Angularities from LEP to FCC-ee

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Abstract: We present a preliminary NNLL$'$ resummation of the event shape *angularities* and compare it to LEP data at $Q = 91.2$ GeV. Our calculation permits a future precision determination of the strong coupling $\alpha_s(m_Z)$ from a fit to the experimental distributions. As the angularities are sensitive to the same non-perturbative parameter $\mathcal{A}$ that shifts the thrust distribution, our analysis may help to lift current degeneracies in the two-dimensional $\alpha_s(m_Z) - \mathcal{A}$ fits.

Introduction

Event-shape variables [1] characterize the geometric properties of a final-state distribution (e.g. dijet, three-jet-like, spherical, etc.) in collider processes. They are generally global observables that do not reject any final-state hadrons. Event shapes can be studied at hadron or e^+e^- colliders, though we focus here on the latter where a wealth of experimental data already exists from the Large Electron-Positron Collider (LEP) and where an e^+e^- Future-Circular-Collider (FCC-ee) could help to alleviate tensions in a number of α_s–determinations that are based on different theoretical methods (cf. contributions from *V. Mateu* and *P. Monni*).

We focus on a class of event shapes generically defined as

$$e(X) = \frac{1}{Q} \sum_{i \in X} |\mathbf{p}_\perp^\mathbf{i}| \, f_e(\eta_i) \,, \tag{1}$$

where η_i is the rapidity of the i'th final-state particle with respect to the thrust axis and $\mathbf{p}_\perp^\mathbf{i}$ its transverse momentum. The function $f_e(\eta)$ determines the specific observable. For example, for the two well-known event shapes *thrust* $T \equiv 1 - \tau$ [2] and (total) *jet broadening* B_T [3], one has $f_\tau(\eta) = e^{-|\eta|}$ and $f_{B_T}(\eta) = 1$, respectively.

Both thrust and broadening can be generalized into a class of observables known as *angularities* [4,5],

$$f_{\tau_a}(\eta) = e^{-|\eta|(1-a)} \qquad \longleftrightarrow \qquad \tau_a(X) = \frac{1}{Q} \sum_{i \in X} E_i \, |\sin\theta_i|^a \, (1 - |\cos\theta_i|)^{1-a} \,, \tag{2}$$

where E_i is the energy and θ_i the angle of the i'th particle with respect to the thrust axis. The angularities thus depend on a continuous parameter a, which fulfils $-\infty < a < 2$ due to infrared (IR) safety. For $a = 0$, the angularity reduces to thrust, $\tau_0 = \tau$, and for $a = 1$, it reduces to broadening $\tau_1 = B_T$.

Like other QCD observables that depend on widely separated energy scales, event shapes are affected by logarithmic enhancements to the perturbative QCD (pQCD) expansion, which must be *resummed* to all orders. Many analyses have been performed to this end, both with standard pQCD and, more recently, also with Soft-Collinear Effective Theory (SCET) [6,7,8,9]. SCET formally separates the relevant scales present in collider processes, and it provides an elegant means to establish factorization theorems. For example, the angularity distribution factorizes in the dijet limit $\tau_a \to 0$ into a *hard* function $H(\mu, \mu_H)$ that encodes the matching of SCET to QCD, two *jet* functions $J(\mu, \mu_J)$ describing the evolution of the coloured partons into collimated jets, and a *soft* function $S(\mu, \mu_S)$ describing low-energy, wide-angle background radiation, all of which live at an associated scale $\mu_H \gg \mu_J \gg \mu_S$ [4,10]. The dependence of H, J, and S on the factorization scale μ is controlled by renormalization group (RG) equations, which can be used to resum large logarithms present in each function. Indeed, many of the most precise event-shape resummations have been achieved with SCET techniques, with thrust and broadening currently resummed to N^{3}LL [11,12] and NNLL [13,14] accuracy, respectively*.

An immediate goal of this note is to use methods from SCET to predict angularity distributions to NNLL$'$ accuracy [15], thereby realizing an improvement on a prior NLL$'$ resummation [10,16]. Our calculation is based on a recent two-loop calculation of the angularity soft function [17], which we use to extract the missing NNLL$'$ ingredients. We are also motivated by the presence of L3 Collaboration data [18], which measured the angularity distributions at 8 different values of $a \in \{-1, -0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75\}$ at both $Q = 91.2$ GeV and $Q = 197.0$ GeV. This will allow for a future extraction of $\alpha_s(m_Z)$ and the non-perturbative (NP) shift parameter $\mathcal{A}$ as discussed below.

Sensitivity to Non-Perturbative Effects

As with any hadronic observable, event shapes are sensitive to low-energy QCD radiation. The importance of these NP effects depends on the domain of τ_a considered. For angularities with $a < 1$ in the (near-)tail region, power corrections from the collinear sector are suppressed with respect to those from the soft sector† [19,20]. The NP effects can then be parameterized into a *shape function* that is convolved with the perturbative distribution [21]. In the tail region, it can be shown rigorously via an operator-product-expansion (OPE) that the dominant NP effect results in a shift of the perturbative distribution [19]‡

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\Big(\tau_a - c_{\tau_a}\frac{\mathcal{A}}{Q}\Big)\,. \tag{3}$$

Here $\mathcal{A}$ is a *universal* NP parameter that is defined as a vacuum matrix element of soft Wilson lines and a transverse energy-flow operator (for details, see [19]), while c_{τ_a} is an exactly calculable

*For a thorough elaboration of the logarithmic enhancements captured in a N^{k}LL ($k \in \{0, 1, ...\}$) resummation and the subtle differences between primed and unprimed accuracies, see [16].

†The endpoint $a = 1$ corresponds to the onset of SCET$_{\text{II}}$ physics. We will not discuss the subtle differences between SCET$_{\text{I}}$ and SCET$_{\text{II}}$ observables, though thrust and angularities are examples of the former (for $a < 1$). At the broadening limit, the angularity reduces to a SCET$_{\text{II}}$ observable and therefore predictions based on a SCET$_{\text{I}}$ factorization theorem should become progressively worse as $a \to 1$. We observe this effect.

‡ In the peak region, the OPE does not apply and a full shape function is required to capture the non-perturbative effects. Furthermore, the result in (3) is not only leading-order in the OPE, it is also subject to other corrections like finite hadron masses and perturbative renormalization effects on the quantity $\mathcal{A}$, as described in [22].

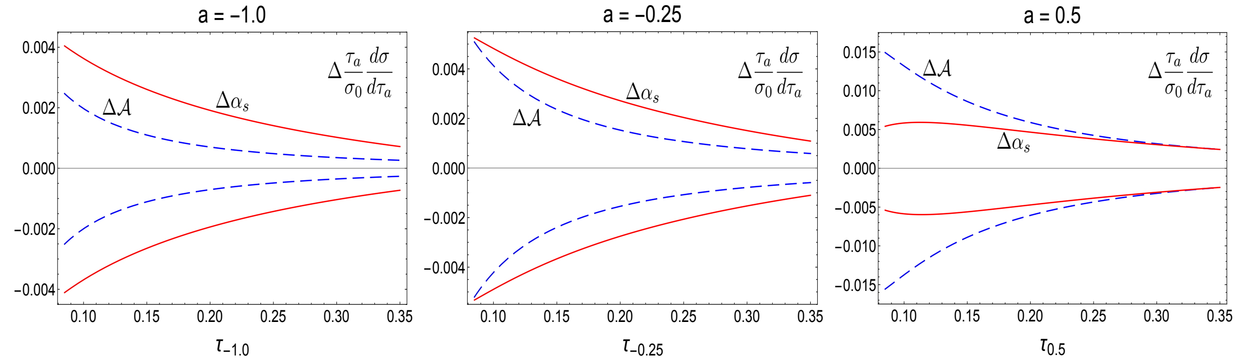

Figure 1: Difference distributions between central curves and curves evaluated with single variations of either $\mathcal{A}$ (dashed, blue) or $\alpha_s(m_Z)$ (solid, red) at three values of $a \in \{-1, -0.25, 0.5\}$. $Q = 91.2$ GeV in all three plots.

observable-dependent coefficient. For the angularities, it is given by[§]

$$c_{\tau_a} = \int_{-\infty}^{\infty} d\eta \, f_{\tau_a}(\eta) = \frac{2}{1-a} \,. \tag{4}$$

Hence, in any attempt to extract a value of the strong coupling by comparing data to theoretical predictions, one is simultaneously sensitive to $\alpha_s(m_Z)$ and $\mathcal{A}$. Indeed, the most precise extractions employing analytic treatments of NP effects [12,23] report values in an $\alpha_s(m_Z) - \mathcal{A}$ plane (cf. contribution from *V.Mateu*). Furthermore, the extracted values of $\alpha_s(m_Z)$ from these analyses are consistently (and often dramatically) lower than the world average, which is currently dominated by lattice-QCD calculations (cf. 0.1123 ± 0.0015 [23] to the world average 0.1181 ± 0.0011 [24]). It can be shown that the event-shape extractions are driven to small values precisely due to NP effects, and so any elucidation of these discrepancies requires a disentangling of perturbative and non-perturbative contributions.

Our proposal is to perform a future extraction of both $\alpha_s(m_Z)$ and $\mathcal{A}$ along the lines of previous SCET treatments, but at multiple values of the angularities a. The critical point is that the leading NP shift in (3) is a-dependent. Therefore, an extraction at a single centre-of-mass energy Q, but different values of a, will have a discriminating sensitivity to $\mathcal{A}$ and $\alpha_s(m_Z)$ in a similar way as varying Q. For example, angularities for $-2 \leq a \lesssim 0.5$ exhibit a factor of six variance in the overall NP shift. This sensitivity is essentially equivalent to measurements made between $Q = 35$ GeV and $Q = 207$ GeV, as analyzed for thrust e.g. in [12]. In Figure 1 we show the difference $(d\sigma/d\tau_a)_{\text{central}} - d\sigma/d\tau_a$ over the range $0.085 \leq \tau_a \leq 0.35$ for $a \in \{-1, -0.25, 0.5\}$, where $(d\sigma/d\tau_a)_{\text{central}}$ is an (unmatched) NNLL$'$ resummed distribution evaluated at $\alpha_s(m_Z) = 0.1161$ and $\mathcal{A} = 0.283$ GeV. For $(d\sigma/d\tau_a)$ we have varied $2\mathcal{A}$ by ± 0.1 GeV and $\alpha_s(m_Z)$ by ± 0.001, corresponding to the blue and red curves, respectively. These plots are analogous to Figure 10 in [12], where the same variations were made but at different values of Q, rather than a. Indeed, we find that varying a (Q) down (up) from high (low) values leads to an enhanced sensitivity of the distributions to the relative effects of $\mathcal{A}$ and $\alpha_s(m_Z)$ variation. We are therefore optimistic that the a-dependence of the angularities can help to lift the degeneracies between $\alpha_s(m_Z)$ and $\mathcal{A}$ in the two-parameter fits.

[§]The expression for c_{τ_a} diverges in the limit $a \to 1$, where the SCET$_\text{I}$ factorization theorem we use breaks down. A careful analysis revealed that the NP effects to the broadening distributions are enhanced by a rapidity logarithm, $c_{B_T} = \ln Q/B_T$ [20].

Angularities at NNLL$'$ Accuracy

The resummed cumulative distribution in τ_a, $\sigma_c(\tau_a) = (1/\sigma_0)\int_0^{\tau_a} d\tau_a'(d\sigma/d\tau_a')$, will ultimately be given by [10,16]

$$\sigma_c(\tau_a) = e^{K(\mu,\mu_H,\mu_J,\mu_S)}\left(\frac{\mu_H}{Q}\right)^{\omega_H(\mu,\mu_H)}\left(\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a}\right)^{2\omega_J(\mu,\mu_J)}\left(\frac{\mu_S}{Q\tau_a}\right)^{\omega_S(\mu,\mu_S)}$$
$$\times H(Q^2,\mu_H)\,\widetilde{J}\Big(\partial_\Omega + \ln\frac{\mu_J^{2-a}}{Q^{2-a}\tau_a},\mu_J\Big)^2\,\widetilde{S}\Big(\partial_\Omega + \ln\frac{\mu_S}{Q\tau_a},\mu_S\Big)\,\frac{e^{\gamma_E\Omega}}{\Gamma(1-\Omega)}\,,$$

where σ_0 is the Born cross-section summed over massless quark flavours $f=\{u,d,s,c,b\}$, H is the hard function, $\tilde{J}$ and $\tilde{S}$ are the Laplace-space jet and soft functions, and K, Ω and $\omega_{H,J,S}$ are evolution kernels that depend on the anomalous dimensions of the functions H, $\tilde{J}$ and $\tilde{S}$. The anomalous dimensions and the fixed-order functions have expansions in α_s, such that resummations of higher logarithmic accuracy require increasingly higher-order terms.

To achieve NNLL$'$ accuracy, one needs all of the ingredients from Table 5 of [16]. In particular, the two-loop jet and soft anomalous dimensions and the respective finite (non-logarithmic) terms were not previously known. Calculating the soft variants has now been achieved in [17] via a generic algorithm for the numerical evaluation of two-loop dijet soft functions. The remaining two-loop jet anomalous dimension can then be calculated using RG consistency relations, and the finite term in the two-loop expansion of $\tilde{J}$ can be extracted via a comparison with a fixed-order code, for which we use the `EVENT2` generator [25] (details will be given in [15]).

Matching

SCET is an effective theory of QCD that predicts the singular terms in the cross section as $\tau_a \to 0$ and resums them to all orders. To obtain a reliable description in the large τ_a domain, one then needs to match the resummed distribution to the fixed-order QCD result. To this end, we utilize `EVENT2` to generate the differential distribution up to $\mathcal{O}(\alpha_s^2)$.

Furthermore, we have designed *profile functions* [12] that smoothly interpolate between the peak region (where $\mu_H \gg \mu_J \gg \mu_S \sim \Lambda_{QCD}$), the tail region (where $\mu_H \gg \mu_J \gg \mu_S \gg \Lambda_{QCD}$) and the far-tail region (where $\mu_H \sim \mu_J \sim \mu_S \gg \Lambda_{QCD}$). In the peak region the soft scale is very nearly NP although, as we do not employ a model shape function in this analysis, we will not show predictions in this region anyway. On the other hand, the scales are well separated in the tail region, which is the region where resummation is most important. Finally, our predictions should match onto fixed-order perturbation theory in the far-tail region. Resummations should therefore be switched off, and the scales should merge at $\mu_{H,J,S} = Q$. While we do not show the explicit functional form of our profile scales, they are similar to those in [26]. The final theory errors presented below reflect independent variations of the hard, jet and soft scales added in quadrature.

Results

Some benchmark *preliminary* results for NNLL$'$ resummed and $\mathcal{O}(\alpha_s^2)$ matched distributions are shown in Figure 2. Our results are for $a \in \{-0.5, 0.5\}$ and $Q = 91.2$ GeV, and we have set $\alpha_s(M_Z) = 0.1161$ and $\mathcal{A}_{\mathrm{NNLL}'} = 0.283$ GeV as in [12]. The plots show the curves without (blue) and with the NP shift (green), and they also display the data points from [18]. We focus here on

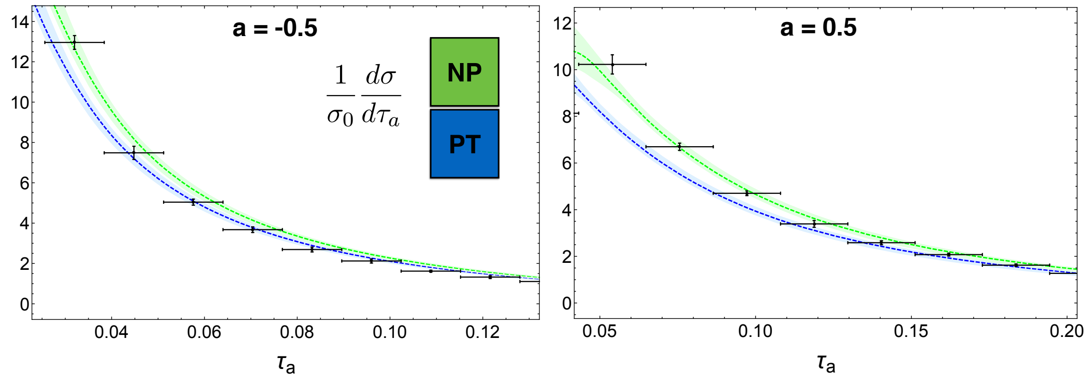

Figure 2: Preliminary NNLL′ resummed and $\mathcal{O}(\alpha_s^2)$ matched angularity distributions at two values of the parameter $a \in \{-0.5, 0.5\}$. The blue (PT) curves represent the purely perturbative result, whereas the green (NP) curves includes the NP shift according to (3). $Q = 91.2$ GeV in both plots.

the central (or tail and far-tail) τ_a domain where the effect of resummation is most relevant. Plots including the peak region will be left for future studies. For $a = -0.5$ the difference between the perturbative (blue) and the NP shifted curve (green) is too small for one to be clearly preferred by the experimental data. For $a = 0.5$, on the other hand, the NP effect is sizeable and, indeed, necessary to accurately describe the data. This is a clear visual confirmation of the leading-order prediction in (3). Note that the error bars in Figure 2 *do not* include any error estimate coming from the `EVENT2` extraction of the two-loop jet constant nor from matching to QCD. This will be addressed in [15].

Moving from LEP to FCC-ee

We argued that an α_s–extraction using angularities could potentially alleviate the current degeneracies in the $\alpha_s(m_Z) - \mathcal{A}$ plane, due to the dependence of the leading NP shift on a. Of course, one also notes from (3) that the power correction is sensitive to the centre-of-mass energy Q. Therefore an FCC-ee operating at different energies could be an even greater probe in disentangling hadronization effects in e^+e^- event-shape distributions. In Figure 3 we have demonstrated the minimization of NP effects as Q increases from $91.2 \to 400$ GeV. Not only does one notice that the distributions are larger and more peaked in the low-τ_a region, one observes that the correction moves from a 9% effect at $Q = 91.2$ GeV to a 2% effect at $Q = 400$ GeV (calculated at $\tau_{0.25} = 0.15$). It is clear that the combined dependence of NP effects on a and Q could be significant. Regardless, precision resummations as presented in this note represent critical first steps in pursuing these goals.

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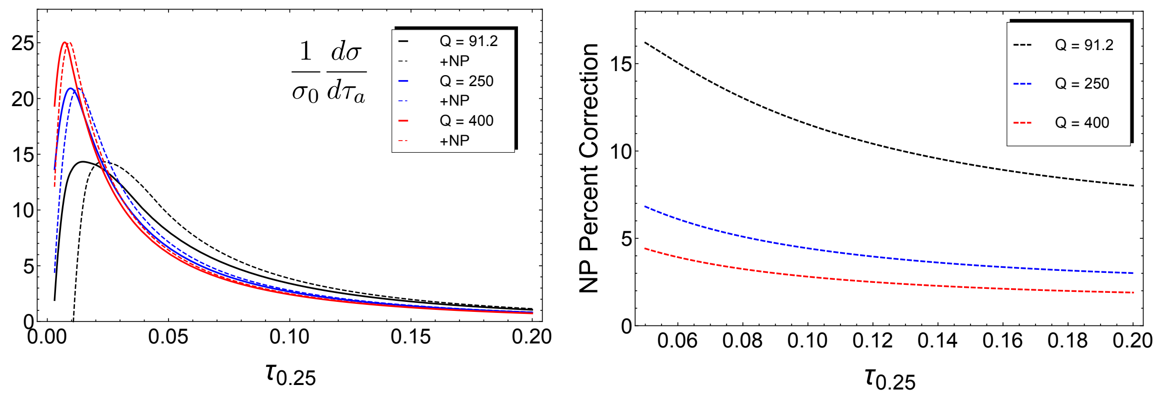

Figure 3: *Left:* Correction to (unmatched) differential angularity distributions from the leading NP power correction at various values of Q (for $a = 0.25$). *Right:* Percent correction from the same effect.

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