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# The Freudenthal triple classification of 3-qubit entanglement and STU black hole entropy

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**Abstract.** We show that the 3-qubit entanglement classification is naturally encoded by that elegant algebraic structure, well known from supergravity, the Freudenthal triple system. In particular, we show that the six entanglement classes: (1) Separable A-B-C, (2a) Biseparable A-BC, (2b) Biseparable B-CA, (2c) Biseparable C-AB, (3) W and (4) Greenberger-Horne-Zeilinger correspond respectively to the Freudenthal triple ranks 1, 2a, 2b, 2c, 3, 4. Based on work done in collaboration with D. Dahanayake, M. J. Duff, H. Ebrahim and W. Rubens.

### 1. Introduction

The interdisciplinary nature of Quantum Theory and Symmetries 6 made for a truly unique and rewarding experience - it was certainly a privilege to be counted amongst its participants. Moreover, as the week went by, it became apparent that not only is this union of disciplines exciting, it is unequivocally of our time. We are living at the dawn of the (quantum) information age whilst, hand-in-hand, pushing the boundaries of fundamental particle physics as we begin in earnest to probe new unchartered energy scales. In such a light it seems nothing but natural to bring together the frontiers of quantum information science and high energy physics.

While this is a tall order to live up to, the present work, at least in some limited sense, fits the bill. On the one hand, its primary focus centres on the classification of quantum entanglement and, in particular, the 3-qubit system. Entanglement lies at the heart of quantum information theory and its many diverse applications, such as quantum computation, cryptography and communication. On the other hand, the mathematical structures employed in this analysis are imported from the seemingly disparate field of M-theory and, in particular, the study of stringy black holes, which have played, and continue to play, an essential role in understanding the non-perturbative aspects of quantum gravity.

Indeed, while logically independent, this work has its provenance in the black-hole/qubit correspondence, reviewed in [1]. In 2006 [2] it was observed that the measure of 3-qubit entanglement, the so called 3-tangle [3, 4], and the entropy of the STU black hole [5, 6] are both given by the same mathematical object, Cayley's hyperdeterminant [7]. Further work [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1, 20, 21, 22] has led to a more complete dictionary translating a variety of phenomena in one language to those in the other. In fact, the black holes of the maximally supersymmetric  $D = 4, \mathcal{N} = 8$  supergravity have been related to a very special tripartite entanglement of seven qubits [11, 12]. It is well known that  $\mathcal{N} = 8$  supergravity is intimately related to the exceptional Jordan algebra of  $3 \times 3$  split-octonionic Hermitian matrices and the Freudenthal triple system (FTS) [23]. Since the STU model may be consistently embedded in the  $D = 4, \mathcal{N} = 8$  theory it is natural to study the application of the Jordan algebras and the FTS to both three qubits and STU black holes [8, 24, 11, 12, 13, 14, 17, 18, 1, 25].

Accordingly, in this contribution we briefly review the FTS classification of 3-qubit entanglement as presented in [26]. Unfortunately, the corresponding treatment of the STU black hole will only be touched upon due to limited space. We begin by recalling the conventional classification of 3-qubit entanglement as first obtained by Dür *et al.* using the paradigm of *stochastic local operations and classical communication* (SLOCC) [27]. We then set up the FTS framework before illustrating how it reproduces the correct entanglement classification. The entanglement orbits obtained in [26] via the FTS are also presented. We conclude by summarising the associated analysis of the STU black hole.

#### 2. Conventional three-qubit entanglement classification

The proper qualitative and quantitative characterisation of multipartite entanglement remains one of the longest standing open problems in quantum information science. While the complete picture still eludes us, substantial progress has been made using the SLOCC framework, as first proposed in [28]. Two quantum states of a composite system are considered SLOCCequivalent if and only if they may be probabilistically interrelated using purely local operations, i.e. operations restricted to act on constituent parts independently, supplemented by the classical communication of any relevant information between said constituents. The essential observation is that while such operations may be used to establish arbitrary classical correlations, they are manifestly unable to create genuine quantum correlations. Hence, two SLOCC-equivalent states possess the same entanglement. For more details see [29, 30] and the references therein.

In [27] it was argued that two states of an *n*-qubit system are SLOCC-equivalent if and only if they are related by  $[SL(2, \mathbb{C})]^{\otimes n}$ , which we will refer to as the *SLOCC-equivalence group*. It may be usefully thought of as the "gauge" group of *n*-qubit entanglement. In particular, the 3-qubit (Alice, Bob and Charlie) state vector is given by,

$$|\psi\rangle = a_{ABC}|ABC\rangle, \qquad A, B, C = 0, 1,$$
(1)

where  $a_{ABC}$  transforms as a  $(\mathbf{2}, \mathbf{2}, \mathbf{2})$  under the 3-qubit SLOCC-equivalence group  $SL_A(2, \mathbb{C}) \times SL_B(2, \mathbb{C}) \times SL_C(2, \mathbb{C})$ .

Dür *et al.* [27] used simple arguments involving the ranks and ranges of the reduced density matrices to show that there are six SLOCC equivalence classes, representing six physically distinct forms of entanglement: (1) Separable A-B-C, (2a) Biseparable A-BC, (2b) Biseparable B-CA, (2c) Biseparable C-AB, (3) W and (4) Greenberger-Horne-Zeilinger (GHZ). These six classes may be distinguished [27] by the three local entropies,

$$S_A = \det \rho_A, \quad S_B = \det \rho_B, \quad S_C = \det \rho_C,$$
 (2)

 $(\rho_A = \text{tr}_{BC} |\psi\rangle \langle \psi|$  and similarly for  $\rho_B$  and  $\rho_C$ ) and Cayley's hyperdeterminant [7, 4],

$$Det a_{ABC} = -\frac{1}{2} \epsilon^{A_1 A_2} \epsilon^{A_3 A_4} \epsilon^{B_1 B_2} \epsilon^{B_3 B_4} \epsilon^{C_1 C_3} \epsilon^{C_3 C_4} a_{A_1 B_1 C_1} a_{A_2 B_2 C_2} a_{A_3 B_3 C_3} a_{A_4 B_4 C_4}, \qquad (3)$$

which is the unique  $[SL(2, \mathbb{C})]^3$  quartic invariant also displaying a permutation symmetry under the interchange of Alice, Bob and Charlie. This classification of classes is summarised in Table 1. It is only the W and GHZ classes that possess genuine tripartite entanglement - "three qubits can be entangled in two inequivalent ways" [27]. The GHZ class is distinguish by non-vanishing hyperdeterminant and has the highest order of entanglement in the sense that it maximally violates the principle of local realism [31, 32]. The non-invertible SLOCC operations (lying outside the SLOCC-equivalence group) may be used to descend through these classes, creating the entanglement hierarchy as depicted on the right of Figure 1. Let us now turn our attention to FTS analysis of three qubits.

Class	Poprogentative	Condition					
	Representative	$\psi$	$S_A$	$S_B$	$S_C$	$\operatorname{Det} a$	
A- $B$ - $C$	$ 000\rangle$	$\neq 0$	= 0	= 0	= 0	= 0	
A-BC	010 angle+ 001 angle	$\neq 0$	= 0	$\neq 0$	$\neq 0$	= 0	
B- $CA$	100 angle+ 001 angle	$\neq 0$	$\neq 0$	= 0	$\neq 0$	= 0	
C-AB	010 angle+ 100 angle	$\neq 0$	$\neq 0$	$\neq 0$	= 0	= 0	
W	100 angle+ 010 angle+ 001 angle	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	= 0	
GHZ	$ 000\rangle + \sqrt{\text{Det }a} 111\rangle$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	

Table 1: The values of the local entropies  $S_A, S_B$ , and  $S_C$  and the hyperdeterminant Det a are used to partition three-qubit states into entanglement classes.

Figure 1: Left: Classification of  $\mathcal{N} = 8$  black holes according to susy. N denotes the number of intersecting D-branes in the microscopic picture. Right: The entanglement classification of 3-qubits. The arrows represent the removal of a D-brane or a non-invertible SLOCC operation.



#### 3. The FTS classification of qubit entanglement

3.1. Cubic Jordan algebras and the Freudenthal triple system

A Jordan algebra  $\mathfrak{J}$  is vector space defined over a ground field  $\mathbb{F}$  equipped with a bilinear product satisfying

$$X \circ Y = Y \circ X, \quad X^2 \circ (X \circ Y) = X \circ (X^2 \circ Y), \quad \forall \ X, Y \in \mathfrak{J}.$$

$$\tag{4}$$

The class of *cubic* Jordan algebras are constructed as follows. Let V be a vector space equipped with a cubic norm, i.e. a homogeneous map of degree three,  $N: V \to \mathbb{F}$  s.t.  $N(\lambda X) = \lambda^3 N(X)$ ,  $\forall \lambda \in \mathbb{F}, X \in V$ , with trilinear linearisation

$$N(X,Y,Z) := \frac{1}{6} [N(X+Y+Z) - N(X+Y) - N(X+Z) - N(Y+Z) + N(X) + N(Y) + N(Z)].$$
(5)

If V further contains a base point  $N(c) = 1, c \in V$  one may define the following three maps,

$$Tr: V \to \mathbb{F}; \quad X \mapsto 3N(c, c, X),$$
  

$$S: V \times V \to \mathbb{F}; \quad (X, Y) \mapsto 6N(X, Y, c),$$
  

$$Tr: V \times V \to \mathbb{F}; \quad (X, Y) \mapsto Tr(X) Tr(Y) - S(X, Y).$$
(6)

A cubic Jordan algebra  $\mathfrak{J}$ , with multiplicative identity  $\mathbb{1} = c$ , may be derived from any such vector space if N is Jordan cubic. That is: (i) the trace bilinear form (6) is non-degenerate (ii)

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the quadratic adjoint map,  $\sharp : \mathfrak{J} \to \mathfrak{J}$ , uniquely defined by  $\operatorname{Tr}(X^{\sharp}, Y) = 3N(X, X, Y)$ , satisfies  $(X^{\sharp})^{\sharp} = N(X)X, \forall X \in \mathfrak{J}$ . The Jordan product is then given by

$$X \circ Y = \frac{1}{2} \left( X \times Y + \operatorname{Tr}(X)Y + \operatorname{Tr}(Y)X - S(X,Y)\mathbb{1} \right), \tag{7}$$

where,  $X \times Y$  is the linearisation of the quadratic adjoint,  $X \times Y = (X + Y)^{\sharp} - X^{\sharp} - Y^{\sharp}$ .

Given a cubic Jordan algebra  $\mathfrak{J}$  defined over a field  $\mathbb{F}$ , one is able to construct an FTS by defining the vector space  $\mathfrak{F}(\mathfrak{J})(=\mathfrak{F})$ ,

$$\mathfrak{F}(\mathfrak{J}) = \mathbb{F} \oplus \mathbb{F} \oplus \mathfrak{J} \oplus \mathfrak{J}.$$
(8)

An arbitrary element  $x \in \mathfrak{F}(\mathfrak{J})$  may be written as a " $2 \times 2$  matrix",

$$x = \begin{pmatrix} \alpha & X \\ Y & \beta \end{pmatrix} \quad \text{where } \alpha, \beta \in \mathbb{F} \quad \text{and} \quad X, Y \in \mathfrak{J}.$$
(9)

The FTS comes equipped with a non-degenerate bilinear antisymmetric quadratic form, a quartic form and a trilinear triple product [33, 34, 35, 36, 37]:

(i) Quadratic form  $\{x, y\}$ :  $\mathfrak{F} \times \mathfrak{F} \to \mathbb{F}$ 

$$\{x, y\} = \alpha \delta - \beta \gamma + \operatorname{Tr}(X, W) - \operatorname{Tr}(Y, Z), \quad \text{where} \quad x = \begin{pmatrix} \alpha & X \\ Y & \beta \end{pmatrix}, \ y = \begin{pmatrix} \gamma & Z \\ W & \delta \end{pmatrix}.$$
(10a)

(ii) Quartic form  $q: \mathfrak{F} \to \mathbb{F}$ 

$$q(x) = -2[\alpha\beta - \text{Tr}(X, Y)]^2 - 8[\alpha N(X) + \beta N(Y) - \text{Tr}(X^{\sharp}, Y^{\sharp})].$$
(10b)

(iii) Triple product  $T: \mathfrak{F} \times \mathfrak{F} \times \mathfrak{F} \to \mathfrak{F}$  which is uniquely defined by

$$\{T(x, y, w), z\} = q(x, y, w, z)$$
(10c)

where q(x, y, w, z) is the full linearisation of q(x) such that q(x, x, x, x) = q(x).

The *automorphism* group is given by the set of invertible  $\mathbb{F}$ -linear transformations preserving the quadratic and quartic forms [33, 34],

$$\operatorname{Aut}(\mathfrak{F}) = \{ \sigma \in \operatorname{Iso}_{\mathbb{F}}(\mathfrak{F}) | q(\sigma x) = q(x), \{ \sigma x, \sigma y \} = \{ x, y \}, \ \forall x, y \in \mathfrak{F} \}.$$
(11)

The automorphism group corresponds to the U-duality group of a variety 4-dimensional supergravities (see for example [38, 39] and the references therein). The conventional concept of matrix rank may be generalised to Freudenthal triple systems in a natural and Aut( $\mathfrak{F}$ ) invariant manner. The rank of an arbitrary element  $x \in \mathfrak{F}$  is uniquely defined by [36, 37]:

$$\begin{aligned} \operatorname{Rank} x &= 1 \Leftrightarrow 3T(x, x, y) + x\{x, y\}x = 0 \ \forall y;\\ \operatorname{Rank} x &= 2 \Leftrightarrow \exists y \text{ s.t. } 3T(x, x, y) + x\{x, y\}x \neq 0, \ T(x, x, x) = 0;\\ \operatorname{Rank} x &= 3 \Leftrightarrow T(x, x, x) \neq 0, \ q(x) = 0;\\ \operatorname{Rank} x &= 4 \Leftrightarrow q(x) \neq 0. \end{aligned}$$
(12)

3.2. FTS representation of three-qubits To define the 3-qubit FTS we use the particularly simple cubic Jordan algebra  $\mathfrak{J}_{ABC} := \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$  with cubic form

$$N(X) = X_1 X_2 X_3$$
, where  $X = (X_1, X_2, X_3) \in \mathfrak{J}_{ABC}$ . (13)

It is not difficult to verify that this defines a *bona fide* cubic Jordan algebra [40, 26].

We are now in a position to define our 3-qubit FTS,

$$\mathfrak{F}_{ABC} := \mathfrak{F}(\mathfrak{J}_{ABC}) = \mathbb{C} \oplus \mathbb{C} \oplus \mathfrak{J}_{ABC} \oplus \mathfrak{J}_{ABC}, \tag{14}$$

as the representation space of three qubits. We identify three qubit wavefunctions  $|\psi\rangle, |\phi\rangle \dots$  with elements  $\Psi, \Phi, \dots$  of the 8-dimensional complex vector space  $\mathfrak{F}_{ABC}$  via,

$$|\psi\rangle = a_{ABC}|ABC\rangle \quad \longleftrightarrow \quad \Psi = \begin{pmatrix} a_{111} & (a_{001}, a_{010}, a_{100}) \\ (a_{110}, a_{101}, a_{011}) & a_{000} \end{pmatrix}, \tag{15}$$

The powerful machinery of the FTS may now be applied to the 3-qubit system.

First, we note that the automorphism group is the SLOCC-equivalence group,  $\operatorname{Aut}(\mathfrak{F}_{ABC}) = \operatorname{SL}_A(2,\mathbb{C}) \times \operatorname{SL}_B(2,\mathbb{C}) \times \operatorname{SL}_C(2,\mathbb{C})$ . Using (10b) one finds that the quartic norm  $q(\Psi)$  is related to Cayley's hyperdeterminant by

$$q(\Psi) = \{T(\Psi, \Psi, \Psi), \Psi\} = 2 \det \gamma^{A} = 2 \det \gamma^{B} = 2 \det \gamma^{C} = -2 \operatorname{Det} a_{ABC},$$
(16)

where, following [41], we have defined the matrix  $\gamma^A$  by

$$(\gamma^A)_{A_1A_2} = \varepsilon^{B_1B_2} \varepsilon^{C_1C_2} a_{A_1B_1C_1} a_{A_2B_2C_2},\tag{17}$$

and similar for  $\gamma^{B,C}$ , which transform respectively as  $(\mathbf{3},\mathbf{1},\mathbf{1}), (\mathbf{1},\mathbf{3},\mathbf{1}), (\mathbf{1},\mathbf{1},\mathbf{3})$  under  $[\mathrm{SL}(2,\mathbb{C})]^3$ . The  $\gamma$ 's are related to the local entropies of section 2 by

$$S_A = 4 \Big[ \operatorname{tr} \gamma^{B\dagger} \gamma^B + \operatorname{tr} \gamma^{C\dagger} \gamma^C \Big], \tag{18}$$

and its cyclic permutations. The triple product maps a state  $\Psi$ , which transforms as a  $(\mathbf{2}, \mathbf{2}, \mathbf{2})$  of  $[\mathrm{SL}(2, \mathbb{C})]^3$ , to another state  $T(\Psi, \Psi, \Psi)$ , cubic in the state vector coefficients, also transforming as a  $(\mathbf{2}, \mathbf{2}, \mathbf{2})$ . Explicitly,  $T(\Psi, \Psi, \Psi)$  may be written as

$$T(\Psi, \Psi, \Psi) = T_{ABC} |ABC\rangle, \quad \text{where} \quad T_{ABC} = \varepsilon^{A'A''} a_{A'BC} (\gamma^A)_{A''A}. \tag{19}$$

Note,  $T_{ABC}$  may also be written using  $\gamma^{B,C}$  in the obvious manner. We may now assign an abstract FTS rank to an arbitrary 3-qubit state  $\Psi$  using (12).

3.3. The FTS rank entanglement classes

Rank 1 and the class of separable states: A non-zero state  $\Psi$  is rank 1 iff

$$\Upsilon := 3T(\Psi, \Psi, \Phi) + \{\Psi, \Phi\}\Psi = 0, \quad \forall \ \Phi,$$
(20)

In particular, (20) implies  $T(\Psi, \Psi, \Psi) = 0$ , from which we deduce that at most one  $\gamma$  is non-vanishing since

$$(\gamma^A)_{A_1A_2}(\gamma^C)_{C_1C_2} = \varepsilon^{B_2B_1} a_{A_1B_1C_1} T_{A_2B_2C_2} + \varepsilon^{B_1B_2} a_{A_2B_2C_1} T_{A_1B_1C_2}$$
(21)

Class	Rank	FTS rank condition			
		vanishing	non-vanishing		
Null	0	$\Psi$	_		
A- $B$ - $C$	1	$3T(\Psi,\Psi,\Phi) + \{\Psi,\Phi\}\Psi$	$\Psi$		
A- $BC$	2a	$T(\Psi,\Psi,\Psi)$	$\gamma^A$		
B- $CA$	2b	$T(\Psi,\Psi,\Psi)$	$\gamma^B$		
$C ext{-}AB$	2c	$T(\Psi,\Psi,\Psi)$	$\gamma^C$		
W	3	$q(\Psi)$	$T(\Psi,\Psi,\Psi)$		
GHZ	4	_	$q(\Psi)$		

Table 2: The entanglement classification of three qubits as according to the FTS rank system.

and similarly for  $(\gamma^B)_{B_1B_2}(\gamma^A)_{A_1A_2}$  and  $(\gamma^C)_{C_1C_2}(\gamma^B)_{B_1B_2}$ . Then, using

$$-\Upsilon_{A_1B_1C_1} = \varepsilon^{A_2A_3} b_{A_3B_1C_1}(\gamma^A)_{A_1A_2} + \varepsilon^{B_2B_3} b_{A_1B_3C_1}(\gamma^B)_{B_1B_2} + \varepsilon^{C_2C_3} b_{A_1B_1C_3}(\gamma^C)_{C_1C_2}$$
(22)

where

$$|\phi\rangle = b_{ABC}|ABC\rangle \quad \longleftrightarrow \quad \Phi = \begin{pmatrix} b_{111} & (b_{001}, b_{010}, b_{100})\\ (b_{110}, b_{101}, b_{011}) & b_{000} \end{pmatrix}, \tag{23}$$

we observe that (20) implies all three  $\gamma$ 's must vanish. Using (18) it is then clear that all three local entropies vanish.

Conversely, from (18),  $S_A = S_B = S_C = 0$  implies that each of the three  $\gamma$ 's vanish and the rank 1 condition is satisfied.

Rank 2 and the class of biseparable states: A non-zero state  $\Psi$  is rank 2 or less if and only if  $T(\Psi, \Psi, \Psi) = 0$ . To not be rank 1 there must exist some  $\Phi$  such that  $3T(\Psi, \Psi, \Phi) + {\Psi, \Phi} \Psi \neq 0$ . It was shown above that this is equivalent to one and only one non-vanishing  $\gamma$  matrix.

Using (18) it is clear that the choices  $\gamma^A \neq 0$  or  $\gamma^B \neq 0$  or  $\gamma^C \neq 0$  give  $S_A = 0, S_{B,C} \neq 0$  or  $S_B = 0, S_{C,A} \neq 0$  or  $S_C = 0, S_{A,B} \neq 0$ , respectively. These are precisely the conditions for the biseparable class A-BC or B-CA or C-AB presented in Table 1.

Conversely, using (18) and the fact that both the local entropies and  $tr(\gamma^{\dagger}\gamma)$  are positive semidefinite, we find that all states in the biseparable class are rank 2, the particular subdivision being given by the corresponding non-zero  $\gamma$ .

Rank 3 and the class of W-states: A non-zero state  $\Psi$  is rank 3 if  $q(\Psi) = -2 \text{ Det } a = 0$  but  $T(\Psi, \Psi, \Psi) \neq 0$ . From (19) all three  $\gamma$ 's are then non-zero but from (16) all have vanishing determinant. In this case (18) implies that all three local entropies are non-zero but Det a = 0. So all rank 3  $\Psi$  belong to the W-class.

Conversely, from (18) it is clear that no two  $\gamma$ 's may simultaneously vanish when all three S's are non-zero (> 0).  $T(\Psi, \Psi, \Psi) = 0$  implies at least two of the  $\gamma$ 's vanish. Consequently, for all W-states  $T(\Psi, \Psi, \Psi) \neq 0$  and, therefore, all W-states are rank 3.

Rank 4 and the class of GHZ-states: The rank 4 condition is given by  $q(\Psi) \neq 0$  and, since for the 3-qubit FTS  $q(\Psi) = -2 \text{ Det } a$ , we immediately see that the set of rank 4 states is equivalent to the GHZ class of genuine tripartite entanglement as in Table 2. Note, the GHZ class is a one complex dimensional space of orbits parameterised the quartic norm q.

These results are summarised in Table 2. The stability groups H for each orbit coset  $[SL(2, \mathbb{C})]^3/H$ , as obtained using the FTS framework in [26], are also presented in Table 3.

Class	FTS Rank	Stabiliser	dim	Projective stabiliser	dim
Separable	1	$[\mathrm{SO}(2,\mathbb{C})]^2\ltimes\mathbb{C}^3$	4	$[\mathrm{SO}(2,\mathbb{C})\ltimes\mathbb{C}]^3$	3
Biseparable	2	$\mathrm{SO}(3,\mathbb{C}) imes\mathbb{C}$	5	$\mathrm{SO}(3,\mathbb{C}) \times (\mathrm{SO}(2,\mathbb{C}) \ltimes \mathbb{C})$	4
W	3	$\mathbb{C}^2$	7	$\mathrm{SO}(2,\mathbb{C})\ltimes\mathbb{C}^2$	6
GHZ	4	$[\mathrm{SO}(2,\mathbb{C})]^2$	7	$[\mathrm{SO}(2,\mathbb{C})]^2$	7

Table 3: Stability groups and complex dimensions of the 3-qubit entanglement orbits

Table 4: Orbits stabilisers of STU black holes in the real case  $\mathfrak{J}_{STU} = \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$ .

Class	FTS Rank	$q(\Psi)$	Stabiliser	dim	Black hole	Susy
Separable	1	= 0	$[\mathrm{SO}(1,1)]^2 \ltimes \mathbb{R}^3$	4	Small	1/2
Biseparable	2	= 0	$\mathrm{SO}(2,1) \times \mathbb{R}$	5	Small	1/2
W	3	= 0	$\mathbb{R}^2$	7	Small	1/2
GHZ	4	> 0	$[U(1)]^2$	7	Large	1/2
GHZ	4	> 0	$[U(1)]^2$	7	Large	0
GHZ	4	< 0	$[SO(1,1)]^2$	7	Large	0

Note, we consider the states as both points in the Hilbert space and rays in its projectivisation. The 3-qubit separable projective coset is just a direct product of three individual qubit cosets  $SL(2, \mathbb{C})/SO(2, \mathbb{C}) \ltimes \mathbb{C}$ . The biseparable projective coset is just the direct product of an entangled 2-qubit coset  $[SL(2, \mathbb{C})]^2/SO(3, \mathbb{C})$  with a 1-qubit coset as one would expect.

# 4. Black holes

As emphasized in section 1 there is a growing dictionary relating various features of qubit systems to aspects of black holes in supergravity. Using the dictionary relating the STU black hole to the 3-qubit system it is clear that this FTS analysis carries through. The results are summarised in Table 4. We see that the physically distinct forms of 3-qubit entanglement are directly related to physically distinct STU black hole solutions [8]. The entangled GHZ class corresponds to "large" black holes with non-vanishing classical horizon, while the remaining classes are related to "small" black holes with vanishing classical horizon. However, there are differences. The black hole charges are real valued (with Jordan algebra  $\mathfrak{J}_{STU} = \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$ ) and really correspond to the case of real qubits or "rebits' which are, as noted in [42, 9], qualitatively different from the complex case. This feature manifests itself in the different real forms appearing in the GHZ class as in Table 4. For details see [23, 38, 43, 44, 1, 26]. A microscopic interpretation of this correspondence may be given by embedding the STU model in type II string theory compactified on a 6-torus [16]. The 3-qubit states are given by the possible wrapping configurations of intersecting D-branes around the three 2-tori of the compact space, explaining the origin of their binary nature. In this case the entanglement classes are related to the amount of supersymmetry preserved by the corresponding intersecting D-brane configuration as in Figure 1. For more details the interested reader is referred to [16, 1].

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