MONOCHROMATIC BREMSSTRAHLUNG FROM THIN CRYSTALS*

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A simple kinematic argument can be used to extend the calculations of Barbiellini, Bologna, Diambrini and Murtas[1] and Uberall[2] to give the effect of measuring only a small angular region of the bremsstrahlung produced from thin crystals. The net result is that it should be possible to change the sharp breaks in the bremsstrahlung, calculated and observed by Barbiellini et al., into very narrow spikes and hence obtain essentially monochromatic bremsstrahlung. See Fig. 1.

The major curve is taken from Ref. 1. The shaded area is the spectrum which should result if an angular diameter of $0.7\ mc^2/E_0$ is accepted. The expected enhancement increases with increasing electron energy.

The approximation is valid for perfect crystals and the extent to which the actual crystal will reduce this effect is not determined. It would seem reasonable since this is only a kinematic extension of the above calculations that it should be valid to the same extent.

The present paper is merely an observation that since the Bragg condition causes the momentum $\vec{q}$ transferred to the crystal lattice to be quantized and normal to a set of crystal lattice planes, the momentum $\vec{p}_1$

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[1]Barbiellini, Bologna, Diambrini, and Murtas, Phys. Rev. Lett. 8, 454 (1962).

[2]H. Uberall, Phys. Rev. 103, 1055 (1956).

of the combined photon and output electron is completely determined by a knowledge of the input momentum $\vec{p}_o$, the angle of incidence to lattice plane, θ_o, and the lattice spacing, d; $\vec{q} = \frac{nh}{d}$ where h is Planck's constant, and n is an integer. The combined energy of the final photon and electron is equal to that of the incident electron since the energy transferred to the lattice can be ignored.

This allows a large number of possible momentum transfers corresponding to different points in the inverse lattice. To obtain a useful narrowing of the spectrum one must also (as in Ref. 1) make use of the fact that the momentum transfer, along the direction of the incoming electron, is of critical importance and the beam must be so oriented with respect to the crystal that many inverse lattice points suffer the same longitudinal momentum transfer.

Since $\vec{p}_1$ is determined, the following kinematic relationships exist (see Figs. 2 and 3) where $\vec{k}$ is the photon momentum and $\vec{p}_2$ the final electron momentum. In the following the velocity of light = 1 and m is the electron mass.

$$p_1^2 = p_o^2 + q^2 - 2\, p_o q \sin \theta_o \tag{1}$$

$$p_2^2 = p_1^2 + k^2 - 2\, p_1 k \cos \theta_k \tag{2}$$

Since the initial and final energies are the same.

$$p_o^2 + k^2 - 2k\, (p_o^2 + m^2)^{\frac{1}{2}} = p_2^2 \tag{3}$$

$$p_o^2 - p_1^2 = 2k \left[(p_o^2 + m^2)^{\frac{1}{2}} - p_1 \cos \theta_k\right] \tag{4}$$

This gives: [eliminating p_1 from (1) and (4),]

$$k = \frac{2p_o q \sin\theta_o - q^2}{2\left[(p_o^2 + m^2)^{\frac{1}{2}} - (p_o^2 + q^2 - 2p_o q \sin\theta_o)^{\frac{1}{2}} \cos\theta_k\right]} \qquad (5)$$

Equations 3 and 4 of reference 1 express the energy spectrum of the coherent bremsstrahlung in terms of the momentum transfer to the inverse lattice.

Equation 3, Ref. 1.

$$I(x,\theta_o) = [1 + (1 - x)]^2 \, [\psi_1^c(\delta) + \psi_1^{0*}(\theta_o,\delta)] - \tfrac{2}{3}(1 - x)[\psi_2^c(\delta) + \psi_2^{0*}(\theta_o,\delta)] \qquad (6)$$

$\frac{k}{p_o} = x$, $\delta = \frac{m}{2p_o}\left(\frac{x}{1 - x}\right)$ is the minimum momentum transferred to the lattice in units of m. $\psi_{12}^{\;c}$ give the incoherent bremmstrahlung and $\psi_{12}^{\;0*}$ the coherent.

Equation 4, Ref. 1.

$$\psi_1^{0*}(\theta_o,\delta) = \frac{(2\pi)^2}{\Delta} 4\delta \sum_{\vec{g}} |F|^2 \frac{\exp - Ag^2}{(\beta^{-2} + g^2)^2} \frac{g^2}{g_2^2\theta_o^2}, \qquad (7)$$

$$\psi_2^{0*}(\theta_o,\delta) = \frac{(2\pi)^2}{\Delta} 24\,\delta^2 \sum_{\vec{g}} |F|^2 \frac{\exp - Ag^2}{(\beta^{-2} + g^2)^2} \frac{g^2}{g_2^4\theta_o^4} (g_2\theta_o - \delta) \qquad (8)$$

Here Δ is the volume of a unit cell in the lattice. $|F|$ is the Laue Bragg structure factor, $\beta = \frac{111}{Z^{1/3}}$, A = 126, and g is the reciprocal lattice vector in units of m. In the case considered which is the same as that of reference 1, we are considering momentum transfers in the $\vec{b}_2 \equiv [1\ \bar{1}\ 0]$ $\vec{b}_3 \equiv [0\ 0\ 1]$ plane of the inverse lattice of diamond (see Fig. 4) and

$g_2 = \vec{g} \cdot \vec{b}_2 > \delta/\theta_o$. It is pointed out by Uberall[2] that while δ is the minimum possible longitudinal momentum transfer the actual minimum $q_{\ell_{min}}$ is a function of the transverse momentum transfer $q_\perp$, or $q_{\ell_{min}} = \delta + \frac{q_\perp^2}{p_o}$. This is a negligible correction and we use $q_{\ell_{min}} \simeq \delta$.

These sums are made over all values of $g_2\theta_o \geq \delta$ and the sharp breaks in the spectra of Barbiellini et al. occur when $g_2\theta_o = \delta$. It can be seen from equations 7 and 8 that the peak energy of each inverse lattice point is determined by $g_2\theta_o$ and not by g. This leads to the orientation used by Barbiellini et al. and illustrated in Fig. 4. The momentum vector of the incoming electron is at a small angle θ_o to the b_1 axis and so inclined that the plane of minimum momentum transfer (at the energy of the first "break") intercepts the first column of inverse lattice points. In the illustration given these all contribute a single energy determined by the longitudinal momentum transfer $g_2\theta_o$. For higher energy photons or smaller angles, θ_o, these inverse lattice points will not contribute at all, while for lower energies or larger angles they will contribute a smaller amount. The shaded area of Fig. 4 would then lie closer to the b_3 axis. In all configurations all lattice centers emitting the same energy suffer the same longitudinal momentum transfer. It is interesting then to express our kinematic relation, Eq. (5), in terms of the longitudinal momentum transfer, $q_\ell = q \sin\theta_o \simeq q\theta_o$

$$k = \frac{2\, p_o q_\ell - \dfrac{q_\ell^2}{\theta_o^2}}{2\left[\left(p_o^2 + m^2\right)^{\frac{1}{2}} - \left(p_o^2 + \dfrac{q_\ell^2}{\theta_o^2} - 2\, p_o q_\ell\right)^{\frac{1}{2}} \cos\theta_k\right]} \tag{9}$$

k is not completely determined by the laboratory angle of emission and q_ℓ since θ_k refers to $\vec{p}_1$ which varies with $\vec{q}$. However, in the case considered the values of $\vec{q}$ are very much restricted. If we examine Eqs. (7) and (8), we see that over half of the intensity comes from the 4 inverse lattice points closest to the $\vec{b}_2$ axis for which q is < 0.04 m. But

$$p_1 \Delta\theta_1 \simeq q \qquad \text{or} \qquad \Delta\theta_1 < 0.04 \frac{m}{p_o}$$

where $\Delta\theta_1$ is the range of variation of θ_1.

Hence if we are considering angles appreciably larger than this, θ_k can be measured with respect to the initial beam direction. For the case considered in Fig. 4

$$\frac{m}{p_o} \simeq \frac{0.51}{1000}$$

$$\theta_o = 11.3 \quad 10^{-3}$$

$$\theta_{1_{min}} = \frac{\delta m}{\theta_o p_o} = 0.02 \frac{m}{p_o} \frac{x}{1 - x}$$

As a result θ_1 will vary only in one plane which is tilted at an angle $0.02 \frac{m}{p_o} \frac{x}{1 - x}$ to the plane defined by the beam and the b_3 axis. Equation 9 can be simplified to give the more approximate relation

$$\frac{k}{p_o} = x = \frac{1}{1 + \frac{1 + \theta_k'^2}{2 g_2 \theta_o'}} \tag{10}$$

where $\theta_o = \frac{m}{p_o} \theta'_o$ $\theta_k = \frac{m}{p_o} \theta'_k$. Here $g_2 = \bar{g} \cdot \vec{b}_2$ has appropriately replaced q_ℓ in order to restrict this relation to momentum transfers allowed by the crystal lattice. We see that in the spectrum of Fig. 1 photons of different energies come at different values of θ_k. Each break comes at $\theta_k = 0$ and lower energies come from successively larger angles.

To find out to what extent the variation in θ_1 can be ignored we must see with what precision θ_k must be controlled to give a precise energy definition. Equation 10 can be expressed as

$$x = \frac{x_o}{1 + \theta_k'^2(1 - x_o)} \tag{11}$$

where x_o is the energy of the "break," or

$$\theta_k'^2 = \frac{x_o - x}{x(1 - x_o)} \tag{12}$$

When $\frac{x_o - x}{x_o} = 0.01$, $\theta'_k = \frac{0.1}{(1 - x_o)^{\frac{1}{2}}}$;

and when $\frac{x_o - x}{x_o} = 0.1$, $\theta'_k = \frac{0.33}{(1 - x_o)^{\frac{1}{2}}}$.

From this it is apparent that θ_1 variations limit us to spectra with widths of .01 or more. The sharpness of the break remains and the limitation applies to the sharpness of the drop on the low energy side.

The spectrum shown in Fig. 1 is for the $\theta'_k \leq \frac{0.33}{(1 - x_o)^{\frac{1}{2}}}$ case.

An additional gain is made by the angular selection since only a small fraction of the incoherently produced background spectrum is transmitted. This causes a reduction of about a factor of 6 in the case considered and the reduction would be larger for a narrower energy resolution. As is pointed out by Barbiellini, Bologna, Diambrini and Murtas[3] the enhancement increases greatly with increasing electron energy and hence for very high energy electron accelerators essentially monochromatic bremsstrahlung should in principle be possible.

It is not clear to what extent such monochromatic beams can be obtained. In the work of Barbiellini *et al.*[1] a collimator diameter of 0.6 reduced angles was used but the incoming beam apparently had angles appreciably larger and was further widened by multiple scattering.

Multiple scattering will make necessary the use of crystals about 10^{-4} radiation lengths thick and beam angular divergences of the order of $0.1\ \frac{m}{p_o}$. The Stanford 1 Gev linear accelerator has an angular divergence of approximately this, and it is hoped that at higher energies the angular divergence will be correspondingly smaller. It will be most difficult to control the crystal axes with sufficient accuracy especially after heating by an intense electron beam. Since the enhancement calculations are approximate other more intrinsic difficulties may appear.

ACKNOWLEDGMENT

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[3]Barbiellini, Bologna, Diambrini, and Murtas, Phys. Rev. Lett. 8, 112(1962).

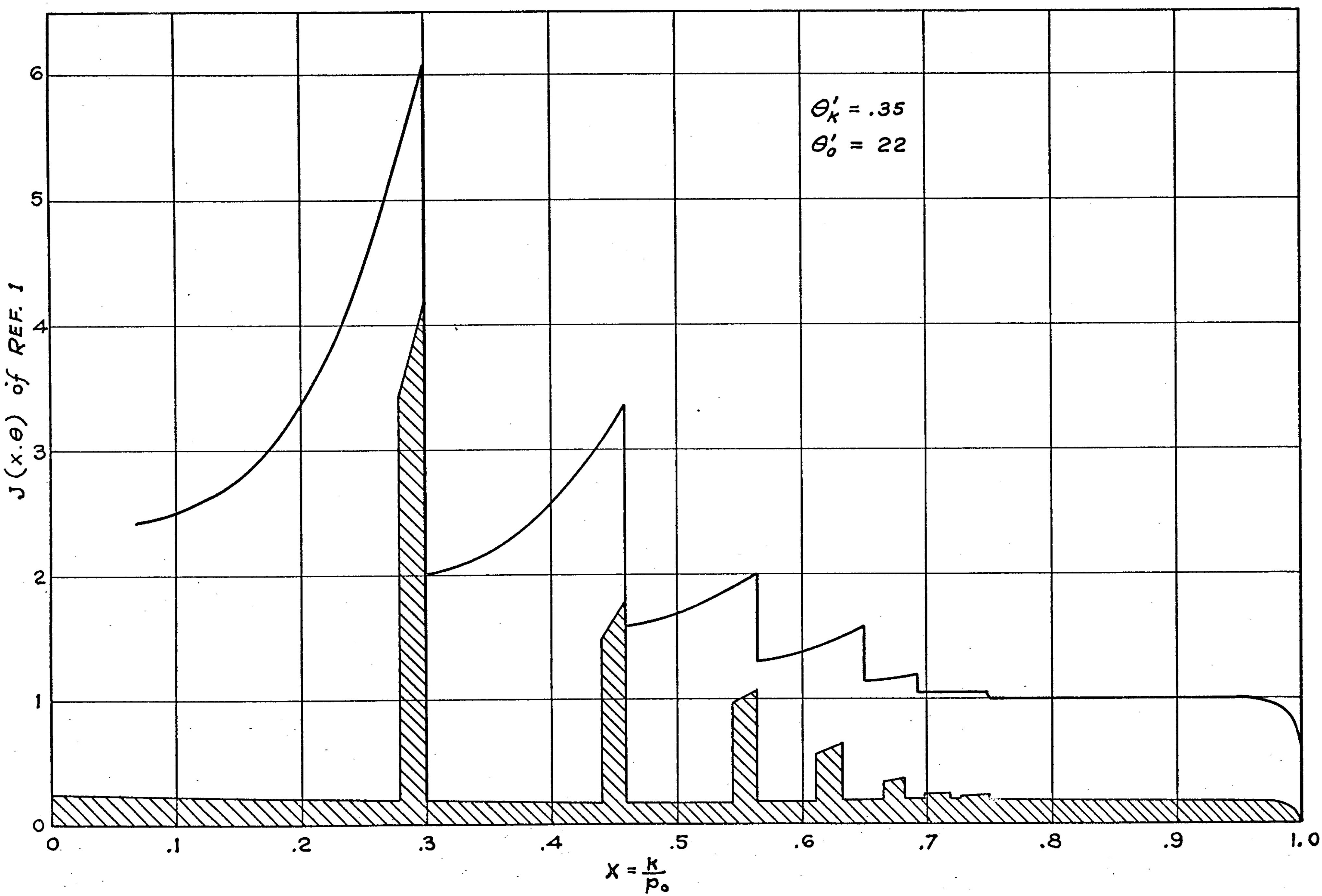
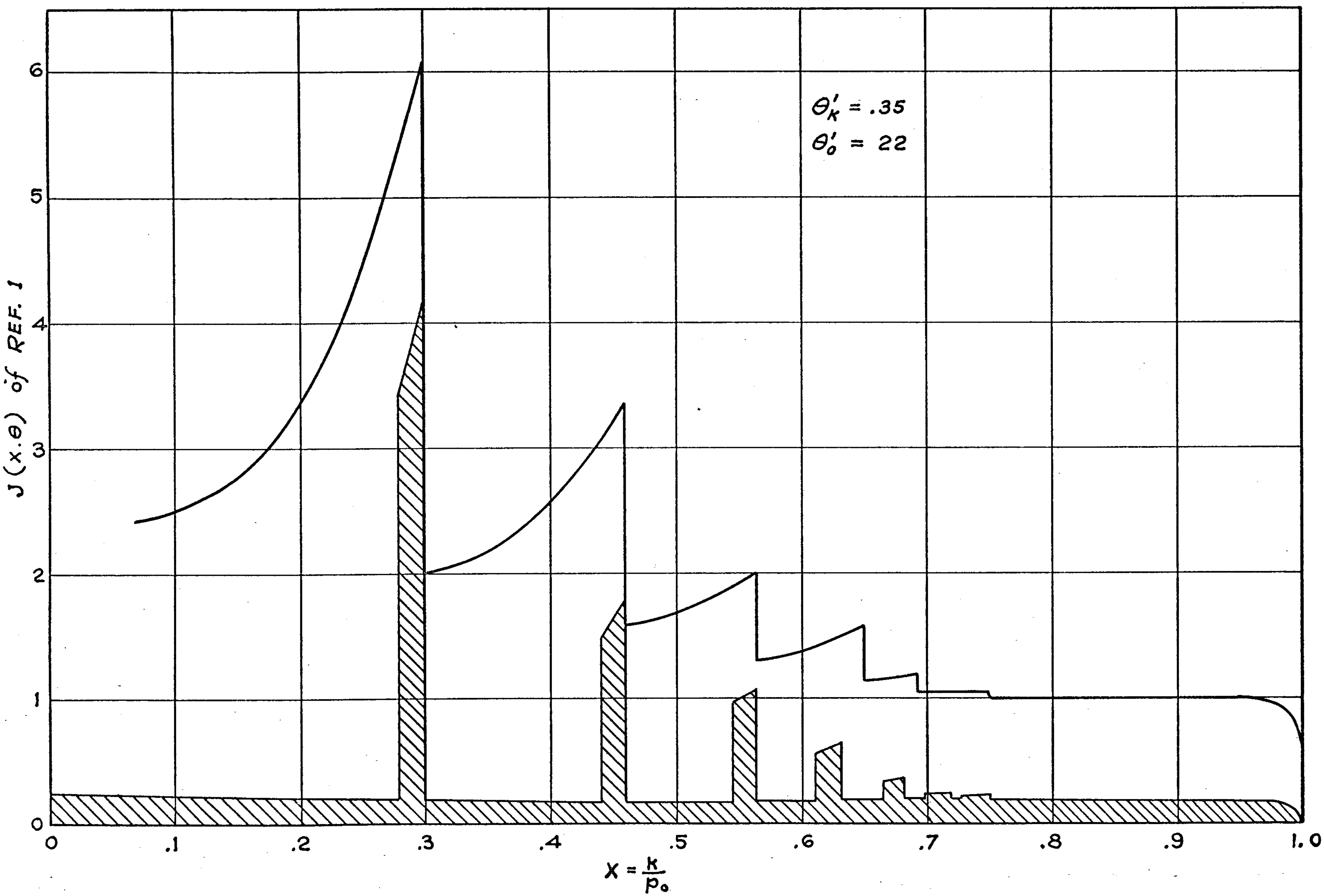


FIG. 1



J (x.θ) of REF. 1
0
1
2
3
4
5
6
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\theta_k' = .35$
$\theta_o' = 22$
$X = \frac{k}{p_o}$

FIG. 1

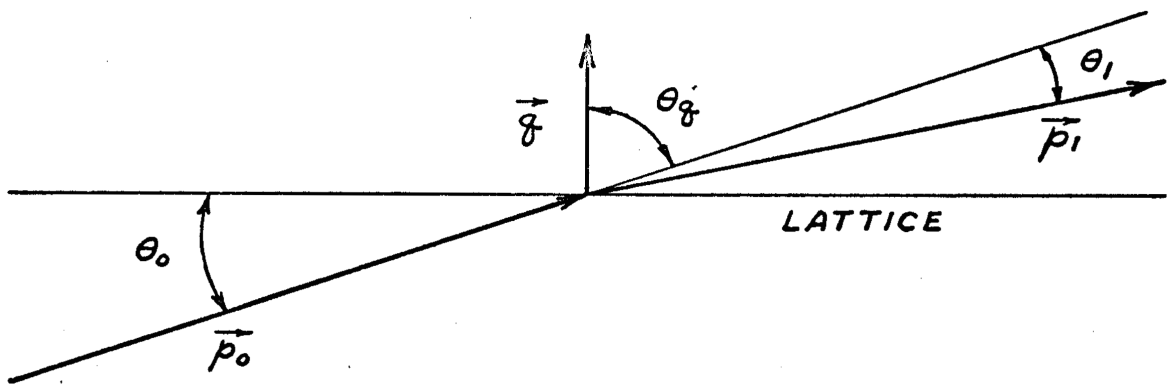


FIG. 2

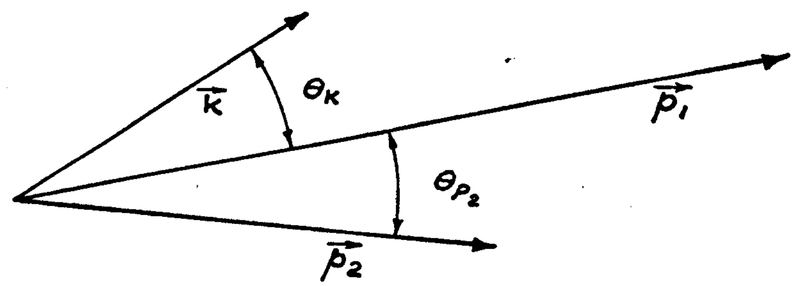
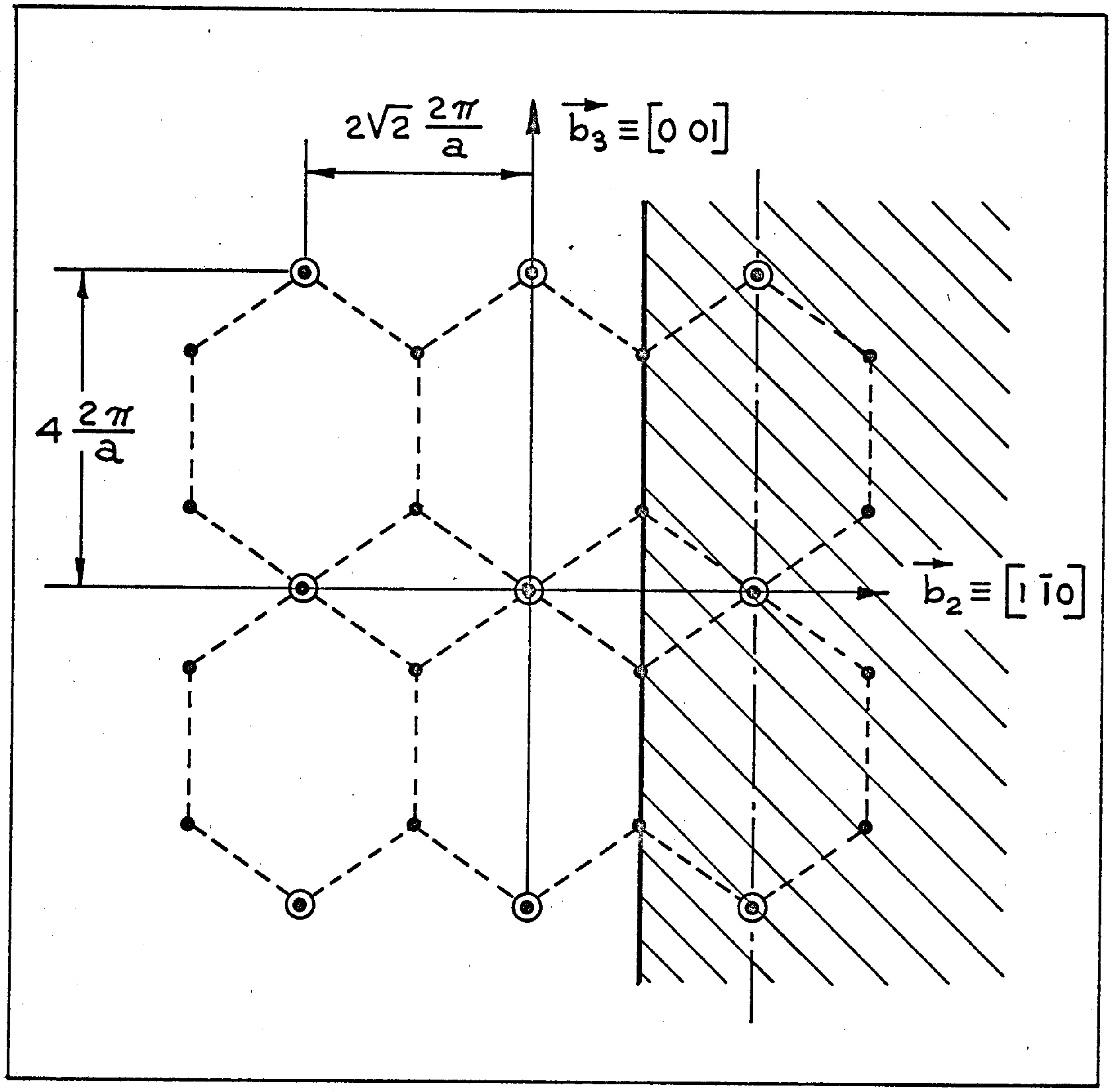


FIG. 3



$2\sqrt{2}\,\frac{2\pi}{a}$
$\vec{b}_3 \equiv [001]$
$4\,\frac{2\pi}{a}$
$\vec{b}_2 \equiv [1\bar{1}0]$

FIG. 4