors have proposed time-dependent forms of DP and derived a differential form of the average scale factor of the model. Alternatively, some authors have chosen the average scale factor and then deduced the time-dependent DP. Eq. (12) can also be written as

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{27}$$

Abdussattar and Prajapati [53] proposed a solution for the time dependent form of q as

$$q=-\frac{\alpha}{t^2}+(\beta-1) \tag{28}$$

The Fig. 1 depicts the behavior of the deceleration parameter with time.
Eq. (12) can be integrated to give scale factor a(t) as

$$a(t)=e^{\delta}\exp\int\frac{dt}{\int(1+q)dt+\gamma} \tag{29}$$

where γ and δ are arbitrary constants of integration. Substituting Eq. (28) into Eq. (29) and integrating the result, Abdussattar and Prajapati [53] derived three different forms of a(t), the simplest form among them is given by

$$a(t)=e^{\delta}\left(t^2+\frac{\alpha}{\beta}\right)^{\frac{1}{2\beta}}. \tag{30}$$

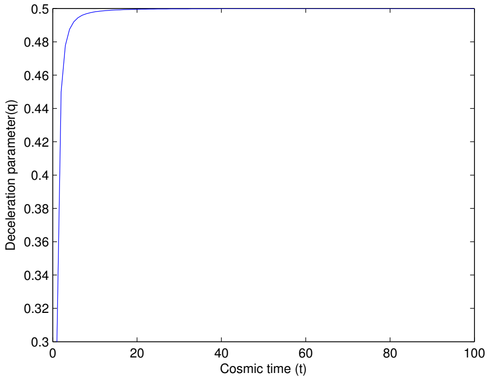

Figure 1. The plot of deceleration parameter q verses cosmic time t,$\beta=\frac{3}{2}$ α=1;

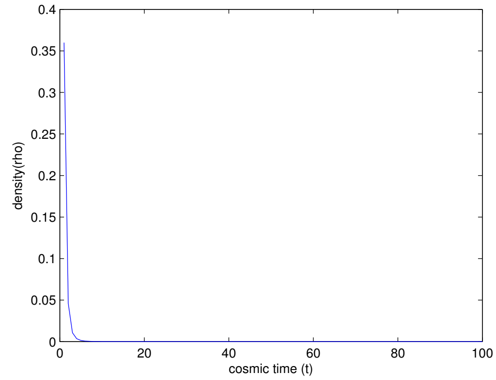

Figure 2. The plot of density ρ verses cosmic time t,λ =1, α=1.

They have also discussed the non-singular bouncing FRW cosmological models with a(t) give by Eq. (30).

Here we use this form of a(t) to determine the scale factors A, B and C from Eqs. (21)-(23). If we use the value of a(t) in Eqs. (21) -(23) the integration is rather difficult. Therefore, we take $\delta = 0$ and $\beta = \frac{3}{2}$ in Eq. (30) so that

$$a(t) = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}}. \tag{31}$$

Substituting Eq. (31) in Eqs. (21) – (23) and integrating, we obtain expression for the metric functions as

$$A = p_1 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_1 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right], \tag{32}$$

$$B = p_2 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_2 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right], \tag{33}$$

$$C = p_3 \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} \exp\left[q_3 \tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}} t\right]. \tag{34}$$

For the model represented by metric functions in (32)-(34) the energy density ρ and the bulk viscous pressure $\bar{p}$ are given by

$$\begin{aligned}\rho =& \frac{1}{9(8\pi+2\lambda)(8\pi+4\lambda)(t^2+\frac{2\alpha}{3})^2} \\ & [t^2[(8\pi+3\lambda)(12+18q_1^2) - (q_1^2+q_3^2)(144\pi+64\lambda) \\ & - 18\lambda(q_2+q_3)] - \lambda(8t+12\alpha(q_2+q_3))],\end{aligned} \tag{35}$$

$$\begin{aligned}\bar{p} =& \frac{1}{9\lambda(t^2+\frac{2\alpha}{3})^2}[(18(q_1^2+q_2^2+q_3^2) - 12 \\ & + \frac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}(12+18q_1^2(8\pi+3\lambda)) \\ & - (q_2^2+q_3^2)(144\pi+64\lambda) - 18(q_2+q_3))t^2 \\ & - \frac{(8\pi+3\lambda)}{(8\pi+2\lambda)(8\pi+4\lambda)}\lambda(8t+12\alpha(q_2+q_3))].\end{aligned} \tag{36}$$

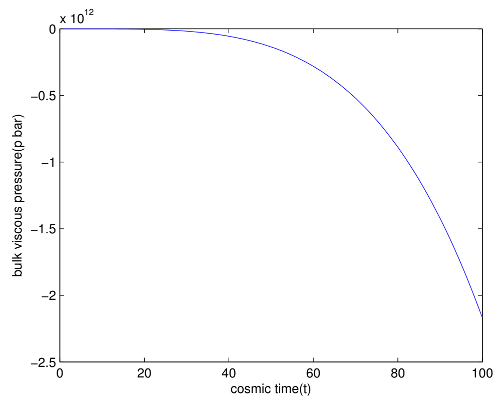

Figure 3. The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t, λ =1,α=1.

The Figs. 2 and 3 depict the behavior of energy density and bulk viscous pressure with cosmic time respectively. The barotropic equation of state parameter may be used to obtain the coefficient of bulk viscosity, which is obtained from Eqs. s(4) and (37) as

$$
\begin{aligned}
\zeta =& \frac{t}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})} * [\epsilon\lambda(8\lambda+3\lambda)(12+18q_1^2) - (q_1^2+q_2^2)(144+64\lambda) - 18\lambda(q_2+q_3) \\
& - (8\pi+3\lambda)(12+18q_1^2(8\pi+3\lambda)) - (q_2^2+q_3^2)(144\pi+64\lambda) - 18(q_2+q_3)] \\
& - \frac{1}{54\lambda(8\pi+4\lambda)(8\pi+2\lambda)t(t^2+\frac{2\alpha}{3})} * [\epsilon t^2(8t+12\alpha(q_2+q_3)) + 18(8\pi+2\lambda)(8\pi+4\lambda)(q_1^2+q_2^2+q_3^2) \\
& - 12(8\pi+2\lambda)(8\pi+4\lambda) + (8\pi+3\lambda)\lambda(8t+12\alpha(q_2+q_3))]
\end{aligned} \tag{37}
$$

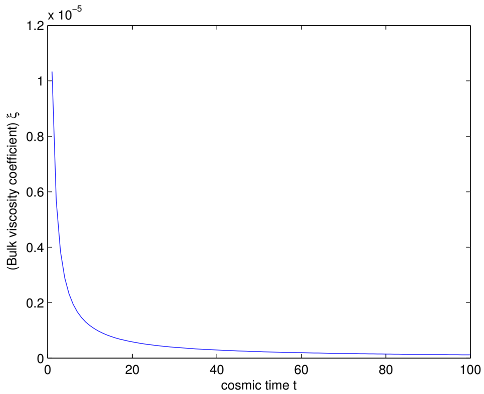

Figure 4. The plot of Bulk viscosity coefficient ζ verses cosmic time t, λ =1,α=1.

Fig. 4 shows behavior of bulk viscosity coefficient with cosmic time.
For the model 1 the energy density conditions $\rho + p \geq 0$ and $\rho + 3p \geq 0$ are identically satisfied as shown, in the Fig. 5.

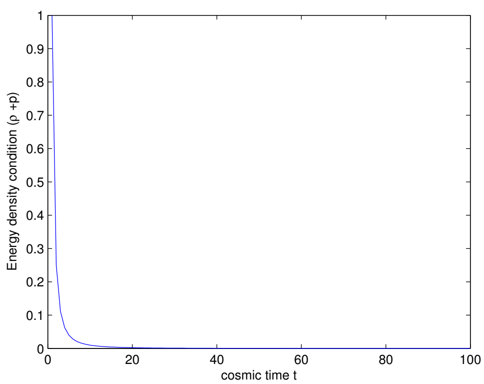

Figure 5. The plot of Energy density condition $\rho + p$ verses cosmic time t, λ =1, α=1.

We now discuss the physical and kinematical behaviours of the Bianchi type-I cosmological model with metric functions given by Eqs. (32)-(34). The directional Hubble parameters and the average Hubble parameter are given by

$$H_1 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_1+1), \tag{38}$$

$$H_2 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_2+1), \tag{39}$$

$$H_3 = \frac{2t}{3\left(t^2+\frac{2\alpha}{3}\right)}(3q_3+1), \tag{40}$$

$$H = \frac{2t}{\left(t^2+\frac{2\alpha}{3}\right)}. \tag{41}$$

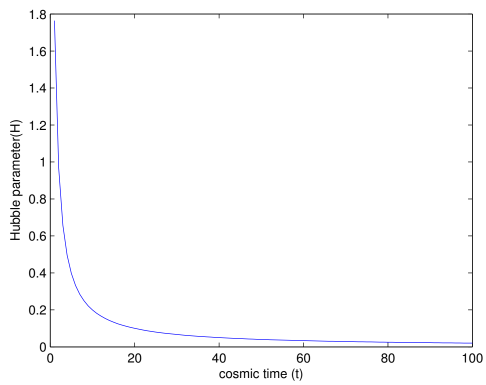

Figure 6. The plot of Hubble parameter H verses cosmic time t, α=1.

The expansion scalar, shear scalar and mean anisotropic parameters are found as

$$\theta = 3H = \left(\frac{6t}{t^2+\frac{2\alpha}{3}}\right). \tag{42}$$

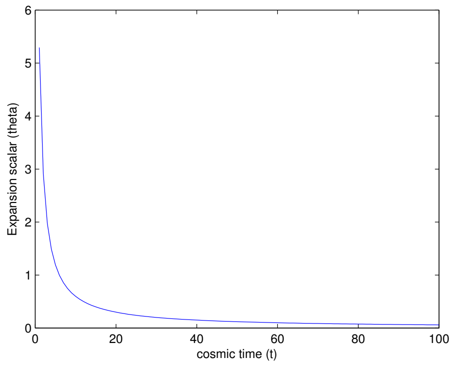

Figure 7. The plot of expansion scalar θ verses cosmic time t, α=1.

$$\sigma^2 = \left(\frac{2t^2}{(t^2 + \frac{2\alpha}{3})^2}\right)\left(q_1^2 + q_2^2 + q_3^2\right). \quad (43)$$

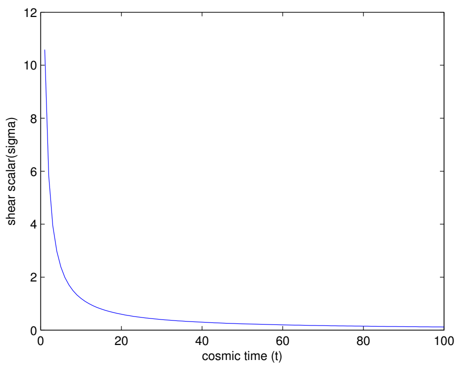

Figure 8. The plot of shear scalar σ verses cosmic time t, α=1.

$$A_m = \frac{1}{3}\left(q_1^2 + q_2^2 + q_3^2\right). \quad (44)$$

Figs. 6, 7 and 8 depict the variation of H, θ and σ respectively. We observe that the model has no initial singularity at t= 0. Also, we see that H, θ, σ, $\bar{p}$, ρ and ξ, are finite at t= 0. These parameters are decreasing functions of time which tend to zero for large values of time. Since $\frac{\sigma^2}{\theta^2} \neq 0$, the model is anisotropic throughout the evolution of the universe.

4. Bianchi type-V model

The diagonal form of the metric of Bianchi type V cosmological model is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}\left[B^2(t)dy^2 + C^2(t)dz^2\right] \quad (45)$$

Here A, B and C are also cosmic scale factors and m is an arbitrary constant.

Using Eqs. (1), (4), (5) and (45) we obtain the following set of equations

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = (8\pi + 3\lambda)\rho - \lambda\bar{p}, \quad (46)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \quad (47)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p}, \quad (48)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \lambda\rho - (8\pi + 3\lambda)\bar{p} \quad (49)$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (50)$$

After integrating Eq. (50), we get A^2 =kBC, where k is an integration constant. Without loss of generality, we take k=1. The same procedure as for the Bianchi type-I solution is used here to solve these equations By making use of Eq. (50), we get the constraint equations as follows:

$$p_1 = 1, \quad p_2 = p_3^{-1} = P, \quad q_1 = 0, \quad q_2 = -q_3 = Q. \quad (51)$$

Then, From Eqs. (46)- (51), we readily obtain

$$A = a, \quad B = aP\exp\left[Q\int\frac{dt}{a^3}\right],$$
$$C = aP^{-1}\exp\left[-Q\int\frac{dt}{a^3}\right] \quad (52)$$

Subsituting the value a(t) given in Eq. (31) into Eqs. (46)-(49) into the equations in (52), we obtain the metric functions A, B and C as follows:

$$A = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}}, \quad (53)$$

$$B = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} P\exp\left[Q\tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}}t\right], \quad (54)$$

$$C = \left(t^2 + \frac{2\alpha}{3}\right)^{\frac{1}{3}} P^{-1}\exp\left[-Q\tan^{-1}\left(\frac{3}{2\alpha}\right)^{\frac{1}{2}}t\right] \quad (55)$$

The energy density and bulk viscous pressure for the Bianchi type-V space -time model have values give as

$$\rho = \frac{1}{9(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})^2}\left[(8\pi+3\lambda)(12-36Q^2)t^2-\lambda(12-t^2(12-t^2(6+34Q^2)))-\frac{3m^2(8\pi+2\lambda)}{(t^2+\frac{2\alpha}{3})^{\frac{1}{3}}}\right], \quad (56)$$

$$\begin{aligned}\bar{p} =& \frac{1}{9(t^2+\frac{2\alpha}{3})^2}\left[\frac{(8\pi+3\lambda)}{\lambda(8\pi+2\lambda)(8\pi+4\lambda)}[(8\pi+3\lambda)(12-36Q)t^2-\lambda(12-t^2(6+34Q^2))]-(12-36Q^2)t^2\right] \\ &-\left[\frac{(8\pi+3\lambda)}{\lambda(8\pi+2\lambda)(8\pi+4\lambda)}-1\right]\frac{m^2}{3(t^2+\frac{2\alpha}{3})^{\frac{2}{3}}}. \end{aligned} \quad (57)$$

Using the equation of state parameter gives the bulk viscosity coefficient

$$\begin{aligned}\zeta =& \frac{1}{36t(8\pi+4\lambda)(8\pi+2\lambda)(t^2+\frac{2\alpha}{3})}*[\lambda(8\pi+3\lambda)(12-36Q^2)\epsilon+\lambda(8\pi+2\lambda)(8\pi+4\lambda)(12-36Q^2) \\ &-(8\pi+3\lambda)(12-36Q^2)]+\frac{1}{36\lambda(8\pi+2\lambda)(8\pi+4\lambda)(t^2+\frac{2\alpha}{3})t}*[\lambda(12-t^2(6+34Q^2))-\lambda^2\epsilon* \\ &(12-t^2(6+34Q^2))]-\frac{m^2}{12(8\pi+2\lambda)(8\pi+4\lambda)t\lambda(t^2+\frac{2\alpha}{3})^{\frac{4}{3}}}*[\epsilon(8\pi+2\lambda)+(8\pi+3\lambda)]+\frac{m^2}{12t(t^2+\frac{2\alpha}{3})^{\frac{4}{3}}} \end{aligned} \quad (58)$$

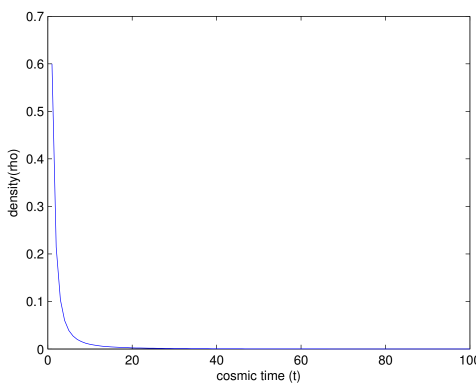

Figure 9. The plot of density ρ verses cosmic time t, Q=1, λ=1, m=0.5, α=1.

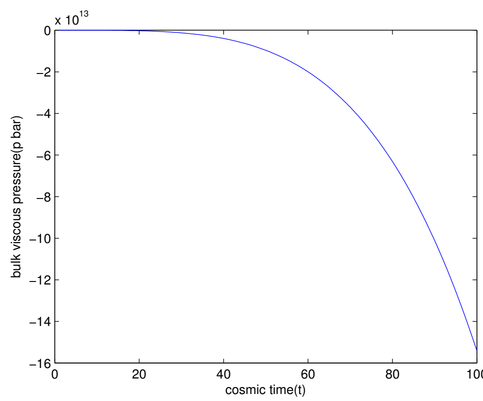

Figure 10. The plot of bulk viscous pressure $\bar{p}$ verses cosmic time t,Q=1,λ=1,m=0.5, α=1.

The directional Hubble parameters H_1, H_2 and H_3 are given the form:

$$H_1 = \frac{2t}{3(t^2+\frac{2\alpha}{3})}, \quad (59)$$

$$H_2 = [3Q+1]\frac{2t}{3(t^2+\frac{2\alpha}{3})}, \quad (60)$$

$$H_3 = [-3Q+1]\frac{2t}{3(t^2+\frac{2\alpha}{3})}. \quad (61)$$

The mean anisotropic parameter A_m has the value

$$A_m = 6\left(1+72Q^2\right). \quad (62)$$

The shear scalar for this model is given by

$$\sigma^2 = \left(\frac{2Qt}{t^2+\frac{2\alpha}{3}}\right)^2. \quad (63)$$

Figs. 9–15 depict the variation of ρ, $\bar{p}$, ζ, $\rho+p$, H, $theta$ and σ with time. From the above results it can be observed

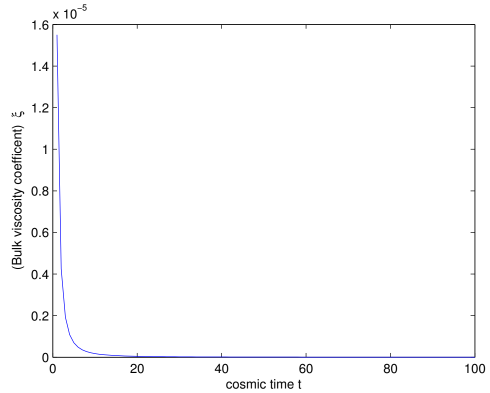

Figure 11. The plot of Bulk viscosity coefficient ζ verses cosmic time t,Q=1,λ=1,m=0.5, α=1.

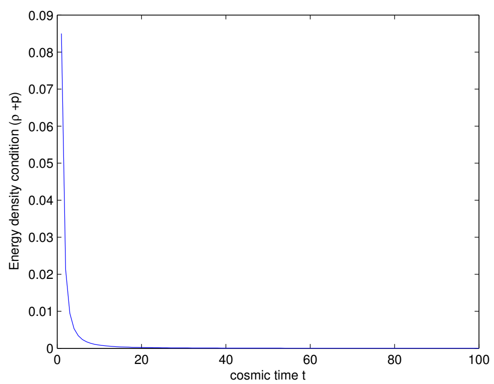

Figure 12. The plot of Energy density condition $\rho + p$ verses cosmic time t, Q=1λ=1, m=0.5, α=1.

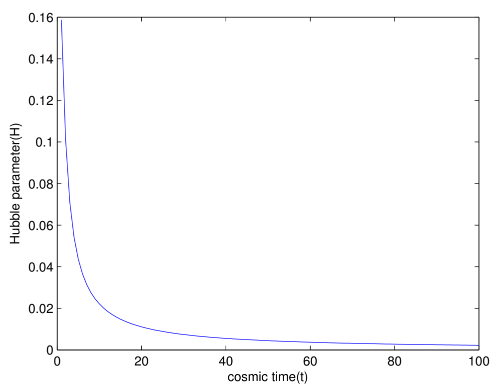

Figure 13. The plot of Hubble parameter H (for second model) verses cosmic time t,Q=1, α=1.

that the model has no singularity at t= 0 and the spatial volume increases as t increases giving the accelerated ex-

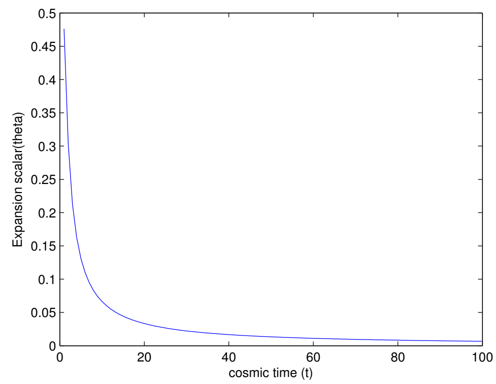

Figure 14. The plot of expansion scalar θ (for model second) verses cosmic time t,Q=1, α=1.

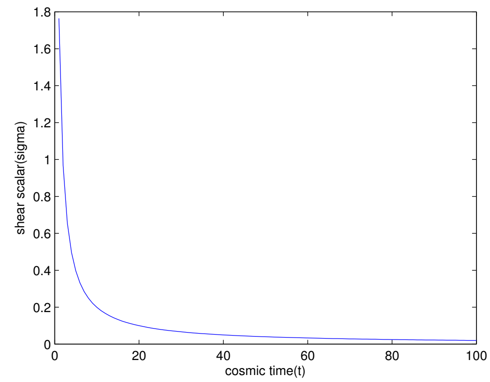

Figure 15. The plot of shear scalar σ cosmic time t,Q=1, α=1.

pansion of the universe. In this model, we also note that σ^2, $\bar{p}$, p, ρ, and ζ are finite at t= 0 while they vanish for infinitely large t. However, $\frac{\sigma^2}{\theta^2} \neq 0$, which shows that the model does not approach isotropy for large time t. From Eq. (28) we see that q< 0 for t $< \sqrt{(2\alpha)}$ and q> 0 for t $> \sqrt{(2\alpha)}$. It is worth mentioning that Shamir et al. [54] have also presented exact solutions of Bianchi type I and V models in f(R, T) gravity theory by applying the law of variation of Hubble's parameter proposed by Berman [51], and Berman and Gomide [52]. Different models were used in that case.

5. Conclusion

In this paper, we have investigated spatially homogeneous and anisotropic cosmological models of Bianchi type I and V, filled with bulk viscous fluid in the framework of f(R,T) gravity theory. The absence of an initial time singularity in both models is a significant feature of the results. The scale factors admit constant values at early times of the universe (t→ 0), after which the scale factors increase with cosmic time without showing any type of initial singularity and finally tend to ∞ as t → ∞. Therefore, the universe represented by both models starts with zero volume in the initial past and expands exponentially approaching infinite volume.

The expansion scalar θ and shear scalar σ are decreasing functions of time and ultimately become zero for large time. The ratio $\frac{\sigma}{\theta}$ tends to a constant as t→ ∞, and therefore the anisotropy in both models are maintained throughout the passage of time. The deceleration parameter q is negative for t< $\sqrt{(2\alpha)}$ and positive for t > $\sqrt{(2\alpha)}$. Therefore, the cosmological models initially accelerate for a certain period of time and thereafter decelerate.

The behavior of the bulk viscosity is illustrated graphically in Figs. 4 and 11. The bulk viscosity decreases with time to give models which are ultimately inflationary (Padmanabhan and Chitre [55]). The matter pressure and energy density are monotonically decreasing functions of time which ultimately tend to zero for large time. Thus, the models would essentially correspond to an empty universe for large time. The conditions (a) $\rho + p \geqslant 0$ (b) $\rho + p \geqslant 0$ are identically satisfied. Models presented in this paper may be useful for understanding the role of bulk viscosity in explaining the decelerating and accelerating behaviors and to understand structure formation in the universe.

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References

[1] A.G. Riess et al., Astrophys. J. 116, 1009 (1998)
[2] S. Perlmutter et al., Astrophys. J. 517, 565 (1998)
[3] D. N. Spergel et al., Astrophys. J. Supple. Ser. 170, 377 (2003)
[4] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004)
[5] S. Nojiri, S.D. Odintsov, Phys. Rev. D 74, 086005 (2006)
[6] S. Nojiri, S.D. Odintsov, Int. J. Geom. Methods Mod. Phys. 4, 115 (2007)
[7] O. Bertolami et al., Phys. Rev. D 75, 104016 (2007)
[8] M.F. Shamir, Astrophys. Space Sci. 330, 183 (2010)
[9] M.F. Shamir, A. Jhangeer Int. J. Theor. Phys. 52, 1326 (2013)
[10] K.S. Adhav, Bulg. J. Phys. 39, 197 (2012)
[11] W. Misner, Nature 214, 40 (1967)
[12] W. Misner, Astrophys. J. 151, 431 (1968)
[13] G.L. Murphy, Phys. Rev. D 8, 4231 (1973)
[14] V.A. Belinski, I.M. Khalatnikov, Sov. Phys. JETP 42, 205 (1976)
[15] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967)
[16] M.R. Setare, A. Sheykhi, Int. J. Mod. Phys. D 19, 171 (2010)
[17] S.R. Roy, O.P. Tiwari, Ind. J. Pure Appl. Math.14, 233 (1983)
[18] M. Syzlowski, M. Heller, Acta Phys. Pol. B 14, 303 (1983)
[19] G. Mohanty, B.D. Pradhan, Int. J. Theor. Phys. 14, 303 (1983)
[20] A. Banerjee, et al. J. Math. Phys. 26, 11 (1985)
[21] F.M. Goenner, F. Kowalewsky, Gen. Rel. Gravit. 21, 467 (1987)
[22] A. Banerjee, A. K.Sanyal, Gen. Rel. Gravit. 20, 103 (1988)
[23] A.A. Coley, Gen. Rel. Gravit. 22, 3 (1990)
[24] A.A.Coley, R.J. Hoogan, J. Math. Phys. 35, 4117 (1994)
[25] A.A. Coley, B.O.J. Tupper, Phys. Rev. D 29, 2701 (1984)
[26] R. Bali, B.L. Meena, Astrophys. Space Sci. 281, 565 (2002)
[27] R. Bali, K. Sharma, Astrophys. Space Sci. 283, 11 (2003)
[28] R. Bali, Anjali, Pramana J. Phys. 63, 481 (2004)
[29] K.S. Adhav et al. Bulg. J. Phys. 34, 260 (2007)
[30] P. S. Baghel, J.P. Singh, Research in Astron. Astrophys. 12, 1457 (2012)
[31] T. Harko et al., Phys. Rev. D 84, 064003 (2011)
[32] R. Myrzakulov, Eur. Phys. J. 72, 2203 (2012)
[33] K.S. Adhav, Astrophys. Space Sci. 339, 365 (2012)
[34] D.R.K. Reddy et al., Astrophys. Space Sci. 342, 249 (2012)
[35] R. Chaubey, A.K. Shukla, Astrophys. Space Sci. 343, 415 (2013)
[36] S. Ram et al., Pramana J. Phys. DOI: 10.1007/s12043-013-0547y (2013)
[37] S. Chandel, S. Ram, Indian J. Phys. DOI: 10.1007/s12648-013-0362-9 (2013)
[38] G. C. Samanta, Int. J. Theor. Phys. 52, 2647 (2013)
[39] D.R.K. Reddy et al., Int. J. Theor. Phys. 51, 249 (2012)
[40] S. Ram, Priyanka, Astrophys. Space Sci. DOI: 10.1007/s10509-013-1517-z (2013)
[41] R.L. Naidu et al., Astrophys. Space Sci. DOI: 10.1007/s1050g-013-1540-o

[42] D.R.K. Reddy et al., Astrophys. Space Sci. DOI: 10.1007/s10.509-013-1417-2 (2013)
[43] N. Ahmed A. Pradhan, Int. J. Theor. Phys. DOI: 10.1007/s10773-013-1809-7 (2013)
[44] S. Chakraborty, arXiv:1212.3050 (2013)
[45] M. Sharif, M. Zubai, Astrophys. Space Sci. 349, 457 (2014)
[46] P.K. Sahoo et al., Eur. Phys. J. Plus 129, 49 (2014)
[47] B. Mishra, P.K. Sahoo, Astrophys. Space Sci. DOI: 10.1007/s10509-014-1914-y
[48] N.K. sharma, J.K. Singh, Int. J. Theor. Phys. DOI: 10.1007/s 10713-019-20896
[49] C.P. Singh, V. Singh, Gen. Relativ. Gravit. 46, 1696 (2014)
[50] N.J. Poplawski, arXiv:gr-qc/0608031.
[51] M.S. Berman, Nuovo Cimento B 74, 182 (1983)
[52] M.S. Berman, F. M. Gomide, Gen. Relativ. Gravit 20, 191 (1988)
[53] Abdussatter, S.R. Prajapati, Astrophys. Space Sci. 331, 657 (2011)
[54] M.F. Shamir, A. Jhangeer, A.A. Bhatti, arXiv:1207.0708 (2012)
[55] T. Padmanabhan, S.M. Chitre, Phys. Lett. A 120, 433 (1987)