Nonlocalized clustering and evolution of cluster structure in nuclei

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Abstract. It is shown that the THSR (Tohsaki-Horiuchi-Schuck-Roepke) wave function describe well not only cluster-gas like structures but also ordinary cluster structures with spatial localization of clusters. Based on this fact, the container model has been proposed as a new model of cluster dynamics. For better description of cluster dynamics, extended version of container model has been introduced. The container model of cluster dynamics teaches us how is the evolution of cluster structure which starts from the ground state having shell-model structure to many kinds of cluster states up to the cluster-gas states.

1. Introduction

The THSR wave function was originally devised for exprssing the 3α-condensate gas nature of the Hoyle state [1]. It was found [2] that the 3α THSR wave function has almost 100 % overlap with the 3α RGM/GCM wave function which had been obtained in 1970's. Surprisingly the THSR wave function for the α+^{16}O system was shown also to be almost 100 % equivalent to the RGM/GCM wave function for the α+^{16}O system [3]. The fact that the THSR wave function which expresses nonlocalized motion of clusters can also describe the localized α+^{16}O clustering in the ^{20}Ne inversion-doublet bands forced us to introduce the container model of cluster dynamics. The container model has been extended to multi-container model in order to get better description of various kinds of cluster structures. The extended container model can now be regarded as being a novel cluster model which gives us a new insight into cluster dynamics. On the basis of the container model we can now discuss how is the evolution of cluster structure which starts from the ground state having shell-model structure to many kinds of cluster states up to the cluster-gas states.

2. Hoyle state and 3α condensate-like structure

The 0_2^+ state of ^{12}C (Hoyle state) has been a mysterious state from the shell-model viewpoint. Even for the no-core shell model it has been difficult to reproduce it. The observed reduced α-decay width $\gamma^2_{\rm obs}$ of this state is very large and is comparable to or larger than the single-α value (= Wigner-limit value γ^2_W) [4]; i.e. $\gamma^2_{\rm obs} \geq \gamma^2_W$. This data means that the Hoyle state should have S-wave dominant structure, namely it has dominantly the structure of ^{8}Be(0^+) - α(S-wave). If the Hoyle state has the structure of 3α linear chain as was postulated by H. Morinaga, the reduced α-decay width $\gamma^2_{\rm chain}$ becomes much smaller than the single-α value. It is because the 3α linear-chain 0^+ wave function is composed of not only ^{8}Be(0^+) - α(S-wave)

component but also the components of ^{8}Be(2^+) - α(D-wave) and ^{8}Be(4^+) - α(G-wave), resulting in the relation $\gamma^2_{\rm chain} \leq 0.5\gamma^2_W$ [4].

The S-wave dominant structure of the Hoyle state was confirmed by the solution of the 3α problem. I solved 3α equation of OCM (orthogonality condition model) [5] and obtained the result that the Hoyle state has the structure of weakly coupled 3α in relative S waves with large spatial extent giving a large α S-factor, $S^2_\alpha = 1.33$. The results by this 3α OCM was confirmed a few years later by 3α GCM[6] and RGM [7]. Both succeeded to reproduce many observed data in ^{12}C.

About 30 years later, the wave functions of the Hoyle state by 3α GCM and RGM were found to be almost 100 % equivalent to single THSR wave function [2]:

$$|\,\langle\Phi(\text{single THSR})|\Phi(3\alpha\text{GCM/RGM})\rangle\,|^2 \approx 100\%. \tag{1}$$

The 3α THSR wave function has a very simple form $\Phi_B(3\alpha) = \mathcal{A}[\prod_{j=1}^3 \exp(-(2/B^2)\tilde{\mathbf{X}}_j^2)\phi(\alpha_j)]$ [1] with $\tilde{\mathbf{X}}_j = \mathbf{X}_j - \mathbf{X}_{\rm C.M.}$ and $\mathbf{X}_j$ standing for the C.M. coordinate of j-th α-cluster. This THSR wave function has a Bose-condensate-like character of α clusters and has clearly an S-wave dominant form.

3. Localized v.s. nonlocalized clustering

The inversion-doublet rotational bands in ^{16}O and ^{20}Ne have been regarded as convincing evidences of spatial localization of clusters [8]. Therefore it was really astonishing that the RGM/GCM wave functions of the inversion-doulet bands of ^{20}Ne were found to be almost 100 % equivalent to single THSR wave functions [3]:

$$|\left\langle\Phi(\text{single THSR})|\Phi(^{16}\text{O} + \alpha\text{ RGM/GCM})\right\rangle|^2 \approx 100\% \qquad \text{(inversion doublet band states)}. \tag{2}$$

Seeming puzzle why the localized clustering states of ^{20}Ne are well described by THSR wave functions which express nonlocalized clustering can be solved by using the fact that the nucleon density distribution of THSR wave function displays localized clustering of ^{16}O and α clusters [9]. Since the THSR wave function without antisymmetrization of nucleons shows no sign of localized clustering, the nucleon density distribution that shows localized clustering should come from the antisymmetrization operation of nucleons. We can conclude that the spatial localization of ^{16}O and α clusters comes from the inter-cluster Pauli repulsion originating from the antisymmetrization of nucleons [9].

4. Container model of cluster dynamics

We explained in the previous section that the clustering is based on the two fundamental ingredients. One is the nonlocalized cluster dynamics and the other is the inter-cluster Pauli repulsion. The container model of cluster dynamics is the model which explains concretely what are the characteristic features of the nonlocalized cluster dynamics. There are two important points in this model. The first point is that clusters make nonlocalized motion in the container(s). The second point is the recognition that the size parameter(s) of the container(s) are the generator coordinates of the evolution of the cluster structure. In Ref. [1], in the case of ^{12}C, we solved the Hill-Wheeler equation and constructed the energy eigenfunction Ψ_j

$$\sum_B \langle\Phi_{B'}(3\alpha)|(H - E_j)|\Phi_B(3\alpha)\rangle f_j(B) = 0, \quad \Psi_j = \sum_B f_j(B)\Phi_B(3\alpha). \tag{3}$$

The lowest-energy 0^+ solution is the ground state (0^+_1) and the second-lowest-energy 0^+ solution is the Hoyle state (0^+_2). Thus we see that the Hill-Wheeler equation describes the evolution of the cluster structure from the ground state to the Hoyle state.

The container model has been extended by introducing two or more containers. In the case of ^{12}C, different values of size parameters B_{kj} were adopted for two Jacobi coordinates $\boldsymbol{\xi}_1 = \boldsymbol{X}_2 - \boldsymbol{X}_1$ and $\boldsymbol{\xi}_2 = \boldsymbol{X}_3 - (1/2)(\boldsymbol{X}_1 + \boldsymbol{X}_2)$; $\Phi_{\boldsymbol{B}_1,\boldsymbol{B}_2}(3\alpha) = \mathcal{A}[\exp(-\sum_j^{x,y,z}\{(1/B_{1j}^2)\xi_{1j}^2 + (3/4B_{2j}^2)\xi_{2j}^2\})\prod_i^3 \phi(\alpha_i)]$ [10, 11]. In Ref. [11] calculated results of the Hoyle-family states by the extended container model were reported. One of the important results is the support to the existence of the 0_3^+ state and its band-like member states in addition to the 0_4^+ state and its band-like member states which have 3α bent-arm structure. The existence of the 0_3^+ state was proposed only by the 3α OCM calculations [12, 13] but has never been supported by fully microscopic calculations including 3α GCM, AMD and FMD calculations.

The extended container model was also successfully applied to the studies of neutron-rich Be isotopes, ^{9}Be and ^{10}Be [14]. In the case of ^{9}Be, it was reported that obtained extended-THSR wave functions for the ground, $5/2^-$, and $7/2^-$ states have large square overlaps with the Brink-GCM wave functions, 0.96, 0.95, and 0.93 %, respectively.

5. Evolution of cluster structure

Clusters make nonlocalized motion in containers under the constraint of inter-cluster Pauli repulsion. The inter-cluster Pauli repulsion originates from the Pauli-forbidden states of the system, $\chi^F(\tilde{\boldsymbol{X}}_1, \cdots, \tilde{\boldsymbol{X}}_N)$ with $\tilde{\boldsymbol{X}}_j = \boldsymbol{X}_j - \boldsymbol{X}_{\text{C.M.}}$, which are defined by

$$\mathcal{A}\{\chi^F(\tilde{\boldsymbol{X}}_1, \cdots, \tilde{\boldsymbol{X}}_N)\phi(C_1)\cdots\phi(C_N)\} = 0. \tag{4}$$

The physical wave function of cluster motion should be orthogonal to all the Pauli-forbidden states χ^F. It has been shown that this orthogonality requirement is equivalent to the requirement that the physical wave function of cluster motion should be orthogonal to all the two-cluster Pauli-forbidden states between any pair of clusters in the system.

In the case of two-cluster systems, the inter-cluster Pauli repulsion generates the spatial localization of clusters. However in three or more cluster systems, the inter-cluster Pauli repulsion does not necessarily generate the spatial localization of clusters.

In the case of two SU(3)-scalar clusters like α and ^{16}O clusters, the Pauli-allowed states have only $(\lambda,\mu)_{\text{SU(3)}} = (p, 0)$ which is of prolate deformation. However for many clusters, Pauli-allowed states can have variety of deformations.

N	$(\lambda,\mu)/\mu_p^{N(\lambda,\mu)}$								
12	(0, 0) 0.6592								
13	(2, 1) 0.4614								
14	(3, 1) 0.3955	(2, 0) 0.6152	(4, 2) 0.3811	(0, 4) 0.3365					
15	(6, 0) 0.3399	(4, 1) 0.4944	(2, 2) 0.4944	(0, 3) 0.4614	(3, 0) 0.5933	(5, 2) 0.3626	(3, 3) 0.3376	(6, 3) 0.3677	(2, 5) 0.2848
16	(5, 1) 0.5324 0.4237	(3, 2) 0.4635	(1, 3) 0.5191	(4, 0) 0.5982 0.5628	(0, 2) 0.6633	(8, 1) 0.3208	(6, 2) 0.4363 0.3252	(4, 3) 0.4366	(2, 4) 0.4390 0.3856
	(7, 3) 0.3845	(5, 4) 0.3450	(3, 5) 0.2957	(8, 4) 0.4082	(4, 6) 0.2765	(0, 8) 0.2295			
17	(8, 0) 0.3816	(6, 1) 0.5435 0.5192 0.4435	(4, 2) 0.5410 0.4979	(2, 3) 0.5730 0.4944	(5, 0) 0.5933	(3, 1) 0.6406	(1, 2) 0.6691	.....	

Figure 1. Pauli-allowed states of 4α system. N is the number of total H.O. quanta. (λ,μ) are the SU(3) labels of the Pauli-allowed states. Numbers below (λ,μ) label are the eigenvalues of the 4α norm kernel of the norm-kernel eigenstates having (λ,μ) label.

In Fig. 1 we show Pauli-allowed states of 4α system which are labeled by the SU(3) label (λ,μ) [15]. N stands for the number of total H.O. quanta of the Pauli-allowed state. The lowest

value of N is 12 which is of course equal to the H.O. quantum number of the closed-shell wave function $\Phi_{\rm C.S.} = (1/\sqrt{16!})\det|(0s)^4(0p)^{12}|$. In more detail, the 4α Pauli-allowed cluster state with lowest quanta $N = 12$ is equivalent to $\Phi_{\rm C.S.}$, which is nothing but the duality property of $\Phi_{\rm C.S.}$. We see clearly for most values of N except $N = 12$ and 13 the Pauli-allowed states have variety of deformations ranging from prolate to oblate.

In Ref. [16] the solution of the Hill-Wheeler equation with 4α THSR wave function is analysed.

$$\sum_B \langle \Phi_{B'}(4\alpha)|(H - E_j)|\Phi_B(4\alpha)\rangle f_j(B) = 0, \quad \Psi_j = \sum_B f_j(B)\Phi_B(4\alpha). \tag{5}$$

The lowest-energy state is almost equal to $\Phi_{\rm C.S.}$, the state around 4α breakup threshold has the property of 4α condensate-like state, and energy eigenstates between these states have $^{12}{\rm C} + \alpha$ character. The table of Pauli-allowed states in Fig. 1 teaches us that the formation and evolution of cluster states are largely governed by the inter-cluster Pauli repulsion. In Refs. [16, 17] the eigenstates of this Hill-Wheeler equation were analyzed and it was found that these eigenstates have almost 100 % squared overlaps with single THSR wave functions. Furthermore the size parameters B of these equivalent single THSR wave functions were found to become gradually larger for the larger excitation energies, which means the evolution of cluster structure along the growth of the excitation energy is in parallel with the increase of the system volume.

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