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A SIMPLE METHOD OF CALCULATING THE EFFECTIVENESS OF HIGH TEMPERATURE RADIATION HEAT SHIELDS, AND ITS APPLICATION IN THE CONSTRUCTION OF A HIGH TEMPERATURE VACUUM FURNACE*

Kimo Welch
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A method is given whereby one may graphically determine the radiation heat transfer properties of a number of closely packed radiation shields. The method uses the gray body formula, taking into account variations in total thermal emissivity of the adjacent shields as a function of temperature. Results were used in the construction of a high temperature vacuum furnace. Excellent correlation was found between graphically predicted and calorimetrically measured results.

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INTRODUCTION

With very large furnace hot zones, power losses are appreciable. At the time construction of a high temperature furnace was considered, we were advised no simple way of calculating radiation losses was known. The literature contained a smattering of empirical data, but not enough to give us complete confidence in determining losses for our particular configuration. Data were available from vendors for particular heater and heat shield configurations. However, these data included "secondary" heat losses resulting from work support structures, power-feed-through penetrations, etc. Data were insufficient to separate secondary losses from primary radiation losses through the ideal radiation shields.

CUSTOMARY METHOD OF CALCULATION

The total power transferred between two shielding surfaces, using notation similar to Crawford,[1] is

$$P = \sigma \left(\frac{1}{\beta\epsilon_0} + \frac{1}{\alpha\epsilon_1}\right)^{-1} \left[\left(A_1 T_1^4/\beta\right) - \left(A_0 T_0^4/\alpha\right)\right] \qquad (1)$$

where A, T, and ϵ denote the area, temperature, and total thermal emissivity, σ is the Stefan-Boltzmann constant, α the probability that radiation leaving the cooler surface will strike the hotter, and β the reverse probability. Subscripts "1" and "0" identify the parameters of the hotter and cooler surfaces, respectively.

At this point, the approximation is usually made that in a multishield package, with high temperature gradients, emissivities will be assumed constant from shield to shield. However, for very large hot zones, with closely spaced heater and heat shields, variations in thermal emissivity of respective shields would seem much more important than shape factors. This is due to the fact that close shield spacings, on the order of 1 mm, result in shield packages (perhaps contained within a water cooled "cold wall") closely approaching an infinite plane approximation. This is particularly true for furnaces with hot zone dimensions greater than 10 cm. The standard equations then take the form

$$Q = a_1\left(T_1^4 - T_0^4\right) = a_2\left(T_2^4 - T_1^4\right) = \cdots = a_n\left(T_n^4 - T_{n-1}^4\right) \tag{2}$$

where

$$a_k = \sigma \frac{\epsilon_k \epsilon_{k-1}}{\epsilon_k + \epsilon_{k-1} - \epsilon_k \epsilon_{k-1}} , \qquad k = 1, 2, \cdots, n \tag{3}$$

and

T_0 = the cold wall temperature

T_k = the temperature of the kth surface

ϵ_k = the total thermal sensitivity of the kth surface

Q = the radiated power density through the package

As a rule, the object of all of this manipulation is to determine the hot zone temperature for Q and T_0. To achieve this, Eq. (2) is manipulated to give

$$T_n^4 = Q \frac{[(a_1 a_2 \cdots a_{n-1}) + (a_1 a_2 \cdots a_{n-2} a_n) + \cdots (a_2 a_3 a_4 \cdots a_n)]}{(a_1 a_2 \cdots a_n)} + T_0^4 \tag{4}$$

which for $a_1 = a_2 = \cdots = a_k = \cdots = a_n$, reduces to

$$T_n^4 = nQ/a_k + T_0^4 \tag{5}$$

The problem here is in the choice of a_k. With emissivities varying as much as an order of magnitude with temperature, selecting the proper a_k is a matter of guess work, and even the maximum a_k will lead to errors of a factor of two in required power for a 2000° C hot zone with half a dozen heat shields.

SIMPLIFIED GRAPHICAL APPROACH

Assume a certain heat flux density Q through the radiation shields (a restriction perhaps imposed by a power supply). The three other parameters known are the hot zone temperature required, cold wall temperature T_0 and emissivity ϵ_0. Let n be some general shield within the shielding package. The gray body equation for heat loss between two closely spaced plane surfaces is:

$$Q = \sigma \left(\frac{1}{\epsilon_{n+1}} + \frac{1}{\epsilon_n} - 1\right)^{-1} \left[T_{n+1}^4 - T_n^4\right] \tag{6}$$

At this point, assume T_n, ϵ_n, and Q are known. T_{n+1} and ϵ_{n+1}, which is a function of T_{n+1}, are unknown. They may be found by trial and error "fitting" into Eq. (6), an operation best suited to a computer. However one may manipulate Eq. (6) to take the form

$$\underbrace{\left[T_{n+1}^4 - \frac{Q}{\sigma \epsilon_{n+1}}\right]}_{\text{LHS}} = \underbrace{\left[T_n^4 + \frac{Q}{\sigma}\left(\frac{1 - \epsilon_n}{\epsilon_n}\right)\right]}_{\text{RHS}} \tag{7}$$

which for a fixed value of Q requires that there be some function F such that

$$F_{LHS}(T_{n+1}) = F_{RHS}(T_n) \tag{8}$$

Using standard references for the ϵ's of a particular material as a function of temperature (tungsten in our case), each side of Eq. (7) may be separately plotted as a function of temperature. In doing this, two curves are obtained which when plotted appear as shown in Fig. 1. The upper curve represents the exact solution of the right-hand side of F, and the lower curve represents the exact solution of the left-hand side of F for $Q = 1$ watt/cm^2. Note that $T_n < T_{n+1}$ in Eq. (7). Also, it should be stressed that Fig. 1 is valid only for tungsten and with a heat flux density of 1 watt/cm^2. If a different power density is prescribed and different shielding material used, a new figure must be drawn using appropriate values of $\epsilon(T)$ and Q.

In order to use an equivalent to Fig. 1 the temperature of the first (i.e., outer-most) shield must be determined. In this case, the RHS of Eq. (7) is solved using selected values for cold wall emissivity and temperature. Note that this calculated value for F has nothing to do with the properties of the tungsten shields. However, once the value of F is found for the boundary, it must also apply to the LHS of Eq. (7). Therefore, projection horizontally from the ordinate intercept of this first F value to the point of interception on the lower function (LHS of Eq. (7)) yields the temperature at which the first shield must be operating. In this application the first shield plays the role of the hotter of two shields in Eq. (7). Holding the temperature of this shield constant and projecting vertically to the upper F function is equivalent to now letting the first shield play the role of the "cooler" of two shields (i.e., the RHS of Eq. (7)).

Once F is established for the first shield, according to Eq. (7), it must be the same for the second shield. Therefore, a horizontal projection (i.e., holding F constant) from the upper function first T_n intercept to the lower function gives the intercept T_{n+1}, or the temperature of the second shield. One may now let the second shield play the role of the "cooler" of the two shields represented by Eq. (7) by again vertically projecting to the upper function. This gives a new F which when projected horizontally intercepts on the lower function at the temperature of the third shield. By further projection of horizontal and vertical lines one is able to determine the temperature of successive shields closer to the hot zone.

A circle, rectangle, and triangle are shown plotted on the upper function in Fig. 1. These represent the temperature of the outer-most heat shield for cold wall emissivities of 0.6, 0.09, and 0.02 respectively. Cold walls, usually copper in very high temperature furnaces, tend to become optically contaminated due to the condensation of metallic vapors originating from shielding, heaters, and work during high temperature cycles. This tends to increase the emissivity of the cold wall on the inner surface with time. The tables included in Fig. 1, 2, and 3, summarize the effect variations in cold wall emissivity have on shielding efficiency and temperature of the outer shield.

At this point, nothing has been said about the temperature of the hot zone. One need only select a hot zone temperature value on the abscissa of Fig. 1, project vertically to the interception point on the lower function, and count the number of shields implied. Figures 2 and 3 provide similar information for power densities of 2 and 4 watts/cm^2 respectively. Figure 4 shows how the outer heat shield temperature will vary for various power densities.

APPLICATION IN CONSTRUCTION OF HIGH TEMPERATURE FURNACE

A high temperature furnace was constructed using the above method to calculate radiation losses through shielding. The inside diameter of the shield package is $\sim$ 23 cm; the length is $\sim$ 34 cm. The heater cylinder, a woven tungsten mesh has an outside diameter of 20 cm and is 30 cm long. A view of the furnace interior, with top cold wall plate and heat shield package removed, is shown in Fig. 5. Six layers of tungsten sheet were used as shielding concentric with the heater, and the equivalent of 8 layers used as top and bottom shielding.

Separate water cooling and temperature monitoring circuits were used in the cold wall cylinder and top and bottom plates. Water flow and temperature of the vacuum vessel and power feed-throughs also were separately monitored. This made it possible to calorimetrically measure power dissipation in each of these circuits and thus determine the mode and source of power exchange. Electrical vs. calorimetric power balance of the total system checked to within 12% in all cases. An optical pyrometer was used to measure temperature of the hot zone, with appropriate corrections made for view port optical transmission.

The cold wall cylinder provides the most ideal reference surface by which to make comparisons between calculated and calorimetrically measured radiation shield efficiency. This cylinder is well isolated thermally from both top and bottom cold wall plates. Table 1 gives the calorimetrically measured cold wall power vs. that which is calculated assuming 0.1 emissivity for the inner cold wall surface, and shield surface area equivalent to the inside diameter of the shield package. Agreement is seen to be excellent between measured and calculated hot zone temperature, certainly far within measurement and drafting errors possible in comparing the two systems. At the time the experiments were conducted, the copper cold wall cylinder was new and optically "clean". Therefore, it is possible that its total emissivity was less than 0.1. This would result in slightly higher temperatures than predicted in Table 1. Predicted values were determined graphically from a composite of numerous figures similar to Fig. 1, but for varying heat flux densities.

SECONDARY LOSSES

Using calorimetrics, it was possible to accurately measure all losses resulting from work support structures, power feed-throughs, and radiation leakage through holes in the cold wall and shielding. These are termed secondary losses, and prove to be of some consequence in high temperature furnaces. For example, power losses to the work support structure, power feed-throughs, and radiation leakage at $\sim 2470^{\circ}$K amounted to 18%, 7.3%, and 12% respectively of total power requirements. From these data, it is apparent that serious consideration should be given these three design aspects when constructing high temperature furnaces.

Total power requirements including secondary losses are given in Fig. 6. Vacuum performance of the furnace during an empty run is also given in this figure. Power requirements shown in this figure were within 10% of predicted values, the major portion of this power discrepancy being found in secondary losses.

CONCLUSION

A simple method is given whereby heat shield efficiency may be determined very accurately for temperature gradients on the order of 2000°K (the limit of our experiments). This graphical technique provides the user design data for specific heat shield packages. The technique also gives the user a better qualitative understanding, through graphical display, of how boundary conditions, and variables including power density and number of shields, affect hot zone temperature.

Acknowledgements

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Reference

1. C. K. Crawford, J. Vac. Sci. Technol., Vol. 9, 1, 23 (1972).

TABLE 1

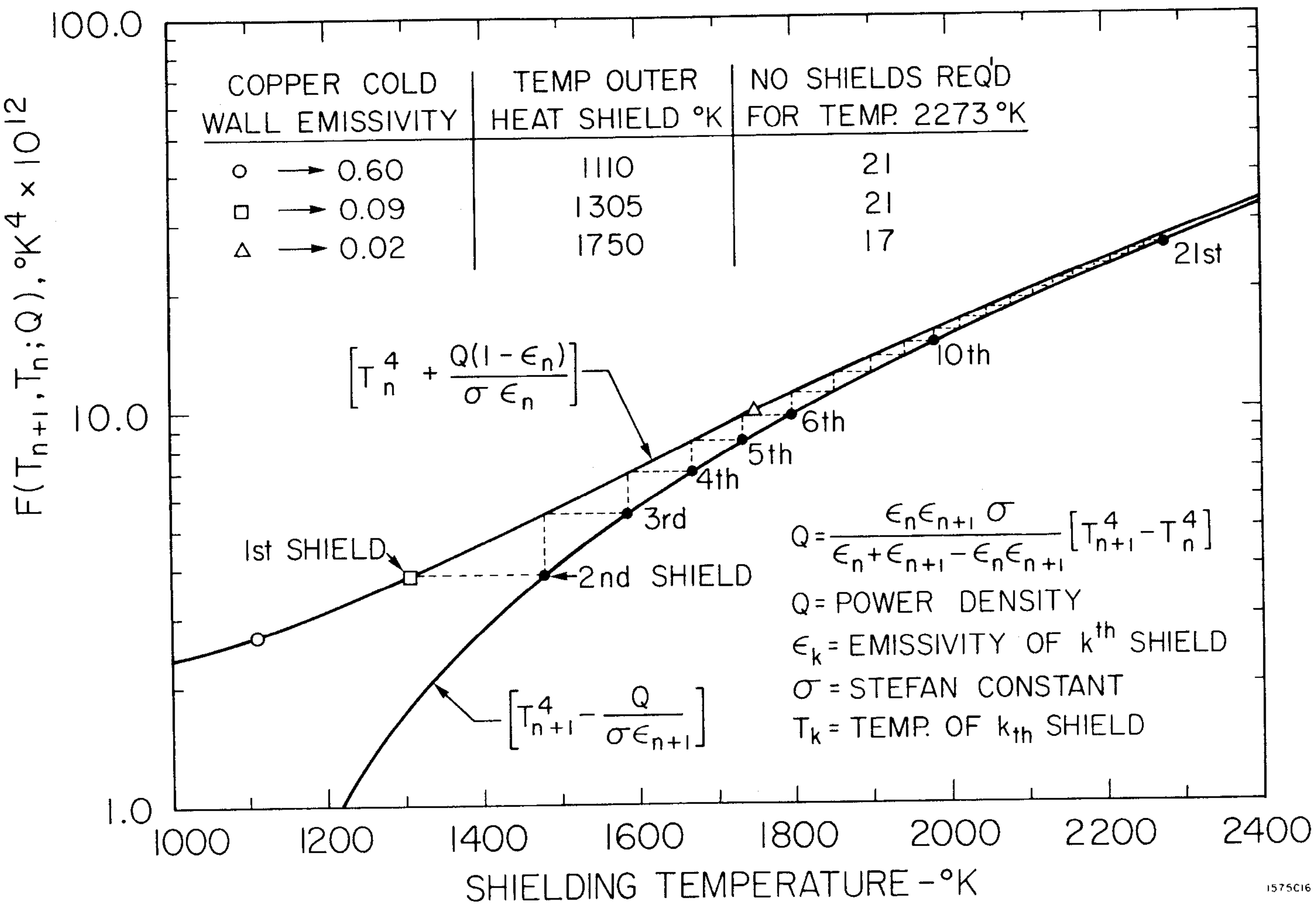
A Comparison of Measured and Predicted Hot Zone Temperatures

for Various Power Densities and Copper Cold Wall Emissivity of 0.1

Power Dissipated in Cold Wall Cylinder	Measured Hot Zone Temperature	Predicted Hot Zone Temperature Assuming Six Radiation Shields	Predicted Hot Zone Temperature Assuming Heater is Seventh Shield
1.9 kW	1673° K	1710° K	1760° K
4.4 kW	2033° K	2000° K	2060° K
6.5 kW	2198° K	2170° K	2235° K
6.9 kW	2216° K	2200° K	2260° K
8.2 kW	2298° K	2285° K	2350° K
9.0 kW	2413° K	2330° K	2400° K
10.0 kW	2473° K	2390° K	2460° K

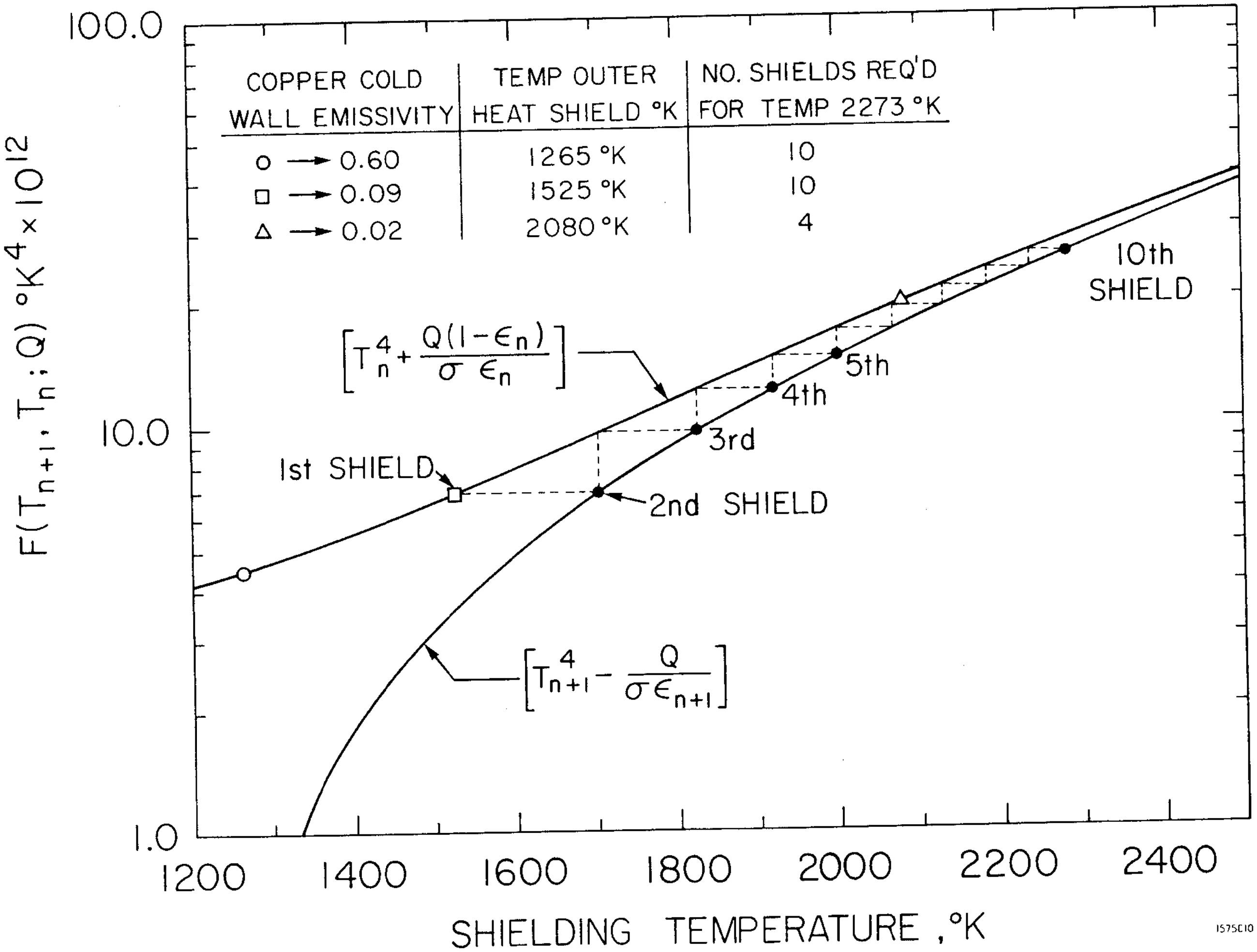
FIGURE CAPTIONS

1. Number and temperature of tungsten heat shields as a function of copper cold wall emissivity for power dissipation of one watt per square centimeter.
2. Two watts per square centimeter.
3. Four watts per square centimeter.
4. Temperature of outer tungsten heat shield as a function of copper cold wall total emissivity for various power densities.
5. High temperature furnace hot zone.
6. Furnace hot zone temperature as a function of heater power.



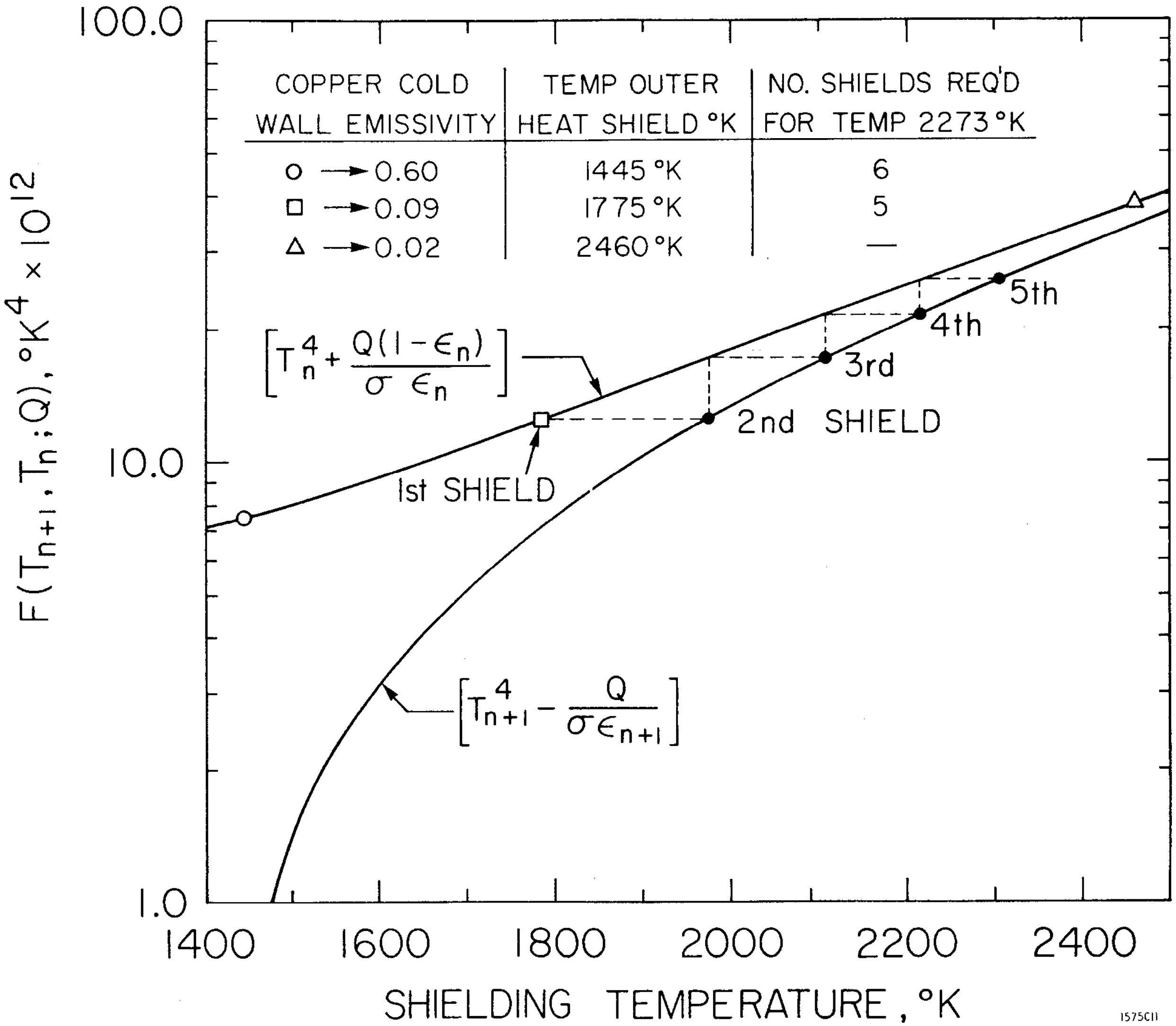
100.0
10.0
1.0
1000
1200
1400
1600
1800
2000
2200
2400
SHIELDING TEMPERATURE -°K
F(T n+1, T n; Q), °K 4 × 10 12
COPPER COLD WALL EMISSIVITY
TEMP OUTER HEAT SHIELD °K
NO SHIELDS REQ'D FOR TEMP. 2273 °K
○ → 0.60 1110 21
□ → 0.09 1305 21
△ → 0.02 1750 17
[T n 4 + Q(1 − ε n) / σ ε n]
[T n+1 4 − Q / σ ε n+1]
1st SHIELD
2nd SHIELD
3rd
4th
5th
6th
10th
21st
Q = ε n ε n+1 σ / (ε n + ε n+1 − ε n ε n+1) [T n+1 4 − T n 4]
Q = POWER DENSITY
ε k = EMISSIVITY OF k th SHIELD
σ = STEFAN CONSTANT
T k = TEMP. OF k th SHIELD
1575C16

Fig. 1



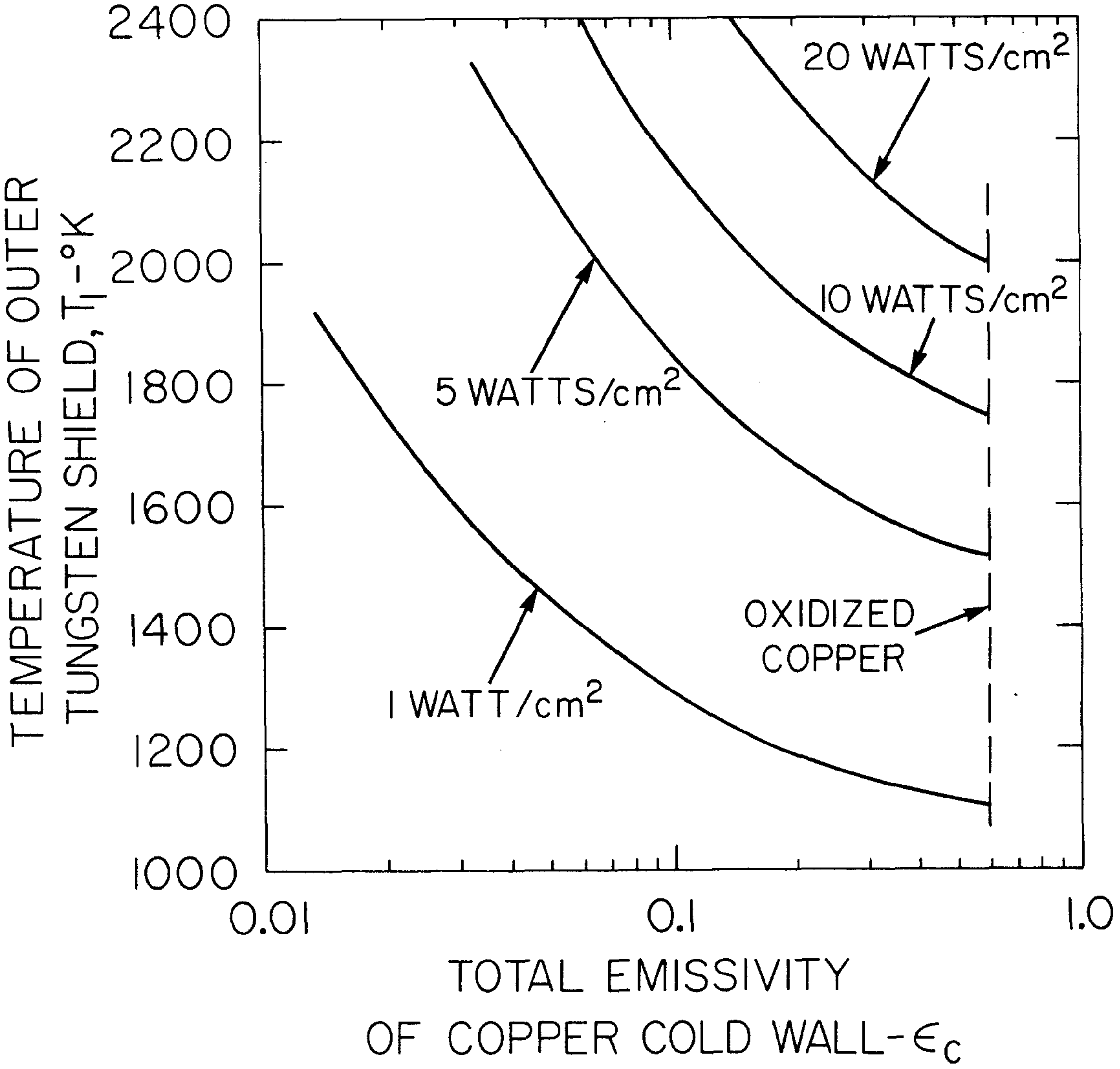
100.0
10.0
1.0
1200
1400
1600
1800
2000
2200
2400
SHIELDING TEMPERATURE ,°K
F(T n+1 , T n ; Q) °K 4 × 10 12
COPPER COLD WALL EMISSIVITY
TEMP OUTER HEAT SHIELD °K
NO. SHIELDS REQ'D FOR TEMP 2273 °K
○ → 0.60 1265 °K 10
□ → 0.09 1525 °K 10
△ → 0.02 2080 °K 4
10th SHIELD
5th
4th
3rd
2nd SHIELD
1st SHIELD
[T n 4 + Q(1−ε n) / σ ε n]
[T n+1 4 − Q / σ ε n+1]
1575C10

Fig. 2



100.0
10.0
1.0
1400
1600
1800
2000
2200
2400
SHIELDING TEMPERATURE, °K
F(Tn+1, Tn; Q), °K4 × 10^12
COPPER COLD WALL EMISSIVITY
TEMP OUTER HEAT SHIELD °K
NO. SHIELDS REQ'D FOR TEMP 2273 °K
○ → 0.60 | 1445 °K | 6
□ → 0.09 | 1775 °K | 5
△ → 0.02 | 2460 °K | —
$\left[T_n^4 + \frac{Q(1-\epsilon_n)}{\sigma \epsilon_n}\right]$
$\left[T_{n+1}^4 - \frac{Q}{\sigma \epsilon_{n+1}}\right]$
1st SHIELD
2nd SHIELD
3rd
4th
5th
1575C11

Fig. 3



$$\epsilon_c \cong \frac{Q\epsilon_W(T_1)}{\epsilon_W(T_1)\left[\sigma T_1^4+Q\right]-Q}, \text{ where}$$

$\epsilon_W(T_1)$ = TOTAL EMISSIVITY OF OUTER SHIELD AS A FUNCTION OF TEMPERATURE T_1

Q = POWER DENSITY

T_1 = TEMPERATURE OF OUTER HEAT SHIELD

1575B4

Fig. 4

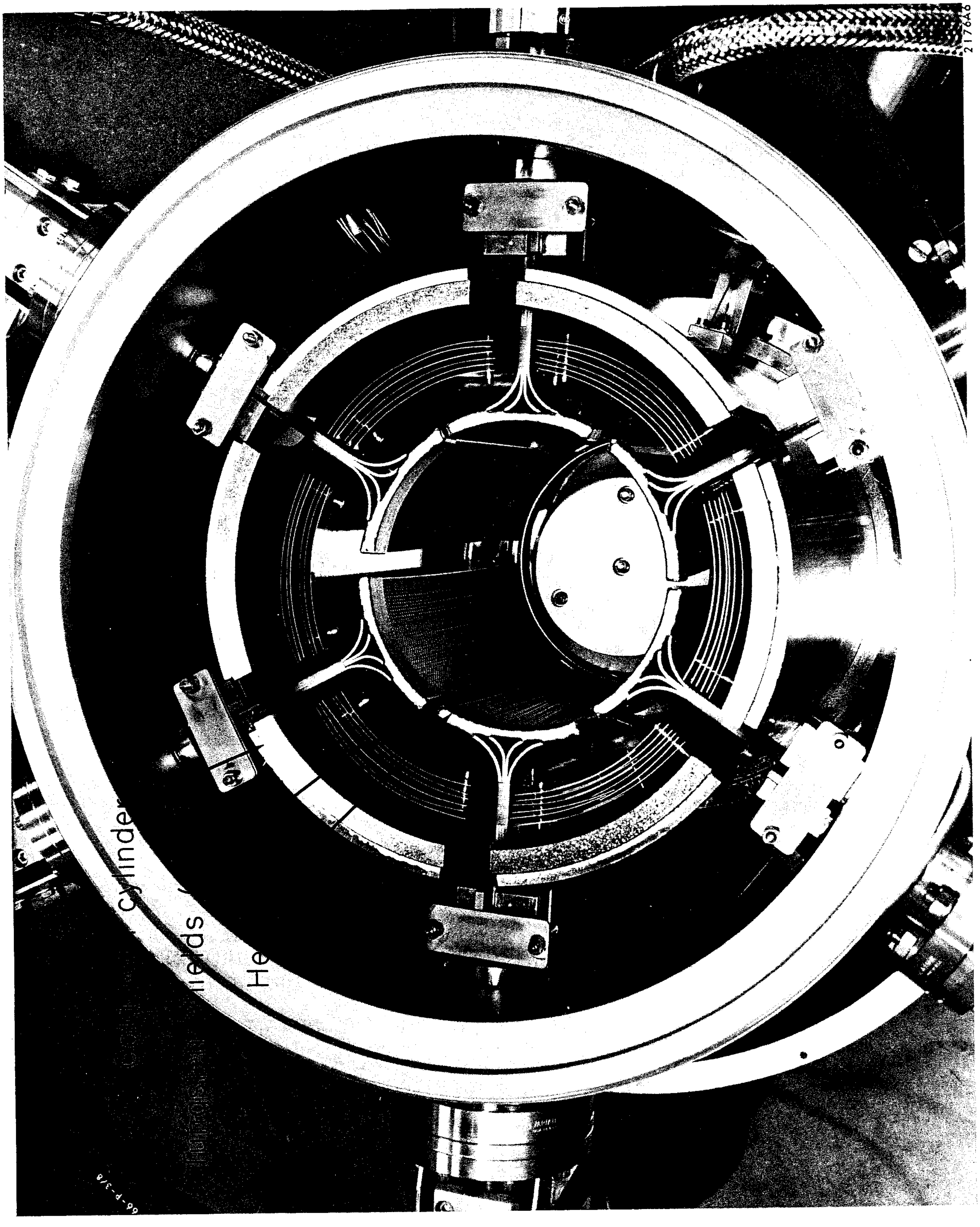
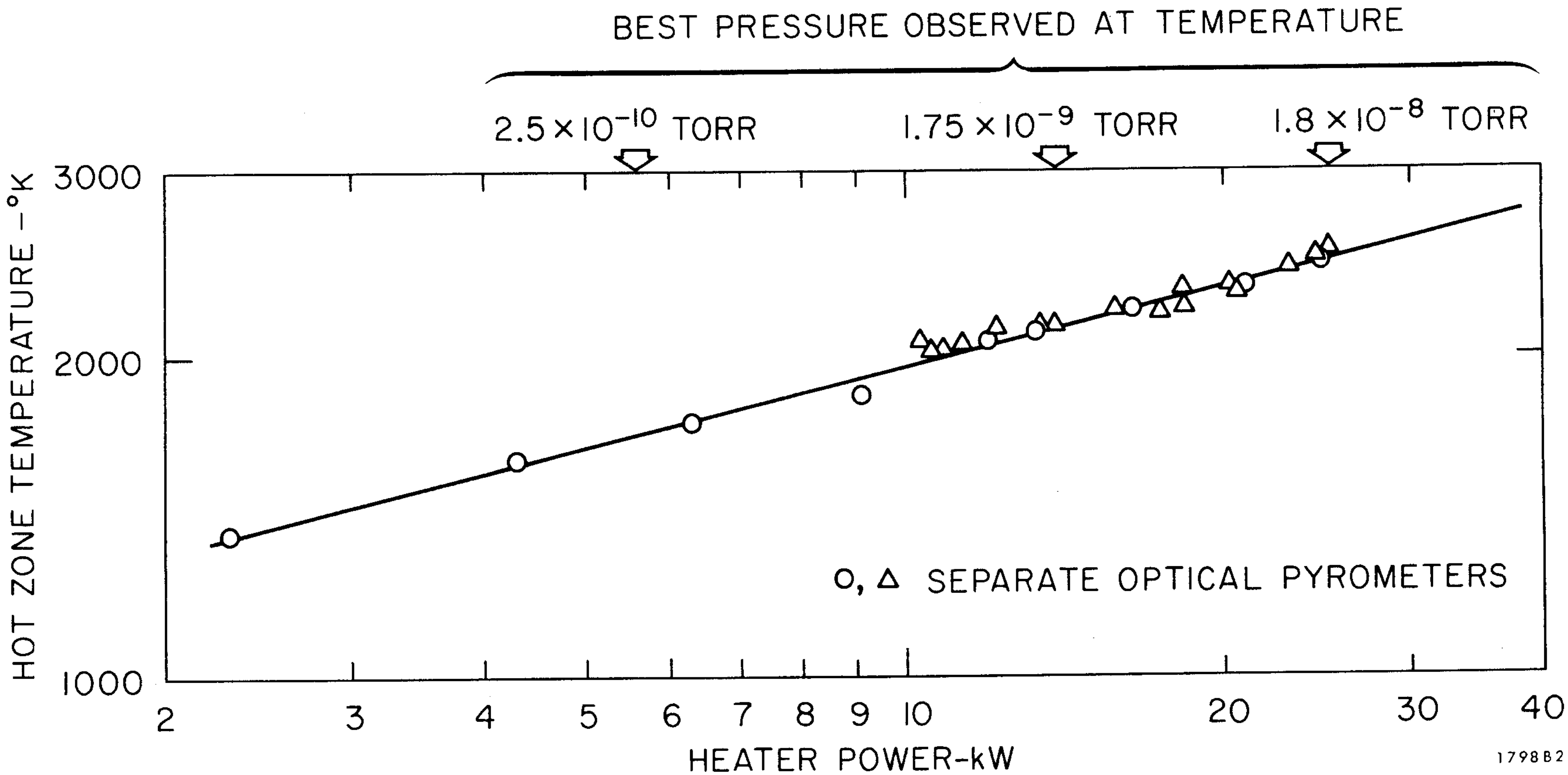


Fig. 5



BEST PRESSURE OBSERVED AT TEMPERATURE
2.5 × 10⁻¹⁰ TORR
1.75 × 10⁻⁹ TORR
1.8 × 10⁻⁸ TORR
HOT ZONE TEMPERATURE – °K
3000
2000
1000
2
3
4
5
6
7
8
9
10
20
30
40
O, Δ SEPARATE OPTICAL PYROMETERS
HEATER POWER–kW
1798B2

Fig. 6