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CREATING THE BARYON ASYMMETRY AT THE ELECTROWEAK PHASE TRANSITION*

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Abstract

At temperatures above the weak phase transition, baryon and lepton number are badly violated. We explore the suggestion that the baryon asymmetry might be produced at the transition, if the transition is first order. We find that the asymmetry is proportional to a CP violating parameter, a large power of the gauge coupling and to factors which depend on the details of the transition. In extensions of the standard model, such as multi-Higgs or supersymmetric models, the result may be consistent with observations.

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It has long been known that baryon and lepton number are not conserved in the standard model, as a consequence of anomalies.[1] States of different baryon number are smoothly connected to one another through different configurations of the gauge and Higgs fields, separated by a potential barrier. At extremely low temperatures, baryon number violation, being a tunneling phenomenon in a weakly coupled theory, is highly suppressed. On the other hand, in the last few years, it has become clear that baryon number is badly violated at temperatures much above M_W ($B-L$ is conserved).[2-5] Indeed, once the temperature is large compared to M_W, the system is well described by classical statistical mechanics. At temperatures below the weak phase transition, the lowest energy barrier is associated with the "sphaleron", a finite energy solution of the classical equations of motion with a single negative mode.[2] The rate for barrier penetration is essentially the Boltzmann factor for this configuration, which, using the computed values of the sphaleron energy, yields a rate

$$\Gamma \sim e^{-\frac{BM_W}{\alpha_W T}} \tag{1}$$

where B is a number which depends rather weakly on the Higgs mass, varying between about 3 and 6. Above the weak phase transition, the situation is more complicated. The classical thermodynamics of the system is equivalent to a three dimensional field theory with no small dimensionless parameter. On dimensional grounds, however, the rate must be given by

$$\Gamma = \kappa(\alpha_W T)^4. \tag{2}$$

A recent simulation gives $\kappa = 0.01 - 1$.[6] While no single classical configuration dominates this rate, we can give a heuristic description in terms of instanton trajectories which will be useful. It is generally believed that the three dimensional field theory has a mass gap, $a\alpha_W T$, where a is a number of order unity. Correspondingly, the correlation length of the high temperature theory (the so-called magnetic screening length) is $\xi = (a\alpha_W T)^{-1}$. Consider now instantons in the high temperature theory. These will exist with arbitrary scale size, from $\rho = 0$ to $\rho \sim \xi$. The instanton represents a particular tunneling trajectory through configuration space. The barrier height associated with such a trajectory is necessarily of the form $E_\rho = \frac{c}{\alpha_W \rho}$, where $c \sim 1$. Clearly, then, the smallest barriers are associated with the largest possible values of ρ, i.e. $\rho \sim \xi$. Such a configuration has a Boltzman factor of order unity, while the prefactor is of order ξ^{-4}.

The large rate of baryon number violation has important implications for any baryon number produced at very early times. For example, if no net $B-L$ is produced, the baryon (and lepton) numbers will completely disappear. It also raises the intriguing possibility that the observed baryon number could arise at temperatures of order the scale of weak interactions. This could have significant implications for our understanding of cosmology. In particular, in inflationary models, one usually requires significant reheating after inflation in order to produce baryons. This would not be necessary if baryons could be produced at such low temperatures.

The possibility that the baryon asymmetry might be produced al the weak phase transition was first discussed by Kusmin, Rubakov and Shaposhnikov,[3] and has been must extensively explored in subsequent papers of Shaposhnikov and collaborators.[7] Other important works on the subject are those of McLerran,[8] Turok and Zadroxny,' and of Cohen, Kaplan and Nelson.[10] The main point is tbat if the phase transition in the Weinberg-Salam model is al kasl mildly first order, then the three conditions enumerated by Sakharov" necessary lo obtain a net asymmetry are satisfied. Baryon violation is provided by the $SU(2)$ gauge interactions themselves. CP violation is already present in the standard model, and extensions of the standard model, such as multi-Higgs systems, supersymmetry or technicolor tend to yield much larger violations of CP. Deviations from equilibrium will automatically arise if the transition is first order.

Many of the specific proposals which have been made for the origin of the baryon asymmetry at the weak phase transition have been based on the minimal standard model. It is clear from the start, however, that unless the dynamics of the bigh temperature theory exhibits certain bizarre features,[7] CP violation in this theory is simply too small to yield anything like the observed asymmetry, whatever the details of the phase transition might be. Moreover, as recently stressed in ref. 12, there is another strong constraint on any such picture of baryon number production, which rules out the minimal standard model. Once the phase transition is compkkd, the Higgs field will have some expectation value ϕ_f. The corresponding sphaleron (free-) energy is proportional to ϕ_f. If ϕ_f is too small, the sphaleron rate will be larger than the expansion rak and any baryon number produced during the phase transition will be washed out. This almost certainly requires that the Higgs boson be so light that it would have shown up in recent LEP experiments.

Since there are numerous possibk extensions of the standard model, it is necessary to make simplifying assumptions. Those we make here are not, we believe, essential; our analysis is easily extended to a wide variety of situations, including supersymmetry, technicolor, and multi-Higgs theories. In particular, we will assume in the discussion which follows that the new physics responsibk for CP violation is associated with energy scales large compared lo T_c, the transition temperature, and that the effective theory at T_c contains the usual quarks and leptons, and a Higgs doublet, ϕ. For reasons which will become clear shortly, we will also allow for the possibility of au additional scalar singlet, s. In the effective lagrangian, CP will be broken not only by the usual phase in the KM matrix, but also by various non-renormalixable operators. We will focus on the dimension-six operator[7,9]

$$\mathcal{O} = \frac{1}{M^2}\frac{g^2}{32\pi^2}|\phi|^2 F\tilde{F} = \frac{1}{3M^2}\partial_\mu|\phi^2|j^\mu. \tag{3}$$

Here j^μ is the baryon current, and we have used the anomaly equation. In theories with singlets, we will consider the dimension-5 operator

$$\mathcal{O}' = \frac{1}{3M'}\partial_\mu s j^\mu. \tag{4}$$

in the minimal supersymmetric standard model, for exampk, $\mathcal{O}$ would be generated al one loop by a diagram with gauginos and higgsinos in the iakrmediate elate. $\frac{1}{M^2}$ would thus be of order some combination d CP violating phases, 6, divided by some typical supersymmetry breaking mam-squared. There are no strong limits on 6. In a non-minimal supersymmetric model with a compkx gauge siagkt geld, S, s could be some component d tbis geld. It could possess tree level, CP violating couplings to the higgsino fields. The coefficient, $\frac{1}{M'}$, would be d order 6 divided by a super-symmetry breaking mass.

Already, we can see the potential for baryon number creation. Indeed, consider two extreme cases. First, suppose the Higgs field is changing very slowly with lime, so that the system can respond adiabatically, in *the* sense that at each instant the baryon number violation rate, $\Gamma(\phi, T)$, is that appropriate to the value d the temperature and Higgs field at tbat moment. Since the dominant processes are associated with gauge boson wavelengths d order ξ, rapid change means change on a lime scale much shorter than ξ. For small ϕ ($g\phi < \alpha_W T$), Γ is unknown. Since Γ falls exponentially for $g\phi \gg \alpha T$, a simple model for Γ is to take $\Gamma = (\alpha_W T)^4$ for $g\phi < \alpha_W T$ and $\Gamma = 0$ for larger ϕ.[†] At each instant, the system will tend to that value of the baryon number which minimizes the free energy. The CP violating operator corresponds to a term in the free energy linear in the baryon number or the corresponding chemical potential. (This is a realization d the idea of "spontaneous baryogenesis."[14]) We are interested in the minimum of the free energy, n_B^0, subject to the constraint that all of the doublet species have equal

† For large ϕ, the rate baa been computed in ref. 13. For $m_H \sim$ my, ud small ϕ, their result is similar to the $\phi = 0$ result with $\kappa \sim 1$.

densities. For three generations, an elementary calculation gives

$$n_B^0 = \frac{T^2}{12M^2}\partial_0|\phi^2| \qquad n_B^0 = \frac{T^2}{12M'}\partial_0 s \tag{5}$$

for the doublet or singlet case, respectively. The baryon number then obeys an equation

$$\frac{dn_B}{dt} = -18\Gamma T^{-3}(n_B - n_B^0) \tag{6}$$

which can be obtained from considerations of detailed balance.[3,4] Because of the four powers of α_W appearing in Γ, we can neglect n_B relative to n_B^0, on the right hand side of this equation, provided $\frac{dn_B^0}{dt}$ is large enough. We will see shortly that this is the case for a broad range of model parametem. Substituting our expression for n_B^0, and using our simple model for Γ yields for the baryon number

$$n_B \sim \kappa\frac{3\alpha_W^6}{2g^2M^2}T^5 \qquad n_B \sim \kappa\frac{3\alpha_W^5}{2gM'}T^4. \tag{7}$$

Here, in the singkt came we have assumed that $gS \sim \alpha T$ when baryon violation turns off.* These numbers need not be so small. In the singlet case, if we suppose the CP violating phase is of order one, and $M' \sim T$, then the baryon to photon ratio is of order 10^{-8}! In models with only doublets, this result is suppressed by an additional power of α_W. These estimates are rather rough. It is already clear, though, that potentially one can obtain a baryon asymmetry as large as that which is observed.

If n_B^0 is changing much more slowly in time, $n_B(t) \approx n_B^0(t)$ until Γ becomes exponentially small. In this case, one obtains a result suppressed by more powers of α_W, due to the time derivative in n_B^0. The extreme case of this type arises if the transition is second order. Then the asymmetry is suppressed by the Hubble constant.[3]

Before describing the case where the transition occurs suddenly, it is helpful to understand these results in another way. Consider the operator $\mathcal{O}$ written in the form containing $F\tilde{F}$. As in our heuristic discussion above, consider a single instanton trajectory, and treat the usual instanton time, τ, as parameterizing a path in configuration space. $\tau = 0$ corresponds to the top of the harrier. If we replace the gauge field in the lagrangian by its classical value as a function of τ, then we obtain a lagrangian for τ for small τ of the form

$$\mathcal{L}(\tau, \dot{\tau}) = c_1\frac{4\pi}{g^2\rho}\dot{\tau}^2 + \frac{4\pi b_1}{g^2\rho^3}\tau^2 \tag{8}$$

where $\rho \sim \xi$ is the instanton scale size and c_1 and b_1 are coefficients of order unity.* For small τ, $\mathcal{O}$ has the form

$$\mathcal{O} = c_2\frac{|\phi|^2}{M^2}\frac{\dot{\tau}}{4\pi\rho} \tag{9}$$

and similarly for $\mathcal{O}'$. In the adiabatic limit, where the field ϕ is essentially constant, τ and $\dot{\tau}$ will be Boltcmann distributed at each instant. The canonical momentum receives a ϕ-dependent contribution from $\mathcal{O}$, eq. (9). This has the effect of skewing the velocity distribution, giving rise to an excess flux over the barrier in one direction. Because of the anomaly, this corresponds to a net production of baryons or antibaryons, depending on the sign of 6. Proceeding in this way one obtains a rate equation of the form eq. (6). In particular, this heuristic argument gives the correct dependence on α_W.

This picture is readily adapted to the case where the field ϕ changes suddenly. Despite the fact that this corresponds to a more violent departure from equilibrium, it does not in general lead to a much larger production of baryons. Before the transition, one has a Boltzmann distribution for τ and $\dot{\tau}$, and this distribution remains essentially unchanged as ϕ changes. However, the system receives a "kick" from the sudden change in ϕ. In the time ϕ changes from 0 to ϕ_0, the value at which baryon number violation turns off, the velocity changes by an amount:

$$\Delta\dot{\tau} = \int dt\, \frac{d}{dt}\left(\frac{c_2 g^2}{c_1 16\pi^2}\frac{|\phi|^2}{M^2}\right) = \frac{c_2 g^2}{c_1 16\pi^2}\frac{\phi_0^2}{M^2}. \tag{10}$$

$\Delta\dot{\tau}$ has a definite sign. If it is large compared to the initial velocity, it will send the system over the barrier in the direction corresponding to the production of (say) baryons rather than antibaryons. If it is small compared to this velocity, it will have no effect on the baryon number. The fraction of the distribution with velocities, $\dot{\tau} < \Delta\dot{\tau}$, is simply of order $\Delta\dot{\tau}$. If Δt is the time it takes for the Higgs

* Whether or not this is the case depends on the details of the phase transition, One can easily imagine that $S \sim |\phi|^2$, for example.

* There is some arbitrariness in these definitions, since the result depends on the gauge choice for the instanton. Here we have indicated the factor of 4π coming from the angular integration. The constant will depend upon the gauge choice.

field to rise to ϕ_0 over a correlation volume, ξ^{-3}, the final baryon number is of order the product of this fraction, Al, and I :

$$n_B \sim \frac{|\phi_0|^2}{M^2}(\alpha_W)^5 \Delta t T^4. \tag{11}$$

Here we have attempted to keep track of g's and 4π's, but not (unknown) coefficients of order unity. We will see shortly that $g\phi_0 \sim \alpha T$, so only if $\Delta t \sim \xi$ is this result comparable to that obtained in the "adiabatic" case. A similar expression holds in the case of the operator $\mathcal{O}'$. The picture described here is close to that described in ref. 9, where the behavior of certain particular field configurations is considered.

In order to actually determine the baryon number produced in this rapid case, we need to determine how the baryon number violating procem turns off as the Higgs field changes. In this case, in contrast to the adiabatic case, where the boson distributions follow the instantaneous value of the Higgs field, the distribution remains essentially unchanged from its initial value. Consider the system in a box of size of order ξ. For wavelengths of order ξ, the gauge fields obey an equation of the form

$$(\partial_t^2 + \xi^{-2} + (g\phi)^2)A_\mu(k) = (\xi^{-\frac{1}{2}}A^2, \xi^{-3}A^3). \tag{12}$$

Thus for $\phi = 0$, the system become8 non-linear for $A \sim \xi^{\frac{1}{2}}$. If we examine eq. (12), however, we see that for such A,,, the equations become linear once $g\phi > a\alpha_W T$. At thii point, no further passage over the barrier can occur; the barrier ha8 simply "grown" and there is not enough energy available in these modes. Thus the process turns off both for slow and rapid changes in ϕ at about the same value of ϕ. In each case, the relevant value of the Higgs field is very small.

It is worth commenting here on our assumption that the scale of CP violation is large compared to T_c. Clearly this is not essential to the analysis. Indeed, T_c cannot be too much smaller than this scale if one is to obtain a large enough asymmetry. If T_c is as large or larger than this scale, then it is necessary to include additional fields. Study of particular examples indicates that these fields in some cases lead to further suppression of the asymmetry, but in many instances one obtain8 a good estimate by replacing the masses which appeared in our formulae above by T_c. T_c itself is constrained by the requirement that the baryon violation rate be extremely small after the phase transition. More detailed analysis of particular models will appear elsewhere.

We now have all the ingredients to estimate the baiyon asymmetry, once the behavior of the Higgs field is known as a function of time. In a first order phase transition, baryon number will be produced near the bubble walls, where the Higgs

field is changing in time. In order to compute the asymmetry, it is thus necessary to know about the shape aud velocity of the walls. We will leave a survey of different models for a future publication. Here we simply illustrate some of the possible behavior8 by considering the minimal standard model,[15] even though this cannot be a realistic model of baryon generatipn. For Higgs masses smaller than m_W, the transition is first order. The effective potential for the ϕ field a8 a function of temperature is given by, for small self-coupling A and setting $\sin^2\theta_W = 0$ to simplify the writing:

$$V(\phi, T) = M^2(T)\phi^2 - \frac{3g^3}{32\pi}T\phi^3 + \lambda\phi^4 \tag{13}$$

where $M^2(T) = \frac{3g^2T^2}{32} - m^2$. When the phase transition occurs, the coefficient of the quadratic term is extremely small, $M^2(T) \sim \frac{\alpha_W^3 T^2}{\lambda}$; otherwise the potential has only a minimum at the origin. We can make a crude estimate of the bubble wall velocity and size (well after the bubble forms) by requiring that in the rest frame of the wall, the pressure is constant. This pressure receives an extra contribution from the motion of the gas in this frame. The momentum change of a particle passing through the wall can he estimated by assuming that the particle's energy is conserved, while its mass changes due to the change in ϕ. This gives $v^2 \sim \frac{\Delta P}{\Delta E}$, where ΔP and AE are the changes in pressure and internal energy across the wall. From eq. (13), $v^2 \sim \frac{\alpha^2}{\lambda}$. The shape of the wall can be inferred from similar considerations. For small ϕ, one finds $\phi \sim e^{M(z-vt)}$, where $M \sim (\frac{\alpha^3}{\lambda})^{\frac{1}{2}}T$. As a result, if A is not too small, the scalar field is changing rather slowly in space and time and the system is in the adiabatic regime described earlier. On the other hand for such a field, n_B^0 is changing quickly enough that the approximations leading to eq. (7) are valid. As one increasea λ, and the transition become8 more second order, the amount of baryon number is reduced; decreasing λ brings us to the "sudden" regime. Considerations of this type apply as well to the minimal supersymmetric standard model, where the quartic coupling8 are of order g^2, and the scalar masses are of order M_W.

In other models, the transition might be strongly first order, with bubbles expanding at nearly the speed of light, and with a wall of microscopic dimensions. This is the regime of rapid change of the Higgs field. Here what is needed is an estimate of the time Δt, appearing in eq. (1 I), required for the zero momentum mode of the field in a correlation volume, ξ^3, to reach ϕ_0. In a multi-Higgs model, we might expect this time to be of order a timessome microscopic (mass) parameter in the Iagrangian. Since the characteristic time for baryon violation is rather long (ξ), this may be a source of additional suppression.

In sum, it seems quite reasonable to think that the baryon number of the universe was created at the electroweak phase transition, in some modest extension of the standard model. There are large uncertainties in the calculations described here, however, particularly in the actual calculation of the rate Γ. Detailed studies of the phase transition in particular models are also essential, including not only the structure of the bubble wall but also flow of baryon number across the wall. One should also reconsider models such as that of ref. 10, in which there are other sources of lepton number violation in the theory, but in which the mechanisms described here may also operate efficiently.

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