

Neutrinoless Double Beta Decay The Nuclear Matrix Elements Revisited

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Abstract. We explore the influence of the deformation on the nuclear matrix elements of the neutrinoless double beta decay (NME), concluding that the difference in deformation -or more generally in the amount of quadrupole correlations- between parent and grand daughter nuclei quenches strongly the decay. We correlate these differences with the seniority structure of the nuclear wave functions. In this context, we examine the present discrepancies between the NME's obtained in the framework of the Interacting Shell Model and the Quasiparticle RPA. In our view, part of the discrepancy can be due to the limitations of the spherical QRPA in treating nuclei which have strong quadrupole correlations. We surmise that the NME's in a basis of generalized seniority are approximately model independent, *i. e.* they are "universal". We discuss as well how varies the nuclear matrix element of the ^{76}Ge decay when the wave functions of the two nuclei involved in the transition are constrained to reproduce the experimental occupancies. In the Interacting Shell Model description the value of the NME is enhanced about 15% compared to previous calculations, whereas in the QRPA the NME's are reduced by 20%-30%. This diminishes the discrepancies between both approaches. In addition, we update the effects of the short range correlations on the NME's in the light of the recently proposed parametrizations obtained by renormalizing the $0\nu\beta\beta$ transition operator at the same footing than the effective interaction.

1. Introduction

The double beta decay is a rare weak process which takes place between two even-even isobars when the single beta decay is energetically forbidden or hindered by large spin difference. The two neutrino beta decay is a second order weak process —the reason of its low rate—, and has been measured in a few nuclei. The $0\nu\beta\beta$ decay is analog but requires neutrinos to be Majorana fermions. With the exception of one unconfirmed claim [1], it has never been observed, and currently there is a number of experiments either taking place or expected for the near future —see e.g. ref. [2]— devoted to detect this process and to set up firmly the nature of neutrinos. Furthermore, the $0\nu\beta\beta$ decay is also sensitive to the absolute scale of the neutrino mass, and hence to the mass hierarchy. Since the half-life of the decay is determined, together with the masses, by the nuclear matrix element for the process, its knowledge is essential to predict the most favorable decays and, once detection is achieved, to settle the neutrino mass scale and

hierarchy.

Two different methods were traditionally used to calculate the NME's for $0\nu\beta\beta$ decays, the quasiparticle random-phase approximation and the shell model in large valence spaces (ISM). The QRPA has produced results for most of the possible emitters since long [3, 4, 5]. The ISM, that was limited to a few cases till recently [6], can nowadays describe (or will do it shortly) all the experimentally relevant decays but one, the decay of ^{150}Nd . Other approaches, that share a common prescription for the transition operator (including higher order corrections), and for the treatment of the short range correlations (SRC) and the finite size effects, are the Interacting Boson Model [7], and the Projected Hartree Fock Bogolyubov method [8].

The expression for the half-life of the $0\nu\beta\beta$ decay can be written as [9]:

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right)^{-1} = G_{01} \left|M^{0\nu\beta\beta}\right|^2 \left(\frac{\langle m_{\beta\beta} \rangle}{m_e}\right)^2, \quad (1)$$

where $\langle m_{\beta\beta} \rangle = |\sum_k U_{ek}^2 m_k|$ is the effective neutrino mass, a combination of the neutrino mass eigenvalues m_k . U is the neutrino mixing matrix and G_{01} is a kinematic factor dependent on the charge, mass and available energy of the process. $M^{0\nu\beta\beta}$ is the nuclear matrix element of the neutrinoless double beta decay operator, which has Fermi, Gamow-Teller and Tensor components. The kinematic factor G_{01} depends on the value of the coupling constant g_A . In addition, some calculations use different values of r_0 in the formula $R=r_0 A^{1/3}$. It is therefore convenient to define:

$$M'^{0\nu\beta\beta} = \left(\frac{g_A}{1.25}\right)^2 \left(\frac{1.2}{r_0}\right) M^{0\nu\beta\beta} \quad (2)$$

In this way the theoretical $M'^{0\nu\beta\beta}$'s are directly comparable among them irrespective of the values of g_A and r_0 employed in their calculation, since they share a common G_{01} factor—the one computed with $g_A = 1.25$ and $r_0 = 1.2$ fm. Thus, the translation of the $M'^{0\nu\beta\beta}$'s into half-lives is transparent.

2. Pairing and Quadrupole; The Influence of Deformation

An important issue regarding the $0\nu\beta\beta$ decay is the role of the correlations; pairing that drives the nucleus toward a superfluid state and quadrupole that favors deformed intrinsic shapes. It has been show recently that the $2\nu\beta\beta$ is hindered by the difference in deformation between the initial and final nuclei [10, 11]. For the neutrinoless mode, the calculations [6] indicate that the pairing interaction favors the decay and that, consequently, the truncations in seniority, which quench the pair breaking action of the quadrupole correlations, produce an overestimation of the values of the NME's. On the other hand, the NME's are also reduced when the parent and grand-daughter nuclei have different deformations [12, 13].

We have chosen to study the (unphysical) transition between the mirror nuclei ^{66}Ge and ^{66}Se in order to have a clearer view of the effect of the deformation in the NME's. This transition has the peculiarity that the wave functions of the initial and final nuclei are identical (provided Coulomb effects are neglected) and consequently it is easier to disentangle the contributions of the $0\nu\beta\beta$ operator and the nuclear wave functions to the NME. The calculations are carried out in the valence space comprising the orbits between the magic numbers 28 and 50 ($r3g$) with the effective interaction gcn28:50. The SRC are modeled by a Jastrow factor with the Spencer and Miller parametrization [14], although it has been shown recently that, once the finite size of the nucleon has been taken into account by a dipole form factor, softer options are more realistic [15, 16]

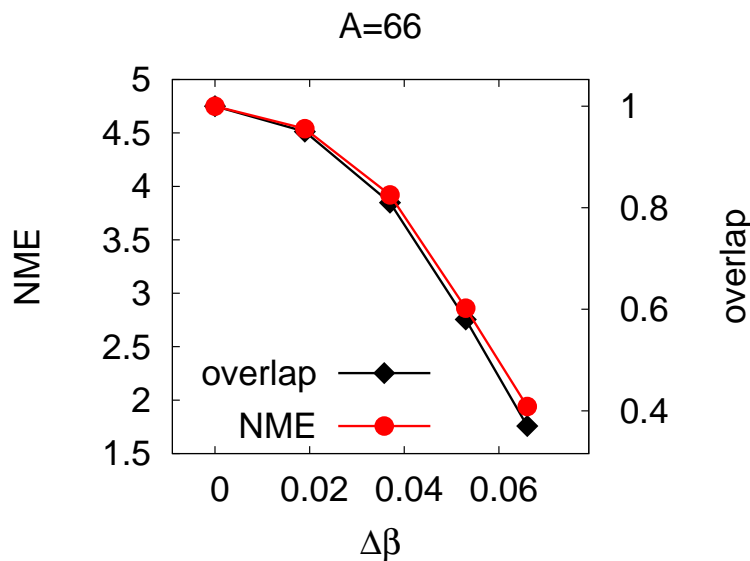


Figure 1. $^{66}\text{Ge} \rightarrow ^{66}\text{Se}$ NME, $M'^{0\nu}$, as a function of the difference in deformation induced by the extra quadrupole interaction added to ^{66}Se .

To increase the deformation of a given nucleus we add to the effective interaction a term $\lambda Q \cdot Q$. Fig. 1 shows the results when the final nucleus has been artificially deformed by adding an extra quadrupole-quadrupole term. Notice in the first place that for $\lambda=0$ both nuclei are deformed with $\beta \sim 0.2$. In spite of that, the NME is a factor of two larger than the values obtained for the $A=76$ and $A=82$ decays in the same valence space and with the same interaction. Hence, even if the two $A=66$ partners are deformed, the fact that their wave functions are identical enhances the decay. Nevertheless, the NME is still far from its expected value in the superfluid limit ($\text{NME} \sim 8$). The figure shows that the reduction of the NME as the difference in deformation increases is very pronounced. For the values of λ between 0.0 and 0.2, the difference in deformation parameter between parent and grand daughter grows from zero to about 0.1. In addition, the NME follows closely the overlap between the wave function of one nucleus obtained with $\lambda=0$ and the wave function of the same nucleus obtained with $\lambda \neq 0$. This means that, if we write the final wave function as: $|\Psi\rangle = a|\Psi_0\rangle + b|\Psi_{qq}\rangle$, the $0\nu\beta\beta$ operator does not connect Ψ_0 and Ψ_{qq} . This behavior of the NME's with respect to the difference of deformation between parent and grand daughter is common to all the transitions between mirror nuclei that we have studied ($A=50$, $A=110$) and to more realistic cases like the $A=82$ decay that we have examined in detail in [17]. Therefore we can submit that this is a robust result. Similar results hold also for the 2ν decays.

3. The NME's and the seniority structure of the nuclear wave functions

We can also analyze the results of the preceding section in terms of the seniority structure of the wave functions of parent and grand daughter nuclei. Indeed when $\Delta\beta=0$ both ^{66}Ge and ^{66}Se have identical wave functions. The probabilities of the components of different seniority are given in table 1. It is seen that changing β from 0.22 (mildly deformed) to 0.30 (strongly deformed) increases drastically the amount of high seniority components in the wave function, provoking a seniority mismatch between the decaying and the final nuclei. This leads to very large cancelations of the nuclear matrix elements of the decay, as shown also in table 1.

Table 1. The seniority structure of the wave functions in the A=66 mirror decay

	$s = 0$	$s = 4$	$s = 6$	$s = 8$	$s = 10$
$\Delta\beta=0$	39	43	7	10	1
$\Delta\beta=0.08$	6	32	21	31	10
	$M_F^{0\nu}$	$M_{GT}^{0\nu}$	$M_T^{0\nu}$	$M^{0\nu}$	
$\Delta\beta=0$	-2.02	3.95	0.08	5.16	
$\Delta\beta=0.08$	-0.76	1.65	0.02	2.12	

ISM: full(squares) , $s_m=4$ (circles) ; QRPA: Tu(bars) , Jy(diamonds)

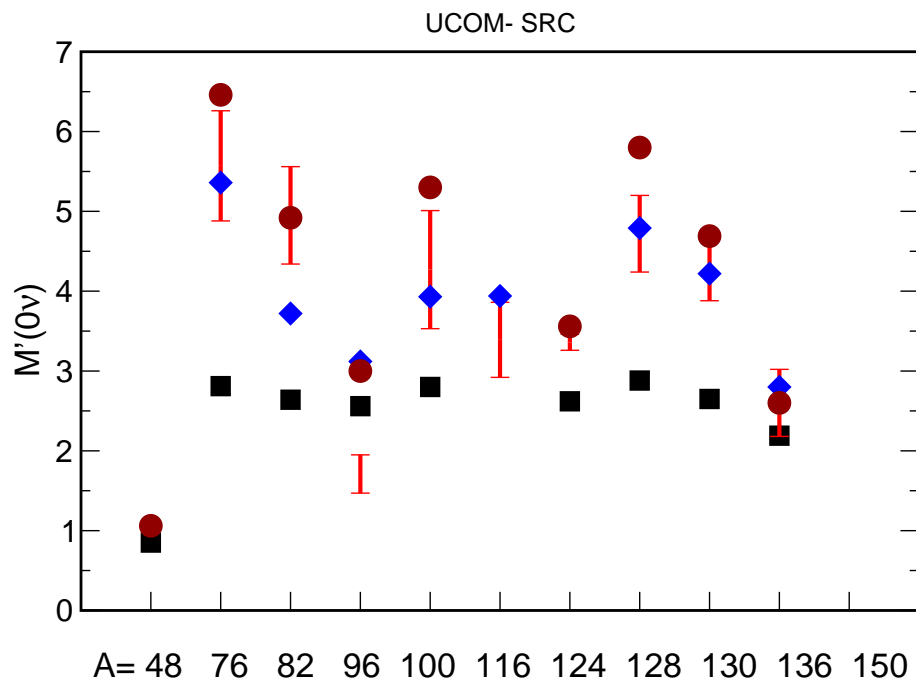


Figure 2. The neutrinoless double beta decay nuclear matrix elements $M'^{0\nu\beta\beta}$ for ISM and QRPA calculations treating the SRC with the UCOM approach. Tu, QRPA results from ref. [18] and Jy, QRPA results from refs. [3, 4]. The ISM results for A=96 and A=100 are preliminary

Coming back to the physically relevant decays, we compare in figure 2 the ISM and QRPA NME's. In both approaches, the SRC are taken into account in the UCOM framework [19] and $g_A=1.25$ is adopted. We have discussed elsewhere that the discrepancies between both approaches show the following trends: when the nuclei that participate in the decay have a low level of quadrupole correlations, as in the decays of ^{96}Zr , ^{124}Sn and ^{136}Xe , the calculations tend to agree. On the contrary, when the correlations are large, the QRPA in a spherical basis seems not to be able to capture them fully. As the effect of the correlations is to reduce the NME's, the QRPA produces NME's that are too large in ^{76}Ge , ^{82}Se , ^{100}Mo , ^{128}Te , and ^{130}Te . Indeed, when the ISM calculations are truncated to maximum seniority $s_m=4$, which is the leading order of the ground state correlations in the QRPA (corresponding to the two quasi-particle contribution), they follow closely the QRPA results, as can be seen also in figure 2. Notice that only when the ISM calculations are converged at this level of truncation the two approaches do produce similar NME's.

Table 2. The seniority structure of the wave functions in the ISM and QRPA

	$s = 0$	$s = 4$	$s = 6$	$s = 8$	$s = 10$	$s = 12$	$s = 14$	$s = 16$
ISM								
^{48}Ca	97	3	-	-	-	-	-	-
^{48}Ti	59	36	4	1	-	-	-	-
^{76}Ge	43	41	7	8	1	-	-	-
^{76}Se	26	41	11	16	4	1	-	-
^{82}Se	50	39	10	1	-	-	-	-
^{82}Kr	44	41	6	8	1	-	-	-
^{128}Te	70	26	3	1	-	-	-	-
^{128}Xe	37	41	9	10	2	-	-	-
QRPA								
^{76}Ge	55	33	-	10	-	2	-	-
^{76}Se	59	31	-	8	-	2	-	-
^{82}Se	56	32	-	9	-	2	-	-
^{82}Kr	54	34	-	11	-	2	-	-
^{128}Te	52	34	-	11	-	3	-	-
^{128}Xe	40	37	-	17	-	5	-	1

We compare in table 2 the seniority structure of the wave functions of the ISM and QRPA, in some of the cases for which the latter are available [20]. It is seen that the differences are important and share a common trend: in the QRPA, the seniority structure of parents and grand daughters is much more similar than in the ISM. According to what we have seen in the $A=66$ case, this is bound to produce larger NME's in the QRPA than in the ISM, as it is actually the case. To make this statement quantitative, we have developed the ISM matrix elements in a basis of generalized seniority

$$M_{F,GT,T} = \sum_{\alpha,\beta} A_{\nu_i(\alpha)} B_{\nu_f(\beta)} \langle \nu_f(\beta) | O_{F,GT,T} | \nu_i(\alpha) \rangle$$

where the A's and B's are the amplitudes of the different seniority components of the wave functions of the initial and final nuclei. Obviously, when we plug the ISM amplitudes in this formula, we recover the ISM NME's. But, what shall we obtain if we put the QRPA amplitudes instead? Indeed, we get approximately the QRPA NME's! (5.73 for $A=76$ and 4.15 for $A=82$). Therefore as we had anticipated, the seniority mismatch of the initial and final wave functions, which is severely underestimated in the QRPA calculations, explains most of the discrepancy between the two descriptions. In addition, this result strongly suggests that there is some kind of universal behavior in the NME's of the neutrinoless double beta decay when they are computed in a basis of generalized seniority. If this is so, the only relevant difference between the various theoretical approaches would reside in the seniority structure of the wave functions that they produce.

Table 3. The GT NME's of the $A=48$ decay in the generalized seniority basis

^{48}Ti	$s = 0$	$s = 4$	$s = 6$	$s = 8$
$^{48}\text{Ca } s = 0$	3.95	-3.68	-	-
$^{48}\text{Ca } s = 4$	0.00	-0.26	0.08	-0.02

A very spectacular example of the cancellation of the NME by the seniority mismatch is provided by the ^{48}Ca decay. In Table 2 we have included also the seniority structures of the two nuclei, and we see that they are very different. If we compute the the matrix elements $\langle \nu_f(\beta) | O_{GT} | \nu_i(\alpha) \rangle$ we find the values listed in Table 3. There are two large matrix elements; one diagonal and another off-diagonal of the same size and opposite sign. If the two nuclei were dominated by the seniority zero components one should obtain $M_{GT} \sim 4$. If ^{48}Ti were a bit more deformed, M_{GT} will be essentially zero. The value produced by the KB3 interaction is 0.75, which represents more than a factor five reduction with respect to the seniority zero limit. Earlier work on double beta decays in a basis of generalized seniority (limited to $s=0$ and $s=4$ components) showing also this kind of cancellations can be found in ref. [21]

4. Benchmarking the NME's with the experimental occupancies: The $A=76$ case

Very recently, there has been a intense experimental effort to extract the occupation numbers of the nuclei ^{76}Ge and ^{76}Se [22, 23] by accurate measurements of one nucleon transfer reactions. At present, both neutron and proton occupancies have been determined. The main motivation to study these nuclei is that they are the initial and final states of a $\beta\beta$ transition. Therefore, we have the possibility to compare these experimental results with the theoretical occupations and, if necessary, detect which modifications would be required in the effective interactions in order to obtain improved agreement with the data. In principle, this would lead to more reliable results when obtaining, for instance, the value of NME's for the $0\nu\beta\beta$ decay process. In the case of the interacting shell model (ISM), the calculations reported so far [6, 24] were performed using the gcn28:50 interaction. This interaction was obtained by a global fit to the region comprised by the $1p_{3/2}$, $1p_{1/2}$, $0f_{5/2}$ and $0g_{9/2}$ orbits — r_3g valence space. In addition, we had produced another interaction based on gcn28:50, aimed to improve locally the quadrupole properties of the nuclei in the $A = 76$ region, which we call RG. When the experimental occupation numbers were published, we decided to compute them with the two available effective interactions, in order to check the stability of the ISM $0\nu\beta\beta$ NME's with respect to this property of the nuclear wave functions..

Table 4. Proton and neutron occupation numbers of nuclei ^{76}Ge and ^{76}Se . Experiment from Refs. [22, 23] vs theoretical results, obtained for the gcn28.50 and RG interactions.

	$1p_{1/2}+1p_{3/2}$	$0f_{5/2}$	$0g_{9/2}$
	Neutrons		
^{76}Ge (exp)	4.87 ± 0.20	4.56 ± 0.40	6.48 ± 0.30
^{76}Ge (gcn28.50)	5.19	5.02	5.79
^{76}Ge (RG)	4.83	4.78	6.39
^{76}Se (exp)	4.41 ± 0.20	3.83 ± 0.40	5.80 ± 0.30
^{76}Se (gcn28.50)	4.86	4.54	4.60
^{76}Se (RG)	4.08	4.06	5.86
	Protons		
^{76}Ge (exp)	1.77 ± 0.15	2.04 ± 0.25	0.23 ± 0.25
^{76}Ge (gcn28.50)	1.70	1.90	0.40
^{76}Ge (RG)	1.34	2.00	0.66
^{76}Se (exp)	2.08 ± 0.15	3.16 ± 0.25	0.84 ± 0.25
^{76}Se (gcn28.50)	2.74	2.27	0.99
^{76}Se (RG)	2.12	2.79	1.08

In Table 4 we compare the experimental occupancies along with the theoretical ones obtained with both the gcn28.50 and RG interactions. The occupancies obtained with the former are

quite close to the experimental ones, specially in the case of ^{76}Ge . However, for ^{76}Se they lie somewhat further from experiment. On the contrary, the interaction RG produces occupancies for ^{76}Se which are almost perfect. The only drawback of this interaction is found on the proton occupancies in ^{76}Ge that slightly overfills the $0g_{9/2}$ orbit against the filling of the p orbits. In any case, the results obtained with both interactions compare reasonably well with the measured ones, while the RG interaction can be said to fit quite successfully the experimental numbers.

The QRPA occupancies deviate more from measurements than our ISM values. In order to cure these discrepancies with the measured occupations, Suhonen *et al.* [25] and Šimkovic *et al.* [26] have adjusted the parameters of their reference Woods-Saxon potential to reproduce the experimental numbers. The former do it such as to obtain agreement at the BCS level while the latter get the experimental numbers only after the QRPA correlations have been included. The changes in occupancies required to match the experiment are much larger in the case of the QRPA calculations, notably for neutrons, than for the ISM. The effect of the new ISM interaction RG is much milder. In the end, all final interactions are able to reproduce the experimental occupations fairly well, with similar accuracies. Once the interactions have been settled to give results as close as possible to experiment, the next step is to look at the NME's. In Table 5 we have collected their values for the ISM and QRPA with the six interactions considered.

Table 5. Values of the NME ($M^{0\nu\beta\beta}$) for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay for ISM and QRPA calculations. QRPA(Jy)-WS and QRPA(Tu)-WS UCOM type SRC's are considered. We take $r_0 = 1.2$ fm and non-quenched axial coupling.

$M^{0\nu\beta\beta}$	GCN	WS	RG	ADJ-WS
ISM	2.81		3.26	
QRPA(Jy)		5.36		4.11
QRPA(Tu)		5.07-6.25		4.59-5.44

In the case of the Jyväskylä's QRPA, the NME suffers a substantial reduction of about 30% when calculated with the adjusted interaction. There is an effect in the same direction, whereas more moderate, present in the Tübingen's results. In this case, the reduction is closer to 20%. The different changes are probably related to the adjustment of experimental occupancies at BCS or QRPA level. These modifications can be traced back to the new values of the QRPA parameters g_{pp} obtained with the modified single particle energies, which are significantly different from those obtained with the original single particle energies originated by Woods-Saxon potentials. As for the ISM, the NME obtained with the RG interaction is enhanced with respect to the previous result obtained with the interaction gc28.50. The increase is of some 15%. This means that the ISM result is reasonably stable when obtained with different effective interactions. Moreover, when adjusting the interactions to agree with the measured occupancies in ^{76}Ge and ^{76}Se , the difference between the ISM and QRPA NME values diminishes, as can be seen more clearly in Fig. 3.

We want to end this section with a caveat; that having the good occupancies is a necessary but not sufficient condition to conclude that the wave functions are close to the physical ones and therefore to trust the NME's that they produce. This is clearly seen in Table 6 where we show that one can obtain good occupation numbers for very different pair structures of the wave function and, as a consequence produce very different NME's.

5. Update on Short Range Correlations

We can study as well the NME of the $A=76$ $0\nu\beta\beta$ decay in the light of very recent treatments of the short range correlations (SRC) [16, 15]. These correlations were parametrized in the past by the Jastrow ansatz. Recently there has been efforts to order to study them consistently, this

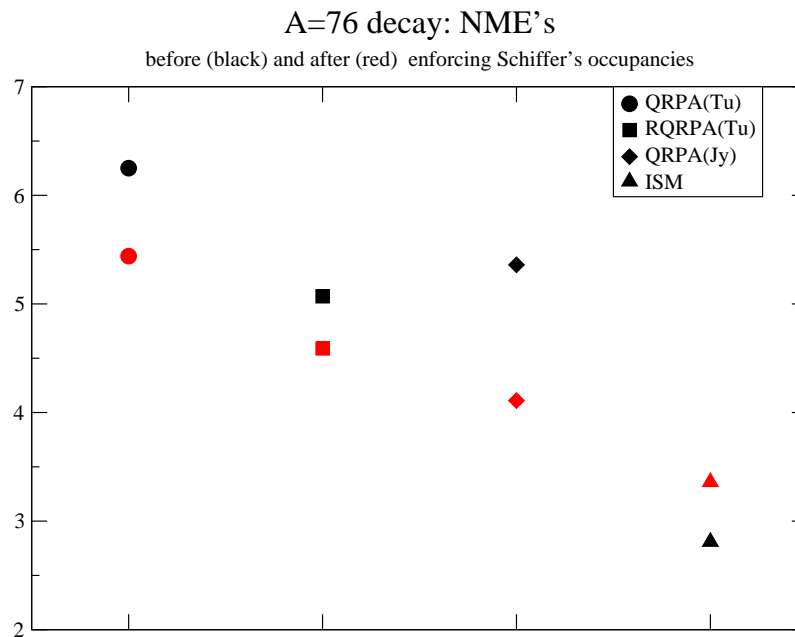


Figure 3. The evolution of the NME's of the A=76 decay when the ISM and QRPA calculations are modified so as to reproduce the experimental occupancies

Table 6. Values of the NME ($M^{0\nu\beta\beta}$) for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay and occupation numbers at different seniority truncations

	Neutrons			Protons			NME
	⁷⁶ Ge						
	1p	0f _{5/2}	0g _{9/2}	1p	0f _{5/2}	0g _{9/2}	
s _m = 0	4.8	5.2	6.1	1.3	2.1	0.6	
s _m = 4	4.8	5.0	6.2	1.3	2.0	0.7	
s _m = 10	4.8	4.8	6.4	1.3	2.0	0.7	
	⁷⁶ Se						
	1p	0f _{5/2}	0g _{9/2}	1p	0f _{5/2}	0g _{9/2}	
s _m = 0	3.9	4.6	5.5	1.8	3.3	0.9	11.85
s _m = 4	4.3	4.4	5.3	2.1	2.6	1.3	7.99
s _m = 14	4.1	4.1	5.9	2.1	2.8	1.1	3.26

is, obtaining them from the regularization of the bare operator in the same way that the bare interaction is regularized into the effective one within the nuclear medium. In both papers the effect of the short range correlations in the $0\nu\beta\beta$ process is found to be negligible, (less than 5%) once the dipole form factor is taken into account in the operators.

Table 7. Values of the NME for the $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay for ISM interactions, using the SRC's proposed in Ref. [16].

Interaction	$M_{no\ SRC}$	$M_{Argonne}^{0\nu\beta\beta}$	$M_{Bonn}^{0\nu\beta\beta}$
gcn28.50	2.89	2.82	3.00
RG	3.40	3.33	3.52

If we compare these results with the two standard parametrizations of the SRC's for this decay, namely the Miller-Spencer parametrization of a Jastrow type function [27, 14] and the UCOM [28, 29] approach, the latter seems to be more adequate, with the Miller-Spencer parametrization leading to a large underestimation of the NME's. Moreover, in ref. [16] these effects are parametrized by two Jastrow-like functions. Within the ISM we can take these two parametrizations and calculate the modification that they cause on the NME's. The results are shown on Table 7. They agree with those of Ref. [16], showing very mild modifications of the NME's by the SRC's, either a small increase—in the case of the parametrization that comes from the Bonn potential—or decrease—when the original potential is Argonne's.

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