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Recent Progress in the Reggeon Calculus*

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ABSTRACT

The theoretical and experimental properties of the strong coupling solution of the Reggeon Calculus are reviewed. The analogy of rising total cross-sections with a critical phenomenon is emphasized as well as the consistency of the solution with both t and s-channel unitarity. The status of other solutions is also briefly discussed.

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The major progress in the Reggeon Calculus within the last year has been the development of a very beautiful self-consistent solution of the high-energy problem. This solution is essentially the "strong-coupling" or "scaling" solution suggested several years ago by Gribov and Migdal.[1] However, recent studies of this solution by Migdal, Polyakov and Ter-Martirosyan[2] and by Abarbanel and Bronzan,[3] using the renormalization group, have shown that this solution predicts that total cross sections will rise asymptotically like $(\ln s)^{\eta}$ and that η can in principle, be calculated exactly. (That this solution can be explicitly constructed, despite difficult infra-red problems has recently been shown by Sugar and myself.[4]) η can be determined exactly because it is a critical exponent in the sense that the whole solution is analagous to a critical phenomenon (the intercept of the Pomeron plays the role of temperature and placing the intercept exactly at one places us right at the critical temperature). The critical phenomenon analogy is part of the beauty and strength of the solution. Universality is a familiar idea in statistical mechanics --that is, the critical exponents of a phase transition (which determine the divergences of thermodynamic quantities at the critical point) are independent of the short-range forces of the system and depend only on the symmetry properties of the system. The analagous statement here is that the "strong-coupling" solution (and in particular η) does not depend on the underlying strong interaction forces between particles. Gribov's original motivation[5] for studying the high-energy Pomeranchuk problem was his belief that the solution of the problem should be independent of the details of strong interaction forces. The possibility that at very high energy we will observe a critical phenomenon, seems to me to be a very attractive realization of Gribov's belief.

There are many important theoretical and experimental features of the strong-coupling solution. I shall devote most of this talk to describing as many as possible of these features. I will briefly discuss the status of other possible solutions and work that has been done on them in the remainder of the talk. The major virtues of the strong-coupling solution that I shall emphasize are:

1. Critical Phenomenon Analogy--Scaling Laws

I will give a very brief discussion of the phase transition analogy which will serve both as an introduction to the Reggeon Field theory and explain why the renormalization group helps solve the problem in the way described by Abarbanel.[7] It also explains why scaling laws are obtained.

2. Multiparticle t-channel Unitarity

I'll emphasize the rigorous background to the solution if we proceed through the angular momentum plane. Since the Pomeron is a pole for positive t[4], and the field theory satisfies multi-Pomeron unitarity, multiparticle t-channel unitarity is automatically satisfied.

3. s-Channel Unitarity--Avoidance of Decoupling Problems

We cannot prove s-channel unitarity directly, but at least it appears that all well-known confrontations of a Regge description of the Pomeron with unitarity in this channel are avoided. These include the Finkelstein-Kajantie problem[8] and the inclusive sum rule decoupling arguments.[9,10]

4. The "Bare" Pomeron Intercept Is Above One by $0\ [g_p(0)^2]$

The renormalized triple Pomeron coupling has an absolute value in the strong coupling solution.[2] The present experimental value is too small. This suggests that the bare parameters of the Pomeron are what we observe at present energies (Fermilab-ISR). To be exactly at the critical point, however, these parameters have to be closely related.[4] Experimentally this relation looks pretty good. A tight-relation between diffractive and non-diffractive production is implied and this could unify various views of rising cross sections.

5. "t-channel" Enforcement of the Froissart Bound

Eikonalization of a "bare" Pomeron pole with intercept above one requires absorptive corrections to the bare Pomeron to be closely inter-related. t-channel renormalization of the bare intercept down to one does not require this, but the bare intercept has to be exactly right. Recent fits to $(\frac{d\sigma}{dt})_{el}$ at the ISR[11] suggest that a single bare Pomeron pole can be isolated and this favors the t-channel process.

6. A Big Bonus--Fermion Parity Doublets Removed

In the strong-coupling solution, rising cross sections seem to correspond to a fixed cut in the angular momentum plane. In the case of Fermion exchange this same branch-point plays a crucial role in the removal of fermion parity doublets.[12] It is very nice that this old phenomenological puzzle can be resolved by the strong-coupling Pomeron.

I. CRITICAL PHENOMENON ANALOGY

It is now fairly well accepted that the bulk of low and medium energy experimental data can be explained in terms of multiperipheral-like models that are short-range in rapidity and cut-off in transverse momentum.[13] Correlations among produced particles are largely short-range in rapidity space. We also know from theoretical consistency arguments[9,10](and experimental data) that eventually long-range interactions must develop at asymptotic energies. The language, of course, already suggests an analogy with the approach to a phase transition and several people have previously tried to make such an analogy explicit.[14] The general idea has been to treat the correlation functions of particle production as the correlation functions of a statistical gas or fluid. I shall proceed differently. My analogy will be ill-defined and capable of considerable improvement.

Consider elastic scattering in rapidity and impact parameter space (rapidity is the logarithm of the energy and impact parameter is the conjugate fourier transform variable to transverse momentum — transverse distance!) There will be an effective interaction volume which we shall treat as analagous to the medium in which initial short-range forces co-operate to produce the long-range order characteristic of a phase transition. Our initial scattering forces we envisage as operating over short distances in rapidity and some finite range in impact parameter.

The Wilson use of the renormalization group[5] in this situation proceeds as follows. We assume that the local short-range interactions can be smoothed out in the sense that the medium can be broken up into blocks and that lump parameters for each block (in solid state physics average spin density is such a parameter) are sufficient to describe the development of long-range order. The parameters of the block define a field for which an effective interaction Hamiltonian is introduced. The renormalization group transformation is implemented by an increase of the block size, with a consequent transformation of the effective Hamiltonian. The development of the truly long-range order characteristic of a phase transition should be insensitive to the block size transformation and so should correspond to a "fixed-point" of the renormalization group transformation, i.e., the effective Hamiltonian for describing the phase transition should be unchanged by the renormalization group transformation.

In fact the mathematics to describe the renormalization group transformation has been developed in momentum space rather than in position space. The block-size is replaced by a cut-off in the momenta (or frequencies) that are considered. The renormalization group transformation is realized by decreasing the cut-off (≡ increasing the block size).

To proceed in the Wilson manner in our problem we clearly should introduce blocks in rapidity/impact parameter space and introduce a field to describe the lump parameters of the block--the Pomeron field. The Pomeron field therefore describes the smoothed out short-range interactions. Care will be necessary in introducing the blocks since our interactions must be between points well-separated in rapidity and at least a finite separation in impact parameter. There is no three-dimensional symmetry to the problem. Therefore, neighboring blocks must be displaced relatively in both variables. Also the interaction volume is bordered by the external hadrons since they define the limits of the interaction region in rapidity space. To treat these borders properly the hadrons must be regarded as acting as sources for any number of Pomerons which then propagate through the interaction volume.

If we now transform to "momentum space" for our field theory, we transform to the conjugate angular momentum and momentum transfer variables. Since hadrons act as sources for Pomerons we can calculate the elastic scattering amplitude if we can calculate the series shown in Fig. 2. That is, we need to calculate all Green's functions for interacting Pomerons. If we are at a critical point we should be able to calculate these Green's functions from an interaction Lagrangian which is a fixed point of the renormalization group transformation.

[Note that in our analogy the Pomeron Green's functions are the equivalent of the correlation functions of the fluid and not the correlation functions of particle production. To relate the long-range oscillations of our Pomeron field to long-range correlations in particle production, it is first necessary to give a description of production amplitudes in terms of Pomeron Green's functions.]

Scaling laws for correlation functions at the critical point are a familiar phenomenon in a phase transition and the renormalization group explains why they occur. In our case we argue as follows: if the Lagrangian is covariant under a change of scale, then similarly the Green's functions are covariant under a change of scale in rapidity space

$$A(\ln S, 0) \underset{\ln S \to \infty}{\propto} A(\lambda \ln S, 0) \Rightarrow A \propto (\ln S)^{\eta}. \tag{1}$$

For small, non-zero t this result generalizes to

$$A(\ln S, t) \xrightarrow[(\ln S) \to \infty]{} (\ln S)^{\eta} A[t(\ln s)^{\nu}] . \tag{2}$$

Both η and ν can be viewed as critical exponents.

This ends the intuitive discussion of the phase transition analogy.

To go any further and in particular to discuss calculating η we must be more formal. We write down a Lagrangian for our Pomeron field. In (x, y) space (impact parameter, rapidity)

$$\mathscr{L}(x,y) = \frac{i}{2}\psi^{+}(x,y)\overleftrightarrow{\frac{\partial}{\partial t}}y\psi(x,y) - \alpha_0'\nabla\psi^{+}\nabla\psi - \Delta_0\psi^{+}\psi$$

$$-\frac{1}{2}ir_0\left[\psi^{+}\psi^{2} + \psi^{+2}\psi\right] + \frac{\lambda_{30}}{2}\left[\psi^{+}\psi^{3} + \psi^{+3}\psi\right] + \ldots \qquad (3)$$

The imaginary triple Pomeron coupling arises from the absorptive character of the Pomeron field.

If the result we were looking for were not simply a fixed-point of the renormalization group transformation we would be lost at this stage, since the result would appear to be critically dependent on the Lagrangian chosen and who tells us what that is. Fortunately the results of Wilson[5] tell us that we obtain the same fixed-point by starting with a large class of Lagrangians. The only determining factors being the symmetry properties of the Lagrangian. In our case there seem no justification for giving ψ more than one component or introducing any new fields into the Lagrangian.

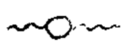
To proceed using Wilson's approach we should now go to momentum space. This procedure has been carried out explicitly by Jengo[15] and is essentially the method used by Migdal, Polyakov and Ter-Martirosyan.[2] The action A can be written as [we write $p \equiv (E, k)$ $E = 1-j$, and $k^2 = -t$ -- j = angular momentum, t = [momentum transfer]^2].

$$A = \int_0^{\Lambda} [dp] \quad \psi^+\psi \,[E - \alpha_0' k^2 + \Delta_0 + \ldots] \;-$$

$$\frac{i}{2} (\psi^{+2}\psi + \psi\psi^{+2}) \; (r_0 + r_2\, p^2 + \ldots) \qquad (4)$$

$$+ \frac{1}{2} (\psi^{+3}\psi + \psi^+\psi^3)(\lambda_0 + \lambda_{0p}^{\;2} + \ldots)$$

The renormalization group transformation is performed by integrating out the range of p from $\frac{\Lambda}{2}$ to Λ and showing that this can be intrepreted as renormalizing the parameters $(\alpha_0', r_0, r_2 \ldots)$. This procedure is repeated until a fixed-point is reached.

More simply we can start with the Lagrangian involving just the triple Pomeron interaction. Since this theory is renormalizable in a conventional sense, we can apply the renormalization group in the way familiar in renormalizable relativistic theories. This is the method previously described by Abarbanel and used by him and Bronzan. The result obtained for η should be the same in either approach. In the last approach the normalization point is varied and a renormalization group equation written for the Green's functions which is then solved e.g., for the propagator $\Gamma^{(1,1)}$ ($\equiv$ )

$$\left\{ E \frac{\partial}{\partial E} - \beta(g)\frac{\partial}{\partial g} + \left[\alpha' - \zeta(\alpha', g)\right]\frac{\partial}{\partial \alpha'} + \gamma(g) - 1 \right\} \qquad (5)$$

$$\Gamma_R^{(1,1)}(E, k, \alpha', g) = 0$$

g is a dimensionless coupling constant. If there exists a $\bar{g}$ such that $\beta(\bar{g}) = 0$ and $\underline{k}^2 = 0$ so that

$$\frac{\partial}{\partial \alpha'} \Gamma_R^{(1,1)} = 0$$

then

$$E \frac{\partial}{\partial E} \Gamma_R = (\eta + 1)\Gamma_R \qquad \eta = -\gamma(\bar{g}) \tag{6}$$

$$\Rightarrow \Gamma_R \propto E^{\eta + 1} \tag{7}$$

which is the result we want. $\beta(g)$ and $\gamma(g)$ can be calculated in perturbation theory and $\bar{g}$ can be consistently determined this way if it is small. It turns out that this can only be done by varying the dimension of the transverse momentum ($\underline{k}$). Writing dimension D = 4 - ϵ and making an "ϵ-expansion" we obtain[16]

$$\eta = \frac{\epsilon}{12} [1 + 0.64\,\epsilon] + 0(\epsilon^3) \tag{8}$$

$$= \frac{2.28}{6} \qquad \epsilon = 2 \tag{9}$$

$$\simeq \frac{1}{3} \tag{10}$$

Although the ϵ-expansion doesn't look very convergent, it doesn't look any better in solid-state.[5] However, $0(\epsilon^2)$ calculations are quite close to the correct result.

The two approaches we have outlined here-using a bare field theory with a cut-off and a renormalizable theory including the lowest coupling, are also used in solid-state calculations.[5]

So far our treatment has been formal only after we have justified introducing the Reggeon Field Theory. Although our intuitive introduction is closely related to Gribov's derivation of the Reggeon field theory from an underlying field theory.[17] There is, however, an alternative way to proceed--

II. MULTIPARTICLE t-CHANNEL UNITARITY

The t-channel unitarity relation--which at fixed t involves only a finite number of terms--can be projected onto (t-channel) partial waves. Assuming only quite general analyticity properties of production amplitudes,[18] the resulting equations can be analytically continued to complex j. These equations can be used to give unitarity corrections to a Regge pole.[19] The generation of the two-Reggeon branch point (or threshold) in the four-particle unitarity relation is illustrated in Fig. 3. More formally the discontinuity in the partial-wave amplitude a (j, t) can be written in terms of the two-particle/two-Pomeron amplitude $A_{\alpha_1\alpha_2}(j, t)$ -

$$\text{Disc}\ a(j,t) = i \sin\frac{\pi}{2} j \underset{\lambda(t,t_1,t_2)>0}{\int dt_1 \int dt_2} \frac{\delta(j-\alpha_1-\alpha_2+1)\,\lambda^{1/2}(t,t_1,t_2)}{t \sin\frac{\pi}{2}\alpha_1 \sin\frac{\pi}{2}\alpha_2}$$

$$\times A_{\alpha_1\alpha_2}(j^+,t)\, A_{\alpha_1\alpha_2}(j^-,t) \qquad (11)$$

Further equations which can be derived are illustrated in Figs. 4

and 5. The set of coupled discontinuity formulae for all multi-Regge cuts can be summarized by writing $E = 1-j$, $k^2 = -t$ and saying that in the j-plane a Reggeon behaves like a quasi-particle with energy $E - \alpha[k^2]$ and momentum k. The coupled Regge cut equations can then be regarded as the unitarity equations controlling the scattering of the quasi-particle. An important point is that the analysis shows that the two Pomeron branch point has a negative imaginary part[20]--so the unitarity relation is also a "quasi-unitarity" relation. This negative sign requires the triple Pomeron coupling to be pure imaginary.

Since the Regge cut discontinuity formulae are derived with some rigor and depend only on quite general analyticity properties they should hold in a wide class of strong interaction theories. Any treatment of the Pomeron that satisfies these equations carries with it the virtue of satisfying full, multi-particle, t-channel unitarity in the j-plane. Our j-plane Pomeron field theory naturally satisfies unitarity in the j-plane and it is attractive to make this the basis for introducing the field theory. The full propagator reduces to the free propagator at large j(-E) and large $t(-k^2)$

$$G_0^{(1,1)} = \frac{i}{E - \alpha_0 k^2 + \Delta_0 + \dots} \tag{12}$$

which (by design!) contains a Pomeron pole. The Reggeon Calculus is the only treatment of high-energy scattering which satisfies full t-channel unitarity. The eikonal formalism neglects it and in fact the corrections to the eikonal formalism which are needed to satisfy t-channel unitarity are believed by several people to substantially modify eikonal calculations.[21]

Introducing the Reggeon Calculus as a field-theoretic solution of Regge Cut discontinuity formulae is unambiguous and aesthetically appealing. It might, however, seem rather mathematical compared to the intuitive picture we presented in I.

III. S-CHANNEL UNITARITY-- AVOIDANCE OF DECOUPLING ARGUMENTS

If we could explicitly prove S-channel unitarity there would be very little scope for doubters of the Reggeon Calculus. Gribov's derivation of the Calculus from an underlying field theory[17] which clearly does satisfy s-channel unitarity shows that the calculus can be consistent with the unitarity condition. The best we can do to check unitarity without going to an underlying strong-interaction theory, is to check the general decoupling arguments that have been used to argue that the Pomeron cannot be just a simple pole. First we consider the old[8]

Finkelstein-Kajantie Argument (simplified version)

If the Pomeron is a pole with $\alpha_p(0) = 1$ then from the inequality shown in Fig. 6 we obtain

$$\sigma_n \gtrsim \frac{(\ln [\ln s])^{n-2}}{\ln s} \qquad (13)$$

and

$$\sigma_T = \Sigma\sigma_n \gtrsim s^{\lambda} \qquad \text{some } \lambda \qquad (14)$$

To discuss how this problem is resolved in the strong-coupling solution we need to be able to calculate the production amplitude shown in Fig. 6. Migdal, Polyakov and Ter-Martirosyan[2] extended the Pomeron field theory to production amplitudes in a straightforward way. The general opinion of their treatment is that it is probably right for the Pomeron near $t = 0$. Soon we should have a derivation of their approach from hybrid Feynman graphs[22] and also from t-channel unitarity.[23] Basically (14) is avoided because of a decoupling suggested by Finkelstein and Kajantie, the vertex shown in Fig. 7 does vanish at $t_1 = t_2 = 0$. Detailed calculations[2] show that Fig. 6 gives

$$\sigma_n \sim (\ln s)^{-\alpha-[n-2]\beta}, \alpha = \frac{5}{6} + 0(\epsilon^2), \quad \beta = \frac{1}{2} + 0(\epsilon^2) \qquad (15)$$

So multiple diffraction cross sections eventually fall with energy.

Inclusive Sum Rule Arguments

It is well-known[9] that the energy sum rule gives the inequality shown in Fig. 8 if the integration is limited to the triple Regge region of the inclusive phase space. If the Pomeron is a simple pole and the triple Pomeron coupling $g_p(0) \neq 0$ and $\alpha_p'(0) \neq \infty$ then we obtain

$$\sigma_T > c \ln \ln s \Rightarrow \sigma_T \not\to \text{const} \tag{16}$$

$$\text{or } g_p(0) = 0, \text{ or } \alpha'_p(0) = \infty$$

In the strong coupling solution the behavior of the inclusive cross section in the triple Pomeron region is quite complicated. The Pomeron propagator is not a simple pole, but if we isolate a pure pole contribution we obtain[3]

$$g_p(0) = 0 \quad \text{and} \quad \alpha'_p(0) = \infty \tag{17}$$

Integrating the complete contribution of the three propagators and the vertex function shown in Fig. 9 we find that the right-hand side of Fig. 8 gives $(\ln s)^\eta$ for large ln s and so consistency is achieved.[2] Figure 9 also gives[2]

$$\frac{1}{8\pi} M^2 \left.\frac{d\sigma}{dM^2 d^2 p_\perp}\right|_{t=0} = \left(\frac{\sigma_T}{8\pi}\right)^{3/2} \frac{\sqrt{\alpha'_p}}{2\pi} \bar{g} \frac{(\ln s)^{\alpha_1}}{(\ln M^2)^{\alpha_2}} \tag{18}$$

$$\alpha_1 = \frac{1}{12} + 0(\epsilon^2) \qquad \alpha_2 = \frac{1}{2} + 0(\epsilon^2)$$

where α'_p is the effective slope measured experimentally. When the strong coupling solution is valid $\bar{g}$ has an absolute value

$$\bar{g} = \sqrt{\frac{4}{3}} + 0(\epsilon^2) \approx 1 \tag{19}$$

The recent deuteron scattering experiment at Fermilab gives[24]

$$\bar{g} \sim 0.4 \tag{20}$$

and so we conclude that at present energies the strong coupling solution cannot be used directly. Before going on to "bare perturbation theory," we note another important property of the strong-coupling solution.

In the Central Region the leading behavior comes from the graph shown in Fig. 10. This gives

$$\frac{1}{\sigma_T}\;\frac{1}{8\pi}\;\frac{d\sigma}{dy\,d^2p_\perp} = f(p_\perp^2)\left(\frac{y(Y-y)}{Y}\right)^{\eta} \qquad \begin{array}{l} Y = \ln s \\ y = \text{rapidity of detected particles} \end{array} \tag{21}$$

This is shown in Fig. 11. At asymptotic energies there is no central plateau.

IV. BARE PERTURBATION THEORY AND PRESENT ENERGIES

The renormalization group results we have presented so far are obtained from the complete renormalized Green's functions. The theory, however, also has a bare perturbation expansion as illustrated in Fig. 12. This is the expansion we would expect to obtain if we followed Gribov's procedure of deriving the calculus from an underlying field theory.[17] The bare propagator which appears in these diagrams will not have intercept one, but the intercept has to be chosen so that the renormalized intercept is one. In fact we can show that[4]

$$\alpha_B(0) = 1 + \frac{r_0^2}{4\alpha_0'}\ln\frac{\Lambda\alpha_0'}{r_0^2} + 0\left(\frac{r_0^2}{\alpha_0'},\lambda_n\right) \tag{22}$$

where r_0 and α_0' are bare quantities but the normalization is such that

$r_0/\surd\,\alpha_0'$ compares directly with $\bar{g}$ above. Λ is the cut-off discussed above, and λ_n represents all other bare Pomeron couplings.

The effective expansion parameter for our bare expansion is $\frac{r_0^2}{4\alpha_0'} \cdot \times$ ln s, which at present energies is small so that we may expect to see only a few terms in this expansion. The renormalization group results apply when $\frac{r_0^2}{4\alpha_0'} \ln s \gtrsim 1$. Another reason for expecting to see only a few terms at low energy is that there must be enough energy available for each Pomeron channel to be well represented by a pole. For example, from Fig. 13 we see that the masses M_1 and M_2 should be well above any important thresholds in these channels,[25] for this process to be well represented by the Pomeron graph. Note that the bare perturbation expansion is very similar to other perturbative approaches to the Pomeron[26] that have been advocated. A major difference is that we take account of absorptive effects which are responsible for the negative sign of the two Pomeron cut. This is why our bare Pomeron intercept is renormalized downwards by t-channel iteration of Pomeron interaction diagrams.

As we mentioned earlier it is only when the renormalized intercept is exactly at one that we obtain the critical behavior--or strong-coupling solution. This requires the bare parameters, $\alpha_B(0)$, r_0, α_0' to be related by the above expression. Now recent fits to the ISR data[11] for $\left(\frac{d\sigma}{dt}\right)_{el}$ have shown that the data are best fitted by a simple pole with intercept above one where

$$\alpha_B(0) - 1 = 0.06 \tag{23}$$

$$\alpha_0' = 0.25 \tag{24}$$

and the recent deuteron scattering experiment at Fermilab gives

$$r_0 \sim 0.2 \tag{25}$$

if we take Λ such that $\ln \frac{\Lambda \alpha_0'}{r_0^2} \sim 1$, (an order of magnitude estimate is clearly insensitive to the exact value of Λ) then

$$\frac{r_0^2}{4\alpha_0'} \ln \frac{\Lambda \alpha_0'}{r_0^2} \sim 0.04 \tag{26}$$

This seems a very encouraging result, since these three parameters could a-priori be of totally different orders of magnitude. Also 0.06 is probably an over-estimate for $\alpha_B(0) - 1$ since some small part of the rise of the total cross section must be coming from cut contributions.

Note that the rise of the effective Pomeron intercept from low to ISR energies has to be unrelated to diffractive production in this picture (it could well be explained by $N\bar{N}$ production, as advocated by several[27] people). Yet the amount the intercept is pushed above one is determined by diffractive production in this picture as measured by the triple Pomeron coupling. Therefore, there has to be a tight relation between diffractive and non-diffractive production and, in this sense, both effects can be regarded as responsible for the rising total cross section, as measured by $\alpha_B(0) - 1$.

V. t-CHANNEL IMPOSITION OF THE FROISSART BOUND

I don't mean to imply here that t-channel unitarity is strong enough to satisfy the Froissart bound (I'll discuss this outside of the strong-coupling solution shortly). I simply wish to compare the t-channel process whereby a particular $\alpha_B(0)$ (> 1) can give (via the Reggeon Calculus) a renormalized intercept of one, with the more familiar process of s-channel eikonalization were an arbitrary $\alpha_B(0) > 1$ can be brought down to one.

Roughly the t-channel effect is achieved by summing the diagrams of Fig. 12, while the eikonal effect is achieved by summing the diagrams of Fig. 14. For the eikonal effect each of the diagrams in this sum must be comparable and all are present at any given energy, so the bare pole can never be isolated. This explains why the eikonal approach has such difficulty fitting $\left(\frac{d\sigma}{dt}\right)_{el}$ at the ISR.[28,29] The t-channel process can work even when all the rescattering diagrams of the eikonal approach are small. Further the bare pole can and should be isolated at low energies. Hence, the fit of Collins, Gault and Martin,[11] which favors a bare pole over an eikonal sum of cuts, suggests that the t-channel process is at work.

We would expect the rise of σ_T initially produced by the bare pole to slow down as diffractive production of two large masses builds up, giving rise to the diagram of Fig. 13. This diagram should produce a partial cancellation of the bare pole. Note that the t-channel renormalization effects of diagrams such as this should fall off sharply in t because of the fall-off of the triple-Pomeron coupling for negative t. Hence if the bare trajectory has the form shown in Fig. 15, then after renormalization we would expect the form shown in Fig. 16. The scaling law mentioned earlier requires[3] the trajectory to be non-analytic at t = 0

$$\alpha_p(t) = 1 + ct^{1/\nu} \qquad \frac{1}{\nu} = \frac{12}{13} + 0(\epsilon^2) \tag{27}$$

Finally we come to

VI. THE REMOVAL OF PARITY DOUBLETS

Since fermion regge trajectories are approximately linear in the Mandelstam variable u then from MacDowell symmetry we expect to see nearly degenerate, positive and negative parity partner trajectories. Experimentally this is apparently not the case. Some time ago Carlitz and Kislinger[30] showed that a natural analytic structure for partial-wave amplitudes was a fixed-cut located at $j = \alpha_F(0)$. This could move the wrong parity trajectory off the physical sheet in the angular momentum plane. Carlitz and Kislinger suggested a Van-Hove like model constructed out of resonances of one parity, as the origin of the fixed-cut. However, the Pomeron corrections to the Fermion propagator coming from Pomeron interactions (as shown in Fig. 17 give rise to multi-Pomeron

Fermion branch-points passing through $j = \alpha_F(0)$. Bartels and Savit have shown that the strong coupling Pomeron can give rise to a fixed-cut[31] which will play a very similar role to that suggested by Carlitz and Kislinger. The analytic structure obtained by Bartels and Savit is slightly different. Their bare Fermion propagator contains both parity trajectories. The renormalized propagator has complex-conjugate partner trajectories for negative u but only one parity trajectory remains on the physical sheet of the fixed j-cut for positive u (see Fig. 18). This is a welcome bonus from the strong coupling Pomeron, which is now seen as the dynamical source of the fixed j-cut.

It is hard to determine the complete analytic structure of both the Pomeron and the Fermion propagators. An explicit form for the scaling functions in the scaling laws can only be derived for small ϵ and it is difficult to check whether the scaling laws are related to a fixed cut in j or u (or conceivably neither). However, it can be proved that for large j there is no fixed cut in u, whereas the converse cannot be proved. So it seems very likely that there is a fixed-j cut.

VII. OTHER SOLUTIONS

Weak Coupling

The renormalization group has also been used to realize the well-known alternative weak-coupling solution of Gribov and Migdal[32] (which Gribov apparently still favors) in which the triple-Pomeron coupling vanishes linearly.[15, 33] As in Gribov and Migdal's original Schwinger-Dyson equation analysis this is achieved by cut-off dependent cancellations and so it is very hard to understand how the further de-couplings arguments of Ref. 10 outlined in Fig. 19 can be avoided. It would seem that cuts corrections to the simple Regge pole exchange have to be vital[34] in avoiding the argument of Fig. 19 and it is hard to see how this can be achieved when cut-off dependent cancellations are invoked to produce the triple Pomeron zero. If the weak-coupling solution can be realized consistently it seems likely that higher-order interactions play a vital role.[34] In the strong coupling solution there should be no difficulty in avoiding the argument of Fig. 19, since cut corrections to the pole are as strong as the pole.

$\underline{\alpha_B(0) - 1 > 0(r_0^2)}$

In this situation we are below the critical point. This looks very much like the negative $(\text{mass})^2$ spontaneous symmetry breaking situation familiar from relativistic field theory or solid-state physics.[35] Unfortunately the pure imaginary triple Pomeron coupling makes the effective potential non-hermitian. This together with the complex nature of the Pomeron field has so far defied attempts to treat this situation from the spontaneous symmetry breaking point of view.

It may be that if $\alpha_B(0) - 1 > 0(r_0^2)$ that the Froissart bound must be imposed by s-channel summation. That is the s-channel sums of the eikonal form shown in Fig. 14 will be more important than the t-channel sums of Fig. 12. Cardy[36] has given a nice discussion of how the eikonal summation could generalize in this situation--see also Bronzan's "weak-coupling" calculation.[37]

Although we don't yet have a completely satisfactory solution with $\alpha_B(0) - 1 > 0(r_0^2)$ it remains an intriguing possibility.

Pomeron Not a Pole, but $\sigma_T \to$ Constant

If a bare Pomeron is used which is not a pole, a constant cross section can be obtained[38] (infra-red freedom). Since the renormalized propagator reduces to the bare propagator at large positive t, the leading singularity in this sort of solution is not a pole and so t-channel unitarity is no longer guaranteed.

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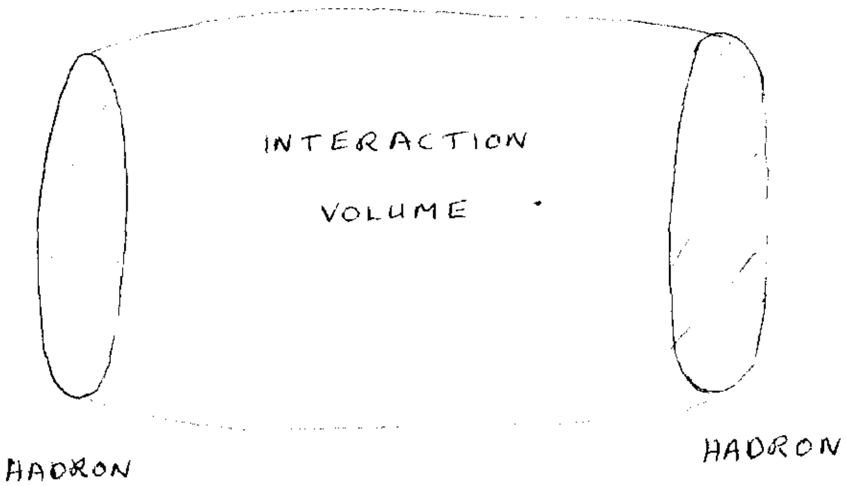


FIG. 1

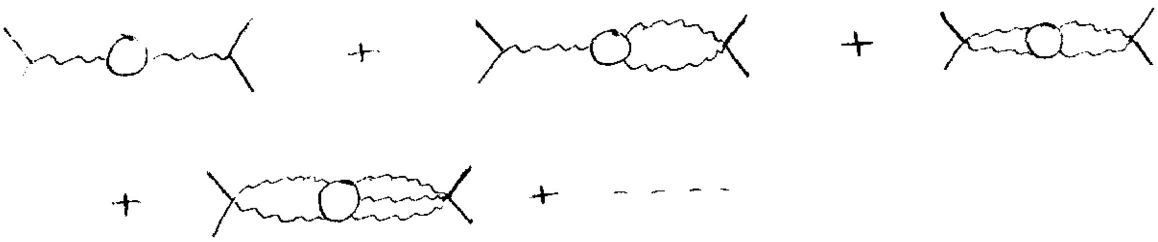
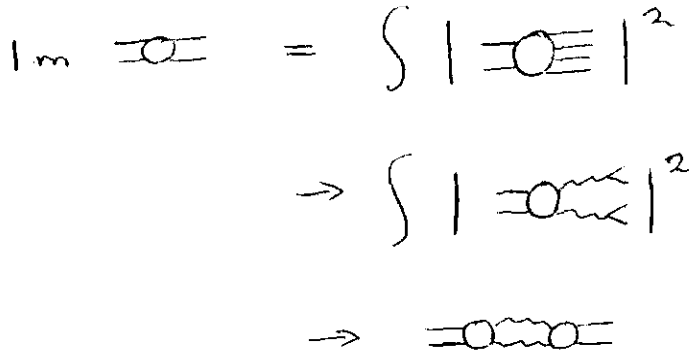


FIG. 2



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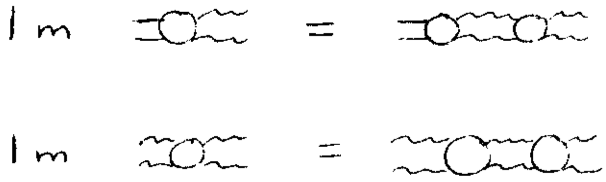
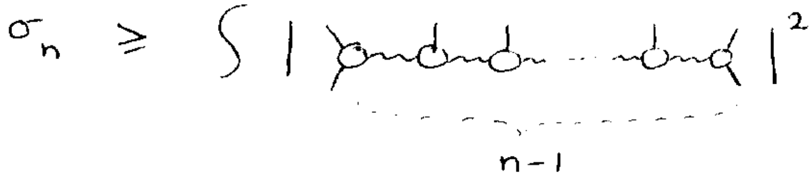


FIG. 4

FIG. 5



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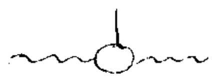

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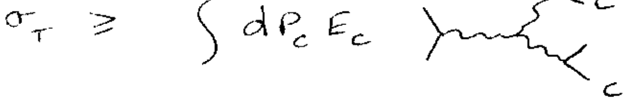

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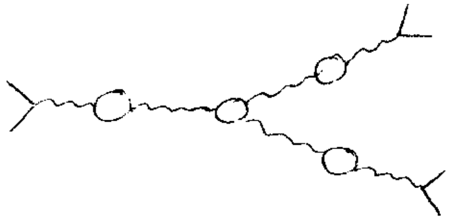

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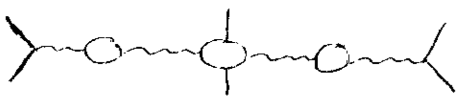

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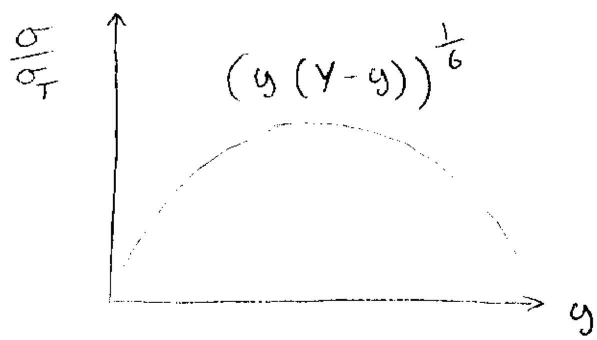

FIG. 11

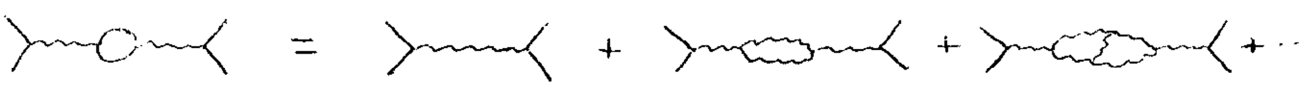

FIG. 12

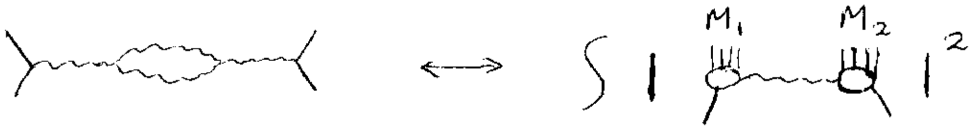

FIG. 13

FIG. 14

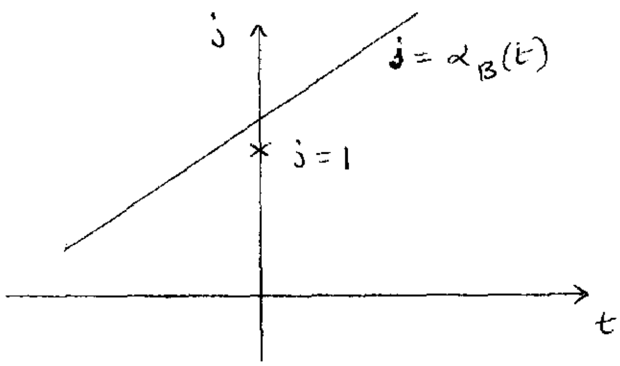

FIG. 15

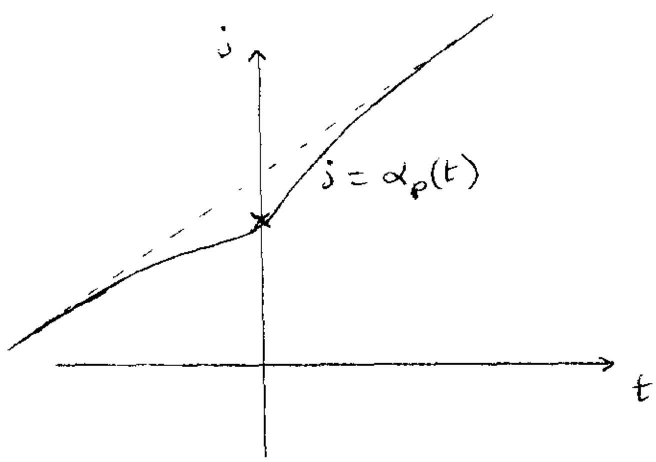

FIG. 16