STRONG DAMPING OF LARGE MOMENTUM TRANSFERS, AND ITS CONSEQUENCES FOR HIGH-ENERGY INELASTIC PROCESSES

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I. INTRODUCTION

Recently some progress has been made towards understanding the properties of cosmic ray events in terms of properties of strong interactions at lower energies. For example, the one-pion exchange model was applied

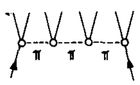

Fig. 1

A highly inelastic event leading to several "clumps" by means of repeated one-pion exchange

to peripheral collisions and then generalized by the SALZMANs [1], GOEBEL [2] and AMATI et al. [3] to a repeated one-pion exchange model which leads to several "clumps" of particles in the final state (Fig. 1). In this generalized approach a very high energy process is reduced to a product of factors, each representing production of one of the clumps at much lower energy where the interactions are better understood.

Meanwhile elastic proton-proton scattering [4] at accelerator energies has been found to decrease exponentially with increasing momentum transfer $|t|$. Over part of the range of experiments, especially at $|t| \lesssim 1\ (\mathrm{GeV})^2$, the observed behaviour may be explained by the exchange of a single dominant Regge pole [5-8], but the exponential falloff persists at larger $|t|$ where the detailed mechanism is not understood.

In the present approach we shall assume, without attempting to understand the underlying reasons or formalism, that the exponential damping of large momentum transfers is a general characteristic of high-energy amplitudes. The rate of damping will be taken from the existing elastic proton-proton results [4] and applied to inelastic $p + p$ and $\pi + p$ events. We also employ a breakdown into low-energy clumps as in the work of the SALZMANs [1], GOEBEL [2] and AMATI et al. [3], whose approach and results we follow in many respects. No restricition is made to one-pion exchange between clumps, however.

Observed features [9, 10], such as "fireballs" and constant transverse momentum of secondary particles, come out in a natural way, with reasonable magnitudes. The relation of these properties of cosmic rays to small momentum transfers has already been noticed by cosmic ray experts [10], so the present approach serves especially to emphasize that momentum transfers are comparably small at machine energies and at higher energies. One result of the present approach is that a definite conception of the fireball, as distinguished from individual particles, emerges. This picture will be discussed in detail, especially in the energy region 10^3 - 10^5 GeV where most of the data on fireballs has been obtained.

II. THE ASSUMPTION ON MOMENTUM TRANSFER DEPENDENCE

At $-1\ (\mathrm{GeV})^2 < t < 0$, elastic proton-proton scattering decreases exponentially with increasing $|t|$ and the width of the exponential peak decreases slowly as the energy rises. The data are consistent with the formula [5, 8]

$$\frac{d\sigma\ (s,t)}{dt} = f\,(t)\left(\frac{s}{2\ M^2}\right)^{2\,\alpha(t)\ -2} \sim f\,(t)\,e^{-2|t|\,\alpha'\ln(s/2M^2)}, \qquad (\mathrm{II}.1)$$

where s is the square of the centre of mass energy, M is the nucleon mass, and α (t) is the spin of the dominant Regge trajectory, rising from about $\alpha = 0$ at $t = -1\ (\mathrm{GeV})^2$ to $\alpha = 1$ at $t = 0$.

At larger $-t$, α (t) seems to stabilize in the region $0.5 < \alpha < 0$, with large errors. If this is true, the factor $\exp\,[-2|t|\,\alpha'\ln(s/2M^2)]$ decreases no further; nevertheless $d\sigma/dt$ still falls with increasing $-t$ at approximately the rate $10^{t/M^2} = \exp[\,2.3\ t/M^2]$, rather independent of energy [4]. The reason for this behaviour is not known.

Our assumption will be that any high-energy amplitude decreases at least as fast as $\exp\,[\,1.1\ t/M^2]$. This is taken directly from the square root of the elastic proton-proton cross-section. If (II.1) is appropriate, the amplitude may decrease faster. For the dominant inelastic processes, however, a simple kinematic analysis shows that the reactions are not in the asymptotic region where (II.1) is valid.

Actually there are two momentum transfers in elastic scattering, the "direct" transfer t and the "exchange" transfer u. They are related by the constraint $s + t + u = \Sigma\, M_i^2$. The distance to the nearest singularity ($t = u^2$ at small $|t|$) is therefore the same in either variable; $|t - u^2| = |u - \Sigma M_i^2 + u^2 + s|$. Thus our assumption can be formulated more generally: in each variable the amplitude falls off exponentially as the distance of the variable from the nearest singularity increases. Naturally it is most convenient to use t at small $|t|$, for then the nearest singularity lies at a small mass fixed independently of s. In the inelastic case where many momentum transfers can be defined, we shall again find it convenient to use a small one.

III. HOW LARGE ARE THE CLUMPS?

To appreciate the effect of the assumption made in Section II, consider Fig. 2 for the reaction $A + B \to C + D$, where C and D are arbitrary clumps

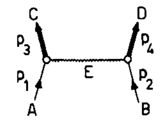

Fig. 2

The reaction $A + B \rightarrow C + D$

of particles with energies M_3 and M_4 respectively in their own centres of mass. Define $s = (p_1 + p_2)^2$ and $t = (p_3 - p_1)^2$ in the usual way. In the centre-of-mass system of the entire reaction, C and D each emerge with momentum p_T transverse to the initial direction of motion. The relation between t and p_T is given by

$$t = -p_T^2 + \frac{1}{4s}\{[M_3^2 - M_4^2 + M_1^2 - M_2^2]^2 - [M_3^2 + M_4^2 - M_1^2 - M_2^2 + 2p_T^2]^2\} \tag{III.1}$$

One sees that $-t$ grows directly with p_T^2. Thus our assumption of exp $[1.1/M^2]$ falloff implies exp $[-1.1\, p_T^2/M^2]$ falloff with increasing transverse momentum. In fact, from (III.1) it is clear that the experimental absence of large p_T directly implies that large $|t|$ are absent. The momentum transfer is somewhat less sensitive to the masses of clumps at high energy s, and the exponential falloff tends to restrict the masses only when they grow at least as fast as $M_3^2 M_4^2 \sim s$.

Before discussing further the dynamical limitation on clump size, we need to agree on a definite way to assign the various particles in a complicated final state to clumps. Consider the centre-of-mass frame for the reaction $A + B \rightarrow$ many particles. Now clump C will be defined to consist of all particles which go forward in the centre of mass, and clump D will be defined to consist of all particles which go backward. This definition yields a relatively small momentum transfer and coincides with the natural experimental division into forward and backward groups.

There are various ways to categorize the exchange that occurs between (A, C) and (B, D) in Fig. 2. It can be described as a one-pion exchange, plus a two-pion exchange, plus an $N\bar{N}$ exchange, and so forth. Or it can be described as the exchange of a succession of Regge poles. In any case the complete amplitude factors into a product of terms:

(1) The amplitude for A + exchanged object E $\rightarrow$ C. (Of course, the amplitude must be continued from the physical square mass of E to a negative square mass.)

(2) A factor involving only E.

(3) The amplitude for B + E $\rightarrow$ D.

The next step is to take amplitude (1) or (3) and again break the final state into two groups of particles. For example (Fig. 3) in the centre-of-mass of (1) we include forward-moving particles in group 5, with energy M_5 in its own rest frame, and backward-moving particles in group 6.

There are now four groups of particles in the final state, and these groups could be sub-divided further to the point where each clump contains

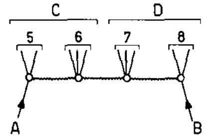

Fig. 3

Breakdown of $A + B \rightarrow C + D$ into 4 clumps

only one particle. But we shall carry the subdivision only down to the point where each clump contains a couple of GeV. At this point it is possible to make some qualitative estimate of what will happen without reducing the energy further, and our assumption on exponential damping of large momentum transfers cannot be used at lower energies. The question then is: how many subdivisions are required before each clump is reduced to a couple of GeV ? If there were no dynamical restrictions, the energies M_3 and M_4 of clumps C and D in their own rest frames could take up all the available centre-of-mass energy $\sqrt{s}$, leaving no relative kinetic energy for the clumps. In this case many subdivisions would be required to reduce the clumps to low masses. However, $M_3^2\ M_4^2$ would then grow as s^2 and $-t$ would grow as s, and here the dynamical assumption of section II which damps large momentum transfers becomes relevant.

Consider first $A + B \rightarrow C + D$(Fig. 2). The cross-section can be expressed in terms of the cross-sections $\sigma_{AEC}(t; M_3^2)$ for $A + E \rightarrow C$ [E has $m^2 = t$] and $\sigma_{BED}(t; M_4^2)$ for $B + E \rightarrow D$ by a slightly modified form of the Salzman relation [1] :

$$\frac{\partial^3 \sigma}{\partial t\ \partial M_3^2 \partial M_4^2} = \frac{1}{2(2\pi)^3\, p_{iL}^2\ M^2}\ [\sigma_{AEC}(t; M_3^2) q_A M^3]\ F(s, t, s_1, s_2)[\sigma_{BED}(t; M_4^2) q_B M_4] \tag{III.2}$$

where p_{iL} is the momentum of A in the lab [rest frame of B], q_c is the momentum of A in the centre of mass of the reaction $A + E \rightarrow C$, and q_B is the momentum of B in the centre of mass of $B + E \rightarrow D$. The factor F(t) refers to the system exchanged; it is $(t - m_\pi^2)^{-2}$ in one-pion exchange and exponentially decreasing in our case. At high energies, with M_1 and M_2 fixed, (III.2) simplifies to

$$\frac{\partial^3 \sigma}{\partial t \partial M_3^2 \partial M_4^2} = \frac{M_3^2 M_4^2}{2\,(2\pi)^3 s^2}\ \sigma_{AEC} F \sigma_{BED}. \tag{III.3}$$

The cross-sections σ_{AEC} and σ_{BED} are expected to remain approximately constant as M_3^2 and M_4^2 respectively increase. Our method is too weak to understand the t dependence of $\sigma_{AEC}\ \sigma_{BED}$ or the dependence of F on s, s_1 and s_2, but provided none of these factors increase exponentially, it is clear that large $|t|$ are restrained by $F \sim \exp(-2.3\,|t|)$, and this is sufficient to establish that $M_3^2 M_4^2 \lesssim s M^{*2}$ where M^* is of order 1 GeV*. The next step is to break clump C down into subgroups with masses M_5 and M_6. The centre-of-mass energy squared for the reaction $A + E \to C$ is M_3^2, and the limitation on momentum transfer leads in this case to $M_5^2 M_6^2 \lesssim M_3^2 M^{*2}$. Similarly the breakdown of clump D leads to $M_7^2 M_8^2 \lesssim M_4^2 M^{*2}$. Altogether one has $M_5 M_6 M_7 M_8 \lesssim M^{*3} s^{\frac{1}{2}}$.

For example, if the lap energy of a proton-proton collision is 10^4GeV, then $s = 2 \times 10^4$ GeV^2 and $M_5 M_6 M_7 M_8 \lesssim 1.4 \times 100 \times M^{*3}$. If we take $M^{*2} = (2.3)^{-1}$ GeV^2[the value for which $\exp[-2.3\ |t|\ ]$ becomes $\exp(-1)$],then $M_5 M_6 M_7 M_8 \lesssim 40$ GeV^4. In case each split was symmetric, $M_5 = M_6 = M_7 = M_8 \lesssim 2.5$ GeV and all 4 clumps have reached the low-energy region where one can make plausible guesses about them without further reductions. Of course non-symmetric splits are also allowed, and in extreme cases larger clumps would require more than two successive reductions at 10^4 GeV.

IV. FIREBALLS

Let us discuss in more detail the 4 clumps obtained in proton-proton reactions at lab energies of 10^4 GeV. Although we have only obtained a maximum size, the experiments suggest that this maximum size is about normal; and we shall confine the discussion to the case where the maximum is attained without attempting to discover why it is usually attained. In the centre of mass one will see a fast clump moving forward along the original direction of A (remember that p_T must be small for a clump) and another moving backward along the original direction of C, each followed by a slower clump moving along the same line (Fig. 4).

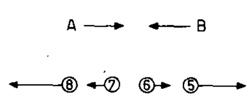

Fig.4

Motion of clumps in centre of mass of reaction $A + B \to C + D$

The damping of large momentum transfers between clumps suggests the dominance of long-range forces, and on this basis one expects that systems of baryon number zero will normally be exchanged between the clumps. Each of the two fast clumps (5 and 8) then carries baryon number one since the incoming particles A and C were baryons, and the two slow clumps (6 and 7) carry baryon number zero. In accordance with cosmic ray terminology the clumps with baryon number one will be called nucleon isobars, and the clumps with baryon number zero will be called "fireballs".

How many fireballs are there in general? We have adopted the procedure of subdividing until reduced scattering events are obtained, each at a rel-

* It might be objected that, as $-t$ becomes very large and far from the nearest singularity at positive $t = \mu^2$, it may approach the nearest singularity at negative $t = \Sigma M_i^2 - s - \mu^2$, and the cross-section may rise again. This possibility is excluded by the definition of t as $(p_0 - p_A)^2$ where $\vec{p_C}$ and $\vec{p_A}$ are both in the forward hemisphere in the centre of mass.

atively low energy. Because of the low energies involved, each of these scatterings produces a relatively isotropic final state in its own centre of mass, though still peaked somewhat forward and backward. It is these relatively isotropic final states which are called nucleon isobars or fireballs. Now as the overall energy of the reaction is increased, the centre-of-mass energy of each "fireball" and "isobar" slowly increases, and each of them becomes more strongly peaked forward and backward. Above some clump mass, of order 5 GeV for the clumps with nucleon number one and perhaps lower for the fireballs, it becomes meaningful to split the clump again into its forward and backward components, each of which has a mass between 1 and 2 GeV and is relatively isotropic again. In summary the mass of fireball always lies between extremes of order 1 and 5 GeV, and as the overall energy of the reaction increases fireballs swell into dumbbell shapes and divide rather than grow beyond their proper sizes [11]. The process is illustrated in Fig. 5*.

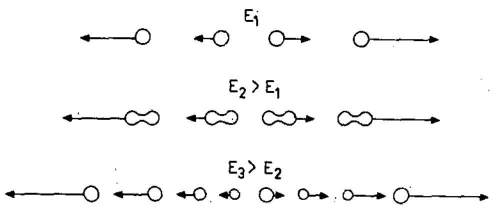

Fig. 5

The growth of clumps along the original direction of motion in the centre of mass, as the energy increases

Although the number of fireballs increases with energy, the increase is slow. At 30 GeV lab energy, proton-proton scattering leads to two "isobars" and no fireballs. At 10^4 GeV lab energy two fireballs have also developed. At 10^8 GeV a symmetric split-up leads to 6 fireballs, each with a mass of about 2.3 GeV. In general n clumps are obtained with repeated applications of the formula

$$M_3^2 M_4^2 / M^{*2} = s \tag{IV.1}$$

leading to

$$(M_5^2 M_6^2 / M^{*2})(M_4^2 / M^{*2}) = s$$

$$\prod_{i=1}^{n} M_i^2 / (M^{*2})^{n-1} = s \tag{IV.2}$$

*It should be mentioned, however, that there is some evidence for fireballs emitting secondaries into a disk pattern peaked perpendicular to the incoming direction, rather than into a dumbbell pattern (e.g. Ref.[10]). Further evidence on this point should be of great importance for the consistency of the multiple fireball picture.

where each of the M_i is a fireball or isobar mass, not greater than 5 GeV. Taking an average square mass for the fireballs and isobars, one finds

$$n = -[\ln (s/M^{*2})] / [\ln (\overline{M_i^2}/M^{*2})] \qquad \text{(IV.3)}$$

so the multiplicity of isotropic clumps increases as ln s.

Asymmetric as well as symmetric split-ups occur, since only the combination $M_3^2 \, M_4^2$ enters into Eq. (III.1) when M_3^2 and M_4^2 are each much greater than 1 GeV^2. The asymmetry is especially noticeable when it occurs in the first split-up, leading to a depletion of secondaries in one hemisphere in the centre of mass. Suppose this happens at 10^4 GeV lab energy, and M_3^2 is large while M_4^2 is only a few GeV^2. Then in the backward hemisphere a nuclear isobar, or perhaps only a single nucleon emerges, while in the forward hemisphere $M_3^2 = M^{*2} s / M_4^2$ can be split into 4 clumps if $M_4^2 = 1\ \mathrm{GeV}^2$ (single nucleon), or 3 to 4 clumps if M_4^2 = several GeV^2 (nucleon isobar). The general nature of the derivation showing that fireball multiplicity rises as ln s ensures that asymmetric split-ups lead to similar multiplicities.

From the foregoing description it is clear that the nucleon isobars and fireballs have a similar origin in the present model. The masses of fireballs and isobars are sufficiently low relative to the nucleon mass, however, to lead to certain differences, and one of these is the multiplicity of particles emitted from the fireball or isobar. Consider the mass 2.5 GeV, for example. A state with this mass and baryon number one is expected to contain one nucleon and one or two pions. A state with this mass and baryon number zero is expected to contain three or four particles which are most likely π, ρ, ω or n. The decay of the ρ, ω or n then leads to a final state with about six pions. This is what happens, for example, in the final state of $p\bar{p}$ annihilation. Thus the fireballs produce pions much more copiously than the nucleon isobars. For an incident lab energy of 10^4 GeV each isobar emits one or two pions and each fireball about six, or a total of about 15 pions.

The logarithmic growth with s of fireball and clump multiplicity Eq. (IV.3) indicates that particle multiplicity increases as ln s at large s where the fireball picture is applicable [3], since an average fireball emits about the same number of particles whatever the original s is. Actually the rate of increase in pp scattering from 30 GeV to 10^4 GeV in the lab is somewhat enhanced because the two fireballs which appear in this energy region provide more particles than the two "nucleon isobars" which were already present at 30 GeV. For example, we expect the total number of particles, N, to increase from about 5 to 17 as the energy rises from 30 GeV (2 nucleon isobars) to 10^4 GeV (15 pions + 2 nucleons) whereas the form N = a ln s would give a rise from about 5 to 13 in this interval. This makes our predictions fairly compatible with the data even though the observed multiplicity is traditionally represented as growing at a rate $N \sim s^{\frac{1}{4}}$ over much of this region [9, 10].

Everything that has been said for proton-proton scattering would also hold for pion-pion scattering, with one of the outside nucleon isobars replaced by a fireball. As a by-product at 30 GeV where only two clumps are typically formed, one expects a somewhat higher particle multiplicity in π N reactions than in NN reactions because one of the clumps contains only pions in the former case.

Since events of arbitrarily high energy reduce to products of events at several GeV, most of the secondaries are pions, and K mesons and baryons will be produced in ratios similar to those found at a few GeV.

V. TRANSVERSE MOMENTA

One of the most persistent phenomena in high-energy and cosmic-ray physics involves the transverse momentum distribution of inelastic secondaries: for any incident energy, the distribution is peaked around $p_T \sim 0.4$ GeV/c. At accelerator energies the tail of large p_T has also been studied quantitatively [12] and is found to fall off exponentially, consistent with $\exp(-p_T/0.2)$ [p_T in GeV/c].

The kinematical dependence of t on p_T^2 (Eq. III.1), together with exponential damping of large $|t|$, damps the transverse momentum of each clump as $\exp(-2.3\, p_T^2)$. As a consequence each fireball or nucleon isobar moves approximately along the line of flight of the particles which initiated the reaction. Then the transverse momentum of each particle has a component due to the motion of its clump (shared among several particles and therefore small), plus the motion of the particle relative to the clump centre of mass. The later contribution refers to a reaction of only a few GeV, so the transverse momentum of individual particles reduces approximately to the low-energy figure no matter what the incoming energy is. Large transverse momenta are strongly damped by dynamical factors, and further damped at a few GeV by competition among the particles in a clump for phase space.

What does this model have to say about the transverse momenta of different kinds of secondaries: π, K, N? Distinctions can appear only in the last stage where a clump is broken down into several particles, and this involves reactions at a few GeV. Here the dynamical damping of cross-sections at large momentum transfer is probably of order $(t - M^2)^{-2}$, where M is the exchanged mass, rather than exponential. The exchanged mass is greater for production of K's and baryons than for pion production, so the dynamical damping of large p_T relative to small p_T may be weaker for K's and baryons. There are also phase space factors to consider, and these strongly inhibit the transverse momentum of any particle from becoming very large, especially in the fireballs because they have more particles than the "nucleon isobars". At accelerator energies pp scattering does not yet produce fireballs, and the strange particles which require a large mass exchange may well have larger p_T than pions have.

VI. INELASTICITY

As a measure of the distribution of energy in the final states, cosmic-ray physicists [9, 10] define the inelasticity K

$$K = \frac{\text{Centre-of-mass energy of secondaries}}{\text{Centre-of-mass energy of initial state}} . \tag{VI.1}$$

The "secondaries" are defined as all particles in the final state except for the fastest particle in each hemisphere.

Let us first consider the contribution of a fireball to K, in the energy range where there are only two fireballs. In section III the relation $M_5^2 M_6^2 M_7^2 M_8^2 \sim M^{*6} s$ was established for this energy range. A particularly low inelasticity is found when the reaction is symmetric in the centre of mass [$M_5 = M_8$, $M_6 = M_7$] and the "nucleon isobar" is simply one nucleon [$M_5 = M$]. Then the fireball mass M_6 is proportional to $s^{\frac{1}{4}}$, as compared with the total centre-of-mass energy $s^{\frac{1}{2}}$, so the inelasticity K can fall off as rapidly as $s^{\frac{1}{4}}$ if the fireball moves only slowly in the centre-of-mass frame (a possibility which is consistent with our conditions). Larger inelasticities are also possible, especially when the nucleon isobar is larger and emits pions. The result depends somewhat on the detection method; for example, only charged secondaries may be detected, and then the question is whether the fast nucleon isobar in the lab emits more than one charged particle. If it does, the inelasticity can easily be 0.5 or greater. At somewhat higher energies where 4 fireballs appear, the inelasticity can be low if the original fireballs have grown large and split in two, or large if the original nucleon isobar has grown and split in two [11]. At all energies, then, the inelasticity will have a broad spread. The average is essentially controlled by the fraction of energy the fastest nucleon isobar shares with pions that get counted as "secondaries". The composition of the nucleon isobar is not very energy-dependent, so the average should be approximately energy-independent [3]. The inelasticity for π N events should behave similarly, but the average should be higher because there are more fast pions.

VII. REGGE POLES

The author began this study of highly inelastic events with the hope that exchange of a dominant Regge trajectory would lead to a simple formula like (II.I). This worked in the case of elastic or nearly elastic scattering [5-8], where the amplitude at fixed t and large s was dominated by a term proportional to $P_\alpha(\cos\theta_t) \sim (\cos\theta_t)^\alpha$. As s increased, $\cos\theta_t$ grew as

$$\cos\theta_t = -1 - 2s/(t - 4M^2), \tag{VII.1}$$

taking equal masses as an example, and the amplitude grew as s^α. Now if all four masses are unequal in the process A + B → C + D, (VII.1) is replaced by

$$\cos\theta_t = \frac{-\{t^2 + t(2s - M_1^2 - M_2^2 - M_3^2 - M_4^2) + (M_1^2 - M_3^2)(M_2^2 - M_4^2)\}}{\sqrt{t-(M_1-M_3)^2}\sqrt{t-(M_1+M_3)^2}\sqrt{t-(M_2-M_4)^2}\sqrt{t-(M_2+M_4)^2}} . \tag{VII.2}$$

We are interested in the cosmic-ray case discussed in the previous sections, where $M_3^2\ M_4^2 = M^{*2} s$ and M_1, M_2 can be neglected, leaving

$$\cos\theta_t \sim -2\,ts/M_3^2\,M_4^2 \sim -2\,t/M^{*2}. \tag{VII.3}$$

at large s and fixed t. So $\cos\,\theta_t$ does not increase with s, and (II.1) cannot be used.

Nevertheless it would be desirable to have a Regge pole formalism applicable to highly inelastic events; it might help to put the very simple considerations of the preceding sections on a more adequate basis. The author does not know how to do this but would like to call attention to a few problems which come up [13].

To begin with, recall the one-pion exchange model for $A + B \rightarrow C + D$. The amplitude is written as the product of

(a) the amplitude for A + exchanged $\pi \rightarrow C$,

(b) the pion propagator,

(c) the amplitude for $B + \pi \rightarrow D$.

So far the unknown amplitude for $A + B \rightarrow C + D$ has simply been reduced to a product of unknown amplitudes. The next step is to calculate $A + \pi \rightarrow C$, which is done by expressing this amplitude as another one-pion exchange. The process is repeated until one has a product of low-energy amplitudes. The incoming objects in these amplitudes (except for the original particles A and B) have spin zero, and their masses are in many cases continued from $t = m^2$ to negative t.

Now if the pion lies on a Regge trajectory, the one-pion exchange procedure still applies for $t = m^2$, and a natural extension is to exchange the pion Regge trajectory (or to be more complete, the sum over all trajectories) at $t \neq m^2$. The original amplitude can still be factored [14, 15] into the products of (a) the amplitude for A + exchanged trajectory $E \rightarrow C$, (b) a term involving only the Regge pole, (c) the amplitude for $B + E \rightarrow D$. Let us assume that the amplitude for $A + E \rightarrow C$, for example, can be expressed in terms of another Regge pole exchange. One again obtains a product of low-energy amplitudes. This time the incoming objects in the low-energy amplitudes (except for A and B) are Regge poles which not only have masses continued to $m^2 = t$ where t may be negative, but also have spins continued to non-integer values which vary with t. In order to construct a theory of repeated Regge pole exchange, then, it will be necessary to construct a theory of amplitudes in which some of the external objects are Regge poles.

Suppose that all this can be done, and consider the case where $s/M_3^2M_4^2$ grows and $\cos\,\theta_t$ becomes large. The exchange of a Regge pole with $\alpha_i(t)$ between clumps C and D gives a factor

$$\left(\frac{-2\,t\,s}{M_3^2M_4^2}\right)^{\alpha_i(t)}; \tag{VII.4}$$

the exchange of a pole with $\alpha_j(t')$ between clumps 5 and 6 (Fig. 3) gives a factor $(-2t'M_3^2/M_5^2\,M_6^2)^{\alpha_j(t')}$, and so forth. The first factor (VII.4) contributes to F in the Salzman formula (III.2), which may however contain further dependence on t, $M_3^2 = s_1$, and $M_4^2 = s_2$. We would like to end with the point that until this further dependence is known one has no idea whether the highest α dominates even the contribution to $d\sigma/dt$ from large $\cos\theta_t$, for $d\sigma/dt$ involves an integration $\iint dM_3^2\, dM_4^2$ and the factor (VII.4) suppresses large M_3^2 and M_4^2 when it favours large s.

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