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# EXCLUSIVE PROCESSES IN QUANTUM CHROMODYNAMICS: EVOLUTION EQUATIONS FOR HADRONIC WAVEFUNCTIONS AND THE FORM FACTORS OF MESONS*

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ABSTRACT

The predictions of quantum chromodynamics for meson form factors at large momentum transfer are given. Evolution equations are derived which determine the structure of hadronic wavefunctions at short distances from their form at large distances. The eigenvalues of the evolution equations appear as exponents in anomalous logarithm corrections to the nominal power law of form factors determined by dimensional counting. The results lead to detailed tests of the spin and scaling structure of QCD at short distances. The predictions for the charged pion, kaon and rho form factors and the $\gamma \to \pi^0$ transition form factor of the photon are absolutely normalized.

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## I. Introduction

Exclusive processes involving large momentum transfer test both the internal dynamics of hadrons and the detailed structure of hadronic wavefunctions at short distances. In this paper we outline a new analysis of exclusive processes in quantum chromodynamics, with emphasis on the meson form factors at large momentum transfer [1]. Further papers [2,3] will be devoted to detailed derivations and extensions of these techniques to the baryon form factors and exclusive scattering processes at large momentum transfer. The same methods can also be used to study the $x \to 1$ dependence of hadronic structure functions and the exclusive-inclusive connection in QCD [1].

The central element of this work involves the derivation of evolution equations which determine the short distance behavior of the Fock components of the hadronic wavefunctions from their structure at large distances. The eigensolutions of the evolution equations determine the form of the meson and nucleon wavefunctions, and are directly related to terms in operator product expansions at short distances of the wavefunctions [3]. The eigenvalues of the evolution equations appear as exponents in anomalous logarithmic corrections to the nominal power law fall-off of exclusive amplitudes at large momentum transfer. Our analysis shows that the dimensional counting rules [4,5] for form factors and other exclusive processes at large momentum transfer are rigorous predictions of quantum chromodynamics up to calculable powers of the running coupling constant $\alpha_s(Q^2)$ or $\log^{-1}(Q^2/\Lambda^2)$.

## II. Evolution Equations for Meson Wavefunctions

A convenient framework for the analysis of hadronic states in QCD is time-ordered perturbation theory in the infinite momentum frame (i.e., quantization on the light cone) [6]. The meson state $\Psi$, for example, can be represented as a column vector of wavefunctions - one for each of the Fock states $q\bar{q}, q\bar{q}g, \ldots$ in the meson. (The two particle $q\bar{q}$ wavefunction is the positive energy projection of the usual Bethe-Salpeter amplitude evaluated at relative "time" $x^+ = x^o + x^3 = 0$.) Components having a finite number of constituents can only exist for color singlet states [7]. In general, $\Psi$ satisfies the bound state equation $\Psi = S K \Psi$ — an infinite set of coupled equations where the matrix K is the completely irreducible kernel, $S_n^{-1} = M^2 - \sum_{j=1}^{n} (k_\perp^2 + m^2)_j / x_j + i\varepsilon$ is the n-particle propagator, and $x_j = (k^o + k^3)_j / (p_\pi^o + p_\pi^3) \geq 0$ are the constituents' fractional longitudinal momenta $\left( \sum_{j=1}^{n} x_j = 1 \right)$. We can separate "hard" from "soft" components of the wavefunction by defining a propagator $S^{(\lambda)}$ which vanishes for virtual states near the energy shell [8]:

$$S^{(\lambda)} = \begin{cases} S \quad \text{if} \quad \left| M^2 - \sum_j (k_\perp^2 + m^2)_j / x_j \right| \geq \lambda^2 & (\text{'hard'}) \\ 0 \quad \text{otherwise} & (\text{'soft'}) \end{cases} \tag{1}$$

We can then write

$$\Psi = (S - S^{(\lambda)}) K \Psi + S^{(\lambda)} K \Psi$$

$$= \Psi_\lambda + S^{(\lambda)} K \Psi$$

where wavefunction $\Psi_\lambda \equiv (S - S^{(\lambda)})\, K\, \Psi$ is non-zero only when its constituents are near energy shell. The full wavefunction can be expressed in terms of $\Psi_\lambda$:

$$
\begin{aligned}
\Psi &= \Psi_\lambda + G^{(\lambda)}\, K\, \Psi_\lambda \\
G^{(\lambda)} &\equiv S^{(\lambda)} + S^{(\lambda)}\, K\, G^{(\lambda)} \\
&= S^{(\lambda)} + S^{(\lambda)}\, K\, S^{(\lambda)} + S^{(\lambda)}\, K\, S^{(\lambda)}\, K\, S^{(\lambda)} + \ldots \qquad (2)
\end{aligned}
$$

By the definition in eq. (1), the Green's function $G^{(\lambda)}$ contains only hard loop momenta, and thus it has a sensible perturbative expansion, at least in asymptotically free theories. In particular, bound state poles cannot develop in $G^{(\lambda)}$ since only far off-shell propagation occurs in intermediate state (e.g., $V(r) = ar\theta(r < 1/\lambda)$ does not bind for $\lambda$ sufficiently large). The soft wavefunction $\Psi_\lambda$ contains all intrinsically non-perturbative effects. Given $\Psi_\lambda$, eq. (2) determines the far off-shell structure of the full hadronic wavefunctions $\Psi$ from perturbation theory.

The meson form factor $F_M(Q^2)$ can now be represented as a sum of matrix elements between initial and final $\Psi_\lambda$ states:

$$
F_\mu = \Psi_\lambda(q\bar{q})\ \Gamma^{(\lambda)}(q\bar{q},q\bar{q})\ \psi_\lambda(q\bar{q}) + \Psi_\lambda(q\bar{q})\ \Gamma^{(\lambda)}(q\bar{q},q\bar{q}g)\ \psi_\lambda(q\bar{q}g) + \ldots
$$

The amplitudes $\Gamma^{(\lambda)}(q\bar{q},q\bar{q})$, etc. consist of all connected diagrams (reducible and irreducible), but with all loop momenta hard as in $G^{(\lambda)}$. In light-cone gauge ($A^+ = 0$), the nominal power law contribution to $F_M(Q^2)$ as $Q^2 \to \infty$ is $F_M(Q^2) \sim 1/(Q^2)^{n-1}$ if n quark or gluon constituents are forced to change direction[1]. Thus only the $q\bar{q}$ component of $\Psi_\lambda$ contributes to the leading ($1/Q^2$) behavior of $F_M(Q^2)$. (Higher Fock states in which constituents annihilate before the exchange of the hard momentum $q^\mu$ can be treated as corrections to the $q\bar{q}$ components of $\Psi_\lambda$.)

The leading logarithmic corrections to this power-law behavior are readily identified in each order of perturbation theory. They are order $(\alpha_s \log Q^2)^n$ in $n^{th}$ order; double logarithmic terms, $(\alpha_s \log Q^2 \log Q^2)^n$, do <u>not</u> appear due to infrared cancellations in the color singlet state. We choose a frame where $q^\mu$ is transverse to the direction of the incident meson $(-q^2 = Q^2 = q_\perp^2)$. To leading order: the dominant momentum flow occurs through the minimal exchange graphs $T_B$; only planar ladder graphs are required (in light-cone gauge); and the transverse momentum integrations are ordered, as indicated in fig. 1. Up to neglected terms of order $\alpha_s(Q^2)$ and m/Q, the meson form factor in QCD now takes the form

$$F_M(Q^2) = \int_0^1 dx_1\, dx_2\, \delta\left(1 - \sum_j x_j\right) \int_0^1 dy_1 dy_2\, \delta\left(1 - \sum_j y_j\right)$$

$$\times\ \phi^\dagger(y_i,Q)\ T_B(y_i,x_i,Q)\ \phi(x_i,Q) \qquad (3)$$

where

$$T_B = 16\pi\, C_F\, \frac{\alpha_s(Q^2)}{Q^2}\, \frac{1}{x_2 y_2} \qquad \left(C_F = \frac{4}{3}\right)$$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \log Q^2/\Lambda^2} \qquad \left(\beta = 11 - \frac{2}{3} n_{flavors}\right) \qquad (4)$$

for mesons with zero helicity, and

$$\phi(x_i,Q) = \left(\log \frac{Q^2}{\Lambda^2}\right)^{-C_F/\beta} \int_0^{Q^2} \frac{dk_\perp^2}{16\pi^2}\, \psi(x_i,k_\perp)$$

$$\equiv x_1 x_2\, \tilde{\phi}(x_i,Q) \qquad (5)$$

is the two-body wavefunction integrated over transverse momenta $k_\perp^2 \lesssim Q^2$. The factor $(\log Q^2)^{-C_F/\beta}$ in $\phi$ is due to vertex and fermion self-energy corrections in $T_B$.

Defining

$$\xi = \frac{\beta}{4\pi} \int_0^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \alpha_s(k_\perp^2) \simeq \log \log \frac{Q^2}{\Lambda^2}$$

we find that, because of the strong ordering of the $\vec{k}_\perp^2$ integrations (see Fig. 1), $\tilde{\phi}$ satisfies an evolution equation[2]

$$x_1 x_2 \left\{ \frac{\partial}{\partial \xi} \tilde{\phi}(x_i,Q) + \frac{C_F}{\beta} \tilde{\phi}(x_i,Q) \right\}$$

$$= \int_0^1 dy_1\, dy_2\, \delta\left(1 - \sum_j y_j\right) V(x_i,y_i)\, \tilde{\phi}(y_i,Q) \qquad \text{(6a)}$$

where

$$V(x_i,y_i) = 2\frac{C_F}{\beta} \left\{ y_1 x_2\, \theta(y_2 - x_2) \left( \delta_{h_1 \bar{h}_2} + \frac{\Delta}{y_2 - x_2} \right) + (1 \leftrightarrow 2) \right\}$$

$$= V(y_i,x_i) \qquad \left( \Delta\tilde{\phi} \equiv \tilde{\phi}(y_i,Q) - \tilde{\phi}(x_i,Q) \right) \qquad \text{(6b)}$$

represents the one-gluon exchange interaction. The quantity $\delta_{h_1 \bar{h}_2}$ is defined to be 0(1) when the $q\bar{q}$ helicities are parallel (anti-parallel). The terms cancelling the infrared divergences at $x_i = y_i$ are due to self-energy corrections to the q and $\bar{q}$ legs.

The evolution equation has a general solution

$$\phi(x_i,Q) = x_1 x_2 \sum_{n=0}^{\infty} a_n\, C_n^{3/2}(x_1 - x_2)\, e^{-\gamma_n \xi} \qquad (7)$$

where the Gegenbauer polynomials $C_n^{3/2}$ are eigenfunctions of $V(x_i,y_i)$. The corresponding eigenvalues are

$$\gamma_n = \frac{C_F}{\beta}\left\{1 + 4\sum_{2}^{n+1}\frac{1}{k} - \frac{2\delta_{h_1\bar{h}_2}}{(n+1)(n+2)}\right\} \geq 0 \qquad (8)$$

The coefficients $a_n$ can be determined from the soft wavefunction:

$$\bar{a}_n\left(\log\frac{\lambda^2}{\Lambda^2}\right)^{-\gamma_n} = \frac{2(2n+3)}{(2+n)(1+n)}\int_{-1}^{1} d(x_1-x_2)\, C_n^{3/2}(x_1-x_2)\,\phi(x_i,\lambda^2) \qquad (9)$$

(If we assume isospin symmetry for the pion wavefunction, $\phi(x_1,x_2) = \phi(x_2,x_1)$ and only n = even terms contribute). Notice that as $Q^2 \to \infty$

$$\phi(x_i,Q) \to \begin{cases} a_0\, x_1\, x_2 & h_1+h_2 = 0 \\ a_0\, x_1\, x_2\left(\log\frac{Q^2}{\Lambda^2}\right)^{-C_F/\beta} & |h_1+h_2| = 1 \end{cases} \qquad (10)$$

where $a_0$ is 6 times the wavefunction at the origin (by eqs. (5) and (9)). For pions this constant can be determined from the weak decay amplitude for $\pi \to \mu\nu$:

$$a_0 = \frac{3}{\sqrt{n_{colors}}}\, f_\pi \qquad (f_\pi \simeq 93\text{ MeV}) \qquad (11)$$

It is remarkable that the eqs. (10) and (11) completely determine the short distance structure of the pion wavefunction. An analogous result is obtained for the kaon. The decay $\rho \to \ell\bar{\ell}$ can be used to normalize the asymptotic $\rho$ wavefunction. If we define $\langle 0|J_\mu|\rho(p,\varepsilon)\rangle = mf_\rho\varepsilon_\mu$, then $a_0$ is $3f_\rho/\sqrt{n_c}$.

The convergence of series (7) is assured if the $q\bar{q}$ wavefunction satisfies the boundary condition

$$\phi(x_i,Q) \lesssim K\, x_i^{\varepsilon} \qquad \text{as} \qquad x_i \to 0 \tag{12}$$

for some $\varepsilon > 0$. This condition is satisfied by wavefunctions representing truly composite system — i.e., by solutions of the homogeneous bound state equations which are regular at high energies[3]. In theories with an elementary field representing (or mixing strongly with) the meson, the bound state equation has a source term corresponding to the bare coupling $\bar{\psi}\gamma_5\psi$, and consequently $\phi$ tends to a constant as $x_i \to 0$. Precisely this type of analysis is required in the case of photon structure functions and transition form factors in QCD.

Because of the boundary condition (12), the singularity in $T_B$ at $x_2 = 0$, $y_2 = 0$ (eq. (4)) does not result in additional factors of $\log Q^2$.[4] The behavior of $F_M(Q^2)$ is thus determined by $T_B$ and the short distance behavior of the wavefunction $\psi(x_i,k_\perp)$ (i.e., $k_\perp \to \infty$, $x_i \neq 0$). Since the wavefunction is essentially $\langle 0|T(\psi(r)\bar{\psi}(0))|M\rangle$, the anomalous dimensions $\gamma_n$ of $\phi(x_i,Q^2)$ are those associated with the twist two operators appearing in the operator product expansion of $\psi(r)\bar{\psi}(0)$ [3,11]. Furthermore, the usual renormalization group arguments imply that the leading logarithms summed by eqs. (3)-(6) are in fact the dominant contribution as $Q^2 \to \infty$. Of course non-leading terms may be relevant at present energies, but these too may be computed in the framework described above.

Combining eqs. (3),(4), and (7), we find the QCD predictions for helicity zero mesons $(\pi, K, \rho_L, \ldots)$ [11,12],

$$F_M(Q^2) = \frac{4\pi\ C_F\ \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \right|^2$$

$$\times \left[1 + O(\alpha_s(Q^2)\ ,\ m/Q)\right] \qquad (13)$$

Asymptotically the $a_0$ term dominates and from eq. (11) [12]

$$F_\pi(Q^2) \rightarrow 16\pi\ \alpha_s(Q^2)\ \frac{f_\pi^2}{Q^2} \quad \text{as} \quad Q^2 \rightarrow \infty \qquad (14)$$

Identical results follow for $F_K$ and $F_{\rho_L}$ if $f_\pi$ is replaced by $f_K$ and $f_\rho$ respectively.

Although eqs. (13) and (14) agree asymptotically, the $n \neq 0$ terms in (13) can result in sizeable corrections to both the normalization and shape of $F_M(Q^2)$ until $Q^2$ is quite large. In general these terms tend to compensate for the fall-off in $\alpha_s(Q^2)$. If we assume that $\phi(x_i,\lambda)$ is sharply peaked at $x_i \sim \frac{1}{2}$, as is characteristic of non-relativistic bound states, then the evolution equation causes $\phi(x_i,Q)$ to broaden, as $Q^2$ increases, out to its asymptotic form $x_1x_2$. Since $T_B$ is maximum at $x_2 = 0$, this effect enhances the form factor. Figure 2 illustrates predictions for $Q^2F_\pi$ assuming that $\phi(x_i,\lambda)$ is either strongly peaked at $x_i = \frac{1}{2}$, or has a smooth $x_1x_2$ dependence (no evolution). In neither case is the normalization arbitrary; both tend to the form given in eq. (14) as $Q^2 \rightarrow \infty$.[5]

The factor $\sum_n a_n\ (\log Q^2/\Lambda^2)^{-\gamma_n}$ in (13) can in fact be measured directly by studying the transition form factor of the photon: $\gamma(Q^2) + \gamma(k^2 \sim 0) \rightarrow \pi^0$. Using the techniques described above, we find a transition form factor

$$F_{\pi\gamma}(Q^2) = \frac{2(e_u^2 + e_d^2)\sqrt{n_c}}{Q^2} \sum_{n=0}^{\infty} a_n \left(\log \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n}$$

$$\rightarrow \frac{10}{3} \frac{f_\pi}{Q^2} \quad \text{as} \quad Q^2 \rightarrow \infty \tag{15}$$

(The $\gamma^* \gamma \pi^0$ vertex is defined $ie^2 F_{\pi\gamma}(Q^2)\, \epsilon_{\mu\nu\sigma\rho}\, p_\pi^\nu q^\rho \epsilon^\sigma$).

For mesons with helicity ±1 (e.g., $\rho_T$) or for transition between mesons of differing helicities (e.g., $\gamma^* \rho_L \rightarrow \rho_T$), $T_B$ vanishes as a power of Q faster than eq. (4).[6] One significant consequence of this is the suppression of reactions $e^+e^- \rightarrow \rho_T\rho_T$, $\rho_L\rho_T$, $\pi\rho$ by a factor $m^2/Q^2$ (in the cross section) relative to $e^+e^- \rightarrow \pi^+\pi^-$, $\rho_L\rho_L$, $K\bar{K}$ [13]. Furthermore, each of the leading processes has a positive form factor at large $Q^2$ relative to its sign at $Q^2 = 0$ [14]. These are all non-trivial consequences of QCD dynamics. By way of comparison, $e^+e^- \rightarrow \rho_T\rho_T$ is not suppressed in theories with either scalar or pseudo-scalar gluons. In addition $F_{\rho_T}$, $F_{\rho_L}$ in scalar theories, and $F_\pi$, $F_K$ in pseudo-scalar theories become relatively negative for large $Q^2$ and thus must vanish at some finite $Q^2$. Current data for $F_\pi$ already rules out the pseudo-scalar theory.

## III. Conclusions

As we have shown in this paper, the testing ground of quantum chromodynamics can be extended to exclusive processes at large momentum transfer. The essential features which are required in the calculation of form factors are: (a) the separation of hard (far-off-shell) and soft regimes of each hadronic Fock component, and (b) the derivation of evolution equations which determine the form of the hadronic wavefunctions at short distances. The eigenvalues of the evolution equations yield the anomalous logarithmic corrections to the leading power behavior of large momentum transfer amplitudes. The dimensional counting rules for hadronic form factors, modulo calculable logarithmic corrections, thus emerge as predictions of perturbative QCD.

The Fock space light-cone gauge description used here, provides an exact description of QCD which is a direct analogue of the parton model. In general, the lowest-particle-number Fock state dominates the power-law behavior of large momentum transfer exclusive reactions and inclusive reactions at $x \to 1$.

It is important to emphasize that power-law scaling of the hadronic form factors directly reflect the scaling behavior of quark interactions within hadrons. The nominal power behavior $Q^{-2}$ in eqs. (13) and (15) is consequence of the underlying scale-invariance of quark-quark interactions in QCD, as well as the existence of a color singlet $q\bar{q}$ component in the meson wavefunction. Further, as we have discussed in Section II, the presence or absence of zeroes as well as the helicity dependence of mesonic form factors, allows a systematic determination of the spin structure of quark-quark scattering. These results, together with the predictions of scale-breaking from the hadronic wavefunction evolution euqations, provide detailed tests of the short distance structure of quantum chromodynamics.

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## FOOTNOTES

1. The analysis described here has also been performed in general covariant gauges (see ref. [2] for details). The final results are demonstrably gauge invariant even though the covariant analysis involves Fock states in $\Psi_\lambda$ containing any number of longitudinally polarized gluons.

2. This result follows by integrating eq. (16) over $k_\perp^2 < Q^2$, where K is approximated by single gluon exchange and the transverse momentum integrals are strongly ordered; i.e.,

$$S^{(\lambda)}(k_\perp)\,K(k_\perp,\ell_\perp)\,\psi(\ell_\perp) \to S^{(\lambda)}(k_\perp)\,K(k_\perp,0_\perp)\,\psi(\ell_\perp)\,\theta(k_\perp^2-\ell_\perp^2) \quad .$$

Differentiating with respect to $\xi$ leads immediately to eq. (6).

3. Bound state equations and formal expansions as in eq. (2) are mathematically undefined until boundary conditions are specified. The choice of acceptable boundary condictions depends upon details of the interaction. However condition (12) is required if the "kinetic energy" operator $M^2 - \sum_i (k_\perp^2+m^2)_i/x_i$ is to be self-adjoint. Furthermore this condition appears in confining theories such as 2-d QCD. Finally, perturbative analyses of the $x_i \to 0$ region in QCD suggest that (12) is correct in QCD (see ref. [2]).

4. In super-renormalizable theories such as 2-d QCD [9] or 4-d $\phi^3$ field theory [10], $T_B$ has <u>quadratic</u> divergences at $x_2,y_2 = 0$. In these theories additional factors of $(Q^2)^{1-\varepsilon}$ come from the x-integrations which can result in substantial modifications [9] to dimensional counting predictions.

5. It should be possible to predict the structure of $\phi(x_i,\lambda)$ directly from bag and other models used in hadronic spectroscopy.

6. This is a consequence of the vector nature of the gluon. The helicity of massless fermions is conserved by vector couplings and thus any helicity-flip amplitude $T_B$ vanishes as the quark masses become negligible. Furthermore, in the Breit frame ($\vec{p}\,' = -\vec{p}$), the change in longitudinal angular momentum for a helicity-conserving amplitude is $\Delta\vec{J} \cdot \hat{p} = 2h$ where h is the helicity of the initial and final hadrons. Since a photon induces the transition, angular momentum conservation requires $|2h| \leq 1$. Consequently form factors for $|h| \geq 1$ hadrons, as do those for transitions with $\Delta h \neq 0$, are suppressed by factors of m/Q relative to the leading form factors.

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FIGURE CAPTIONS

1. Leading logarithmic contributions to the meson form factors. The dominant momentum flow is through $T_B$. The light-cone gauge ladder and self-energy insertions yield the evolution equation (6).

2. QCD prediction for the meson form factor for two exteme cases: (a) $\tilde{\phi}(x_i,\lambda) \propto \delta(x_1 - \frac{1}{2})$ or (b) $\tilde{\phi}(x_i,\lambda) \propto x_1 x_2$. In the latter case the wavefunction is unchanged under evolution. The asymptotic predictions are absolutely normalized, according to eq. (14). The bands correspond to $\pm\alpha_s(Q^2)/\pi$. We take $\Lambda^2 = 1\ \mathrm{GeV}^2$; notice that because of momentum sharing the natural argument of $\alpha_s$ is ~ $Q^2/4$ so this value is equivalent to $\Lambda^2_{eff} \sim .25$. The determination of a value for $\Lambda^2$ requires the computation of the order $\alpha_s(Q^2)$ terms in eq. (13). The data are from the analysis of electroproduction $e^- p \to e^- + \pi^+ + n$ [15].

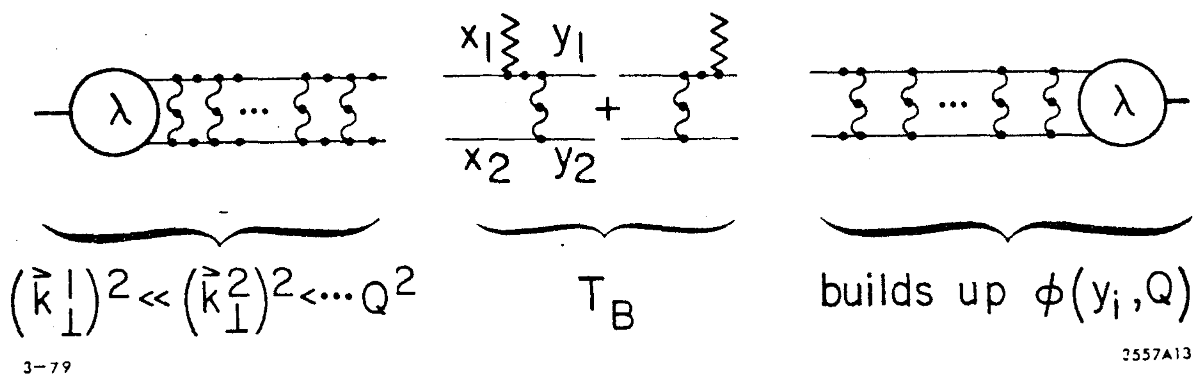
λ
$x_1$ $y_1$
$x_2$ $y_2$
+
λ
$(\vec{k}_\perp^1)^2 \ll (\vec{k}_\perp^2)^2 < \cdots Q^2$
$T_B$
builds up $\phi(y_i, Q)$
3-79
3557A13

Fig. 1

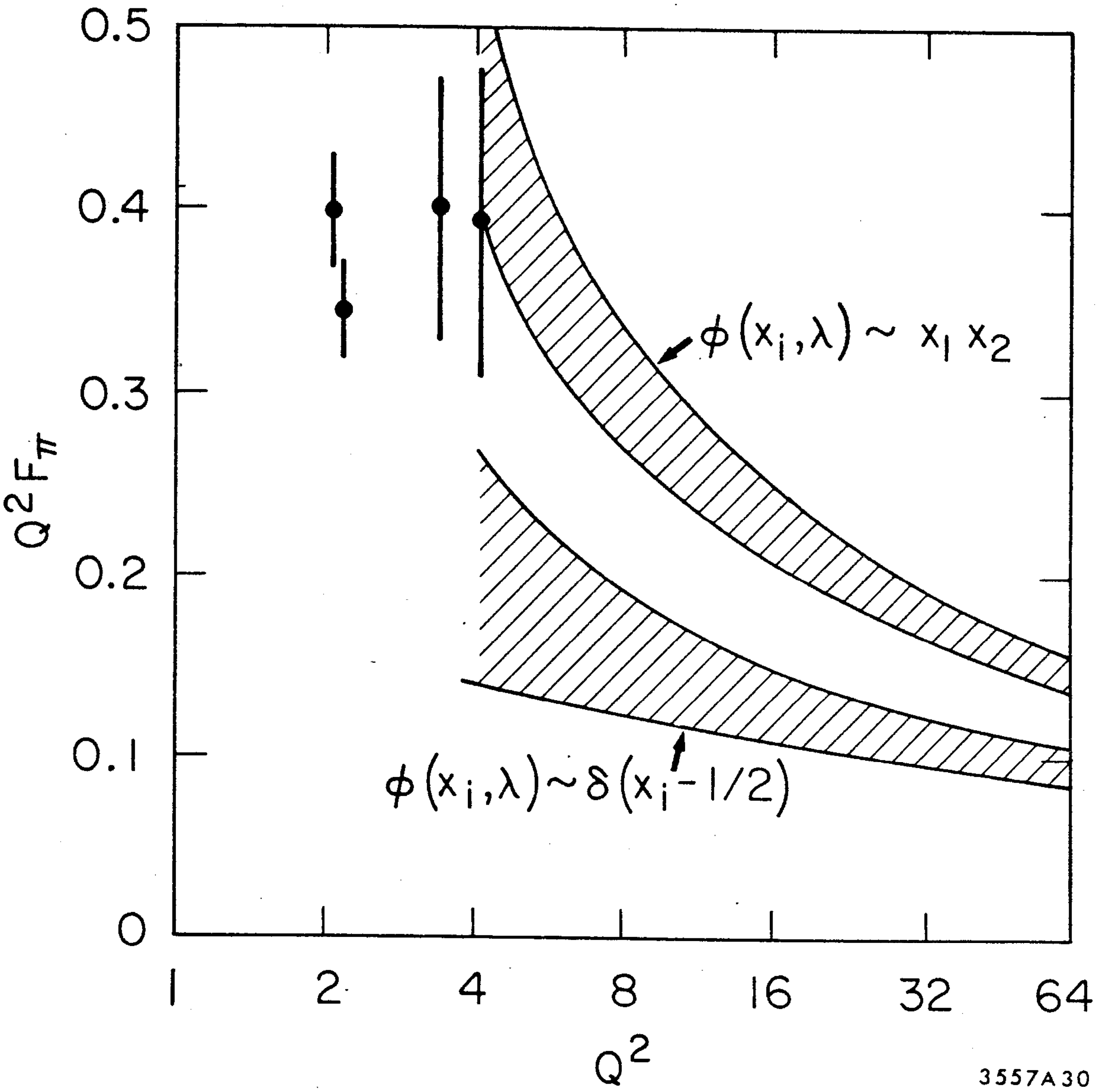
0.5
0.4
0.3
0.2
0.1
0
$Q^2 F_\pi$
1
2
4
8
16
32
64
$Q^2$
$\phi(x_i, \lambda) \sim x_1 x_2$
$\phi(x_i, \lambda) \sim \delta(x_i - 1/2)$
3557A30

Fig. 2