$e^+e^- \to 3$ jets and event shapes at NNLO

A. Gehrmann-De Ridder
Institute for Theoretical Physics, ETH, CH-8093 Zürich, Switzerland

We report on the calculation of NNLO corrections to the 3-jet cross section and related event shape distributions in electron-positron annihilation. The corrections are sizable for all variables, however their magnitude is substantially different for different observables. We observe that inclusion of the NNLO corrections yields a considerably better agreement between theory and experimental data both in shape and normalization of the event shape distributions. A new extraction of α_s using the event shape variables up to NNLO yields a considerably better consistency between the observables indicating a stabilization of the perturbative corrections at this order.

1 Introduction

Jet observables in electron–positron annihilation play an outstanding role in studying the dynamics of the strong interactions, described by the theory of quantum chromodynamics (QCD). In addition to measuring multi-jet production rates, more specific information about the topology of the events can be extracted, using variables which characterize the hadronic structure of an event. With the precision data from LEP and SLC, experimental distributions for such event shape variables have been extensively studied [1,2] and have been compared with theoretical calculations based on next-to-leading order (NLO) parton-level event generator programs [3,4], improved by resumming kinematically-dominant leading and next-to-leading logarithms (NLO+NLL) [5] and by the inclusion of non-perturbative models of power-suppressed hadronisation effects [6].

The precision of the strong coupling constant determined from event shape data has been limited up to now largely by the scale uncertainty of the perturbative NLO calculation. We report here on the first calculation of NNLO corrections to the 3-jet cross section and related event shape variables. The knowledge of the NNLO corrections to the event shape distributions has important phenomenological impacts. We discuss those on the extraction of α_s from LEP data.

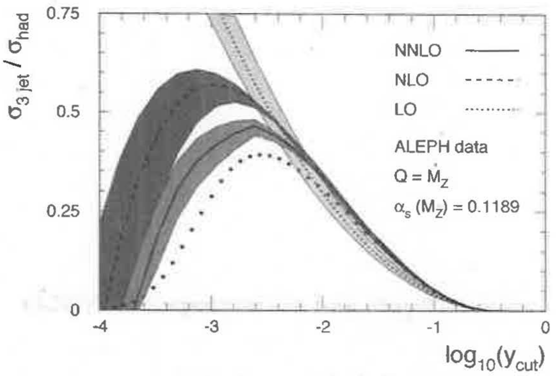

Figure 1: Perturbative fixed-order description of the three-jet rate at $Q = M_Z$, compared to data obtained with the ALEPH experiment [2]

2 The 3-jet cross section at NNLO

Jets are defined using a jet algorithm, which describes how to recombine the momenta of all hadrons in an event to form the jets. These algorithms are used in the experimental analysis and in the parton-level event generators to combine hadrons respectively partons into jets. Among those algorithms, the Durham procedure [7] has been widely used by experiments at LEP and SLD. Here we report on the first calculation of NNLO corrections to the three-jet production rate at parton-level in e^+e^- annihilation using this Durham jet algorithm.

The calculation of the α_s^3 corrections for three-jet production is carried out using a newly developed parton-level event generator program [8]EERAD3 which contains the relevant matrix elements with up to five external partons. Besides explicit infrared divergences from the loop integrals, the four-parton and five-parton contributions yield infrared divergent contributions if one or two of the final state partons become collinear or soft. In order to extract these infrared divergences and combine them with the virtual corrections, the antenna subtraction method [9] was extended to NNLO level [10] and implemented for $e^+e^- \to 3\,\text{jets}$[11] and related event-shape variables [12] into EERAD3.

Figure 1 displays the three-jet rate at LEP1 energy $Q = M_Z$ as function of the jet resolution y_{cut} at LO, NLO, NNLO. The theoretical uncertainty band is defined by varying the renormalization scale μ in the coupling constant in the interval $M_Z/2 < \mu < 2\,M_Z$, and the world average value [13] $\alpha_s(M_Z) = 0.1189$ is used, consistently evolved to other scales at each order. The fixed-order theoretical predictions for three-jet rate become negative for small values of y_{cut}, where fixed order perturbation theory is not applicable due to the emergence of large logarithmic corrections at all orders, requiring resummation [7,14]. We therefore restrict our comparison to $y_{cut} > 10^{-4}$, although data at lower jet resolution parameters are available.

For large values of y_{cut}, $y_{cut} > 10^{-2}$, the NNLO corrections turn out to be very small, while they become substantial for medium and low values of y_{cut}. The maximum of the jet rate is shifted towards higher values of y_{cut} compared to NLO, and is in better agreement with the experimental observation. The theoretical uncertainty is lowered considerably compared to NLO. Especially in the region $10^{-1} > y_{cut} > 10^{-2}$, which is relevant for precision phenomenology, one observes a reduction by almost a factor three, down to below two per cent relative uncertainty.

The fixed-order NNLO description is still above the data at low jet resolution, where the convergence of the perturbative series is spoilt by large logarithms of y_{cut} at all orders. Furthermore, the theoretical parton-level prediction is compared to hadron-level data, thereby neglecting hadronization corrections, which may also account for part of the discrepancy.

3 Event shape variables

In order to characterize hadronic final states in electron-positron annihilation, a variety of event shape variables have been proposed in the literature. For a review see e.g. [15,16]. In the following we shall consider only variables for three-particle final states which are thus closely related to three-jet final states.

Among those shape variables, six variables were studied in great detail [1]: the thrust T, the normalized heavy jet mass ρ, the wide and total jet broadenings B_W and B_T, the C-parameter and the transition from three-jet to two-jet final states in the Durham jet algorithm Y_3.

The perturbative expansion for the distribution of a generic observable up to NNLO at e^+e^- centre-of-mass energy $\sqrt{s}$, for a renormalization scale μ^2 involves perturbative coefficients [12] which only depend on the event shape variable y Those coefficients are computed by a fixed-order parton-level calculation, which includes final states with three partons at LO, up to four partons at NLO and up to five partons at NNLO.

For small values of the event shape variable y, the fixed-order expansion fails to converge, because the fixed-order coefficients are enhanced by powers of $\ln(1/y)$. In order to obtain reliable predictions in the region of $y \ll 1$ it is necessary to resum entire sets of logarithmic terms at all orders in α_s. A detailed description of the predictions at next-to-leading-logarithmic approximation (NLLA) can be found in Ref. [16].

The precise size and shape of the NNLO corrections depend on the observable in question. Common to all observables is the divergent behaviour of the fixed-order prediction in the two-jet limit, where soft-gluon effects at all orders become important, and where resummation is needed. For several event shape variables (especially T and C) the full kinematical range is not yet realised for three partons, but attained only in the multi-jet limit. In this case, the fixed-order description is also insufficient since it is limited to a fixed multiplicity (five partons at NNLO). Consequently, the fixed-order description is expected to be reliable in a restricted interval bounded by the two-jet limit on one side and the multi-jet limit on the other side.

In this intermediate region, we observe that inclusion of NNLO corrections (evaluated at the Z-boson mass, and for fixed value of the strong coupling constant) typically increases the previously available NLO prediction. The magnitude of this increase differs considerably between different observables[12], it is substantial for T (18%), B_T (17%) and C (15%), moderate for ρ and B_W (both 10%) and small for Y_3 (6%). For all shape variables, we observe that the renormalization scale uncertainty of the NNLO prediction is reduced by a factor 2 or more compared to the NLO prediction. Inclusion of the NNLO corrections modifies the shape of the event shape distributions. We observe that the NNLO prediction describes the shape of the measured event shape distributions over a wider kinematical range than the NLO prediction, both towards the two-jet and the multi-jet limit.

4 Determination of the strong coupling constant

Using the newly computed NNLO corrections to event shape variables, we performed [17] a new extraction of α_s from data on the standard set of six event shape variables, measured by the ALEPH collaboration [2] at centre-of-mass energies of 91.2, 133, 161, 172, 183, 189, 200 and 206 GeV. The combination of all NNLO determinations from all shape variables yields

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008\,(\text{stat}) \pm 0.0010\,(\text{exp}) \pm 0.0011\,(\text{had}) \pm 0.0029\,(\text{theo}).$$

We observe a clear improvement in the fit quality when going to NNLO accuracy. Compared to NLO the value of α_s is lowered by about 10%, but still higher than for NLO+NLLA [2], which shows the obvious need for a matching of NNLO+NLLA for a fully reliable result. Work is in progress in this direction [18]. The scatter among the α_s-values extracted from different shape variables is lowered considerably, and the theoretical uncertainty is decreased by a factor 2 (1.3) compared to NLO (NLO+NNLA), showing the improvements gained from the inclusion of the NNLO corrections.

5 Outlook

Our results for the NNLO corrections open up a whole new range of possible comparisons with the LEP data. The potential of these studies is illustrated by a new determination of α_s reported here, which can be further improved by the matching NLLA+NNLO, currently in progress.

Acknowledgments

This work was supported by the Swiss National Science Foundation (SNF) under contract Nr. PP002-118864.

References

1. G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **40** (2005) 287; P. Achard *et al.* [L3 Collaboration], Phys. Rept. **399** (2004) 71; J. Abdallah *et al.* [DELPHI Collaboration], Eur. Phys. J. C **29** (2003) 285.
2. A. Heister *et al.* [ALEPH Collaboration], Eur. Phys. J. C **35** (2004) 457.
3. R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B **178** (1981) 421.
4. Z. Kunszt and P. Nason, in *Z Physics at LEP 1*, CERN Yellow Report 89-08, Vol. 1, p. 373; W. T. Giele and E.W.N. Glover, Phys. Rev. D **46** (1992) 1980;
5. S. Catani, L. Trentadue, G. Turnock and B.R. Webber, Nucl. Phys. B **407** (1993) 3;
6. Y.L. Dokshitzer and B.R. Webber, Phys. Lett. B **352** (1995) 451; **404** (1997) 321;
7. S. Catani, Y.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Phys. Lett. B **269** (1991) 432;
8. A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover and G. Heinrich, JHEP **0711** (2007) 058.
9. D.A. Kosower, Phys. Rev. D **57** (1998) 5410; Phys. Rev. D **71** (2005) 045016; J. Campbell, M.A. Cullen and E.W.N. Glover, Eur. Phys. J. C **9** (1999) 245.
10. A. Gehrmann-De Ridder, T. Gehrmann and E.W.N. Glover, JHEP **0509** (2005) 056;
11. A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **100** (2008) 172001.
12. A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, Phys. Rev. Lett. **99** (2007) 132002, A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, JHEP **0712** (2007) 094.
13. S. Bethke, Prog. Part. Nucl. Phys. **58** (2007) 351.
14. A. Banfi, G.P. Salam and G. Zanderighi, JHEP **0201** (2002) 018.
15. R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press (Cambridge, 1996); G. Dissertori, I.G. Knowles and M. Schmelling, *Quantum Chromodynamics: High Energy Experiments and Theory*, Oxford University Press (Oxford, 2003).
16. R.W.L. Jones, M. Ford, G.P. Salam, H. Stenzel and D. Wicke, JHEP **0312** (2003) 007.
17. G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich and H. Stenzel, JHEP **0802**, 040 (2008).
18. T. Gehrmann, G. Luisoni and H. Stenzel, arXiv:0803.0695 [hep-ph], G. Dissertori, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich, G. Luisoni and H. Stenzel, in preparation.