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RESEARCH ARTICLE

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Slowly rotating charged black holes in anti-de Sitter third order Lovelock gravity

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Abstract In this paper, we study slowly rotating black hole solutions in Lovelock gravity ($n = 3$). These solutions are obtained in uncharged and charged cases, respectively. Up to the linear order of the rotating parameter a, the entropy and gyromagnetic ratio of black holes keep invariant after introducing the Gauss-Bonnet and third order Lovelock interactions.

Keywords Third order Lovelock gravity, Slow rotation, Black hole, Thermodynamics

1 Introduction

It is believed that Einstein's gravity is a low-energy limit of a quantum theory of gravity. Considering the fundamental nature of quantum gravity, there should be a low-energy effective action which describes gravity at the classical level [1]. In addition to Einstein-Hilbert action, this effective action also involves higher derivative terms, and these higher derivative terms can be seen in the renormalization of quantum field theory in curved spacetimes [2; 3; 4; 5; 6], or in the construction of the low-energy effective action of string [7; 8; 9]. In AdS/CFT correspondence, the higher derivative terms can be regarded as the corrections of large N expansion in the dual conformal field theory. In general, the higher powers of curvature can give rise to a fourth or even higher order differential equation for the metric, which will introduce ghosts and violate unitarity. However, Zwiebach and Zumino [10; 11] found that the ghosts can be avoided if the higher derivative terms only consist of the dimensional continuations of the Euler densities, leading to second order field equations for the metric [12]. This higher derivative theory is the so-called Lovelock gravity [13], and the equations of motion contain the

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most symmetric conserved tensor with no more than second derivative of the metric. In this paper, we indulge ourselves with the first four terms of the Lovelock gravity, corresponding to the cosmological constant, Einstein term, Gauss-Bonnet and third order Lovelock terms respectively. So far, the exact static and spherically symmetric black hole solutions in third order Lovelock gravity were first found in [14; 15; 16; 17; 18; 19; 20], and the thermodynamics have been investigated in [12; 17; 18; 19; 20; 21; 22; 23; 24; 25].

On the other hand, a great many attentions have been focused on the rotation effect of static and spherically symmetric black hole solutions. In the AdS/CFT correspondence, the rotating black holes in AdS space are dual to certain CFTs in a rotating space [26], while charged ones are dual to CFTs with chemical potential [27; 28; 29]. In general relativity, the higher dimensional rotating black holes have been recently studied and some exact analytical solutions of Einstein's equation are found in [30; 31; 32; 33; 34; 35].

Since the equations of motion of Lovelock gravity are highly nonlinear, it is rather difficult to obtain the explicit rotating black hole solutions. In order to find rotating black hole solutions in the presence of dilaton coupling electromagnetic field in Einstein-Maxwell theory, Horne and Horowitz [36] first developed a simple perturbative method by introducing a small angular momentum into a non-rotating system, and obtained slowly rotating dilaton black hole solutions. Until now, this approach has been extensively discussed in general relativity [37; 38; 39; 40; 41; 42; 43]. Taking advantage of this crucial tool, Kim and Cai [44] studied slowly rotating black hole solutions with one nonvanishing angular momentum in the Gauss-Bonnet gravity. Recently, some numerical results about the existence of five-dimensional rotating Gauss-Bonnet black holes with angular momenta of the same magnitude have been presented in [45]. In addition, it is worth to mention that some rotating black brane solutions have been investigated in the second (Gauss-Bonnet) and third order Lovelock gravity [46; 47; 48]. Nevertheless, these solutions are essentially obtained by a Lorentz boost from corresponding static ones. They are equivalent to static ones locally, although not equivalent globally. In this paper, we will analyze slowly rotating black hole solutions in third order Lovelock gravity. Generally, the rotating parameters could be more than one in high dimensional Einstein-Maxwell theory [33]. Following the Horne and Horowitz's perturbative method, we limit on one small rotating parameter a case here. The slowly rotating black hole solutions will be studied in both uncharged and charged cases, and then we analyze some physical properties of these black holes.

The outline of this paper is as follows. In Sect. 2, we review the $(n = 3)$ Lovelock gravity, and derive the equations of gravitation and electromagnetic fields. Then, we explore slowly rotating uncharged black holes and obtain the slowly rotating black hole solution $f(r)$ and expression for function $p(r)$ by putting a new form metric into these equations. Moveover, we discuss some related physical properties of the black holes. In Sect. 3, we set about learning slowly rotating black holes in charged case. Section 4 is devoted to conclusions and discussions.

2 Slowly rotating black holes in uncharged case

2.1 Action and black hole solutions

The third order Lovelock gravity coupled to an electromagnetic field is given by

$$\mathscr{I} = \frac{1}{16\pi G}\int d^D x\sqrt{-g}(-2\Lambda + \alpha_1\mathscr{L}_1 + \alpha_2\mathscr{L}_2 + \alpha_3\mathscr{L}_3 - 4\pi G F_{\mu\nu}F^{\mu\nu}), \quad (1)$$

where α_i is the i-th order Lovelock coefficients, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is electromagnetic field tensor with potential A_μ. The Einstein term $\mathscr{L}_1$ equals to R, and the second order Lovelock(Gauss-Bonnet) term $\mathscr{L}_2$ is $R_{\mu\nu\sigma\kappa}R^{\mu\nu\sigma\kappa} - 4R_{\mu\nu}R^{\mu\nu} + R^2$. The third order Lovelock term $\mathscr{L}_3$ reads

$$\begin{aligned}\mathscr{L}_3 =& 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}_{\mu} \quad (2)\\ &+3RR^{\mu\nu\sigma\kappa}R_{\mu\nu\sigma\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3.\end{aligned}$$

Varying the action with respect to $g_{\mu\nu}$ and $F_{\mu\nu}$, the equations for gravitation and electromagnetic fields are

$$\Lambda g_{\mu\nu} + \alpha_1 G^{(1)}_{\mu\nu} + \alpha_2 G^{(2)}_{\mu\nu} + \alpha_3 G^{(3)}_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (3)$$

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0, \quad (4)$$

where $T_{\mu\nu} = F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$ is the energy-momentum tensor of electromagnetic field. The $G^{(1)}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, $G^{(2)}_{\mu\nu}$ and $G^{(3)}_{\mu\nu}$ are the Einstein tensor, second order Lovelock(Gauss-Bonnet) and third order Lovelock tensors respectively:

$$\begin{aligned}G^{(2)}_{\mu\nu} =& 2(R_{\mu\sigma\kappa\tau}R_\nu{}^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}_{\nu} + RR_{\mu\nu}) - \frac{1}{2}\mathscr{L}_2 g_{\mu\nu},\\ G^{(3)}_{\mu\nu} =& 3R_{\mu\nu}R^2 - 12RR_\mu^\sigma R_{\sigma\nu} - 12R_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 24R_\mu^\alpha R_\alpha^\beta R_{\beta\nu}\\ &-24R_\mu^\alpha R^{\beta\sigma}R_{\alpha\beta\sigma\nu} + 3R_{\mu\nu}R_{\alpha\beta\sigma\kappa}R^{\alpha\beta\sigma\kappa} - 12R_{\mu\alpha}R_{\nu\beta\sigma\kappa}R^{\alpha\beta\sigma\kappa} - 12RR_{\mu\sigma\nu\kappa}R^{\sigma\kappa}\\ &+6RR_{\mu\alpha\beta\sigma}R_\nu{}^{\alpha\beta\sigma} + 24R_{\mu\alpha\nu\beta}R_\sigma^\alpha R^{\sigma\beta} + 24R_{\mu\alpha\beta\sigma}R_\nu^\beta R^{\alpha\sigma} + 24R_{\mu\alpha\nu\beta}R_{\sigma\kappa}R^{\alpha\sigma\beta\kappa}\\ &-12R_{\mu\alpha\beta\sigma}R^{\kappa\alpha\beta\sigma}R_{\kappa\nu} - 12R_{\mu\alpha\beta\sigma}R^{\alpha\kappa}R_{\nu\kappa}{}^{\beta\sigma} + 24R_\mu{}^{\alpha\beta\sigma}R_\beta{}^\kappa R_{\sigma\kappa\nu\alpha}\\ &-12R_{\mu\alpha\nu\beta}R^\alpha_{\sigma\kappa\rho}R^{\beta\sigma\kappa\rho} - 6R_\mu{}^{\alpha\beta\sigma}R_{\beta\sigma}{}^{\kappa\rho}R_{\kappa\rho\alpha\nu} - 24R_{\mu\alpha}{}^{\beta\sigma}R_{\beta\rho\nu\lambda}R_\sigma{}^{\lambda\alpha\rho}s - \frac{1}{2}\mathscr{L}_3 g_{\mu\nu}.\end{aligned}$$

Usually, the action Eq. (1) is supplemented with surface terms (a Gibbons-Hawking surface term) whose variation will cancel the extra normal derivative term in deriving the equation of motion Eq. (3). However, these surface terms is not necessary in our discussion and will be neglected. Note that for third order Lovelock gravity, the nontrivial third term requires the dimension(D) of spacetime satisfying $D \geq 7$.

Assume the metric of slowly rotating spacetime to be [44]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \sum_{i=j=3}^{D} r^2 h_{ij}dx^i dx^j - 2ar^2 p(r)h_{44}dtd\phi, \quad (5)$$

where $h_{ij}dx^i dx^j$ represents the metric of a $(D-2)$-dimensional hyper-surface with constant curvature scalar $(D-2)(D-3)k$ and volume Σ_k, here k is a constant. Without loss of generality, one can take $k=0$ or ± 1. When $k=1$, one has $h_{ij}dx^i dx^j = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega^2_{D-4}$ and $h_{44} = \sin^2\theta$; when $k=0$, $h_{ij}dx^i dx^j = d\theta^2 + d\phi^2 + dx^2_{D-4}$ and $h_{44}=1$; when $k=-1, h_{ij}dx^i dx^j = d\theta^2 + \sinh^2\theta d\phi^2 + \cosh^2\theta d\Omega^2_{D-4}$ and $h_{44} = \sinh^2\theta$, where dx^2_{D-4} is the line element of a $(D-4)$-dimensional Ricci flat Euclidian surface. While $d\Omega^2_{D-4}$ denotes the line element of a $(D-4)$-dimensional unit sphere. For the convenience future, we introduce new parameters $\tilde{\alpha}_i$

$$\tilde{\alpha}_0 = \frac{2\Lambda}{(D-1)(D-2)}, \qquad \tilde{\alpha}_i = \alpha_i \prod_{l=1}^{2i-2}(D-2-l), \quad (i=1,2,3). \tag{6}$$

Firstly, we consider pure gravity case; namely $T_{\mu\nu}=0$. substituting Eq. (5) into Eq. (3) and discarding any terms involving a^2 or higher powers, we find that the rr-component of the equations of motion

$$\begin{aligned} 0 &= (D-7)\tilde{\alpha}_3(f(r)-k)^3 - (D-5)\tilde{\alpha}_2(f(r)-k)^2 r^2 + (D-3)\tilde{\alpha}_1(f(r)-k)r^4 \\ &\quad + [3\tilde{\alpha}_3 r(f(r)-k)^2 + 2\tilde{\alpha}_2(f(r)-k)r^3 + \tilde{\alpha}_1 r^5] f'(r) + (D-1)\tilde{\alpha}_0 r^6, \end{aligned} \tag{7}$$

where a prime denotes the derivative with respect to r. Note that the angular momentum parameter a does not appear in the rr-component. Thus, the slowly rotating black hole solutions $f(r)$ is identical to the static one in form. Generally, the Eq. (7) has one real and two complex solutions. (It may has three real solutions under some conditions.) Here, we only take the real one. This general solution $f(r)$ for D-dimensional slowly rotating black hole is

$$f(r) = k + \frac{r^2}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma+\kappa^2(r)}+\kappa(r)} - \sqrt[3]{\sqrt{\gamma+\kappa^2(r)}-\kappa(r)}\right], \tag{8}$$

where

$$\gamma = (3\tilde{\alpha}_1\tilde{\alpha}_3 - \tilde{\alpha}_2^2)^3, \quad \kappa(r) = \tilde{\alpha}_2^3 - \frac{9\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3}{2} - \frac{27\tilde{\alpha}_3^2}{2}\left[\tilde{\alpha}_0 + \frac{16\pi G M}{(D-2)\Sigma_k r^{D-1}}\right]. \tag{9}$$

The integral constant M is the gravitational mass. Hereafter, for simplicity, we take notation $m = \frac{16\pi G M}{(D-2)\Sigma_k}$. It is easy to find that the solution is asymptotically flat for $\Lambda=0$, AdS for negative value of Λ and dS for positive value of Λ. For the asymptotically AdS solution, putting $\tilde{\alpha}_0 = -1/l^2$ in Eq. (9), we obtain

$$\begin{aligned} \varphi &= (k-f(r))/r^2 = -\frac{1}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma+\kappa^2(r)}+\kappa(r)} - \sqrt[3]{\sqrt{\gamma+\kappa^2(r)}-\kappa(r)}\right], \\ \kappa(r) &= \tilde{\alpha}_2^3 - \frac{9\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3}{2} - \frac{27\tilde{\alpha}_3^2}{2}\left[-\frac{1}{l^2} + \frac{m}{r^{D-1}}\right]. \end{aligned} \tag{10}$$

Meanwhile, there exists off-diagonal $t\phi$-component of equations of motion, which is concerned with function $p(r)$. A tedious computation leads to a following equation

$$\frac{A(r)}{2}p''(r) + \frac{[3A(r)+(D-3)B(r)]}{2r}p'(r) = 0, \tag{11}$$

where

$$\begin{aligned} A(r) &= \tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2, \\ B(r) &= \tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2 + \frac{2r\tilde{\alpha}_2\varphi'}{D-3} + \frac{6r\tilde{\alpha}_3\varphi\varphi'}{D-3}. \end{aligned} \tag{12}$$

It can be changed into a closed form

$$\begin{aligned} [\log p'(r)]' &= -\left[\frac{D}{r} + \frac{(\tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2)'}{\tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2}\right] \\ &= -[\log(r^D(\tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2))]'. \end{aligned} \tag{13}$$

Therefore, the formal expression for function $p(r)$ in third order Lovelock gravity is given by

$$p(r) = \int \frac{C_2 dr}{r^D(\tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi + 3\tilde{\alpha}_3\varphi^2)} + C_1, \tag{14}$$

where the C_1 and C_2 are two integration constants.

In fact, the Eq. (7) is same as one in the static black hole solution [17; 18; 19; 20]

$$\left[\frac{r^{D-1}}{l^2} + r^{D-1}\varphi(\tilde{\alpha}_1 + \tilde{\alpha}_2\varphi + \tilde{\alpha}_3\varphi^2)\right]' = 0, \tag{15}$$

and φ is a real root of the following 3th-order polynomial equation

$$\tilde{\alpha}_1\varphi + \tilde{\alpha}_2\varphi^2 + \tilde{\alpha}_3\varphi^3 = \frac{m}{r^{D-1}} - \frac{1}{l^2}. \tag{16}$$

With the help of Eq. (14) and taking $C_2 = m(D-1),\ C_1 = 0$, one can find the function $p(r)$

$$p(r) = -\varphi = \frac{1}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma + \kappa^2(r)} + \kappa(r)} - \sqrt[3]{\sqrt{\gamma + \kappa^2(r)} - \kappa(r)}\right]. \tag{17}$$

2.2 Physical properties

As shown in Eq. (8), the slowly rotating black hole solution $f(r)$ is independent of a. Though most interesting physical properties also depend only on a^2, one can still extract some useful information from it. Based on discussions in the last subsection, we will investigate physical properties of slowly rotating black holes in this subsection.

According to the solution $f(r)$, the gravitational mass of the solution can be expressed as

$$M = \frac{(D-2)\Sigma_k r_+^{D-7}}{16\pi G}(r_+^6/l^2 + k\tilde{\alpha}_1 r_+^4 + k^2\tilde{\alpha}_2 r_+^2 + k^3\tilde{\alpha}_3) \tag{18}$$

and the Hawking temperature of the black hole is

$$\begin{aligned} T &= \frac{f'(r_+)}{4\pi} \\ &= \frac{(D-1)r_+^6/l^2 + (D-3)k\tilde{\alpha}_1 r_+^4 + (D-5)k^2\tilde{\alpha}_2 r_+^2 + (D-7)k^3\tilde{\alpha}_3}{4\pi r_+(\tilde{\alpha}_1 r_+^4 + 2k\tilde{\alpha}_2 r_+^2 + 3k^2\tilde{\alpha}_3)}. \end{aligned} \tag{19}$$

Thus, the angular momentum of the black hole

$$J = \frac{2aM}{D-2} = \frac{a\Sigma_k r_+^{D-7}}{8\pi G}(r_+^6/l^6 + k\tilde{\alpha}_1 r_+^4 + k^2\tilde{\alpha}_2 r_+^2 + k^3\tilde{\alpha}_3). \tag{20}$$

Another important thermodynamic quantity is black hole entropy. Usually, the entropy of black hole satisfies the so-called area law of entropy that the black hole entropy equals to one-quarter of the horizon area [49; 50; 51]. It applies to all kinds of black holes and black strings of Einstein gravity [52; 53]. However, in higher derivative gravity, the area law of the entropy is not satisfied in general [54]. Since black hole can be regarded as a thermodynamic system, it obeys the first law of thermodynamics $dM = TdS + \omega_H dJ$. Through the angular velocity ω_H, one can get the entropy of black hole.

For the slowly rotating solution, the stationarity and rotational symmetry metric Eq. (5) admits two commuting Killing vector fields

$$\xi_{(t)} = \frac{\partial}{\partial t}, \quad \xi_{\phi} = \frac{\partial}{\partial \phi}. \tag{21}$$

The various scalar products of these Killing vectors can be expressed through the metric components as follows

$$\begin{aligned} \xi_{(t)} \cdot \xi_{(t)} &= g_{tt} = -f(r), \\ \xi_{(t)} \cdot \xi_{(\phi)} &= g_{t\phi} = -ar^2 p(r) h_{44}, \\ \xi_{(\phi)} \cdot \xi_{(\phi)} &= g_{\phi\phi} = r^2 h_{44}. \end{aligned}$$

To examine further properties of the slowly rotating black holes, as well as physical processes near such a black hole, we introduce a family of locally non-rotating observers. The coordinate angular velocity for these observers that move on orbits with constant r and θ and with a four-velocity u^μ such that $u \cdot \xi_{(\phi)} = 0$ is given by [34; 35; 44]

$$\begin{aligned} \Omega &= -\frac{g_{t\phi}}{g_{\phi\phi}} = ap(r) \\ &= \frac{a}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma + \kappa^2(r)} + \kappa(r)} - \sqrt[3]{\sqrt{\gamma + \kappa^2(r)} - \kappa(r)}\right]. \end{aligned} \tag{22}$$

In contrast to the case of an ordinary kerr black hole in asymptotically flat spacetime, the angular velocity does not vanish at spatial infinity

$$\Omega_\infty = \frac{a}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma + \tilde{\kappa}^2} + \tilde{\kappa}} - \sqrt[3]{\sqrt{\gamma + \tilde{\kappa}^2} - \tilde{\kappa}}\right] = a\Delta. \tag{23}$$

where $\tilde{\kappa} = \tilde{\alpha}_2^3 - \frac{9\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3}{2} + \frac{27\tilde{\alpha}_3^2}{2l^2}$ and Δ a constant.

When approaching the black hole horizon, the angular velocity turns to be $\Omega_H = ap(r_+) = -a\varphi(r_+) = -\frac{ak}{r_+^2}$. This Ω_H can be thought as the angular velocity of the black hole. The relative angular velocity with respect to a frame static at infinity is defined by

$$\omega_H = \Omega_H - \Omega_\infty = -a\left(\frac{k}{r_+^2} + \Delta\right). \tag{24}$$

Therefore, we get the entropy of slowly rotating black hole up to the linear order of the rotating parameter a

$$S = \frac{\Sigma_k}{4G} r_+^{D-2}\left[\tilde{\alpha}_1 + \frac{2(D-2)k\tilde{\alpha}_2}{(D-4)r_+^2} + \frac{3(D-2)k^2\tilde{\alpha}_3}{(D-6)r_+^4}\right], \tag{25}$$

which recovers the results in [17; 18; 19; 20].

3 Slowly rotating black holes in charged case

In this section, we consider slowly rotating black hole solution with charge. In charged case, the situation is dramatically altered. Since the black hole rotates along the direction ϕ, it will generate a magnetic field. Considering this effect, the gauge potential can be chosen

$$A_\mu dx^\mu = A_t dt + A_\phi d\phi. \tag{26}$$

Here we assume $A_\phi = -aQc(r)h_{44}$. As a result, the electro-magnetic field associated with the solution are

$$F_{tr} = -A_t', \quad F_{r\phi} = -aQc'(r)h_{44}, \quad F_{\theta\phi} = -aQc(r)\frac{d(h_{44})}{d\theta}. \tag{27}$$

where Q, an integration constant, is the electric charge of the black hole and a prime denotes the derivative with respect to r. From t-component of electromagnetic field equation $\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0$, one can find $F_{tr} = \frac{Q}{4\pi r^{D-2}}$, which is the same as the static form. Unlike the static case, there exist the ϕ-component of the electromagnetic field equation, and then the equation for function $c(r)$ reads

$$(r^{D-4}f(r)c'(r))' - 2k(D-3)r^{D-6}c(r) = \frac{p'(r)}{4\pi}. \tag{28}$$

To find the black hole solution, one may use any components of the equations of motion Eq. (3). While, these equations are influenced by charge and the rr-component reads

$$\begin{aligned} -\frac{Q^2G}{2(D-2)\pi}r^{10-2D} = {} & [3\tilde{\alpha}_3 r(f(r)-k)^2 - 2\tilde{\alpha}_2(f(r)-k)r^3 + \tilde{\alpha}_1 r^5]f'(r) \\ & +(D-7)\tilde{\alpha}_3(f(r)-k)^3 - (D-5)\tilde{\alpha}_2(f(r)-k)^2r^2 \\ & +(D-3)\tilde{\alpha}_1(f(r)-k)r^4 + (D-1)\tilde{\alpha}_0 r^6. \end{aligned} \tag{29}$$

Setting $\tilde{\alpha}_0 = -1/l^2$, we take the general charged solution $f(r)$ of D-dimensional slowly rotating black hole in third order Lovelock gravity

$$f(r) = k + \frac{r^2}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} + \kappa_*(r)} - \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} - \kappa_*(r)}\right], \quad (30)$$

where

$$\gamma_* = (3\tilde{\alpha}_1\tilde{\alpha}_3 - \tilde{\alpha}_2^2)^3, \quad \kappa_*(r) = \tilde{\alpha}_2^3 - \frac{9\tilde{\alpha}_1\tilde{\alpha}_2\tilde{\alpha}_3}{2} - \frac{27\tilde{\alpha}_3^2}{2}\left[-1/l^2 + \frac{m}{r^{D-1}} - \frac{q^2}{r^{2D-4}}\right].$$

We also introduce $f(r) = k - r^2\varphi_*$ with

$$\varphi_* = -\frac{1}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} + \kappa_*(r)} - \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} - \kappa_*(r)}\right]. \quad (31)$$

The integration constant $M = \frac{(D-2)\Sigma_k}{16\pi G}m$ is also gravitational mass and the charge Q^2 is expressed as $Q^2 = \frac{2\pi(D-2)(D-3)}{G}q^2$.

It is worthy to point out that φ_* satisfies the following equation

$$\frac{1}{l^2} + \tilde{\alpha}_1\varphi_* + \tilde{\alpha}_2\varphi_*^2 + \tilde{\alpha}_3\varphi_*^3 = \frac{m}{r^{D-1}} - \frac{q^2}{r^{2D-4}}, \quad (32)$$

which is exact form for charged static black hole in Lovelock gravity [55; 56; 57; 58; 59; 60]. It is not strange. In Einstein equation, the rotating effect is from the energy-momentum tensor. As shown in Eq. (27), there exist two non-vanishing $F_{r\phi}$ and $F_{\theta\phi}$ which are proportional to parameter a. Discarding all terms involve a^2 and higher power, $F_{\mu\nu}F^{\mu\nu}$ in action Eq. (1) reduces to $F_{tr}F^{tr}$ which is the same as the counterpart in static case. Hence, the diagonal components of Einstein equation keeps invariant.

In addition, the off-diagonal $t\phi$-component of the equation of motion is concerned with functions $p(r)$ and $c(r)$

$$r^D(\tilde{\alpha}_1 + 2\tilde{\alpha}_2\varphi_* + 3\tilde{\alpha}_3\varphi_*^2)p(r)' = 4GQ^2c(r) + C_3, \quad (33)$$

where C_3 is a constant.

Thanks to the Eqs. (28), (32) and (33), we eventually find these explicit solutions for functions $p(r)$ and $c(r)$

$$\begin{aligned} c(r) &= -\frac{1}{4\pi(D-3)r^{D-3}} \\ p(r) &= -\varphi_* \\ &= \frac{1}{3\tilde{\alpha}_3}\left[\tilde{\alpha}_2 + \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} + \kappa_*(r)} - \sqrt[3]{\sqrt{\gamma_* + \kappa_*^2(r)} - \kappa_*(r)}\right]. \end{aligned} \quad (34)$$

In the rest of this section, let us explore some physical properties of charged black holes. From Eq. (30), the charged solutions get no corrections from the rotation up to linear order of a, and the introduction of charged Q does not alter asymptotic behavior of the metric. Therefore, the expressions for the mass and angular momentum for two cases do not change. Another particular characteristic of charged black hole is its gyromagnetic ratio. In general relativity, one of the remarkable facts about a Kerr-Newman black hole in asymptotically flat spacetime is that it can be assigned a gyromagnetic ratio g, just as an electron in the Dirac theory [34; 35; 61]. For example, the gyromagnetic ratio g of a charged rotating black hole is $g=2$ in four-dimensional spacetime [62; 63]. For slowly rotating third order Lovelock black holes, the magnetic dipole moment is $\mu = Qa$. According to $J=\frac{2aM}{D-2}$, the gyromagnetic ratios is obtained

$$g=\frac{2\mu M}{QJ}=D-2.$$

It is clear that the value of g is the same as the case in general relativity [34; 35] and in Gauss-Bonnet gravity [44], and it only depends on the number of spacetime dimensions.

4 Conclusion and discussion

Based on the non-rotating charged black hole solutions, we have successfully derived the slowly rotating (charged) black hole solutions by introducing a small rotating parameter a in third order Lovelock gravity. In the new metric, we choose $g_{t\phi}=-ar^2p(r)h_{44}$ and discard any terms involving a^2 and higher powers, and then get the expression for function $p(r)$, while the function $f(r)$ still keeps the static form. In charged case, the vector potential has an extra non-radial component $A_\phi=-aQc(r)h_{44}$ due to black hole rotation. Since the off-diagonal component of the stress-tensor of electro-magnetic field was related to $c(r)$, the equations for $p(r)$ and $c(r)$ become two coupled differential equations. However, the exact solution for $c(r)$ and $p(r)$ can be expressed as $c(r)=-\frac{1}{4\pi(D-3)r^{D-3}}$ and $p(r)=-\varphi_*$. Up to the linear order of the rotating parameter a, the expressions of the mass, temperature, and entropy for the black holes got no correction from rotation in both uncharged and charged cases. The above discussion is based one rotating parameter. In general, there exist some black hole solutions with more than one rotating parameters in higher dimensional gravity [33; 34; 35]. We hope such kind of solution could be studied in a same procedure.

In third order Lovelock gravity, the Lagrangian $\mathscr{L}_3$ involves eight terms constituted by the Ricci and the Riemann curvature tensors. Moreover, the resulting field equations, obtained after variation with respect to the metric tensor, have thirty-four terms. Considering a higher order Lovelock term, for instance the quartic Lovelock tensor, it involves twenty-five terms and each contains the product of four curvature terms. A general expression of the corresponding field equations was obtained in [64], while this work is very complicated. Therefore, taking into account all the relevant terms of the Lovelock action, then obtaining slowly rotating black hole solutions by solving the field equations for general space-times in high dimensions, is a formidable task. Note that the exact static and spherically symmetric black hole solutions of the Gauss-Bonnet gravity have been found by working directly in the action [14; 15; 16; 65; 66; 67; 68; 69], even higher order

Lovelock gravity [55; 56; 57; 58; 59; 60]. Then, this simple method has been popularized in studying slowly rotating black holes in third order Lovelock gravity [17; 18; 19; 20]. However, the metric should be taken a proper form. By using the same approach, the generalization of the present work may be further simplified and is now under investigation.

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