GLUEBALLS

(Presented by Stephen Pinsky)

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The G(1440) qualitatively satisfies all criteria for a glueball: It is an isosinglet preferentially produced in hard gluon channels which mediate OZI inhibited processes in an SU(3) symmetric way. A simple pole model is used to predict $G \rightarrow \delta\pi$, $\rho\gamma$, $\omega\gamma$, $\varphi\gamma$, $\gamma\gamma$, $\rho\pi\pi$, and $\eta\pi\pi$. The small $G \rightarrow \eta\pi\pi$ rate is explained by a cancellation between $G \rightarrow \delta\pi \rightarrow \eta\pi\pi$ and $G \rightarrow \eta\varepsilon \rightarrow \eta\pi\pi$ amplitudes, which has also been observed in the corresponding η' and s(1275) amplitudes. While the G doesn't fit naturally into a pure radially excitation nonet, standard octet-singlet mixing with $\theta_R = -18°$ gives results consistent with all existing data. Axial Ward identities accommodate glueball contributions and appear to be consistent with glueball parameters.

I. INTRODUCTION

The possibility of quarkless states in the meson spectrum presents theorists and experimentalists with a unique challenge. In current theoretical thinking on the confinement of colored gluons, these states are a necessity; they must be there or our ideas about confinement are wrong. Their experimental observation, accordingly, would represent as direct a confirmation as any other of the reality of gluons and QCD.

The leading questions are thus both experimental and theoretical ones: How does one know a quarkless state or glueball when one sees one? Where should one look? What does it mean if they are not there?

Let us begin first with the question of experimental signature. Since the glueball is to date a hypothetical strong coupling object in QCD, a theory which is far from being solved in the non-perturbative regime, its properties are ambiguous. They can only be inferred from crude models which seem to correlate what we know about the meson spectrum with what we believe to be true of the binding properties of quarks in QCD. In these models quarks and antiquarks combine, with the help of confining color forces, to produce the flavor singlet and adjoint presentations ($1 \oplus 8$, or a nonet, if there are three flavors). The same forces which confine the color of quarks should also confine the color of gluons, producing quarkless flavor singlet bound states of gluons, or glueballs. Thus, the zeroth order spectrum expected consists of nonets plus an undetermined number of singlets which do not fit into the nonet structure.

The glueballs can be divided into two classes: those with J^{PC} quantum numbers allowed also for quark-antiquark states, and all others ("exotics" or "oddballs": $J^{PC} = 0^{--}$, $(\text{odd})^{+-}$ and $(\text{even})^{-+}$).[1] A variety of studies have predicted the spectrum of masses and quantum numbers.[2,3] Most models predict many exotic and non-exotic states in the 1 - 3 GeV energy range. All of these predictions are extremely model dependent and have been discussed at length elsewhere. Since there are no experimental candidates for glueballs with exotic quantum numbers, they will not be further discussed in this review. They do, however, represent the most unambiguous signal of meson states which are not $q\bar{q}$ bound systems.

Let us consider now a nonet of mesons with a single nearby glueball. In the absence of interaction between the quark states and the glueball, one has the ideally mixed spectrum expected of two light and nearly degenerate u, d quarks and a heavier s quark: The flavor free states form a light isosinglet $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ degenerate with the isotriplet $\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$ and a heavy $s\bar{s}$ isosinglet, obeying the mass formulas

$$m^2_{(s\bar{s})} = 2m^2_{(s\bar{u})} - m^2_{\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)} \tag{1.1}$$

$$m^2_{\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)} = m^2_{\left(\frac{uu-d\bar{d}}{\sqrt{2}}\right)} \tag{1.2}$$

and the mixing angles

$$|\frac{u\bar{u} + d\bar{d}}{\sqrt{2}}\rangle = \cos\theta\,|8\rangle - \sin\theta\,|1\rangle \tag{1.3}$$

$$|s\bar{s}\rangle = \sin\theta\,|8\rangle + \cos\theta\,|1\rangle \tag{1.4}$$

where $\tan\theta = -\sqrt{2}$.

This 'ideally mixed' nonet exhibits the "OZI"[4] rule in zeroth order. A good example is the $J^{PC} = 1^{--}$ nonet, ρ, ω, $K^{*\pm}$, K^{*o}, $\bar{K}^{*o}$, and Φ, with ω and ρ nearly degenerate and $m^2_\Phi \approx 2m^2_{K^*} - m^2_\rho$. As a result of the OZI rule, Φ couples to $K\bar{K}$ rather than $\pi\pi$, by means of continuous quark line diagrams. Thus, the $\pi\pi$ rate is OZI forbidden.

This picture is perturbed by the presence of annihilation diagrams such as $q\bar{q} \to gg$ which couple quark-antiquark states of different flavor to each other and to any glueballs in the vicinity in a flavor-blind (SU(3) symmetric) way. The glueballs may be considered a strong resonance in the gg channel which dominates the OZI forbidden reaction $q_1\bar{q}_1 \to gg$ (glueball) $\to q_2\bar{q}_2$, where q_1 and q_2 are quarks of different flavors. This mechanism also disturbs the ideal quark content and mass formula of ideal mixing. Thus significant deviation from ideal mixing or OZI rules is an indication that the gluon annihilation is strong in a process and a glueball may be nearby.

This determines the J^{PC} channels in which nearby glueballs are likely. The question then is where -- in what reactions -- to look for them. Clearly they should be most prominent in OZI forbidden channels where hard gluons must mediate the production of the final state. The prime candidate for a production mechanism of this kind is $\psi \to \gamma X$ where X is any state composed of light quarks. Apart from SU(3) breaking effects, which we will argue are not expected to be unusually large, branching ratios observed in the decay of X should be SU(3) symmetric.

The best known nonets are those with $J^{PC} = 0^{-+}$, 1^{--}, and 2^{++}. The 0^{++} system, after a clouded past, is increasingly respectable as a nonet. Here we discuss each of these multiplets from the point of view of glueball searches.

Vector Mesons 1^{--}

As already mentioned, this multiplet is well known for its nearly perfect mixing and strong suppression of OZI forbidden transitions. Thus one does not expect to find a glueball here, and to date there are no candidates.

The strong suppression in the 1^{--} channel inhibits $\psi \to \Phi$ or ω and then Φ or $\omega \to \gamma X$; if this rate were appreciable, there would be no guarantee that X was formed in a hard gluon, OZI suppressed channel, another reason that $\psi \to \gamma X$ is the ideal glueball hunting ground.

Tensor Mesons 2^{++}

At first glance the tensors seem as good an example of ideal mixing as the vector mesons, with strong OZI suppression and no indications for a nearby glueball. There are however some experimental reasons to reconsider this conclusion. Consider first the experimental limit[5]

$$\frac{\Gamma(\psi \to \gamma f')}{\Gamma(\psi \to \gamma f)} \leq 0.12 \pm 0.05 \ . \tag{1.5}$$

On the basis of ideal mixing one expects

$$\frac{\Gamma(\psi \to \gamma f')}{\Gamma(\psi \to \gamma f)} = \frac{1}{2}\,(x^2)(0.9) = 0.45\, x^2 \ , \tag{1.6}$$

where the phase space factor is 0.9, the ideal mixing factor is 1/2, and x is an enhancement factor for the annihilation into strange instead of light quarks. ($x = 1$ in the limit of exact SU(3) when $m_u = m_d = m_s$.) This SU(3) breaking effect has been conjectured by some authors to be greater than unity,[1,6,7] which makes the disagreement above between experiment and theory even worse. This may be a suggestion that in the 2^{++} channel we have something other than an ideally mixed nonet. However, one must be careful, because x may also be less than unity, as we shall argue in a later section; with $x \approx 0.5$, not implausible, agreement is restored between theory and experiment.

There is, however, an additional state in $\psi \to \gamma X$, $\theta(1640)$,[8,9] which has the following properties:

$$\begin{aligned} &M(\theta) = 1640 \pm 50 \text{ MeV}; \quad \Gamma = 220\,^{+100}_{-70} \text{ MeV}; \\ &B(\psi \to \gamma\,\theta)\,B(\theta \to \eta\eta) = (4.9 \pm 2.4) \times 10^{-4}; \\ &B(\psi \to \gamma\,\theta)\,B(\theta \to \pi\pi) < 2 \times 10^{-4} \ . \end{aligned} \tag{1.7}$$

Note that if SU(3) were exact and θ were an SU(3) singlet, the branching ratio for $\theta \to \pi\pi$ should be three times the branching ratio for $\theta \to \eta\eta$.

Is there a common explanation for θ and the discrepancy between Eq.(1.5) and Eq.(1.6)? Is θ a glueball which disturbs the ideal mixing result in Eq.

(1.6)? The answer seems to be no. A glueball at 1640 would badly destroy ideal mixing, mix significantly with $f'(1514)$ and heavily favor $\psi \to \gamma f'$ over $\psi \to \gamma f$, in contradiction with the data.

For completeness we note a mixing scheme proposed by Rosner,[10] in which the tensors have a state in the 1400-1800 mass range which is not allowed to mix with the f'. While his model has many interesting features, it appears not to explain Eq.(1.5).

A possible explanation for the anomalously large $\psi \to \gamma f$ branching ratio, which preserves many aspects of ideal mixing, is to hide the glueball in this channel under the f meson. With very strong f-glueball mixing, f would be copiously produced in $\psi \to \gamma f$, without appreciably mixing the glueball with f'. This scheme has been studied by Donoghue.[11] It explains discrepancies such as why the f mass is 30-40 MeV low in $\pi^o\pi^o$ reactions.[12] Moreover, there is some indication in high momentum transfer $\pi^- p \to K^+K^- n$ reactions that the f is split.[13] These possibilities are promising, but need further experimental and theoretical study.

Hiding the glueball under the f successfully accounts for a 2^{++} glueball state, but leaves a mystery concerning the status of $\theta(1640)$. It has been conjectured that θ is a four quark state,[7] a possibility beyond the scope of this review; in any case it certainly does not appear to be a glueball.

Scalar Multiplet 0^{++}

We mention the scalars here in connections with some interesting theoretical work which relates their dynamics to the fundamental dynamics of the QCD vacuum. The MIT bag model suggest that the naive mass of the 0^{++} glueball is negative.[14] Thus a vacuum with energy lower than the naive (perturbative) vacuum can be constructed out of a close-packed configuration of 0^{++} glueballs. Hadron states are then bubbles of naive vacuum in the soup of glueballs. A physically realizable 0^{++} glueball is an excited state of one of the vacuum glueballs.[15] If this picture is correct, then it is somewhat puzzling to note that this mechanism seems not to disturb the ideal mixing as indicated by $s^*(980)$ and $\delta(980)$ being so nearly degenerate.

Pseudoscalars 0^{--}

This J^{PC} channel, containing a nonet which is far from ideally mixed, is clearly an excellent hunting ground for glueballs. Moreover, OZI suppression is known to be weak in this channel. Thus the G(1440)[16,17,18] recently reported in $\psi \to \gamma X$, $X \to KK\pi$, must be regarded as a prime suspect for glueball status. The experimental parameters are

	Mark II	Crystal Ball	
Mass	1440^{+10}_{-15} MeV	1440^{+20}_{-15} MeV	
Γ	50^{+30}_{-20} MeV	60^{+20}_{-30} MeV	(1.8)
$B(\psi\to\gamma G)\,B(G\to K\bar{K}\pi)$	$(4.3\pm1.7)\times10^{-3}$	$(4.0\pm1.2)\times10^{-3}$	

The decay $G\to K\bar{K}\pi$ goes primarily through $\delta\pi$, with[19]

$$\frac{B(G\to\delta\pi)B(\delta\to K\bar{K})}{B(G\to K\bar{K}\pi)} = 0.8 \pm .2 \tag{1.9}$$

There is also indication of $G\to\eta\pi\pi$ but $B(\psi\to\gamma G)\,B(G\to\eta\pi\pi)$ is much smaller than the corresponding rate for the $K\bar{K}\pi$ final state.[19] The spin-parity has been established to be 0^- with a probability of less than 1% that it is not 0^-.[20]

Various possibilities present themselves concerning the role G(1440) plays in hadron dynamics:

(1) G(1440) is a radial excitation of η'.

(2) G(1440) is a $q\bar{q}q\bar{q}$ state.

(3) G(1440) is a glueball.

(1)[7,21-24] and (2)[7] have been discussed by a number of authors. We will discuss (1) in greater detail in Section II. Possibility (2) would seem to be rules out since there is no reason why a $q\bar{q}q\bar{q}$ state should be produced more strongly than a $q\bar{q}$ state in $\psi\to\gamma X$. Possibility (3) will be considered in greater detail in Section III.

In Section IV, we will discuss constraints of axial Ward identities on pseudoscalar states. In Section V we will present our summary and conclusions.

II. RADIAL EXCITATIONS

Since there is a conspicuous need for a ninth member, now missing, of the radially excited pseudoscalar nonet, the possibility must be entertained that G(1440) fills the bill. The present members of this nonet are the isotriplet $\pi'(1270)$, the isodoublet $K'(1450)$ and the isoscalar $s(1275)$[26,27] (also known as $\zeta(1275)$).

Since π' and s are nearly degenerate, ideal mixing is suggested, with the corresponding mass formula $m^2_{\pi'} = m^2_s$ and $m^2_G = 2m^2_{K'} - m^2_{\pi'}$, and the mixing angle $\tan\,\theta = -\sqrt{2}$. This would put m^2_G, if it were the missing member, at 1600 GeV. No simple perturbation away from ideal mixing will change this result. The conclusion is that G does <u>not</u> fit nicely into the present radial excitations. However, more sophisticated models such as those mixing ground state and radial excitation[23] might accommodate a quark state at the G mass. It should be pointed out however that this type of mixing requires the wave function at the origin for the radial excitation to be comparable to that of the ground state.[24] This probably is incorrect for the pseudoscalars since chiral perturbation theory indicates that the decay constants and therefore the wave function at the origin for the radial excitations are proportional to the current algebra quark mass.[25] (See Sec. IV for more details.)

There is another argument against radial excitation status for the G,[7,28] which we would like to comment upon. It goes like this: We do not see s(1275) in $\psi \to \gamma X$, and thus it is an SU(3) octet state. Then if G is its partner in the radial excitation nonet, it is a singlet; but this implies (a detailed calculation is given below) that

$$\frac{\sigma(\pi^- p \to Gn) B(G \to \eta\pi\pi)}{\sigma(\pi^- p \to sn) B(s \to \eta\pi\pi)} \approx 5 \ . \tag{2.1}$$

The measurement of Stanton, et al., however, indicates that this ratio is ≤ 0.4.[29] The conclusion is that G is not a radial excitation.

We do not believe, however, that this argument stands up to closer scrutiny. There is no a priori reason to suggest that the G is a singlet, so we must rely on the data and ask the question: Is there a singlet-octet mixture for G consistent both with the SPEAR limit on $\psi \to \gamma s$ and the Stanton limit on $\pi^- p \to Gn$?

At SPEAR an s(1275) signal might have been seen along with G in

$$\psi \to \gamma \begin{Bmatrix} s \\ G \end{Bmatrix} \to \gamma(\delta\pi) \to \gamma(K\bar{K}\pi) \tag{2.2}$$

which we estimate as follows. On the basis of phase space, $\psi \to \gamma s$ is favored over $\psi \to \gamma G$, but $G \to \delta\pi$ is favored over $s \to \delta\pi$. The net result including masses and total width factors favors the G production in this channel by $(.86)^{-1}$. Since the production is via the singlet parts of the s and G (now both assumed members of a radially excited nonet with mixing angle θ_R the net effect is

$$\frac{B(\psi \to \gamma s) B(s \to K\bar{K}\pi)}{B(\psi \to \gamma G) B(G \to K\bar{K}\pi)} = 0.86(\tan\,\theta_R)^2 (\tan(\theta_R + 54.7°))^2 \ . \tag{2.3}$$

Since only the light quark content of G and s contribute to production data, we have

$$\frac{\sigma(\pi^- p \to Gn)}{\sigma(\pi^- p \to sn)} = (\tan(\theta_R + 54.7))^2 \ . \tag{2.4}$$

(In our notation ideal mixing corresponds to $\theta_R = -54.7°$.) We must now estimate the branching ratios $s \rightarrow \eta\pi\pi$, because this is the final state which Stanton puts his limit on. As discussed in the next section, s and G decay to $\eta\pi\pi$ via $\delta\pi$ and $\eta\epsilon$. These amplitudes partially cancel, making the $\eta\pi\pi$ rather anomalously small, and difficult to estimate. On the basis of two body phase space ($\delta\pi$), the G decay is favored by a factor of 1.33. This factor combined with the mixing effect yields

$$\frac{\sigma(\pi^- p \rightarrow Gn)B(G \rightarrow \eta\pi\pi)}{\sigma(\pi^- p \rightarrow sn)B(s \rightarrow \eta\pi\pi)} = 1.33(\tan(\theta_R + 54.7))^4 \ . \tag{2.5}$$

$\theta_R = 0$ reproduces the naive ratio of 5. However, for $\theta_R = -18°$, nearly the mixing in the ground state nonet, we find

$$\frac{B(\psi \rightarrow \gamma s)B(s \rightarrow K\bar{K}\pi)}{B(\psi \rightarrow \gamma G)B(G \rightarrow K\bar{K}\pi)} = 0.05 \tag{2.6}$$

which is small enough to be overlooked at SPEAR,[30] and

$$\frac{\sigma(\pi^- p \rightarrow Gn)B(G \rightarrow \eta\pi\pi)}{\sigma(\pi^- p \rightarrow sn)B(s \rightarrow \eta\pi\pi)} = 0.4 \tag{2.7}$$

which is at the limits of Stanton, et al.[27]

In summary, the evidence against the G(1440) being a radial excitation does not seem overwhelming. While the G does not conspicuously fill the role of a radial excitation - its mass is somewhat low - that possibility cannot now be ruled out.

These questions may well be resolved by looking at $\gamma\gamma$ decays of G and s. The glueball calculations of the next section predict a substantial rate for $G \rightarrow \gamma\gamma$. It is difficult theoretically to make a reliable estimate for the $\gamma\gamma$ rate if G is a radial excitation. Clearly any charmonium calculation must be suspect since in this mass region one is dealing with a relativistic strongly coupled bound state, not a non-relativistic perturbative weak coupling bound state. PCAC methods are also questionable when applied to radial excitations, with the problem further compounded by the large mass extrapolations involved. Moreover, PCAC methods require the wave function of the origin to be small, of order quark mass compared to QCD scale, while charmonium calculations predict quite large wave functions at the origin.[25] The theoretical crystal ball is here somewhat clouded.

One need not, however, rely on theory; the ratio $s \rightarrow \gamma\gamma/\eta \rightarrow \gamma\gamma$ should be similar to $G \rightarrow \gamma\gamma/\eta' \rightarrow \gamma\gamma$ if G is the radial excitation of the η'. A measurement of the decay of either s or G to $\gamma\gamma$ will be useful in sorting these questions out. (Experiments at JADE[39] look at $\eta' \to \gamma\gamma$ and seem to cover the region of $s(1275)$, showing a strong η' signal but no s signal, suggesting that the radials indeed have smaller 2γ branching ratios. The problem here, as well as with similar experiments at TASSO,[40] is that the branching ratio for the G and s tend to be very small, 10^{-2} (because of the large total widths), implying a small data sample.

III. PSEUDOSCALAR GLUEBALL

As we have emphasized above, there is no simple, direct, and unambiguous test for establishing the existence of a glueball in a channel where ordinary quark-antiquark states are allowed. Links in the chain of circumstantial evidence strongly suggesting the glueball "modus operandi" include:

(1) An "extra" isosinglet state not accounted for in a nonet pattern.
(2) Production in hard gluon channels.
(3) Mediation of OZI inhibited processes in production and decay.
(4) SU(3) singlet status, possibly broken by a slightly different coupling to heavy quarks.

In this section a simple pole model is used to correlate (1) $\psi \to \gamma\eta'$ and $\psi \to \gamma\eta$, (2) $\psi \to \gamma G(1440)$, and (3) $G(1440) \to \delta\pi$. In this model the glueball mediates production of η' and η in the hard gluon channels of $\psi \to gg\gamma$; η and η' mediate G decay into light hadrons; and the rate $\psi \to \gamma G$ is used to fix the basic coupling of glue to quark states.

Freund and Nambu[31] first suggested that "0 mesons" or "closed strings" (glueballs in current parlance) mediate transitions forbidden by the OZI rule; and their model has been refined and applied to 0^+, 0^-, 1^-, and 2^+ OZI forbidden processes.[32,33] Here the $\iota(1440)$ or $G(1440)$ is proposed as the 0^- glueball, a quarkless state coupling strongly to two hard gluons as shown in Fig. 1(a), where it mediates a transition between charmed and light quarks. As shown in Fig. 1(b), (short hand notation in Fig. 1(c)), the phenomenological factors f_{η_c} and $f_{\eta'}$ measure the mixing between quark states and the glueball. They are related to the wave functions of the G, η', and η_c, as indicated by the shaded bubbles. The interaction of the glueball is SU(3) symmetric in zeroth order but the annihilation diagram may be quark mass dependent, so deviations from SU(3) should be expected. Any gluon annihilation process near the G mass should be dominated by this diagram, on purely quantum mechanical grounds, so long as the G couples more strongly to glue than quark states do. The amplitude for the OZI forbidden process shown is

$$\frac{f_{\eta_c} f_{\eta'}}{s - m_G^2} \tag{3.1}$$

where the propagator enhancement factor should be noted. A simple theory of the processes $\psi \to \gamma\eta$ and $\psi \to \gamma\eta'$ is thus given by the diagrams of Fig. 2 yielding

$$\frac{B(\psi\to\gamma\eta)}{B(\psi\to\gamma\eta')} = \left(\frac{p_\eta}{p_{\eta'}}\right)^3\left(\frac{f_\eta}{f_{\eta'}}\right)^2\left(\frac{m_G^2 - m_{\eta'}^2}{m_G^2 - m_\eta^2}\right)^2 = 1.2\left(\frac{f_\eta}{f_{\eta'}}\right)^2\left(\frac{m_G^2 - m_{\eta'}^2}{m_G^2 - m_\eta^2}\right)^2 \tag{3.2}$$

(The f_{η_c} factors cancel if their mass shell dependence is neglected, as we do here.) Recent data of the Crystal Ball Collaboration presented by K. Konigsmann (this conference)[34] gives 0.213 ± 0.027 for this branching ratio, fixing the relative magnitude of $f_\eta/f_{\eta'}$. In the limit of exact SU(3) symmetry when η is a pure octet, we have $f_\eta = 0$. However on the basis of $\pi^- p \to \eta n$ and $\pi^- p \to \eta' n$, one concludes the physical η and η' are mixed states as follows:

$$\eta' = \cos\theta\left[\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}\right] + \sin\theta\left[\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}\right] \tag{3.3}$$

$$\eta = -\sin\theta\left[\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}\right] + \cos\theta\left[\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}\right] \tag{3.4}$$

where $\theta = -15°$. Now if we assume that glue couples to strange quarks with a strength x relative to light quarks then the ratio of the amplitudes for $\psi \to \gamma\eta$ to $\psi \to \gamma\eta'$ is

$$\frac{A(\psi\to\gamma\eta)}{A(\psi\to\gamma\eta')} = \frac{(1-\sqrt{2}\tan\theta) - x(1+1/\sqrt{2}\tan\theta)}{(\tan\theta+\sqrt{2}) + x(1/\sqrt{2}-\tan\theta)} \tag{3.5}$$

$$= \pm .42 \text{ (experiment)} .$$

Setting $\theta = -15°$[35] we find two solutions corresponding to the + and - sign respectively $x = .75$ and $x = 4.6$. We reject the $x = 4.6$ solution since it would be contrary to our initial assumption that glue couples in a nearly flavor blind way: It introduces implausible SU(3) breaking into the quark mass dependence of the annihilation process. This leads to the additional result that the relative sign of the amplitudes is positive and yields

$$\frac{f_\eta}{f_{\eta'}}\,\frac{(m_G^2 - m_{\eta'}^2)}{(m_G^2 - m_\eta^2)} = 0.42 \pm 0.027 . \tag{3.6}$$

It is important to note that the above phenomenological conclusion, that strange quarks couple more weakly to glue than light quarks, seems to be in contradiction to the predictions of perturbative QCD.[1,6] If the G(1440) is a

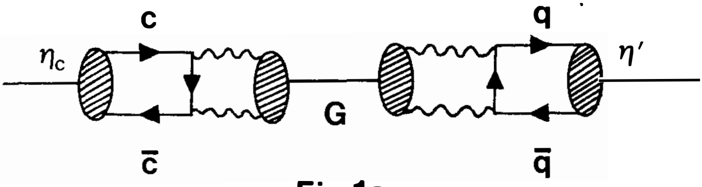

Fig.1a

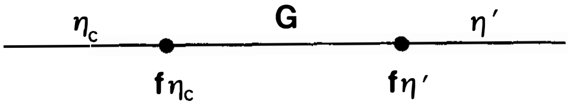

Fig.1b

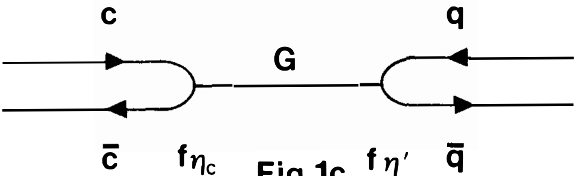

Fig.1c

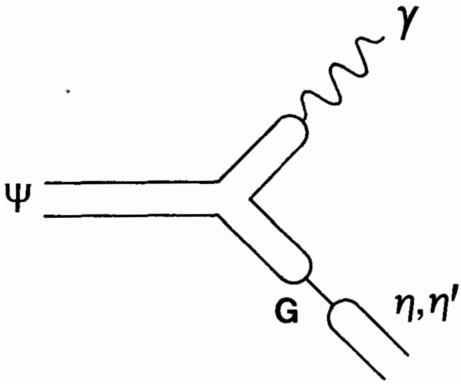

Fig. 2

glueball, the same problem arises with the total width of the G, which is measured to be of order 55 MeV, while perturbative QCD predicts 1 to 3 MeV. It should be noted, however, that these conclusions are extremely sensitive to the quark mass inputs (see Fig. 1 of Ref. 1). Indeed, some authors report an enhancement factor for glue balls decaying into light quarks.[36]

The absolute strength of the glueball couplings can be calculated from $\psi \to \gamma\eta'$ and $\psi \to \gamma G$ as shown in Fig. 2 and 3, yielding

$$\frac{\Gamma(\psi \to \gamma\eta')}{\Gamma(\psi \to \gamma G)} = \left(\frac{p_{\eta'}}{p_G}\right)^3 \frac{f^2_{\eta'}}{(m_G^2 - m_{\eta'}^2)^2} = 1.5 \frac{f^2_{\eta'}}{(m_G^2 - m_{\eta'}^2)^2} , \tag{3.7}$$

where again we assume $f_{\eta_c}(m_\eta^2) = f_{\eta_c}(m_G^2)$, that is neglect the mass shell variation in the coupling parameters. From the data for G production and decay,[20]

$$B(\psi \to \gamma G)\, B(G \to K\bar{K}\pi) = (4.1 \pm 1.5) \times 10^{-3} , \tag{3.8}$$

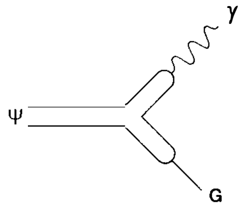

Fig. 3

as well as $B(\psi \to \gamma\eta') = (3.8 \pm .8) \times 10^{-3}$ (an average of Ref. 20 and Mark II Ref. 19), we conclude

$$\frac{f^2_{\eta'}}{(m_G^2 - m_{\eta'}^2)^2} = (0.62 \pm 0.30) B(G \to K\bar{K}\pi) . \tag{3.9}$$

(The error estimate is conservative. The basic mixing strength is plausibly small, about 1/5 if $B(G \to K\bar{K}\pi)$ is 30%.)

Now that the glueball mixing parameters are determined, we may check the consistency between the production and decay of the G within the framework of the pole model. The decay $G \to \delta\pi$ may proceed either through η' or η channels as shown in Fig. 4(a) and may be compared to $\delta \to \eta\pi$ as shown in Fig. 4(b).

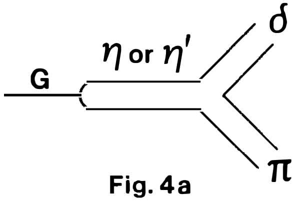

Fig. 4a

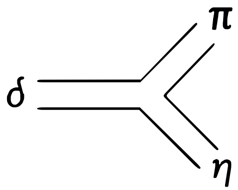

Fig. 4b

In terms of the dimensionless coupling g_{PPS}/m where m is the mass of the decaying particle, we have

$$\frac{\Gamma(G\to\delta\pi)}{\Gamma(\delta^o\to\eta\pi)} = 3\,\frac{p_\delta}{p_\eta}\,\frac{\left(\dfrac{f_{\eta'}}{m_G^2 - m_{\eta'}^2}\,\dfrac{g_{\eta'\pi\delta}}{m_G} + \dfrac{f_\eta}{m_G^2 - m_\eta^2}\,\dfrac{g_{\eta\pi\delta}}{m_G}\right)^2}{\dfrac{g_{\eta\pi\delta}^2}{m_\delta^2}} \tag{3.10}$$

$$\frac{\Gamma(G\to\delta\pi)}{\Gamma(\delta^o\to\eta\pi)} = 3\,\frac{p_\delta}{p_\eta}\,\frac{m_\delta^2}{m_G^2}\,\frac{f_{\eta'}^2}{(m_G^2 - m_{\eta'}^2)^2}\left[\frac{g_{\eta'\pi\delta}}{g_{\eta\pi\delta}} + \frac{f_\eta}{f_{\eta'}}\,\frac{m_G^2 - m_{\eta'}^2}{m_G^2 - m_\eta^2}\right]^2 . \tag{3.11}$$

Assuming an octet-singlet mixing angle of $\theta = -15°$, and that the $\delta\pi$ couples only to light quarks, we find

$$\frac{g_{\eta'\pi\delta}}{g_{\eta\pi\delta}} = \tan(\theta + 54.7) = 0.83 \tag{3.12}$$

All other factors in Eq.(3.11) can be evaluated in terms of Eq.(3.6) and Eq.(3.7). The result is

$$\frac{\Gamma(G\to\delta\pi)}{\Gamma(\delta\to\eta\pi)} = \frac{\Gamma(\psi\to\gamma_{\eta'})}{\Gamma(\psi\to\gamma G)}\left[0.83 + 0.9\sqrt{\frac{B(\psi\to\gamma\eta)}{B(\psi\to\gamma\eta')}}\right]^2 \tag{3.13}$$

In terms of the experimentally measured G parameters this can be written

$$B(\psi\to\gamma G)\,B(G\to\delta\pi) = \frac{\Gamma(\delta\to\eta\pi)}{\Gamma_T(G)}\,B(\psi\to\gamma\eta')\left[0.83 + 0.9\sqrt{\frac{B(\psi\to\gamma\eta)}{B(\psi\to\gamma\eta')}}\right]^2 \tag{3.14}$$

This relation should be regarded as a theoretical prediction of the G decay parameters in terms of other measured OZI forbidden rates. It tests the role of the G as a glueball and a mediator of OZI forbidden processes. In terms of experimental data Eq.(3.14) is evaluated as (taking $B(\delta\to\eta\pi) = .6 \pm .3$)[38]

$$(3.3 \pm 2.0)10^{-3} = (3.6 \pm 2.0) \times 10^{-3} \tag{3.15}$$

indicating consistency with the single pole model of G as OZI mediator. Alternatively, we can simple evaluate the R.H.S. of Eq.(3.11) in terms of our OZI parameters. Then

$$\frac{\Gamma(G\to\delta\pi)}{\Gamma(\delta\to\eta\pi)} = (1.5)(.62 \pm 3)B(G\to K\bar{K}\pi)(1.56) \tag{3.16}$$

or equivalently

$$\frac{\Gamma(G\to\delta\pi)}{\Gamma(G\to K\bar{K}\pi)} = 0.88\ , \tag{3.17}$$

consistent with present observation,[19] keeping in mind the large errors associated with these numbers both theoretically as well as experimentally. We now use this model to predict various decays of G, as shown in Table I.

The single photon rates are described by the diagrams of Fig. 5. Since the vector-vector-pseudoscalar vertex has the dimension of inverse mass, we use for the vertex a coupling $g_{VVP}m$(initial particle). Then we have

$$\frac{\Gamma(G\to\rho^o\gamma)}{\Gamma(\eta'\to\rho^o\gamma)} = 29.5\,\frac{f^2_{\eta'}}{(m_G^2 - m_{\eta'}^2)^2}\,\frac{m_G^2}{m_{\eta'}^2}\left[1 + \frac{g_{\eta\rho\rho}}{g_{\eta'\rho\rho}}\,\frac{f_\eta}{f_{\eta'}}\,\frac{m_G^2 - m_{\eta'}^2}{m_G^2 - m_\eta^2}\right]^2 \tag{3.18}$$

where the numerical factor represents the effect of phase space. Using the earlier result for the relative coupling of η and η' to light quarks, we obtain

$$\frac{\Gamma(G \to \rho^0 \gamma)}{\Gamma(\eta' \to \rho^0 \gamma)} = 93.0 \; B(G \to K\bar{K}\pi) \tag{3.19}$$

or

$$\frac{B(G \to \rho^0 \gamma)}{B(G \to K\bar{K}\pi)} = 93.0 \; \frac{\Gamma(\eta' \to \rho^0 \gamma)}{\Gamma_T(G)} = 14\% \; . \tag{3.20}$$

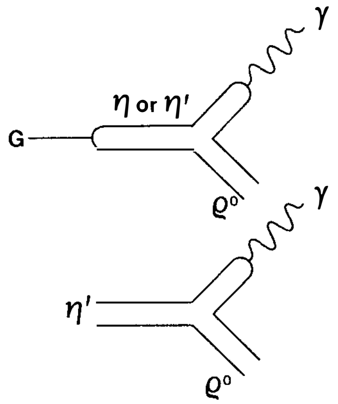

Fig. 5

A similar calculation (but the coupling of ω to a photon is 1/9 the ρ coupling)[37] yields

$$\frac{B(G \to \omega\gamma)}{B(G \to K\bar{K}\pi)} = 1.6\% \; . \tag{3.21}$$

For $G \to \varphi\gamma$ we have

$$\frac{\Gamma(G \to \varphi\gamma)}{\Gamma(\eta' \to \rho\gamma)} = 9.5 \; \frac{f_{\eta'}^2}{(m_G^2 - m_{\eta'}^2)^2} \; \frac{g_{\eta'\varphi\varphi}^2}{g_{\eta'\rho\rho}^2} \; \frac{m_G^2}{m_{\eta'}^2} \; \frac{2}{9} \left[1 + \frac{g_{\eta\varphi\varphi}}{g_{\eta'\varphi\varphi}} \; \frac{f_\eta}{f_{\eta'}} \; \frac{m_G^2 - m_{\eta'}^2}{m_G^2 - m_\eta^2} \right]^2 \tag{3.22}$$

The ratio $g_{\eta\varphi\varphi}/g_{\eta'\varphi\varphi}$, measuring the relative amount of strangeness in the η

and η', is -0.83. A similar treatment of $g_{\eta'\varphi\varphi}/g_{\eta'\rho\rho}$ yields 1.7 for this ratio. The factor 2/9 is the relative strength of the vector dominance coupling. Equation (3.22) can then be expressed as

$$\frac{\Gamma(G\to\varphi\gamma)}{\Gamma(G\to K\bar{K}\pi)} = 0.6\% \ . \tag{3.23}$$

Combining all three single photon rates we have

$$\frac{\Gamma(G\to\rho\gamma) + \Gamma(G\to\omega\gamma) + \Gamma(G\to\varphi\gamma)}{\Gamma(G\to K\bar{K}\pi)} \cong 16\% \ . \tag{3.24}$$

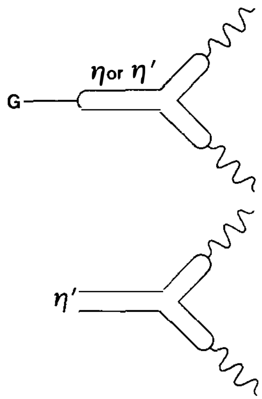

Fig. 6

Let us now consider the process $G\to\gamma\gamma$, which we compare with $\eta'\to\gamma\gamma$, as shown in Fig. 6, yielding

$$\frac{\Gamma(G\to\gamma\gamma)}{\Gamma(\eta'\to\gamma\gamma)} = \left[\frac{m_G}{m_{\eta'}}\right]^3 \frac{f^2_{\eta'}}{(m_G^2 - m_{\eta'}^2)^2} \left[\frac{f_\eta}{f_{\eta'}}\,\frac{(m_G^2 - m_{\eta'}^2)}{(m_G^2 - m_\eta^2)}\,\frac{A(\eta\to\gamma\gamma)}{A(\eta'\to\gamma\gamma)} + 1\right]^2 \ . \tag{3.25}$$

The amplitude ratio $A(\eta\to\gamma\gamma)/A(\eta'\to\gamma\gamma)$ is given by the data[38] up to a sign to be 0.57. This yields for + or - relative phase respectively

$$\frac{\Gamma(G\to\gamma\gamma)}{\Gamma(\eta'\to\gamma\gamma)} = \begin{Bmatrix}3.2\\1.2\end{Bmatrix} B(G\to K\bar{K}\pi) \tag{3.26}$$

or

$$\Gamma(G\to\gamma\gamma) = \begin{Bmatrix}17\\6\end{Bmatrix} \text{KeV} \times B(G\to K\bar{K}\pi) \tag{3.27}$$

The present experimental upper limit on this rate is 10 KeV.[30]

We now consider some three body final states of interest, starting with $G\to\rho\pi\pi$, which we can estimate by comparing it with our previous calculations of $G\to\rho\gamma$. Our model is a very simple isobar picture in which we calculate the phase space by integrating over two two-body phase space factors with a simple Breit-Wigner factor for the internal propagator. No threshold, cross-channel, or symmetrization effects are included here. A careful and detailed treatment will be presented elsewhere. The pole diagrams are given in Fig. 7. The photon in the $G\to\rho^o\gamma$ diagram diminishes the diagram by a factor[37]

$$\frac{3\Gamma_{\rho\to e^+e^-}}{\alpha m_\rho} = 3.6\times10^{-3}\ . \tag{3.28}$$

Thus we have

$$\frac{\Gamma(G\to\rho^o\pi^+\pi^-)}{\Gamma(G\to\rho^o\gamma)} = \frac{0.012}{3.6\times10^{-3}} = 3.5 \tag{3.29}$$

where 0.012 represents the results of our simple isobar model. The result for all $\rho\pi\pi$ charge states can also be written as

$$B(G\to\rho\pi\pi) = 150\%\ B(G\to K\bar{K}\pi)\ ,$$

a significant rate. This final state in the G mass region will be studied at TASSO.[40]

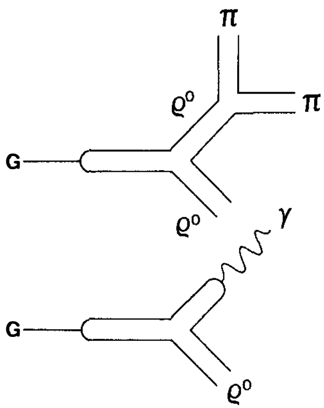

Fig. 7

Finally, we consider the puzzling $G \to \eta\pi\pi$ decay. Since δ is a prominent part of the $K\bar{K}$ in $G \to K\bar{K}\pi$, and since the branching fractions of $\delta \to K\bar{K}$ and $\delta \to \eta\pi$ are similar, one naively expects to see considerable $G \to \eta\pi\pi$ in $\psi \to \gamma\eta\pi\pi$, where experimentally none is seen.[19]

We can understand this as a cancellation between two amplitudes which populate the $\eta\pi\pi$ state,

$$\begin{aligned} G &\longrightarrow \delta\ \pi \\ &\qquad \llcorner\!\longrightarrow \eta\pi \\ G &\longrightarrow \eta\ \epsilon(700) \\ &\qquad \llcorner\!\longrightarrow \pi\pi \end{aligned} \tag{3.30}$$

which parallels an analogous mechanism in the decay of the radial excitation of the η, s(1275)[27] and in the decay of η'.[29] Phase shift analysis of $\eta\pi\pi$ data in the s region show clear $\delta\pi$ and $\eta\epsilon$ signals, with <u>opposite</u> phases, so that the amplitudes partially cancel. A similar effect has been noted in $\eta' \to \eta\pi\pi$. In the s(1275) data the $\eta\epsilon$ signal appears half the size of $\delta\pi$. As in the $\rho\pi\pi$ channel, a final calculation must await a careful treatment of phase space and overlapping off-shell resonances. Now we estimate the process by assuming a cancelling phase as suggested above and indicated in Fig. 8. The phase space for each diagram relative to $\eta' \to \eta\pi\pi$ yields a factor of 20. To gain a rough idea of how big $G \to \eta\pi\pi$ might be, we take the factor of 20. Since the cancellation referred to above is expected to occur in both $\eta' \to \eta\pi\pi$ and $G \to \eta\pi\pi$ channels, the $G \to \eta\pi\pi$ is scaled accordingly, with the OZI suppression given by our earlier analysis

$$\frac{\Gamma(G \to \eta\pi\pi)}{\Gamma(\eta' \to \eta\pi\pi)} = 20 \frac{f^2_{\eta'}}{(m^2_G - m^2_{\eta'})^2} \left[1 + \frac{f_\eta}{f_{\eta'}} \frac{(m^2_G - m^2_{\eta'})}{(m^2_G - m^2_\eta)} \right]^2$$

$$= (20)\ 0.62\ B(G \to K\bar{K}\pi)(2)\ . \tag{3.31}$$

Thus we have

$$\frac{B(G \to \eta\pi\pi)}{B(G \to K\bar{K}\pi)} = 25 \frac{\Gamma(\eta' \to \eta\pi\pi)}{\Gamma_T(G)} = 9\% \tag{3.32}$$

This calculation - rough and uncertain as it is - does indicate that the branching ratio for $G \to K\bar{K}\pi$ and $G \to \eta\pi\pi$ can be very different. A more detailed calculation is in progress.

Collecting all of the partial rates we have calculated for the decay of the G, we can account for a branching ratio which is 175% of that of the $G \to K\bar{K}\pi$,

$$B(G \to \rho\gamma, \omega\gamma, \varphi\gamma, \gamma\gamma, \rho\pi\pi, \eta\pi\pi) = 175\%\ B(G \to K\bar{K}\pi) \tag{3.33}$$

If we conjecture that 90% of the decays have been accounted for, $B(G \to K\bar{K}\pi)$ is about 33%.

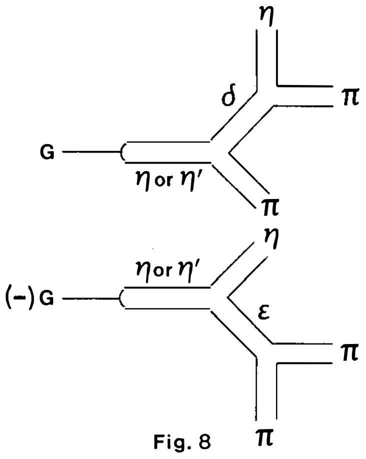

Fig. 8

Table I

G(1440) Decays Based on Pole Model

KK^*	Suppressed by generalized G-parity
$\delta\pi$	Input (Eq. 1.9) 80% ± 20% of $K\bar{K}\pi$
$\rho\gamma$	$\frac{\Gamma(G\to\rho^o\gamma)}{\Gamma(G\to K\bar{K}\pi)} = 14\%$
$\omega\gamma$	$\frac{\Gamma(G\to\omega\gamma)}{\Gamma(G\to K\bar{K}\pi)} = 1.6\%$
$\varphi\gamma$	$\frac{\Gamma(G\to\varphi\gamma)}{\Gamma(G\to K\bar{K}\pi)} = 0.6\%$
$\gamma\gamma$	(6 to 17) KeV $\times$ $B(G\to K\bar{K}\pi)$
$K\bar{K}\pi$	Input
$\rho\pi\pi$	$\frac{\Gamma(G\to\rho\pi\pi)}{\Gamma(G\to K\bar{K}\pi)} = 150\%$
$\eta\pi\pi$	$\frac{\Gamma(G\to\eta\pi\pi)}{\Gamma(G\to K\bar{K}\pi)} = 9\%$

IV. GLUEBALL, CURRENT ALGEBRA, AND THE U(1) PROBLEM

The U(1) problem, at the crossroads of current algebra and QCD, of chiral perturbation theory and the 1/N expansion, has received recent theoretical interest.[41-43] In practical terms, we are concerned with the saturation of anomalous Ward identities by pseudoscalar mesons, including glueballs. These identities provide relations among the matrix elements of the QCD anomaly and other matrix elements measured in $\eta\to 3\pi$, $\psi\to\eta\gamma$, $\psi\to\eta'\gamma$, $\psi\to\gamma G$, $\psi'\to\psi\eta$, $\psi'\to\psi\pi^o$ and and the 2γ decays of η, η', π, and G. The details of the approach are described elsewhere.[41]

Briefly, the saturation is perturbative in powers of the current algebra quark mass δ and the 1/N parameter, where N is the number of colors. We consider the following Ward identities

$$i\int d^4x\left\{T\langle\partial_\mu A^a_\mu(x)\,\partial_\nu A^b_\nu(0)\rangle - \delta^{ao}\delta^{bo}T\langle\sqrt{\frac{2}{3}}\,\frac{3\alpha_s}{4\pi}\,F\tilde{F}(x)\sqrt{\frac{2}{3}}\,\frac{3\alpha_s}{4\pi}\,F\tilde{F}(0)\rangle\right\}$$

$$= -\langle\left[Q^a_5,\left[Q^b_5,H'\right]\right]\rangle \qquad (4.1)$$

where H' is the chiral symmetry breaking part of the Hamiltonian and we saturate by the pseudoscalar (P) states η, η' and G only. The decay constants are defined according to

$$\langle 0|\partial_\mu A^a_\mu|P\rangle = m_P^2 F_{aP} \qquad a = 0,8 \tag{4.2}$$

and

$$\langle 0|\partial_\mu A^o_\mu - \sqrt{\frac{2}{3}}\,\frac{3\alpha_s}{4\pi}\,\mathrm{Tr}\, F\tilde{F}|P\rangle = m_P^2 \tilde{F}_{oP} \quad . \tag{4.3}$$

The decay constants for the radial excitations (R) are defined similarly

$$\langle 0|\partial_\mu A^a_\mu|R\rangle = F_{aR} m_R^2 \tag{4.4}$$

In the chiral (or $N \to \infty$) limit radial excitations are not Goldstone particles and thus their masses do not vanish. Since $\partial_\mu A^a_\mu$ goes like δ (a quark mass) in this limit, we conclude that F_{aR} is proportional to δ. This justifies neglecting radial excitations when saturating chiral Ward identities because their contribution is down by a power of δ. In contrast $F_\pi = O(\delta)$.

The decay constants are directly related to the wave function at the origin for the pseudoscalars,[25]

$$\psi(0) \propto \sqrt{m}\, F \tag{4.5}$$

from which we conclude that

$$\psi_R(0) \propto \delta \tag{4.6}$$

while

$$\psi_P(0) \propto \delta^{\frac{1}{4}} \tag{4.7}$$

That is, the wave function at the origin of the radial excitations is small relative to the ground state wave function.

Returning to Eq.(4.1) with $a,b = 0,8$, we find

$$\frac{4}{3}\, m_K^2 F_K^2 - \frac{1}{3}\, m_\pi^2 = \sum_P m_P^2 F_{8P}^2 \tag{4.8}$$

$$m_\pi^2 = \sum_P m_P^2 F_{8P}(F_{8P} + \sqrt{2}\,\tilde{F}_{oP}) \tag{4.9}$$

$$3m_\pi^2 = \sum_P m_P^2\left[(F_{8P} + \sqrt{2}\,\tilde{F}_{oP})^2 - 2(F_{oP} - \tilde{F}_{oP})^2\right] \tag{4.10}$$

$$0 = \sum_P (F_{oP} - \tilde{F}_{oP}) m_P^2 F_{8P} \tag{4.11}$$

where all decay constants have been normalized to $F_\pi = 1$. These equations are generalizations of our previous work[41] in that the glueball has been included in all pertinent identities, and we have added a fourth sum rule,[43] Eq.(4.11).

If the decay constants $F_{8\eta}$, $F_{8\eta'}$, F_{8G}, $F_{o\eta}$, $F_{o\eta'}$, F_{oG}, $\tilde{F}_{o\eta}$, F_K, $\tilde{F}_{o\eta'}$, $\tilde{F}_{oG}$ are regarded as unknowns, we have four equations in ten unknowns. For the time being we assume $F_K = F_{8\eta} = F_\pi = 1$, an assumption to be tested below. We are left with eight unknowns and four equations.

At this point we appeal to experiment to provide additional constraints. While we will not completely succeed in solving for all the unknowns, we will find an interesting relation involving $\Gamma(G \to \gamma\gamma)$ and $\Gamma(G \to K\bar{K}\pi)$.

First we take the ITEP[44] approach to $\psi \to \gamma P$, mediating the OZI forbidden decay with the anomaly operator $\mathrm{tr}\, F\tilde{F}$:

$$\frac{B(\psi \to \gamma\eta')}{B(\psi \to \gamma\eta)} = \left(\frac{P_{\eta'}}{P_\eta}\right)^3 \left| \frac{\langle 0 | \sqrt{\frac{2}{3}}\, \frac{3\alpha_s}{4\pi}\, \mathrm{tr}\, F\tilde{F} | \eta' \rangle}{\langle 0 | \sqrt{\frac{2}{3}}\, \frac{3\alpha_s}{4\pi}\, \mathrm{tr}\, F\tilde{F} | \eta \rangle} \right|^2 . \tag{4.12}$$

Using branching ratio data we find

$$\left(\frac{F_{o\eta} - \tilde{F}_{o\eta}}{F_{o\eta'} - \tilde{F}_{o\eta'}}\right)^2 = 1.64 \pm 0.24 \tag{4.13}$$

A similar approach to $\psi \to \gamma G$ yields

$$\left(\frac{F_{oG} - \tilde{F}_{oG}}{F_{o\eta'} - \tilde{F}_{o\eta'}}\right)^2 = \frac{0.32 \pm 0.16}{B(G \to K\bar{K}\pi)} \tag{4.14}$$

(We are left with the branching ratio factor because only the product $B(\psi \to \gamma G)\, B(G \to K\bar{K}\pi)$ has been measured.

The $P \to 2\gamma$ rates provide another constraint. Using $\partial_\mu A_\mu^8$ and $\partial_\mu A_\mu^o$ as interpolation fields for η, η' and G, we have

$$\partial_\mu A_\mu^8 = \sum_P m_P^2 F_{8P} \Phi_P + \frac{1}{3\sqrt{3}} \frac{\alpha_{em}}{4\pi} \text{ tr } F_{em} \tilde{F}_{em} \tag{4.15}$$

$$\partial_\mu A_\mu^o = \sum_P m_P^2 F_{oP} \Phi_P + \frac{2}{3}\sqrt{\frac{2}{3}} \frac{\alpha_{em}}{4\pi} \text{ tr } F_{em} \tilde{F}_{em} \tag{4.16}$$

where F_{em} is the electromagnetic field strength tensor. Taking matrix elements between $\langle 0|$ and $|2\gamma\rangle$ states and using Sutherland's theorem and standard current algebra techniques, we have

$$1.12 = 0.53\, F_{o\eta} + F_{o\eta'} + F_{oG} R \tag{4.17}$$

$$-0.131 = F_{8\eta'} + F_{8G} R \tag{4.18}$$

where R is the following amplitude ratio

$$R = \frac{A(G \to 2\gamma)}{A(\eta' \to 2\gamma)} \quad . \tag{4.19}$$

At this point we have eight equations involving eight unknowns in addition to the two missing data R and $B(G \to K\bar{K}\pi)$. We proceed as follows.

Equations (4.8) and (4.11) can be written in terms of $x = F_{8\eta'}$ and $y = F_{8G}$ as

$$\Delta m^2 = x^2 m_{\eta'}^2 + y^2 m_G^2 \tag{4.20}$$

$$B \frac{m_\eta^2}{m_{\eta'}^2} = x + A \frac{m_G^2}{m_{\eta'}^2} y = 0 \tag{4.21}$$

where

$$\Delta m^2 = \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 - m_\eta^2 \tag{4.22}$$

$$B = \left(\frac{F_{o\eta} - \tilde{F}_{o\eta}}{F_{o\eta'} - \tilde{F}_{o\eta'}} \right) \tag{4.23}$$

$$A = \left(\frac{F_{oG} - \tilde{F}_{oG}}{F_{o\eta'} - \tilde{F}_{o\eta'}} \right) \tag{4.24}$$

(A and B are given by Eqs. (4.13) and (4.14) up to a sign.) If the system of equations could be solved for $F_{8\eta'}$ and F_{8G}, they would determine R and hence $G \to 2\gamma$ by Eq.(4.18). (This points out that the remaining equations are underdetermined.) The condition that the ellipse and straight line of Eq.(4.20) and Eq.(4.21) intersect is

$$A^2 \geq B^2 \frac{m_\eta^4}{m_G^2 \Delta m^2} - \frac{m_{\eta'}^2}{m_G^2} \tag{4.25}$$

which by Eq.(4.14) is a constraint on $B(G \to K\bar{K}\pi)$,

$$\frac{0.32 \pm 0.16}{B(G \to K\bar{K}\pi} \geq \left\{ \left(\frac{F_{o\eta} - \tilde{F}_{o\eta}}{F_{o\eta'} - \tilde{F}_{o\eta'}} \right)^2 \frac{m_\eta^4}{m_G^2 \Delta m^2} - \frac{m_{\eta'}^2}{m_G^2} \right\} \tag{4.26}$$

$$\geq \frac{0.072}{\Delta m^2} - 0.44 \quad .$$

Δm^2 is a sensitive term which was originally given by (F_π is scaled to unity).

$$\Delta m^2 = \frac{4}{3} F_K^2 m_K^2 - \frac{1}{3} m_\pi^2 - F_{8\eta}^2 m_\eta^2 \ . \tag{4.27}$$

If we take $F_K^2 = 1$, then

$$\Delta m^2 = 0.325 - F_{8\eta}^2 (0.301) \tag{4.28}$$

Then varying $F_{8\eta}$ from 1.00 to 0.95 causes the limit on $B(G \to K\bar{K}\pi)$ to vary from 12 ± 6% to 36 ± 18%. On the other hand there is evidence that $F_K = 1.15$, which by itself would lead to no restriction on $B(G \to K\bar{K}\pi)$.[45]

The conclusion is that the equations determining $G \to \gamma\gamma$ are quite sensitive to small variations of the parameters. Additional experimental constraints are needed to stabilize the solution.

For amusement and possible instruction, let us assume that the ellipse and straight line of Eq.(4.20) and (4.21) just touch so that we have a unique solution, corresponding to the equal signs in the above inequalities. Then picking a value of Δm^2 (i.e., picking a value of F_K and $F_{8\eta}$) fixes A and therefore $B(G \to K\bar{K}\pi)$. x and y are given by

$$x = \frac{-\Delta m^2}{B m_\eta^2} \qquad y = Ax \tag{4.29}$$

and the rate $\Gamma(G\to\gamma\gamma)$ is determined by

$$\Gamma(G\to\gamma\gamma) = \Gamma(\eta'\to\gamma\gamma)\,\frac{m_G^3}{m_{\eta'}^3}\left(\frac{x+.131}{y}\right)^2 \tag{4.30}$$

Varying Δm^2 from 0.024 to 0.074, which corresponds to extremely small variations in the decay constants, causes the limits on $B(G\to K\bar{K}\pi)$ to vary from 12% ± 6% to 60% ± 30%. The rate on $\Gamma(G\to\gamma\gamma)$ then varies from 8.6 KeV to zero, values not inconsistent with those determined more precisely in earlier sections by other means. The Ward identities accommodate the presence of glueball contributions and should be useful in the future in correlating glueball parameters with other pseudoscalar decay constants.

V. SUMMARY AND CONCLUSIONS

The G(1440) qualitatively satisfies all criteria for a glueball: It is an isosinglet preferentially produced in hard gluon channels which mediate OZI inhibited processes in an SU(3) symmetric way. $\psi\to\gamma\eta'$, $\psi\to\gamma\eta$, $\psi\to\gamma G$, and $G\to\delta\pi$ are all related by a singlet glueball coupling mechanism. A simple pole model predicts $G\to\delta\pi$, $\rho\gamma$, $\omega\gamma$, $\gamma\gamma$, $\rho\pi\pi$, and $\eta\pi\pi$, accounting for a partial width into these channels 1.75 times the $G\to K\bar{K}\pi$ partial width. If, as conjectured, 90% of all decays have been accounted for, we have $B(G\to K\bar{K}\pi) = 33\%$. The $\gamma\gamma$ partial width is estimated to be (6 to 17) KeV $\times$ $B(G\to K\bar{K}\pi)$. The small $G\to\eta\pi\pi$ rate is explained by a cancellation between $G\to\delta\pi\to\eta\pi\pi$ and $G\to\eta\epsilon\to\eta\pi\pi$ amplitudes which has been observed in the corresponding η' and s(1275) amplitudes.

Thus all lights are green for glueball status, but other possibilities should be entertained. While the G does not fit naturally into a pure radially excited nonet (its mass is too low) more sophisticated configuration mixing schemes can accommodate it. For a standard octet singlet mixing angle of $\theta_R = -18°$, its role as a pure radial excitation is consistent with not being seen by Stanton, et al., in $\pi^- p\to Gn$ and with not seeing s(1275) at SPEAR in $\psi\to\gamma X$.

Finally, axial Ward identities accommodate glueball contributions and appear to be consistent with glueball parameters.

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