PAPER • OPEN ACCESS

Thermal properties for an ensemble of polymer Fermi oscillators

To cite this article: Guillermo Chacón-Acosta et al 2015 J. Phys.: Conf. Ser. 654 012002

View the article online for updates and enhancements.

Related content

- Thermal properties of (AI,Gd)O3 doped uranium dioxide S A Pokrovskiy, V G Baranov and A V Tenishev
- Thermal properties of nonstoichiometry uranium dioxide R Kavazauri, S A Pokrovskiy, V G Baranov et al.
- Effect of Co doping on structural, optical, electrical and thermal properties of nanostructured ZnO thin films Sonet Kumar Saha, M. Azizar Rahman, M. R. H. Sarkar et al.

IOP ebooks[™]

Start exploring the collection - download the first chapter of every title for free.

Thermal properties for an ensemble of polymer Fermi oscillators

Guillermo Chacón-Acosta

Applied Mathematics and Systems Departament, Universidad Autónoma Metropolitana-Cuajimalpa, Av. Vasco de Quiroga 4871, México D. F. 05348, Mexico

E-mail: gchacon@correo.cua.uam.mx

Angel A. García-Chung

Physics Department, Universidad Autónoma Metropolitana-Iztapalapa, San Rafael Atlixco 186, México D. F. 09340, Mexico

E-mail: alechung@xanum.uam.mx

Héctor H. Hernandez-Hernandez

Engineering Faculty, Nuevo Campus Universitario Universidad Autónoma de Chihuahua, Chihuahua 31125, Mexico

E-mail: hhernandez@uach.mx

Abstract.

Polymer quantum mechanics is a model inspired on loop quantum gravity in which one can study, in a simplified way, some properties of certain quantum mechanical models. There is a length parameter in this model, known as the polymer scale, comprising the discreteness introduced in this particular quantization. There is a recent analysis on quantum fields where the Fermi oscillator is *polymerized* by means of a non-analytic representation of the corresponding Weyl super-algebra, its energy spectrum acquires modifications by the introduction of polymer parameters, which turn out to be super-numbers. In this work we present the first step in studying thermostatistical properties of an ensemble of Fermi oscillators. As an initial approximation we consider the polymer parameters as real deviations from their usual values. We obtain modifications to the thermal properties of the system in terms of polymer parameters. In the last section we critically discuss the possible physical significance of the results.

1. Introduction

The scheme of Loop Quantum Gravity (LQG) offers a diffeomorphism invariant and non perturbative quantum representation of gravity [1] at the kinematical level. This scheme, originally applied purely to the gravitational sector, has been extended to include matter fields like the scalar, gauge and fermion fields [2, 3]. Due to the intricate mathematical apparatus of the full LQG, a toy model, called "Polymer quantum mechanics" (PQM) [4], has served as a guide to bypass aspects related to the infinite degrees of freedom within the field theory, and to single out some of its aspects, specially as a system with a finite number of degrees of freedom.

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution (cc of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Published under licence by IOP Publishing Ltd 1

Among the systems that PQM has been able to analyze are the study of the problematics to incorporate a Galilean symmetry to it [5], an analysis of singular potentials [6, 7], a particle in the half real line [8], the time diffraction [9] and the tunnel effect of polymer particles [10].

The statistical mechanics of systems built up by a bunch of particles has also been studied in [11, 12]. In that work corrections to the thermodynamics were found, induced by the so called polymer length scale, a parameter arisen in the polymer quantization of bosonic fields [4]. Some other thermal systems, such as hydrogenoid gases [13], have been analyzed. Particularly the study of a polymer Bose-Einstein condensate gives us bounds in the polymer length [14]; it was also possible to obtain modification in the mass of a compact star due to polymer effects by studying a completely degenerate Fermi gas at zero temperature [15, 16].

There have also been attempts to study field theory in this context, resulting in what is known as Polymer Quantum Field Theory (PQFT). The idea of replacing the tower of harmonic oscillators in which the field can be decomposed by polymer oscillators was advanced in [17]. The result is that the propagator has Planck-scale corrections. When the energy of the tower of polymer harmonic oscillators is close to the Planck energy, the propagator departs considerably of the standard propagator and admits tachyonic mass poles. The standard propagator is obtained as a low energy limit. In the same vein a study of the high energy behavior of the Dirac field in the polymer framework has been recently proposed. The idea was to apply PQM to each of the Fermi oscillators into which the Dirac fiel can be decomposed [18]. The result can be found in [19], where an unconventional modification on the free propagator for the Dirac field, which involves Grassmann-type quantities, was obtained.

These results demand a deeper analysis, not only for the transition amplitudes derived in the coupling to other matter fields like the U(1) field within the standard model, but also for their statistical implications and their perspective into phenomenological modeling. This exploration is the main intent of this contribution.

In the first section, we briefly review the main aspects of the PQM for the Fermi oscillator. In the second section we present the thermal properties of an ensamble of polymer Fermi oscillators that is essentially a gas built up of two-level polymer particles. In the third section a discussion of the possibilities of interpreting the super-polymer parameters using the statistical results is presented. As a first exploration of their meaning, we treat them as real perturbations, near to their standard real value without polymer corrections. This was done following previous work on phenomenological models [11]-[16]. The main difference is that in those models the polymer parameter was easily interpretable as a fundamental length scale, while in the fermionic case it cannot be done. This test, however, gives us an idea of the possible effects that the polymer corrections may have on the thermal properties of a simple statistical system. This work is not intended to be a definitive analysis, as we mentioned above it is a first approach aiming to have a point of comparison with the corresponding super-corrections; subsequent considerations will be performed on a later work in preparation.

2. Polymer Fermi oscillator

2.1. The Fermi oscillator

The Fermi oscillator [20] is a mechanical system whose configuration space is \mathbb{R}_a^2 , the two dimensional real anticommutative space. The Dirac formalism for constrained systems [21] can be applied to it, yielding a quantum model with a Hamiltonian of the form

$$\widehat{H} = \frac{\omega}{2i} (\widehat{x}_1 \widehat{x}_2 - \widehat{x}_2 \widehat{x}_1).$$
(1)

The corresponding canonical anticommutation relations (CAR) take the form

$$[\widehat{x}_i, \widehat{x}_j]_+ = \widehat{x}_i \widehat{x}_j + \widehat{x}_j \widehat{x}_i = \delta_{ij} \hbar \widehat{1}, \qquad (2)$$

where $\widehat{1}$ and \widehat{x}_i (with $i = \{1, 2\}$) denote the generators of the quantum algebra. An irreducible representation of this algebra can be realized on a super Hilbert space to allow multiplication by super numbers. In our case, we consider the super Hilbert space \mathcal{H}_s formed by superanalytic functions [20]. A vector state $\Psi \in \mathcal{H}_s$ is a superfunction $\Psi : \mathbb{R}_a \to \Lambda_\infty; x \mapsto \Psi(x)$. Here Λ_∞ is the space of supernumbers [22]. The representation of the CAR is thus of the form

$$\widehat{x}_1 \Psi(x) = \frac{\sqrt{2\hbar}}{2} \Psi(x) \left(x + \frac{\mathrm{d}^R}{\mathrm{d}x} \right), \qquad \widehat{x}_2 \Psi(x) = \frac{\sqrt{2\hbar}}{2i} \Psi(x) \left(x - \frac{\mathrm{d}^R}{\mathrm{d}x} \right), \tag{3}$$

where $\frac{\mathrm{d}^{R}}{\mathrm{d}x}$ is the right derivative. Because any superfunction in \mathcal{H}_{s} may be decomposed as $\Psi(x) = \Psi_{0} + \Psi_{1}x$, where $\Psi_{0} = \Psi(x=0)$ and $\Psi_{1} = \left[\Psi(x)\frac{\mathrm{d}^{R}}{\mathrm{d}x}\right]\Big|_{x=0}$, the stationary Schrödinger equation (1) takes an algebraic form

$$\widehat{H}\Psi(x) = -\frac{\hbar\omega}{2}(\Psi_0 - \Psi_1 x) = E\Psi(x).$$
(4)

There are only two eigenvalues for (4) which are $E_{\pm} = \pm \frac{\hbar\omega}{2}$ and their respective basis of eigenfunctions is $\Psi_{-}(x) = 1$ and $\Psi_{+}(x) = x$. It is worth mentioning that the advantages of a representation on a super Hilbert space is that it allows multiplication (by the left or the right) of supernumbers, as well as for the algebra. Notice that, in this case, the energy spectrum is real and corresponds to a two level system. This does not happen in the polymer case, so the interpretation of the polymer superparameters is not as direct as in this case.

2.2. Polymerization of the Fermi oscillator

Let us review the main ideas in reaching the polymer version of this mechanical system which, basically, resembles the construction of the polymer version of the harmonic oscillator [23, 24]. The first step is to construct the *super* version of the Weyl algebra, which can be performed using the standard representation (3) as follows

$$\widehat{W}(\theta_1, \theta_2)\Psi(x) := e^{\theta_1 \widehat{x}_1 - i\theta_2 \cdot \widehat{x}_2}\Psi(x) := [1 + \theta_1 \cdot \widehat{x}_1 - i\theta_2 \cdot \widehat{x}_2 - i\theta_1\theta_2 \cdot \widehat{x}_1 \widehat{x}_2]\Psi(x).$$
(5)

where the parameters θ_1 and θ_2 are real anticommutative supernumbers. Using (3) and (5), the product of two generators \widehat{W} is well defined and takes the form

$$\widehat{W}(\theta_1, \theta_2) \cdot \widehat{W}(\theta_1', \theta_2') = e^{\frac{\hbar}{2}(\theta_1 \theta_1' + \theta_2 \theta_2')} \widehat{W}(\theta_1 + \theta_1', \theta_2 + \theta_2').$$
(6)

Equation (5) defines a regular representation of the Weyl superalgebra whose multiplication is given by (6). Its polymer version is just a nonregular, i.e. non super analytic representation of this superalgebra [23]. To get such a representation we discretize the real anticommutative space and then define an abstract super vector space basis $|x_j\rangle$. The linear combinations

$$|\Psi\rangle = \sum_{j}^{N} |x_{j}\rangle \Psi_{x_{j}}.$$
(7)

are the super vectors which span the super vector space \mathcal{F}_p after giving it a vector superspace structure [23].

Super Hilbert spaces do not have an inner product structure but rather a bijective map between the vector superspace and its dual [20]. The dual space \mathcal{F}_p^* is formed by elements

$$\langle \Psi | = \sum_{j}^{N} \Psi_{x_j} \langle x_j |.$$
(8)

Journal of Physics: Conference Series 654 (2015) 012002

In the polymer case the bijection is constructed as

$$\langle x|x'\rangle = \delta_{x,x'},\tag{9}$$

where $\delta_{x,x'}$ is the Kronecker delta. Then \mathcal{H}_p denote the super Hilbert space formed with \mathcal{F}_p together with the bijection defined above, where a representation of the Weyl superalgebra generators can be realized as

$$\widehat{W}(\theta_1, \theta_2) |x\rangle := e^{\frac{\hbar}{2}\theta_1 \theta_2} e^{\frac{\sqrt{2\hbar}}{2}(\theta_1 - \theta_2)x} \left| x + \frac{\sqrt{2\hbar}}{2}(\theta_1 + \theta_2) \right\rangle.$$
(10)

Incorporating (9) with (10) makes the Weyl superalgebra non regular in the sense that superanalyticity is not available and hence we cannot recover the generators of the Clifford superalgebra (3).

Finally one can analyze the dynamics of this system. Because the Hamiltonian (1) is not directly related to the Weyl superalgebra generators (6), due to the lack of superanalyticity, a Hamiltonian that can be expressed in terms of Weyl generators is proposed in [23]

$$\widehat{H} \to \widehat{H}_p = \frac{\hbar\omega}{2} \left[2 - \widehat{W}(\theta_1, \theta_2) - \widehat{W}(-\theta_1, -\theta_2) \right].$$
(11)

Notice its similarity with the bosonic case [4].

We now proceed to solve the Schrödinger equation $\hat{H}_p \Psi = E \Psi$, but first we introduce a momentum representation and a Fourier transform, as in the bosonic case

$$\langle \Psi|k\rangle := \tilde{\Psi}(k) = \sum_{x_j} \Psi_{x_j} \langle x_j|k\rangle, \quad \text{where} \quad \langle k|x\rangle := e^{-kx} = \overline{\langle x|k\rangle}.$$
(12)

Let us denote by \mathcal{H}_{poly} , rather than \mathcal{H}_p , the Hilbert space with the structure

$$\mathcal{H}_{poly} = \bigoplus_{x_0} \mathcal{H}_{poly}^{x_0}.$$
(13)

Any $|\Psi_{x_0}\rangle \in \mathcal{H}_{poly}^{x_0}$ takes the form

$$|\Psi_{x_0}\rangle = \sum_j |x_0 + \frac{\sqrt{2\hbar}}{2}j(\theta_1 - \theta_2)\rangle \Psi_{x_0}^j.$$
(14)

The Schrödinger equation for $x_0 = 0$ reads

$$\Psi_0^{j-1} e^{-\frac{\hbar}{2}\theta_1\theta_2} e^{-\hbar(j-1)\theta_1\theta_2} + \Psi_0^{j+1} e^{-\frac{\hbar}{2}\theta_1\theta_2} e^{+\hbar(j+1)\theta_1\theta_2} - 2\left(1 - \frac{E\theta_1\theta_2}{\omega}\right)\Psi_0^j = 0, \quad (15)$$

where E denotes the energy eigenvalue. Let us now perform a Fourier transformation to go to momentum representation using (12). To do so we multiply by $e^{\frac{\sqrt{2\hbar}}{2}jk(\theta_1-\theta_2)}$ and sum over j. This gives us finally

$$\left[\frac{\hbar\omega}{2} - E\right]\theta_1\theta_2\tilde{\Psi}_0 - \left[\frac{\hbar\omega}{2} + E\right]\theta_1\theta_2\tilde{\Psi}_1k = 0.$$
(16)

Here, $\tilde{\Psi}_0 = \tilde{\Psi}(k=0)$ and $\tilde{\Psi}_1 = \tilde{\Psi}(k) \frac{d^R}{dk}\Big|_{k=0}$ are the coefficients of the wave function in *momentum* representation. By using the following property of the wave functions

$$\sum_{x} \Psi_{x} e^{(a+k)x} = \tilde{\Psi}(k) + \left(\frac{\mathrm{d}^{R}}{\mathrm{d}k}\tilde{\Psi}(k)\right)a,\tag{17}$$

we obtain that eq. (16) possesses the eigenvalues

$$E_{\pm} = \pm \frac{\hbar\omega}{2} + (e_{\pm})\hbar\omega(\hbar\theta_1\theta_2).$$
(18)

In this expression, e_{\pm} are two commutative dimensionless arbitrary parameters. When we fix $e_{\pm} = 0, \theta_1, \theta_2 = 0$ we obtain the standard energy spectrum of the Fermi oscillator.

3. Thermal properties of an ensamble of polymer Fermi oscillators

Considering the energy spectrum (18) we can immediately calculate the canonical partition function under the assumption that the statistics does not change [12]. As there are only two possible values for the energy, Z becomes

$$Z = \exp\left(-\frac{E_+}{kT}\right) + \exp\left(-\frac{E_-}{kT}\right) = \exp\left(-\frac{\hbar\omega}{2kT}\Delta_+\right) + \exp\left(\frac{\hbar\omega}{2kT}\Delta_-\right),\tag{19}$$

or in terms of hyperbolic functions, as is commonly written

$$Z = \cosh\left(\frac{\hbar\omega}{2kT}\Delta_{+}\right) - \sinh\left(\frac{\hbar\omega}{2kT}\Delta_{+}\right) + \cosh\left(\frac{\hbar\omega}{2kT}\Delta_{-}\right) + \sinh\left(\frac{\hbar\omega}{2kT}\Delta_{-}\right).$$
(20)

In all previous expressions k is the Boltzmann constant, T is the gas temperature and the following quantities are defined

$$\Delta_+ := 1 + 2\hbar e_+ \theta_1 \theta_2, \quad \Delta_- := 1 - 2\hbar e_- \theta_1 \theta_2, \tag{21}$$

comprising all the polymer effects discussed in the previous section. Also notice that, by making $\Delta_+ = \Delta_- = 1$, we recover the usual partition function for a two level system (as a magnetic dipole) $Z = 2 \cosh\left(\frac{\hbar\omega}{2kT}\right)$,[25].

Just as in [11], we can compute all the thermodynamic functions recalling that the partition function for a system of N particles is Z^N . Thus, the free energy is

$$F = -NkT\ln Z = -NkT\ln \left[e^{-\frac{\hbar\omega}{2kT}\Delta_{+}} + e^{\frac{\hbar\omega}{2kT}\Delta_{-}}\right].$$
(22)

The internal energy and heat capacity are as follows

$$U = NkT^{2}\frac{\partial Z}{\partial T} = \frac{N\hbar\omega}{2} \left[\frac{\Delta_{+}\exp\left(-\frac{\hbar\omega}{2kT}\Delta_{+}\right) - \Delta_{-}\exp\left(\frac{\hbar\omega}{2kT}\Delta_{-}\right)}{\exp\left(-\frac{\hbar\omega}{2kT}\Delta_{+}\right) + \exp\left(\frac{\hbar\omega}{2kT}\Delta_{-}\right)}\right],\tag{23}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{N \,\hbar^2 \omega^2}{16kT^2} \left(\Delta_+ + \Delta_-\right)^2 \operatorname{sech}^2 \left[\frac{\hbar\omega}{4kT} \left(\Delta_+ + \Delta_-\right)\right]. \tag{24}$$

The entropy can be obtained directly by subtracting the free energy (22) to the internal energy:

$$S = \frac{U - F}{T} = \frac{N \hbar \omega}{2T} \left[\frac{\Delta_+ e^{-\frac{\hbar \omega \Delta_+}{2kT}} - \Delta_- e^{\frac{\hbar \omega \Delta_-}{2kT}}}{e^{-\frac{\hbar \omega \Delta_+}{2kT}} + e^{\frac{\hbar \omega \Delta_-}{2kT}}} \right] + Nk \ln \left[e^{-\frac{\hbar \omega \Delta_+}{2kT}} + e^{\frac{\hbar \omega \Delta_-}{2kT}} \right].$$
(25)

Although expressions (22)-(25) give us the form of the thermodynamic quantities of an ensemble of Fermi oscillators, it is still necessary to have a correct interpretation of the parameters Δ_{\pm} since, as pointed out in the previous section, these are super-parameters. However, as a first approximation for this analysis, we suppose Δ_{\pm} behaves as a perturbative

quantity. The behavior of the thermodynamic variables under this assumption is shown in Figures 1-4. Suppose the effect of the polymer parameters is to reduce or slightly increase the real non-polymer value, so what we see in the figures is a graphical comparison of the functions (22)-(25) for values of Δ_{\pm} close to one, for which we recover the values of the functions without polymer effects.

At low temperature the system is in the lower energy level and its internal energy U is $-\hbar\omega\Delta_{-}/2$, this appears shifted due to polymer effects. At high temperatures both energy states are equally likely to be occupied, so that the internal energy tends to a minimum value $\hbar\omega(\Delta_{+} - \Delta_{-})/2$, Fig. 1. The polymer effect is to spread the energy values into a band centered in the non-polymer value, so that, at high temperatures, the internal energy has a nonzero value that depends exclusively on the polymer effects present. Note that the same thing occurs with the free energy due to its functional form, Fig. 2.

At T = 0, the entropy is exactly zero, even with polymer modifications, which cancel out each other. The entropy increases with temperature reflecting the freedom that the system acquires in different states. Amusingly, the entropy tends to $k \ln 2$ exactly, as usual for high temperature, even with polymer corrections. Certainly it seems that the polymer effect on the entropy is to change the way it tends towards the usual limits, provided that $\Delta_+ \neq \Delta_-$, Fig. 3.

Something similar occurs with the heat capacity. The main difference is that the heat capacity tends to zero at both low and high temperature limits. Actually, C_V has a maximum at a temperature approximately equal to $T_c \approx \frac{\hbar\omega}{4k}(\Delta_+ + \Delta_-)$, to be precise at the roots of $X - \tanh \frac{1}{X} = 0$, with $X = \frac{4kT}{\hbar\omega(\Delta_+ + \Delta_-)}$. This maximum, known as the Schottky anomaly, separates two behaviors of the system, namely when only the lowest energy state is occupied and when both levels are occupied. Notice that the polymer effect when $\Delta_+ \neq \Delta_-$ is to shift the critical temperature, keeping the same value for the maximum. This behavior can be seen in Fig. 4.



Figure 1. Internal energy U with polymer corrections. Black solid curve is the usual value with $\Delta_{\pm} = 1$. Blue and purple (dashed) curves have values close to one (from below and above respectively). Red and orange (dot-dashed) curves have values a little bit far from 1.

Figure 2. Free energy F with polymer corrections. Black solid curve is the usual behavior with with $\Delta_{\pm} = 1$. Blue and purple (dashed) curves have values close to one (from below and above respectively). Red and orange (dot-dashed) curves have Δ values deviating slightly more from 1.



Figure 3. Entropy S with polymer corrections. Black solid curve is the usual behavior for $\Delta_{\pm} = 1$. Blue and green (dotted) curves have values close to one (from below and above respectively), but maintaining the same sum. Red and orange (dashed) curves have $\Delta_{+} \neq \Delta_{-}$. Note that the behavior of the entropy changes depending on the value of the polymer corrections, although its limits for high and low temperatures are the same.



Figure 4. Heat capacity C_V with polymer corrections. Black solid curve is the usual behavior for $\Delta_{\pm} = 1$. Blue and green (dotted) curves have values close to one (from below and above respectively), but with the same sum. Red and orange (dashed) curves have $\Delta_{+} \neq \Delta_{-}$. The behavior of C_V is modified by the polymer corrections, although it tends to the same values for high and low temperatures. The dot-dashed vertical line indicates the critical temperature T_c with no corrections. Notice that in the case $\Delta_{+} \neq \Delta_{-}$, T_c is shifted with respect to that value.

4. Discussion

In this work we studied a simplified model for the thermostatistical properties of an ensemble of polymer Fermi oscillators. Thermal variables are modified due to polymer terms. As a first guess for the polymer parameters we treat them perturbatively near to the non-polymer real value. Under such assumption we found that polymer modification for energy variables, such as U and F, make them to slightly vary from the non corrected value. For low temperatures, it modifies the minimum value of the internal energy while for high temperature makes U reach a nonzero value.

On the other hand we observe for the entropy and heat capacity that the behavior at low and high temperatures is not affected by the polymer corrections. However, in the special case where $\Delta_+ \neq \Delta_-$ they exhibit a change in its behavior at an intermediate temperature. Indeed, the critical temperature at which the C_V exhibits a maximum is shifted by polymer effects.

It should be emphasized that this discussion is made under the assumption that polymer effects, encoded in the quantities Δ_{\pm} , are real quantities that slightly vary from the usual nonpolymer value. This assumption is very stringent in light of what we discussed in the first section, where it was found that the polymer parameters are actually super-parameters. Eq. (19) and expressions derived thereforth are actually formal constructs with no *a priori* physical meaning. For example, the logarithm of a supernumber must be defined in terms of a power series: it is in that sense that one must treat other functions. Because the super-corrections were replaced with real perturbations, the figures shown may not necessarily have the complete physical content, however, they serve as a reference for when polymer modifications are taken as actual super-numbers. In this sense, the first thing one need to do is write the partition function (19) in terms of Grassmanian numbers and to establish which additional criteria is needed to consider corrections.

It might happen that in some special representation the influence of the spectrum (18) in phenomenology is not straightforward. Certainly, we may think in a system whose microscopic variables can be accurately measured, so one can bound and understand their effects. Indeed, recall that θ_1 and θ_2 appears when we write the Weyl super algebra in the fermionic case. These parameters are equivalent to the polymer length in the bosonic case, which is fully identifiable with a length scale; but not so in this case. Moreover, the e_{\pm} parameter appears when solving equation (16) as another polymer super-parameter. It is possible that both parameters can be fixed when the polymer Dirac field is coupled to other matter fields. The main difficulty in the present construction is that there is not a clear definition of a Grassmanian observable. For example, the definition of a physical observable given in [20], does not prevent or limit the Grassmannian contribution to the thermodynamic quantities, so a reinterpretation of the parameters is required that could occur through a generalization of the concept of observable taking into account the singular representations appearing in loop quantum gravity and polymer quantum mechanics [24].

5. Acknowledgements

Partial support from the grant CONACYT CB-2008-01/101774 is acknowledged.

References

- [1] A. Ashtekar, Phys. Rev. D 36, 1587 (1987).
- [2] C. Rovelli, Quantum Gravity, (Cambridge University Press, Cambridge, England, 2004).
- [3] T. Thiemann, Introduction to Modern Canonical Quantum General Relativity, (Cambridge University Press, Cambridge, England, 2007).
- [4] A. Ashtekar, S. Fairhurst and J. L. Willis, Class. Quant. Grav. 20, 1031 (2003).
- [5] W. D. Chiou, Classical Quantum Gravity 24 (2007), 2603 2620.
- [6] V. Husain, J. Louko and O. Winkler, Phys. Rev. D 76 (2007), 084002.
- [7] G. Kunstatter, J. Louko and J. Ziprick, Phys. Rev. A 79 (2009), 032104.
- [8] G. Kunstatter and J. Louko, J. Phys. A 45, 305302 (2012).
- [9] A. Martín-Ruiz, Phys. Rev. D **90**, 125027 (2014).
- [10] D. A. Demir, O. Sargin, Phys. Lett A 378, 3237 (2014).
- [11] G. Chacón-Acosta, E. Manrique, L. Dagdug and H. A. Morales-Técotl, SIGMA 7, 110 (2011).
- [12] G. Chacón-Acosta, L. Dagdug and H. Morales-Técotl, AIP Conf. Proc. 1396, 99-103 (2011).
- [13] G. Chacón-Acosta, AIP Conf. Proc. 1473, 153-162 (2012).
- [14] E. Castellanos and G. Chacón-Acosta, Phys. Lett. B 722, 119 (2013).
- [15] G. Chacón-Acosta and H. H. Hernández, AIP Conf. Proc. 1548, 179-182 (2013).
- [16] G. Chacón-Acosta and H. H. Hernandez-Hernandez. Int. J. Mod. Phys. D 24, 1550033 (2015).
- [17] G. M. Hossain, V. Husain and S. S. Seahra, Phys. Rev. D 82, 124032 (2010).
- [18] A. A. García-Chung and H. A. Morales-Técotl, AIP Conf. Proc. 1473, 163 (2011).
- [19] A. A. García-Chung and H. A. Morales-Técotl, Phys. Rev. D 89, 065014, (2014).
- [20] B. S. DeWitt, Supermanifolds, (Cambridge University Press, Cambridge, England, 1984) 1992.
- [21] M. Henneaux and C. Teitelboim, Quantization of Gauge Systems, (Princeton University Press 1991)
- [22] A. Khrennikov, *Superanalysis*, (Kluwer Academic Publishers 1999)
- [23] A. A. García-Chung, H. A. Morales-Técotl and J. D. Reyes, AIP Conf. Proc. 1548, 161 (2013).
- [24] A. A. García-Chung, H. A. Morales-Técotl and J. D. Reyes, "On the GNS formalism for polymer Fermi oscillator" (In preparation).
- [25] W. Greiner, Ludwig Neise, H. Stöcker, Thermodynamics and Statistical Mechanics, (Springer 1995).