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ELASTIC SCATTERING OF LEPTONS ON DEUTERONS AND T NON-INVARIANCE

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A B S T R A C T

We have studied the elastic scattering of polarized leptons (muons or electrons) on polarized deuterons. The electromagnetic vertex of the deuteron is represented by four form factors, one of them violates T invariance. Using the one - photon exchange approximation, we express the differential cross-section in terms of these form factors and the polarizations (initial and final) of leptons and deuterons.

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1. INTRODUCTION

Since the discovery of CP non-invariance in K_L decay [1], the experimental foundation of all discrete symmetries has been re-examined for all the interactions [2]. It has been known for a long time that the effect of T non-invariance introduces one additional form factor in the electromagnetic vertex of a spin 1 particle, for example, the deuteron [3]. The possibility of observing T non-invariance in electromagnetic interaction of hadrons from the elastic electron-deuteron scattering has been discussed in many papers [4]-[8]. Using the one-photon exchange approximation and treating the electron relativistically, Schildknecht [4] has studied the polarization effects in elastic electron-deuteron scattering in detail and has given the expressions for the polarization of the deuterons after scattering and for the dependence of the differential cross-section on the polarization before scattering. The motivation of this paper is to generalize Schildknecht's work to the case where the deuterons are polarized both before and after scattering and to the case of elastic muon-deuteron scattering, where the mass of the muon cannot be ignored. We have considered all different cases of polarizations of leptons and deuterons, before and/or after scattering, and expressed the differential cross-section in terms of the electromagnetic form factors of the deuteron and the initial and final polarizations of leptons and deuterons. T invariance is not assumed in our derivation. In the case of electron-deuteron scattering, the mass of the electron can be ignored and the corresponding expressions simplify greatly.

The differential cross-sections for the elastic muon-deuteron and electron-deuteron scattering are given in Sections 2 and 3, respectively. The detail of our calculation is given in the Appendix.

2. ELASTIC MUON-DEUTERON SCATTERING

In this Section, we shall consider the polarization effects of the elastic scattering of muons (spin $\frac{1}{2}$, mass m) and deuterons (spin 1, mass M). The four momenta of the muon and the deuteron before (after) scattering are denoted by p_i (p_f) and P_i (P_f), respectively. We use the one-photon exchange approximation to calculate the matrix element in the laboratory system.

The differential cross-section in the laboratory system is given by [9]

$$\frac{d\sigma}{d\Omega} = \frac{m^2 M}{4\pi^2}\frac{p'}{p}\;\frac{|B|^2}{M+E-pE'(p')^{-1}\cos\theta}\,, \tag{1}$$

where we set $p_i = (E,p)$, $p_f = (E',p')$, θ is the scattering angle, and B is the Lorentz invariant matrix element.

In the one-photon exchange approximation, we have [10]

$$B = [\bar{u}(p_f)\gamma_\lambda u(p_i)]\, e^2 q^{-2} [\epsilon^+(P_f) R^\lambda \epsilon(P_i)]\,, \tag{2}$$

where

$$q = p_i - p_f = (q_0, \underset{\sim}{q}),$$

$$\begin{aligned} M R_{\alpha\beta}{}^{\lambda} = {} & (P_i + P_f)^\lambda [F_1(q^2) g_{\alpha\beta} - (2M^2)^{-1} F_2(q^2) q_\alpha q_\beta] \\ & + G_1(q^2)(q_\alpha g_\beta{}^\lambda - q_\beta g_\alpha{}^\lambda) \\ & + i M^{-2} G_2(q^2)[q^\lambda q_\alpha q_\beta - \tfrac{1}{2} q^2 (q_\alpha g_\beta{}^\lambda + q_\beta g_\alpha{}^\lambda)]\,, \end{aligned}$$

u is the spinor of the muon, ϵ is the polarization vector of the deuteron, and the electromagnetic form factors of the deuteron are given by F_1, F_2, G_1 and G_2. The form factor $G_2(q^2)$ differs from zero only if T invariance does not hold [3]. It has been shown by Gourdin [11] that linear combinations of these form factors correspond to the charge (F_C), quadrupole (F_Q), and magnetic (F_M) interaction :

$$F_C = F_1 + \tfrac{2}{3}\eta[F_1 - G_1 + F_2(1+\eta)]\,, \tag{3}$$

$$F_Q = F_1 - G_1 + F_2(1+\eta),$$

$$F_M = G_1 ,$$

where $\eta = -q^2(4M^2)^{-1}$. We define $F_T \equiv 2\eta G_2$ 12). Following Schildknecht, our final result is given in terms of F_C, F_Q, F_M, and F_T.

After a straightforward calculation (see Appendix), we find

$$\begin{aligned} j_{\lambda\mu} &\equiv \bar{u}(p_f)\gamma_\lambda u(p_i)\bar{u}(p_i)\gamma_\mu u(p_f) \\ &= (\varphi_i\varphi_f)^\dagger(a_{\lambda\mu} + im\,\varepsilon_{\lambda\mu\alpha\beta}q^\alpha T^\beta)(\varphi_i\varphi_f)(4m^2)^{-1}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} a_{\lambda\mu} = {} & (1-t_i\cdot t_f)[(p_i)_\lambda(p_f)_\mu + (p_i)_\mu(p_f)_\lambda - 2\eta M^2 g_{\lambda\mu}] \\ & - g_{\lambda\mu}(t_i\cdot p_f)(t_f\cdot p_i) - 2\eta M^2[(t_i)_\lambda(t_f)_\mu + (t_i)_\mu(t_f)_\lambda] \\ & + (t_i\cdot p_f)[(t_f)_\lambda(p_i)_\mu + (t_f)_\mu(p_i)_\lambda] \\ & + (t_f\cdot p_i)[(t_i)_\lambda(p_f)_\mu + (t_i)_\mu(p_f)_\lambda] , \end{aligned}$$

$$(t_i)_0 = m^{-1}\underset{\sim}{\sigma}\cdot\underset{\sim}{p} ,$$

$$(t_f)_0 = m^{-1}\underset{\sim}{\sigma}'\cdot\underset{\sim}{p}' ,$$

$$\underset{\sim}{t_i} = \underset{\sim}{\sigma} + (Em^{-1}-1)(\underset{\sim}{\sigma}\cdot\hat{p})\hat{p} ,$$

$$\hat{p} = p^{-1}\underset{\sim}{p} ,$$

$$\underset{\sim}{t_f} = \underset{\sim}{\sigma}' + (E'm^{-1}-1)(\underset{\sim}{\sigma}'\cdot\hat{p}')\hat{p}' ,$$

$$T = t_i + t_f = (T_0, \underset{\sim}{T}) ,$$

$$\epsilon_{0123} = 1 ,$$

φ_i (φ_f) is the Pauli spinor of the muon before (after) scattering, and $\underset{\sim}{\sigma}$ ($\underset{\sim}{\sigma}'$) is the Pauli matrice for φ_i (φ_f).

$$
\begin{aligned}
|B|^2 &= e^4 q^{-4} j_{\lambda\mu} [\epsilon^\dagger(P_f) R^\lambda \epsilon(P_i)][\epsilon^\dagger(P_i) R^{\dagger\mu} \epsilon(P_f)] \\
&= e^4 q^{-4} (4m^2)^{-1} (\varphi_i \varphi_f \psi_i \psi_f)^\dagger A (\varphi_i \varphi_f \psi_i \psi_f) , \qquad (5)
\end{aligned}
$$

where

$$
\begin{aligned}
A = {} & \tfrac{4}{3} a_{00} [F_C{}^2 + \tfrac{8}{9}\eta^2 F_Q{}^2 + \tfrac{2}{3}\eta (F_M{}^2 + F_T{}^2)] \\
& - \tfrac{2}{9} a_\lambda{}^\lambda Q^2 (F_M{}^2 + F_T{}^2) \\
& + \tfrac{4}{3} \eta M^{-1} [\underset{\sim}{q} \times (\underset{\sim}{S} - \underset{\sim}{S}')]^i a_{0i} F_Q F_T \\
& - \tfrac{4}{3} m M^{-1} [\underset{\sim}{q} \times (\underset{\sim}{S} + \underset{\sim}{S}')] \cdot (\underset{\sim}{q} \times \underset{\sim}{T}) F_M (F_C + \tfrac{1}{3}\eta F_Q) \\
& - \tfrac{1}{3} m\, \underset{\sim}{q} \cdot (\underset{\sim}{S} - \underset{\sim}{S}') (T_0 Q^2 - q_0 \underset{\sim}{T} \cdot \underset{\sim}{q} M^{-2}) (F_M{}^2 + F_T{}^2) \\
& - \tfrac{8}{3} m M^{-1} (\Sigma + \Sigma')^{ij} q_i (\underset{\sim}{q} \times \underset{\sim}{T})_j F_T (F_C + \tfrac{1}{3}\eta F_Q) \\
& + \tfrac{1}{3} q_i q_j (\Sigma + \Sigma')^{ij} M^{-2} [a_k{}^k (F_M{}^2 + F_T{}^2) - \\
& \qquad - 4 a_{00} (1+\eta)^{-1} F_Q (F_C + \tfrac{1}{3}\eta F_Q)] \\
& + \tfrac{1}{3} (\Sigma + \Sigma')^{ij} (2 q_0 q_i a_{0j} M^{-2} - Q^2 a_{ij}) (F_M{}^2 + F_T{}^2) \\
& + \tfrac{2}{3} q_i (\Sigma - \Sigma')^{ij} (Q^2 a_{0j} - q_0 q_j a_{00} M^{-2}) M^{-1} (1+\eta)^{-1} F_M F_Q \\
& + m M^{-1} [\underset{\sim}{S} \cdot \underset{\sim}{q} (\underset{\sim}{S}' \cdot \underset{\sim}{T} \times \underset{\sim}{q}) + \underset{\sim}{S}' \cdot \underset{\sim}{q} (\underset{\sim}{S} \cdot \underset{\sim}{T} \times \underset{\sim}{q})] F_T (F_C - \tfrac{2}{3}\eta F_Q) \\
& + m (\underset{\sim}{S} \times \underset{\sim}{S}' \cdot \underset{\sim}{q}) (Q^2 T_0 - q_0 \underset{\sim}{T} \cdot \underset{\sim}{q} M^{-2}) F_T F_M \\
& + \tfrac{1}{2} (\underset{\sim}{S} \times \underset{\sim}{q})_i a^{ij} (\underset{\sim}{S}' \times \underset{\sim}{q})_j M^{-2} (F_M{}^2 - F_T{}^2) \\
& + 2 a_{00} (\underset{\sim}{S} \times \hat{q}) \cdot (\underset{\sim}{S}' \times \hat{q}) (F_C - \tfrac{2}{3}\eta F_Q)(F_C + \tfrac{4}{3}\eta F_Q)
\end{aligned}
$$

$$+a_{oi}(S^i\, \underset{\sim}{S}'\cdot\underset{\sim}{q} - S'^i\, \underset{\sim}{S}\cdot\underset{\sim}{q})M^{-1}F_M(F_C-\tfrac{2}{3}\eta F_Q)$$

$$+2a_{00}(\underset{\sim}{S}\cdot\hat{q})(\underset{\sim}{S}'\cdot\hat{q})(F_C-\tfrac{2}{3}\eta F_Q)\big[(F_C-\tfrac{2}{3}\eta F_Q)(1+2\eta-Q^2)-$$

$$-2\eta^2(1+2\eta)F_M\big]$$

$$+4a_{00}\Sigma'^{ij}\Sigma_{ij}(F_C-\tfrac{2}{3}\eta F_Q)^2$$

$$+a_{00}(1+\eta)^{-2}q_i q_j\Sigma^{ij}q_m q_n\Sigma'^{mn}M^{-4}(F_T{}^2+F_Q{}^2)$$

$$-4a_{00}(1+\eta)^{-1}q^i\Sigma_{ij}\Sigma'^{jk}q_k M^{-2}F_Q(F_C-\tfrac{2}{3}\eta F_Q)$$

$$+q_i q_j a_{mn}(\Sigma^{ij}\Sigma'^{mn}+\Sigma'^{ij}\Sigma^{mn})M^{-2}(F_M{}^2+F_T{}^2)$$

$$+4q_i\big[\Sigma'^{ij}\Sigma_{jk}-\Sigma^{ij}(\Sigma')_{jk}\big]a^{ok}M^{-1}F_M(F_C-\tfrac{2}{3}\eta F_Q)$$

$$-4mM^{-1}\big[\Sigma_{ij}\Sigma'^{jk}+(\Sigma')_{ij}\Sigma^{jk}\big]q_k(\underset{\sim}{q}\times\underset{\sim}{T})^i F_T(F_C-\tfrac{2}{3}\eta F_Q)$$

$$-4m\,q_i\Sigma^{ij}\Sigma'^{kn}q_n\epsilon_{jkm}(q^m T_0-q_0T^m)M^{-2}F_TF_M$$

$$-2q_i\Sigma^{ij}a_{jk}\Sigma'^{km}q_m M^{-2}(F_M{}^2-F_T{}^2)$$

$$+2(1+\eta)^{-1}q_i q_j\Sigma^{ij}q_m a_{on}\Sigma'^{mn}M^{-3}(F_MF_Q-F_T{}^2)$$

$$-2(1+\eta)^{-1}q_i q_j\Sigma'^{ij}q_m a_{on}\Sigma^{mn}M^{-3}(F_MF_Q+F_T{}^2)$$

$$-2m(1+\eta)^{-1}q_i q_j\Sigma^{ij}q_m(\underset{\sim}{T}\times\underset{\sim}{q})_n\Sigma'^{mn}M^{-3}F_T(F_Q+F_M)$$

$$-2m(1+\eta)^{-1}q_i q_j\Sigma'^{ij}q_m(\underset{\sim}{T}\times\underset{\sim}{q})_n\Sigma^{mn}M^{-3}F_T(F_Q-F_M)$$

$$+2mM^{-1}(\underset{\sim}{q}\times\underset{\sim}{T})_i\big[(\underset{\sim}{q}\times\underset{\sim}{S})_j\Sigma'^{ij}+(\underset{\sim}{q}\times\underset{\sim}{S}')_j\Sigma^{ij}\big]F_M(F_C-\tfrac{2}{3}\eta F_Q)$$

$$+2mQ^{-2}(\underset{\sim}{S}\cdot\underset{\sim}{q}\,\Sigma'^{ij}-\underset{\sim}{S}'\cdot\underset{\sim}{q}\,\Sigma^{ij})q_i q_j M^{-2}(Q^2T_0-q_0\underset{\sim}{T}\cdot\underset{\sim}{q}M^{-2})F_T{}^2$$

$$+m(1+\eta)^{-1}(\mathbf{T}\times\mathbf{q})\cdot(\mathbf{S}\times\mathbf{q}\,\Sigma'^{ij}+\mathbf{S}'\times\mathbf{q}\,\Sigma^{ij})\,q_i q_j M^{-3}F_Q F_M$$

$$+2mM^{-1}q_i\left[\Sigma^{ij}(q_j\,\mathbf{S}'\cdot\mathbf{T}-T_j\,\mathbf{S}'\cdot\mathbf{q})+\right.$$

$$\left.+\Sigma'^{ij}(q_j\,\mathbf{S}\cdot\mathbf{T}-T_j\,\mathbf{S}\cdot\mathbf{q})\right]F_M(F_C-\tfrac{2}{3}\eta F_Q)$$

$$-m\,q_i\left[\Sigma^{ij}(S')_j-\Sigma'^{ij}S_j\right](Q^2T_0-q_0\,\mathbf{T}\cdot\mathbf{q}\,M^{-2})(F_M^2-F_T^2)$$

$$+2q_i a_{jk}\left[\Sigma^{ij}(\mathbf{S}'\times\mathbf{q})^k+\Sigma'^{ij}(\mathbf{S}\times\mathbf{q})^k\right]M^{-2}F_M F_T$$

$$+2a_{0i}\left[\Sigma^{ij}(\mathbf{q}\times\mathbf{S}')_j-\Sigma'^{ij}(\mathbf{q}\times\mathbf{S})_j\right]F_T(F_C-\tfrac{2}{3}\eta F_Q)$$

$$-(1+\eta)^{-1}q_i q_j\Sigma^{ij}a_{0k}(\mathbf{S}'\times\mathbf{q})^k M^{-3}F_T[F_M-F_C+(1+\tfrac{2}{3}\eta)F_Q]$$

$$+(1+\eta)^{-1}q_i q_j\Sigma'^{ij}(\mathbf{S}\times\mathbf{q})^k a_{0k}M^{-3}F_T(F_Q-F_M)$$

$$-2(1+\eta)^{-1}a_{00}\,q_i\left[\Sigma'^{ij}(\mathbf{S}\times\mathbf{q})_j-\Sigma^{ij}(\mathbf{S}'\times\mathbf{q})_j(1+\eta)\right]F_T(F_C-\tfrac{2}{3}\eta F_Q)$$

$$+(1+\eta)^{-1}(\mathbf{S}'\cdot\mathbf{q})\,q^i\Sigma_{ij}\,\epsilon^{jmn}q_m a_{0n}M^{-3}F_T(F_C-\tfrac{2}{3}\eta F_Q)$$

$$+2q_i\left[\Sigma'^{ij}\epsilon_{jmn}S^m a^{0n}-(1+2\eta)\Sigma^{ij}\epsilon_{jmn}S'^m a^{0n}\right]$$

$$\times F_T(F_C-\tfrac{2}{3}\eta F_Q)\quad,$$

$$a_\lambda{}^\lambda = 2m^2(1 - t_i \cdot t_f) - 4\eta M^2 ,$$

$$Q^2 = |\underset{\sim}{q}|^2 M^{-2} = 4\eta(1+\eta) ,$$

$$q_0 = 2\eta ,$$

$$(\underset{\sim}{p} \times \underset{\sim}{q})^i = \epsilon^{ijk} p_j q_k ,$$

$$\Sigma^{ij} = \frac{1}{2}(S^i S^j + S^j S^i) - \frac{2}{3}\delta^{ij} ,$$

ψ_i (ψ_f) is the polarization vector of the deuteron before (after) scattering at the rest frame, S (S') is the spin operator of the deuteron before (after) scattering.

3. ELASTIC ELECTRON-DEUTERON SCATTERING

We consider the polarization effects of the elastic electron-deuteron scattering in this Section. The electron is treated as extremely relativistic and correction terms proportional to the electron mass are neglected.

The differential cross-section in the laboratory system is

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2}\,\frac{E'}{E}\,\frac{|M|^2}{1+2EM^{-1}\sin^2\frac{\theta}{2}}\,, \tag{6}$$

and we have

$$\eta = EE'M^{-2}\sin^2\frac{\theta}{2} = \left(EM^{-1}\sin\frac{\theta}{2}\right)^2\left(1+2EM^{-1}\sin^2\frac{\theta}{2}\right)^{-1}, \tag{7}$$

$$E' = E - 2\eta M, \tag{8}$$

$$j_{\lambda\mu} = (4m^2)^{-1}(\varphi_i\varphi_f)^{\dagger}\left[d_{\lambda\mu}\left(1+\underset{\sim}{\sigma}\cdot\hat{p}\,\underset{\sim}{\sigma}'\cdot\hat{p}'\right) + i\left(\underset{\sim}{\sigma}\cdot\hat{p}+\underset{\sim}{\sigma}'\cdot\hat{p}'\right)\varepsilon_{\lambda\mu\alpha\beta}(p_i)^{\alpha}(p_f)^{\beta}\right]\varphi_i\varphi_f\,, \tag{9}$$

$$d_{\lambda\mu} = (p_i)_{\lambda}(p_f)_{\mu} + (p_i)_{\mu}(p_f)_{\lambda} - 2\eta M^2 g_{\lambda\mu}, \tag{10}$$

$$m\,\underset{\sim}{q}\times\underset{\sim}{T} = \left(\underset{\sim}{p}\times\underset{\sim}{p}'\right)\left(\underset{\sim}{\sigma}\cdot\hat{p}+\underset{\sim}{\sigma}'\cdot\hat{p}'\right), \tag{11}$$

$$m\left(T_0Q^2 - q_0\,\underset{\sim}{T}\cdot\underset{\sim}{q}\,M^{-2}\right) = 2\eta(E+E')\left(\underset{\sim}{\sigma}\cdot\hat{p}+\underset{\sim}{\sigma}'\cdot\hat{p}'\right), \tag{12}$$

$$|M|^2 = e^4q^{-4}(4m^2)^{-1}(\varphi_i\varphi_f\psi_i\psi_f)^{\dagger}A(\varphi_i\varphi_f\psi_i\psi_f), \tag{13}$$

where

$$A = \left[1+(\underset{\sim}{\sigma}\cdot\hat{p})(\underset{\sim}{\sigma}'\cdot\hat{p}')\right]A_1 + \left(\underset{\sim}{\sigma}\cdot\hat{p}+\underset{\sim}{\sigma}'\cdot\hat{p}'\right)A_2\,,$$

$$A_1 = \frac{8}{3}EE'\cos^2\frac{\theta}{2}\left\{F_C^{\,2} + \frac{8}{9}\eta^2F_Q^{\,2} + \frac{2}{3}\eta(F_M^{\,2}+F_T^{\,2})\left[1+2(1+\eta)\tan^2\frac{\theta}{2}\right]\right\}$$

$$+\frac{4}{3}\eta(E+E')M^{-1}\left[(\underset{\sim}{S}-\underset{\sim}{S}')\cdot\underset{\sim}{p}\times\underset{\sim}{p}'\right]F_TF_Q$$

$$-\frac{2}{3}EE'q_i q_j(\Sigma+\Sigma')^{ij}M^{-2}\Big[(1+\sin^2\frac{\theta}{2})(F_M^2+F_T^2)+$$

$$+4(1+\eta)^{-1}\cos^2\frac{\theta}{2}F_Q(F_C+\frac{1}{3}\eta F_Q)\Big]$$

$$-\frac{4}{3}\eta(\Sigma+\Sigma')_{ij}(E p'^i p'^j-E'p^i p^j+2p^i p'^j M)M^{-1}(F_M^2+F_T^2)$$

$$+\frac{8}{3}\eta(1+\eta)^{-1}q^i(\Sigma-\Sigma')_{ij}\Big[(1+\eta)M\delta^{0j}-q^j EE'\cos^2\frac{\theta}{2}\Big]M^{-2}F_M F_Q$$

$$+\frac{1}{2}(\mathbf{S}\times\mathbf{q})_i\delta^{ij}(\mathbf{S}'\times\mathbf{q})_j M^{-2}(F_M^2-F_T^2)$$

$$+4EE'\cos^2\frac{\theta}{2}(\mathbf{S}\times\hat{q})\cdot(\mathbf{S}'\times\hat{q})(F_C-\frac{2}{3}\eta F_Q)(F_C+\frac{4}{3}\eta F_Q)$$

$$-(\mathbf{S}\cdot\mathbf{q}\,S'^i-\mathbf{S}'\cdot\mathbf{q}\,S^i)\delta_{0i}M^{-1}F_M(F_C-\frac{2}{3}\eta F_Q)$$

$$+4EE'\cos^2\frac{\theta}{2}(\mathbf{S}\cdot\hat{q})(\mathbf{S}'\cdot\hat{q})(F_C-\frac{2}{3}\eta F_Q)$$

$$\times\Big[(1+2\eta-Q^2)(F_C-\frac{2}{3}\eta F_Q)-2\eta^2(1+2\eta)F_M\Big]$$

$$+8EE'\cos^2\frac{\theta}{2}\Sigma'^{ij}\Sigma_{ij}(F_C-\frac{2}{3}\eta F_Q)^2$$

$$+2(1+\eta)^{-2}EE'\cos^2\frac{\theta}{2}q_i q_j\Sigma^{ij}q_m q_n\Sigma'^{mn}M^{-4}(F_T^2+F_Q^2)$$

$$-8(1+\eta)^{-1}EE'\cos^2\frac{\theta}{2}q^i\Sigma_{ij}\Sigma'^{jk}q_k M^{-2}F_Q(F_C-\frac{2}{3}\eta F_Q)$$

$$+q_i q_j\delta_{mn}(\Sigma^{ij}\Sigma'^{mn}+\Sigma'^{ij}\Sigma^{mn})M^{-2}(F_M^2+F_T^2)$$

$$+4q_i\Big[\Sigma'^{ij}\Sigma_{jk}-\Sigma^{ij}(\Sigma')_{jk}\Big]\delta^{0k}M^{-1}F_M(F_C-\frac{2}{3}\eta F_Q)$$

$$
\begin{aligned}
&-2q_i\Sigma^{ij}d_{jk}\Sigma'^{km}q_m M^{-2}(F_M{}^2-F_T{}^2)\\
&+2(1+\eta)^{-1}q_iq_j\Sigma^{ij}q_m d_{0n}\Sigma'^{mn}M^{-3}(F_MF_Q-F_T{}^2)\\
&-2(1+\eta)^{-1}q_iq_j\Sigma'^{ij}q_m d_{0n}\Sigma^{mn}M^{-3}(F_MF_Q+F_T{}^2)\\
&+2q_i d_{jk}[\Sigma^{ij}(\underset{\sim}{S}'\times\underset{\sim}{q})^k+\Sigma'^{ij}(\underset{\sim}{S}\times\underset{\sim}{q})^k]M^{-2}F_MF_T\\
&+2d_{0i}[\Sigma^{ij}(\underset{\sim}{q}\times\underset{\sim}{S}')_j-\Sigma'^{ij}(\underset{\sim}{q}\times\underset{\sim}{S})_j]F_T(F_C-\tfrac{2}{3}\eta F_Q)\\
&+(E+E')(1+\eta)^{-1}q_iq_j\Sigma^{ij}(\underset{\sim}{S}'\times\underset{\sim}{p}\cdot\underset{\sim}{p}')M^{-3}F_T\\
&\qquad\times[F_M-F_C+(1+\tfrac{2}{3}\eta)F_Q]\\
&-(E+E')(1+\eta)^{-1}q_iq_j\Sigma'^{ij}(\underset{\sim}{S}\times\underset{\sim}{p}\cdot\underset{\sim}{p}')M^{-3}F_T(F_Q-F_M)\\
&-4EE'\cos^2\tfrac{\theta}{2}(1+\eta)^{-1}q_i[\Sigma'^{ij}(\underset{\sim}{S}\times\underset{\sim}{q})_j-\\
&\qquad-\Sigma^{ij}(\underset{\sim}{S}'\times\underset{\sim}{q})_j(1+\eta)]F_T(F_C-\tfrac{2}{3}\eta F_Q)\\
&+(E+E')(1+\eta)^{-1}(\underset{\sim}{S}'\cdot\underset{\sim}{q})q^i\Sigma_{ij}(\underset{\sim}{p}\times\underset{\sim}{p}')^jM^{-3}F_T\\
&\qquad\times(F_C-\tfrac{2}{3}\eta F_Q)\\
&+2q_i[\Sigma'^{ij}\epsilon_{jmn}S^m d^{0n}-(1+2\eta)\Sigma^{ij}\epsilon_{jmn}S'^m d^{0n}]\\
&\qquad\times F_T(F_C-\tfrac{2}{3}\eta F_Q)\,,
\end{aligned}
$$

$$
\begin{aligned}
A_2=&\tfrac{4}{3}[(\underset{\sim}{S}+\underset{\sim}{S}')\times\underset{\sim}{q}]\cdot\underset{\sim}{p}\times\underset{\sim}{p}'\,M^{-1}F_M(F_C+\tfrac{1}{3}\eta F_Q)\\
&-\tfrac{2}{3}\eta(E+E')(\underset{\sim}{S}-\underset{\sim}{S}')\cdot\underset{\sim}{q}\ (F_M{}^2+F_T{}^2)
\end{aligned}
$$

$$-\tfrac{8}{3} q_i (\Sigma+\Sigma')^{ij} (\mathbf{p}\times\mathbf{p}')_j M^{-1} F_T (F_C + \tfrac{1}{3}\eta F_Q)$$

$$-[\mathbf{S}\cdot\mathbf{q}(\mathbf{S}'\cdot\mathbf{p}\times\mathbf{p}') + \mathbf{S}'\cdot\mathbf{q}(\mathbf{S}\cdot\mathbf{p}\times\mathbf{p}')] M^{-1} F_T (F_C - \tfrac{2}{3}\eta F_Q)$$

$$+2\eta(E+E')(\mathbf{S}\times\mathbf{S}'\cdot\mathbf{q}) F_T F_M$$

$$-4[\Sigma_{ij}\Sigma'^{jk} + (\Sigma')_{ij}\Sigma^{jk}] q_k (\mathbf{p}\times\mathbf{p}')^i M^{-1} F_T (F_C - \tfrac{2}{3}\eta F_Q)$$

$$-4 q_i \Sigma^{ij}\Sigma'^{kn} q_n \epsilon_{jkm} (E' p^m - E p'^m) M^{-2} F_T F_M$$

$$+2(1+\eta)^{-1} q_i q_j \Sigma^{ij} q_m \Sigma'^{mn} (\mathbf{p}\times\mathbf{p}')_n M^{-3} F_T (F_Q + F_M)$$

$$+2(1+\eta)^{-1} q_i q_j \Sigma'^{ij} q_m \Sigma^{mn} (\mathbf{p}\times\mathbf{p}')_n M^{-3} F_T (F_Q - F_M)$$

$$+2(\mathbf{p}\times\mathbf{p}')_i [(\mathbf{q}\times\mathbf{S})_j \Sigma'^{ij} + (\mathbf{q}\times\mathbf{S}')_j \Sigma^{ij}] M^{-1} F_M (F_C - \tfrac{2}{3}\eta F_Q)$$

$$+(E+E')(1+\eta)^{-1} q_i q_j (\mathbf{S}\cdot\mathbf{q}\,\Sigma'^{ij} - \mathbf{S}'\cdot\mathbf{q}\,\Sigma^{ij}) M^{-2} F_T^2$$

$$-(1+\eta)^{-1}(\mathbf{p}\times\mathbf{p}')\cdot(\mathbf{S}\times\mathbf{q}\,\Sigma'^{ij} + \mathbf{S}'\times\mathbf{q}\,\Sigma^{ij}) q_i q_j$$
$$\times M^{-3} F_Q F_M$$

$$+2q^i[\Sigma_{ij}(p^j\,\mathbf{S}'\cdot\mathbf{p}' - p'^j\,\mathbf{S}'\cdot\mathbf{p}) +$$
$$+(\Sigma')_{ij}(p^j\,\mathbf{S}\cdot\mathbf{p}' - p'^j\,\mathbf{S}\cdot\mathbf{p})] F_M (F_C - \tfrac{2}{3}\eta F_Q)$$

$$-2\eta(E+E') q_i [\Sigma^{ij}(S')_j - \Sigma'^{ij} S_j](F_M^2 - F_T^2).$$

The first five terms of A_1 have been calculated by Schildknecht [4].

4. DISCUSSION

In both muon-deuteron and electron-deuteron scattering, we find a term of the form which violates T invariance :

$$\frac{4}{3}\eta(E+E')M^{-1}\left[(\underset{\sim}{S}-\underset{\sim}{S}')\cdot\underset{\sim}{p}\times\underset{\sim}{p}'\right]F_T F_Q .$$

The simplest way to measure T non-invariance is either to polarize the deuterons before scattering [4),5)] or to measure the vector polarization of the deuterons after scattering.

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APPENDIX

We shall give the detail of our calculation in this Appendix. We set $M = 1$ for simplicity. The condition of current conservation $j_{\lambda\mu} q^{\lambda} = 0$ implies that, in practical calculations, the electromagnetic vertex of deuteron can be taken to be

$$R_{\alpha\beta}{}^{\lambda} = P^{\lambda}(2F_1 g_{\alpha\beta} - F_2 q_\alpha q_\beta) + G_1(q_\alpha g_\beta{}^{\lambda} - q_\beta g_\alpha{}^{\lambda}) + i2\eta G_2(q_\alpha g_\beta{}^{\lambda} + q_\beta g_\alpha{}^{\lambda}). \tag{A.1}$$

To calculate $j_{\lambda\mu}$, we write

$$j_{\lambda\mu} = \bar{u}(p_f) m_{\lambda\mu} u(p_f) = \frac{1}{2} \varphi_f{}^{\dagger}(a + b\cdot t_f)\varphi_f ,$$
$$m_{\lambda\mu} \equiv \gamma_\lambda u(p_i)\bar{u}(p_i)\gamma_\mu (\not{p}_f + m)(2m)^{-1}, \tag{A.2}$$

where [13]

$$u(p_i)\bar{u}(p_i) = \frac{1}{2}\varphi_i{}^{\dagger}(1 + \gamma_5 \not{t}_i)(\not{p}_i + m)(2m)^{-1}\varphi_i ,$$
$$a = tr(m_{\lambda\mu}) ,$$
$$b_\alpha = tr(\gamma_5 \gamma_\alpha m_{\lambda\mu}) .$$

The coefficients a and b can be calculated in a straightforward manner. The final result is given by Eq. (4).

Similarly, we have [4),14)]

$$\epsilon^{\dagger}(P_f) N \epsilon(P_f) = \psi_f{}^{\dagger}\left[\frac{1}{3} tr N - \frac{1}{2} J_\alpha tr(W^\alpha N) + \Omega_{\alpha\beta} tr(W^\alpha W^\beta N)\right]\psi_f , \tag{A.3}$$

where

$$N = R^{\lambda}\epsilon(P_i)\epsilon^{\dagger}(P_i)R^{\dagger\mu}\bar{P} ,$$
$$\bar{P}^{\alpha\beta} = -g^{\alpha\beta} + (P_f)^{\alpha}(P_f)^{\beta} ,$$

$$P_f = (1+2\eta, \underset{\sim}{q}),$$

$$J_0 = \underset{\sim}{S}' \cdot \underset{\sim}{q},$$

$$\underset{\sim}{J} = \underset{\sim}{S}' + 2\eta \underset{\sim}{S}' \cdot \hat{q}\, \hat{q},$$

$$J \cdot P_f = 0,$$

$$[\epsilon(P_i)\epsilon^\dagger(P_i)]^{\alpha\beta} = 0 \quad \text{if } \alpha = 0 \text{ or } \beta = 0,$$

$$= \psi_i^\dagger (\tfrac{1}{3}\delta^{\alpha\beta} + \tfrac{1}{2} i S_k \epsilon^{k\alpha\beta} - \Sigma^{\alpha\beta})\psi_i \quad \text{if } \alpha \neq 0 \text{ and } \beta \neq 0,$$

$$\Omega_{\alpha\beta} = \frac{1}{2}(J_\alpha J_\beta + J_\beta J_\alpha) - \frac{2}{3}\bar{P}_{\alpha\beta},$$

$$(W^\alpha)^{\beta\gamma} = -i\,\epsilon^{\alpha\beta\gamma\delta}(P_f)_\delta .$$

The covariant spin operator W satisfies the following identities :

$$(\bar{P}_{\alpha\beta} W^\alpha W^\beta)^{\gamma\delta} = 2\bar{P}^{\gamma\delta}, \qquad \text{(A.4)}$$

$$(J\cdot W\, J\cdot W)^{\alpha\beta} = J^\alpha J^\beta - 2\bar{P}^{\alpha\beta}. \qquad \text{(A.5)}$$

Using Eqs. (A.4) and (A.5), one can prove easily that

$$\Omega_{\alpha\beta}\, tr(W^{\alpha} W^{\beta} N) = -\Omega_{\alpha\beta}\left[R^{\lambda}\epsilon(P_i)\epsilon^{\dagger}(P_i) R^{\dagger\mu}\right]^{\alpha\beta}_{.} \tag{A.6}$$

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