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An information theory model for dissipation in open quantum systems

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Abstract. This work presents a general model for open quantum systems using an information game along the lines of Jaynes' original work. It is shown how an energy based reweighting of propagators provides a novel moment generating function at each time point in the process. Derivatives of the generating function give moments of the time derivatives of observables. Aside from the mathematically helpful properties, the ansatz reproduces key physics of stochastic quantum processes. At high temperature, the average density matrix follows the Caldeira-Leggett equation. Its associated Langevin equation clearly demonstrates the emergence of dissipation and decoherence time scales, as well as an additional diffusion due to quantum confinement. A consistent interpretation of these results is that decoherence and wavefunction collapse during measurement are directly related to the degree of environmental noise, and thus occur because of subjective uncertainty of an observer.

1. Introduction

This paper presents a new, simple ansatz for adding dissipation to arbitrary stochastic forcing of a quantum dynamical system. For Gaussian random forces, its Lindblad equation is identical to the Caldeira-Leggett model [1, 2, 3] up to order β^2 , where the same term is present with a different prefactor. The system-centric, phase space picture here shows that this term represents a quantum confinement effect.

This ansatz is significant because nearly all other models for dissipation rely on the Feynman-Vernon influence functional approach to modeling a linearly coupled environment [4]. Because those approaches are based on a model of the environment containing infinitely many harmonic oscillators, they are complicated to carry over to other settings – for example the reconstruction problem in quantum field theory [5].

Work on the problem of quantum decoherence showed that there is a fundamental connection ($\tau/\theta = (\Delta q/\lambda_{db})^2$) between the timescale of energy dissipation, τ , and the decoherence timescale, θ [6, 7, 8]. Those two scales are separated by the squared distance per de Broglie wavelength, $\lambda_{db} = \hbar\sqrt{\beta/4m}$. However, these connections can be modified by changing length scales and models for the environment [9, 10]. The proper representation of the environment is not just an academic question, since the form of environmental coupling is related to different mechanisms for decoherence in laboratory experiments [11, 12], and even give information on the early universe and gravitational background [7, 13].

Investigation of the crossover from quantum to classical chaotic dynamics has been difficult, since classical constraints on energy flows do not have direct translations in the quantum theory [14]. Zurek used the Moyal bracket to relate the two and found that classical noise



causes decoherence that limits the severity of quantum corrections [15]. The maximum entropy approach introduced here provides an alternative, straightforward route to derive force/flux relationships of the Onsager-Machlup type – via a ‘kinetic’ partition function for average fluxes. The associated fluctuation-dissipation theorem links energy exchange with the system to classical and quantum information loss from the system.

This work introduces the energy-based reweighting ansatz for modeling dissipation in several steps. Section 2 begins with a Bayesian framework for analyzing stochastic quantum dynamics. Sec. 3 shows that the forward-backward propagator, applied to the Wigner distribution, has a close connection to the classical Langevin equation. By applying a pre-averaging procedure, we illustrate how stochastic dynamics is caused by loss of phase information. That section briefly recalls key results of the classical moment generating function derived from maximum transition entropy [16]. The energy-based reweighting is finally introduced in Sec. 4 and its associated Wigner propagator is derived in Sec. 5. There, the moment generating function is found to provide an explicit definition for the local quantum temperature.

2. Information Game

For a closed quantum system, the full wavefunction is determined via the Schrödinger equation for all times from a known initial condition and a Hamiltonian. However, probability enters the description of the dynamics in an essential way at the point where the system interacts with its external environment.

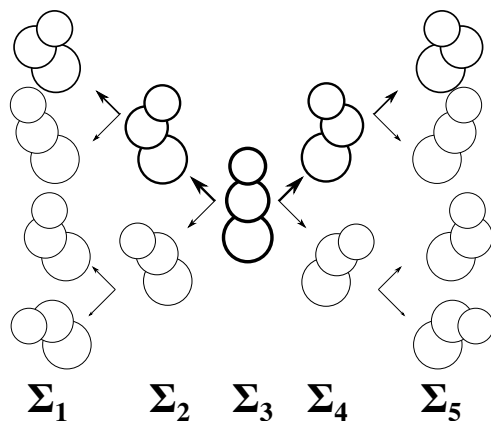


Figure 1. Set of possible histories as inferred from an observer with complete knowledge of the state at surface Σ_3 . The action of a random environment is represented by two possibilities for the propagator at each step (up and down arrows, respectively). The rules of classical probability lead to decoherence.

In the information game model, every possible “state” of an open quantum system is assigned a probability using the rules of classical, Bayesian statistics [17]. In theory, it makes no difference whether the system “state” is defined indirectly as a set of local observables or as a wavefunction covering a complete spacelike (Cauchy) surface at a single instant in some timeframe. The important aspect of the model is that all predictions are made in a consistent way from classical information known to the observers. Each observer uses their information on the quantum state along with a quantum-mechanical Green’s function to generate a classical probability distribution over the set of histories.

To fix these ideas, suppose that five different observers are each able to completely determine the density matrix of a single particle on a different spacelike Cauchy surface, Σ , and that they do so without interfering with the particle’s dynamics. Order the observers by their separation in time, so that $\Sigma_1 < \Sigma_2 < \dots$. Assume further that all observers are able to compute the full time evolution according to Schrödinger’s equation and know the particle’s

Hamiltonian as well as the locations of all five surfaces. In this case, each observer is able to fully determine the information about the particle available to every other observer.

Now suppose that, in addition to everything above, a random environment is also present. The observers can no longer fully predict one another’s measurements. Instead, observer i can only predict a probability distribution over the density matrix, ρ_j that will be available to observer j . The prediction process is straightforward if the information available to observer i is sufficient to define a measure, $(dP[G])$, assigning a probability to all possible propagators, G .

That measure can be used to construct the measure over the space of possible density matrices on Σ_j as a sum of point measures. The machinery of quantum mechanics appears only as a way to transform information about G into its image on the set of possible density matrices. Logical consistency is guaranteed by correct use of inference [18].

Figure 1 illustrates the situation above from the perspective of the third observer. Time evolution of the state is represented by motion of the figure as time progresses along the horizontal axis. In this schematic drawing, there are two possible Hamiltonians, giving rise to slightly different motions. The lighter shading of the lower figures at each timestep illustrates the situation where the corresponding Hamiltonian has a smaller classical probability ($dP[G_{\text{lower}}]$).

The sections that follow examine the inferred process, ρ_t , as viewed from a single fixed observer, and study the average, $\rho(x^1, x^2, t) = \int dP[G] G(t) * \rho_0$, which is appropriate since quantum mechanical operators are linear in ρ .

3. Incipient Classical Langevin Equation

The Wigner distribution [19] is defined as the Weyl symbol of the density matrix, ρ ,

$$W_t(x, p) \equiv \mathcal{W}[\rho_t] = h^{-d} \iint_{\Sigma} dx_1 dx_2 \rho_t(x_1, x_2) \delta(x - \frac{x_1 + x_2}{2}) e^{ip(x_2 - x_1)/\hbar}, \quad (1)$$

where d is the dimension of the position vector, $x \in \mathbb{R}^d$. Recognizing that \mathcal{W} is a Fourier transform, we denote its inverse by \mathcal{W}^{-1} .

For compact notation, we use an underline to cast an operator, A , on Hilbert space into a superoperator, \underline{A} (acting on other operators, B) as $\underline{A}[B] \equiv ABA^\dagger$, where A^\dagger denotes the conjugate transpose. Note that if A, B are bounded, and B has real, non-negative eigenvalues, then $\underline{A}[B]$ does also.

It is well-known that the propagator for the wavefunction, $U^t \equiv e^{-it\hat{H}/\hbar}$, can be expressed as a path integral [20]. The time-slicing procedure for summation over histories has been rigorously defined for Cartesian and for general covariant spatial coordinates, and has been extended to field and point-particle systems in special relativity [21]. More refined derivations of continuous-time path integrals have also been given for particles in curved spacetime [22]. For simplicity in introducing the ideas here, our treatment will be confined to the Cartesian form.

The propagator for the density matrix is given by the forward-backward path integral [23],

$$\rho(x_N^1, x_N^2) = \prod_{n=0}^{N-1} \left[C^{-d} \iint dx_n^1 dx_n^2 \right] e^{\frac{i}{\hbar}(\mathcal{A}[x_0^1 \rightarrow x_N^1] - \mathcal{A}[x_0^2 \rightarrow x_N^2])} \rho(x_0^1, x_0^2) \quad (2)$$

$$C \equiv \epsilon\hbar/m \quad (2)$$

$$\mathcal{A}[x_0 \rightarrow x_N] = \sum_{n=1}^N \frac{m(x_n - x_{n-1})^2}{2\epsilon} - \epsilon V \left(\frac{x_n + x_{n-1}}{2} \right) \quad (3)$$

over all pairs of paths, x^1 , and x^2 , connecting points on $x_0^1, x_0^2 \in \Sigma'$ and $x_N^1, x_N^2 \in \Sigma$. This goes over to continuous time in the limit $N \rightarrow \infty$ with $\epsilon N \rightarrow t$.

We use the path integral and the definition of the Wigner distribution [Eqs. (2) and (1)] to write the ‘Wigner propagator’ from (x', p') to (x, p) in the form,

$$-iL \equiv \lim_{t \rightarrow 0^+} t^{-1} (\mathcal{W} \underline{U}^t \mathcal{W}^{-1} - \underline{I})$$

$$= \lim_{\epsilon \rightarrow 0^+} \epsilon^{-1} \left(C^{-d} \iint dy dy' e^{-\frac{i}{\hbar}\phi_0} - \underline{I} \right) \quad (4)$$

$$\phi_0 = \mathcal{A}[x' - \frac{y'}{2} \rightarrow x - \frac{y}{2}] - \mathcal{A}[x' + \frac{y'}{2} \rightarrow x + \frac{y}{2}] + py - p'y'. \quad (5)$$

In the limit, the action consists only of a single time-step, so that

$$\phi_0 = -\frac{m\Delta x\Delta y}{\epsilon} + \epsilon(V(\bar{x} + \bar{y}/2) - V(\bar{x} - \bar{y}/2)) + \Delta p\bar{y} + \Delta y\bar{p}, \quad (6)$$

where we have used the obvious notations, $\bar{x} = (x + x')/2$ and $\Delta x = x - x'$, etc. and noted that the transformations factor as $py - p'y' = \Delta p\bar{y} + \Delta y\bar{p}$.

The integral over Δy can be performed immediately. The short-time expansion of the evolution operator is thus,

$$e^{-iL\epsilon} = h^{-d}\delta\left(\Delta x - \frac{\epsilon\bar{p}}{m}\right) \int d\bar{y} e^{-\frac{i}{\hbar}\phi_1} \quad (7)$$

$$\phi_1(\bar{x}, \bar{y}) \equiv \bar{y}\Delta p + \epsilon V(\bar{x} + \bar{y}/2) - \epsilon V(\bar{x} - \bar{y}/2)$$

This form shows the equivalence of the Wigner propagator with the Moyal bracket [24]. It was introduced by Schmidt [25].

The physical content of this equation can be seen assuming y is near zero [25] or using a ‘‘pre-averaging’’ method [26]. Pre-averaging makes a computational prescription out of the method of stationary phase by applying the identity $\int f(y) dy = \int (f * g)(y) dy$ [when $\int dy g(y) = 1$] in the form,

$$\int dy e^{-\frac{i}{\hbar}\phi_1(x,y)} = (2\pi l^2)^{-d/2} \iint dydz e^{-\frac{i}{\hbar}\phi_1(x,y-z) - z^2/2l^2}, \quad (8)$$

which holds for any width, l . If l is small enough, the exponential weighting allows expanding about $z = 0$ to good approximation.

$$\phi_1(x, y - z) = \phi_1(x, y) - z \cdot \delta_x \bar{\mathcal{A}} - \frac{1}{2} z \cdot \partial_y z \cdot \delta_x \bar{\mathcal{A}} + O(z^3), \quad (9)$$

$$\delta_x \bar{\mathcal{A}} \equiv \frac{1}{2} \left(\frac{\delta \mathcal{A}(x + y/2)}{\delta x} + \frac{\delta \mathcal{A}(x - y/2)}{\delta x} \right)$$

Retaining the term linear in z exhibits exponential fall-off away from Newton’s law,

$$\int dy e^{-\frac{i}{\hbar}\phi_1(x,y)} \simeq \int dy e^{-\frac{i}{\hbar}\phi_1(x,y)} e^{-\frac{l^2(\delta_x \bar{\mathcal{A}})^2}{2\hbar^2}}. \quad (10)$$

Note that the factor $(\delta_x \bar{\mathcal{A}})^2$ generates the Langevin equation in the classical limit, $\bar{y} \rightarrow 0$.

$$P(x, p|x', p') = \mathcal{Z}_{\text{kin}}^{-1}(x', p') \delta(\Delta x - \epsilon \bar{p}/m) \exp \left\{ -\frac{(\delta_x \mathcal{A})^2}{2\sigma^2 \epsilon} + \frac{\beta}{2m} \bar{p} \cdot \delta_x \mathcal{A} \right\} \quad (11)$$

Eq. (11) clearly has a maximum entropy structure, with constraints on $\langle (\delta_x \mathcal{A})^2 | x', p' \rangle$ and $\langle dE | x', p' \rangle = \langle -\Delta x / \epsilon \cdot \delta_x \mathcal{A} \rangle$ – a connection that was first introduced in Ref. [16]. That derivation identified a normalization factor $\mathcal{Z}_{\text{kin}}(x', p')$ which is termed the kinetic partition function. Averages obtained by differentiating $(\log \mathcal{Z}_{\text{kin}})$ are identical to the results that would be obtained from the usual Stratonovich form of the stochastic calculus for the Langevin equation.

The structure of the Langevin equation is thus embedded in quantum mechanics itself. Quantum mechanics is still reversible because of the phase factor. Classically, however, the loss of the phase information should be modeled with random jumps in momentum on the scale of \hbar/l . Since z^3 and higher terms depend on 3rd order and higher derivatives of the potential. $l = l(x, y)$ can be safely set to the largest length over which the action functional remains quadratic [25, 27].

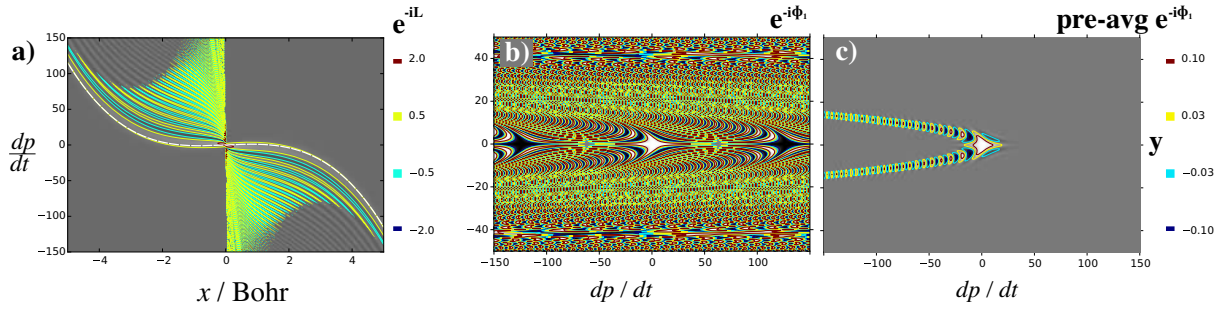


Figure 2. Wigner propagator (a), direct integrand (b), and pre-averaged integrand (c) for the quartic potential energy landscape $V(x) = x^4/4 - x^2$. The white dotted line in (a) shows the force, $-V'(x)$. The pre-averaged integrand in (c) was computed as the convolution of Eq. (8) using the expansion of Eq. (9) up to z^2 .

Figure 2a plots the Wigner propagator for the symmetric quartic potential, $V(x) = x^4/4 - x^2$ at $\epsilon = 10^{-1}$ in atomic units ($\hbar = \text{electron mass} = 1$). Although the full propagator is 4-dimensional, Eq. (7) shows that it only depends on the combination \bar{x} and $\Delta p/\epsilon$. The integrals over \bar{y} were carried out numerically using 200-point Gauss-Hermite quadrature after pre-averaging (integrating over z in Eq. (8)) using the expansion of Eq. (9) up to z^2 (rather than the first-order Eq. (10)). It was verified that the result was not affected by the particular choice of width, l (for which we used 1 Bohr). The dramatic effect of pre-averaging is shown in Fig. 2b and c, which plot the integrand before and after.

The phase structure visible in Airy-function like oscillations of Δp (vertical slices of Fig. 2a) is physically smoothed by the uncertainty principle. Although it is not shown here, propagating a coherent state (a minimum uncertainty wavepacket) is identical to smearing the propagator with a Gaussian function. Further smearing over $\Delta p/\epsilon$ is achieved by averaging over random forces (introduced in Sec. 4).

4. The quantum moment generating function

Following the classical weighting of ΔE , we introduce the following ansatz for the stochastic Wigner propagator,

$$-iL_M \equiv \lim_{t \rightarrow 0^+} t^{-1} \left(\int dP(k) \mathcal{W} \underline{M}[U(k)^t] \underline{Z}_{\text{kin}}^{-1/2} \mathcal{W}^{-1} - \underline{I} \right). \quad (12)$$

The super-operator \mathcal{W} denotes the transformation of an operator into its Weyl symbol, and \underline{M} is some maximum entropy weighting on the propagators, $U(k) = e^{-i\hat{H}_k/\hbar}$, that accounts for the environment's tendency to dissipate energy from the system. By analogy with the exponential weighting of energy changes in the classical case, we introduce dissipation using $M \equiv e^{-\beta\hat{H}/4}$, where $\beta = 1/k_B T$, and T is the temperature of the interacting thermal environment.

Each observer uses their information on $dP(k)$ and T to estimate the time-evolution as a series of applications of Eq. (12). This perspective makes a whole class of paradoxes disappear. The measurement of an observable at a future moment in time does not cause collapse of the wavefunction. Instead, both the preparation of the initial state and the measurement give distinct pieces of information on the possible influence of the environment, through the set of possible propagators (here $dP(k)$ and \hat{H}, T).

The exponential dependence on $\Delta\hat{H}$ makes sense in view of the fact that transitions are only generated by random deviations of $U(k)$ away from $e^{-i\hat{H}/\hbar}$. The energy weighting of M parallels the KMS condition on the symmetry of the corresponding transitions in a thermal bath [28].

In order for the propagator to be normalized we must have,

$$\mathcal{Z}_{\text{kin}} = \lim_{t \rightarrow 0^+} \int dP(k) \underline{M}^\dagger[U(k)^{-t}][I]. \quad (13)$$

The dependence of the kinetic partition function on the starting configuration in the classical case translates to an operator valued \mathcal{Z}_{kin} in the quantum case. Its derivatives generate moments,

$$\left. \frac{\partial^n \mathcal{Z}_{\text{kin}}}{\partial(-\beta/2)^n} \right|_{\beta=0} = \sum_{m=0}^n \binom{n}{m} \sum_{j=0}^m 2^{-m} \binom{m}{j} \lim_{t \rightarrow 0^+} \int dP(k) \hat{H}^j (-\hat{H}_k(t))^{n-m} \hat{H}^{m-j} \quad (14)$$

where $\hat{H}_k(t) \equiv U(k)^{-t} \hat{H} U(k)^t$. When \hat{H} commutes with the density matrix, ρ , the sums can be carried out to give

$$\frac{\partial^n \text{tr}[\mathcal{Z}_{\text{kin}} \rho]}{\partial(-\beta/2)^n} = \int dP(k) \text{tr}[(H_k(t) - H(0))^n \rho]. \quad (15)$$

To make an explicit comparison to the Langevin equation, we let $k \in \mathbb{R}$ be the strength of a random impulsive force so that the set of possible propagators for the system is described by a Wiener-process distribution for the forces,

$$U(k) = e^{-\frac{i}{\hbar}(\hat{H} - k\hat{x})}, \quad dP(k) = (2\pi\sigma^2/\epsilon)^{-d/2} e^{-k^2\epsilon/2\sigma^2} dk. \quad (16)$$

With this choice, it can be shown that $-iL_M$ is the generator of a quantum dynamical semigroup with Lindblad form,

$$\begin{aligned} \mathcal{W}^{-1}[-iL_M[\mathcal{W}\rho]] &= -\frac{i}{\hbar}[\hat{H}, \rho] - \frac{\sigma^2}{4\hbar^2}[\underline{M}[\hat{x}^2] - \underline{M}^\dagger[\hat{x}^2], \rho] \\ &\quad - \frac{\sigma^2}{\hbar^2} \left(\underline{M}[\hat{x}]\rho\underline{M}^\dagger[\hat{x}] - \frac{1}{2}\{\underline{M}^\dagger[\hat{x}]\underline{M}[\hat{x}], \rho\} \right). \end{aligned} \quad (17)$$

The proof is by expanding $e^{-i\hat{H}_k\epsilon/\hbar}$ and retaining all terms of order ϵ , and noting that $\mathcal{Z}_{\text{kin}}^{-1/2} = I - \frac{1}{2}(\mathcal{Z}_{\text{kin}} - I) + O(\epsilon^2)$. Expanding $\underline{M}[\hat{x}]$ in powers of β proves the $O(\beta^2)$ equivalence with the Caldeira-Leggett master equation mentioned earlier.

5. The stochastic Wigner propagator

We proceed to simplify the Wigner propagator for the stochastic quantum mechanical evolution defined in Section 4. From its connection to the Caldeira-Leggett equation, it is expected that this propagator will recover the Langevin form at high temperature. Our purpose is to show how this comes about and to investigate the unique properties of the kinetic partition function.

We follow the derivation of Ref. [25] starting from the unnormalized propagator,

$$-iL'_M = \lim_{t \rightarrow 0^+} t^{-1} \left(\int dP(k) \mathcal{W} \underline{M}[U(k)^t] \mathcal{W}^{-1} - \underline{I} \right) \quad (18)$$

$$\begin{aligned} \underline{M}[U(k)^t] &= e^{-\beta\hat{H}/4} e^{-it(\hat{H}+k\hat{x})/\hbar} e^{\beta\hat{H}/4} \\ &= e^{-it(\hat{H}+k\underline{M}[\hat{x}])/\hbar}. \end{aligned} \quad (19)$$

To proceed, we expand $\underline{M}[\hat{x}]$ as a power series in β and drop terms higher than $(\beta\hbar)^2$,

$$\begin{aligned} \underline{M}[\hat{x}] &= \hat{x} - \frac{\beta}{4}[\hat{H}, \hat{x}] + \frac{\beta^2}{32}[\hat{H}, [\hat{H}, \hat{x}]] - O((\beta\hbar)^3) \\ &= \hat{x} + \frac{i\beta\hbar}{4m}\hat{p} + \frac{(\beta\hbar)^2}{32m}\hat{V}' - O((\beta\hbar)^3) \end{aligned} \quad (20)$$

Note that truncating the series at any order still leads to a propagator that is completely positive. This parallels the procedure by Diósi of truncating the expansion of $\beta\hbar\omega/2 \coth(\beta\hbar\omega/2)$ [2]. The ratio of \hat{x} and \hat{p} coefficients in Eq. (20) shows the origin of the factor λ_{DB}^{-2} in the decoherence time-scale.

The presence of $i\hat{p}$ in the effective Hamiltonian technically requires us to re-derive the momentum part of the action [21]. However, the next step shows the same result even if \hat{p} is directly substituted for $m\Delta x/\epsilon$ in writing down the action.

Using this damped Hamiltonian in the action functional, we note that the integral over Δy (see Ref. [25]) is unaffected. Thus, to order $(\beta\hbar)^2$,

$$e^{-iL'_M\epsilon} = h^{-d}\delta\left(\Delta x - \frac{\epsilon\bar{p}}{m}\right) \int d\bar{y} \int (2\pi\sigma^2/\epsilon)^{-d/2} dk e^{-\frac{i}{\hbar}\phi_k - \frac{k^2\epsilon}{2\sigma^2}} \quad (21)$$

$$\phi_k \equiv \bar{y}\Delta p + \epsilon V(\bar{x} + \bar{y}/2) - \epsilon V(\bar{x} - \bar{y}/2) + k\epsilon \left[\bar{y} + \frac{i\beta\hbar}{2\epsilon}\Delta x + \frac{(\beta\hbar)^2}{32m}(V'(\bar{x} + \bar{y}/2) - V'(\bar{x} - \bar{y}/2)) \right]$$

Performing the Gaussian integral over k leads to,

$$e^{-iL'_M\epsilon} = h^{-d}\delta\left(\Delta x - \frac{\epsilon\bar{p}}{m}\right) \int d\bar{y} e^{-\frac{i}{\hbar}\phi_2 - \frac{\sigma^2\epsilon}{2\hbar^2}\Theta(\bar{x},\bar{y})^2 + \frac{\beta^2\sigma^2}{8\epsilon}\Delta x^2} \quad (22)$$

$$\phi_2 = \bar{y}\Delta p + \epsilon V(\bar{x} + \bar{y}/2) - \epsilon V(\bar{x} - \bar{y}/2) + \frac{\beta\sigma^2}{2}\Delta x\Theta(x, \bar{y})$$

$$\Theta(x, y) = y + \frac{(\beta\hbar)^2}{32m}(V'(x + y/2) - V'(x - y/2))$$

We derive corrections to the Langevin equation (Eq. (27)) by expanding in powers y (which scales as \hbar).

$$V'(x + y/2) - V'(x - y/2) = yV''(x) + \frac{y^3}{2^2 3!}V^{(4)}(x) + O(y^5) \quad (23)$$

$$\Theta(x, y) = y \left(1 + \frac{(\beta\hbar)^2}{32m}(V''(x) + \frac{y^2}{2^2 3!}V^{(4)}(x)) \right) + O(y^5) \quad (24)$$

Neglecting the terms of order y^3 and higher, $\Theta \simeq \bar{y}u$ in

$$e^{-iL'_M\epsilon} \simeq \delta\left(\Delta x - \frac{\bar{p}\epsilon}{m}\right) (2\pi\sigma^2 u^2 \epsilon)^{-d/2} \exp\left\{ \frac{\beta^2\sigma^2}{8\epsilon}\Delta x^2 - \frac{1}{2\sigma^2 u^2 \epsilon} \left(\Delta p + \epsilon V'(\bar{x}) + \frac{\beta\sigma^2 u}{2}\Delta x \right)^2 \right\} \quad (25)$$

$$u(\bar{x}) = 1 + \frac{(\beta\hbar)^2}{32m}V''(\bar{x}) \quad (26)$$

The normalization and the counter-term canceling Δx^2 in constructing $\beta dE/2$ both come out exactly. This is equivalent to the Langevin equation,

$$\Delta p = - \left(V'(x) + \frac{\beta\sigma^2 u}{2} \frac{\Delta x}{\epsilon} \right) \epsilon + \sigma u(\bar{x})k(t)\epsilon, \quad (27)$$

interpreted in the Stratonovich sense.

Even to first order in β , there is a clear difference from the classical case due to the appearance of the Hessian, V'' , in $u(x)$. This can be seen as effectively an expansion term countering quantum confinement. The confinement term becomes important when the Hessian is on the order of $1/\hbar^2$. It then multiplies any noise present from the environment, increasing the

diffusion rate. Ref. [1] (Eq. (9.33) of Ref. [29]) identifies a tunneling contribution to crossing an energy barrier of $(\beta\hbar)^2(V''(\text{min}) + V''(\text{barrier}))/24m$. This compares remarkably with the present derivation of $(\beta\hbar)^2V''/32m$ from a minimal model not based on specific properties of the environment.

This term also compares with the $[\hat{p}, [\hat{p}, \rho]]$ term put forward in Ref. [2]. There, expansion of $\coth(\beta\hbar\omega/2)$ and integration over an Ohmic frequency distribution showed that the coefficient should be the decoherence timescale times an extra factor of $\frac{4}{3}\lambda_{DB}^4\hbar^{-2} = \beta^2\hbar^2/12m^2$. To connect this to the Hessian, note that the \hat{p}^2 operator would effect an energy change on the order of $-mV''\delta x^2$. A similar expansion was noted in Ref. [3].

Because u appears in Eq. (27) as a scale for the temperature, it affects the low-temperature stationary distribution. Thus, the present work falls in-line with Ref. [30], which noted that a completely positive, translation invariant environment cannot achieve the Gibbs distribution at low temperature. We do not view this as a defect, since there is strong evidence that the unattainability of absolute zero is a real physical effect caused by continuous interaction with the environment [31, 32]. All of the previous works that find [33, 2, 30] and use [29] terms of this order include this effect.

Rather than analyze the dynamics as a classical or quantum Langevin equation, it is simpler still to study the properties of the normalization constant. Defining the normalization as in Eq. (13), it must have a Weyl symbol,

$$\mathcal{Z}_{\text{kin}}(x', p') = \iint dpdx e^{-iL'_M\epsilon} = \left(1 + \frac{\epsilon\beta\sigma^2u}{2m}\right)^{-d/2} \exp\left(\frac{\epsilon\beta^2\sigma^2p'^2}{8m^2}\right) \quad (28)$$

The energy exchange is,

$$\langle dE|x', p'\rangle = -\frac{\partial \log \mathcal{Z}_{\text{kin}}}{\partial \beta/2} = \frac{\sigma^2\epsilon}{m} \left(\frac{ud}{2} - \beta\frac{p'^2}{2m}\right). \quad (29)$$

In deriving Eq. (28), it is important to expand $\bar{p} = p' + \Delta p/2$ and retain all terms which contribute to order ϵ . The final result supports the choice of M from Eq. (12) and identifies the local quantum temperature with a scaled kinetic energy. Specifically, if the expected energy exchange during propagation is zero, then the observer can infer that the environment has an energy scale of T satisfying $k_B T u = \langle \frac{p^2}{m} \rangle$. In this formalism, temperature is a subjective, statistical property of the environment rather than an objective property of the system.

6. Conclusions

We have developed a maximum entropy formalism for describing the statistics of open quantum systems. The emergence of statistics has been framed in the context of multiple observers who have incomplete information on the time course of the system's Hamiltonian. This ansatz reproduces the Caldeira-Leggett equation, the connection between dissipative and decoherence timescales, and adds a quantum correction to the classical Langevin equation. It demonstrates how the density matrix of the system can collapse as a result of the inference process. This provides a method for attacking several open problems such as the reconstruction problem for relativistic theories of quantum mechanics [5]. Explicitly, local observables emerge because averaging over the noise process destroys coherences.

The analysis of Eq. (17) shows separate origins for decoherence and dissipation in the quantum Langevin equation. Decoherence comes about from random environmental noise, and this uncertainty carries an inherent time-scale. It plays the major role in determining the pointer states as those that commute with the environmental coupling [6]. However, dissipation comes from a separate source – the overall tendency of an infinite environment to take energy away

from the system [7]. The dissipation used here (M) is characterized by an energy rather than a time-scale. Without random forces, M commutes with the dynamics, and has no opportunity to act. When randomness is present, both decoherence and dissipation work together to remove memory from the system. The decoherence timescale is much shorter because the uncertainty in the force destroys the phase structure (as a function of the momentum jumps, Δp) in the propagator.

This work can be adapted to more specific models for the stochastic environmental forces, including history-dependent processes. Our derivation offers a system-centric alternative to derive the fluctuation-dissipation theorem (as second derivatives of $\log \mathcal{Z}_{\text{kin}}$), and highlights the essential, physical features of the FDT and the relation between information and decoherence.

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