SLAC-PUB-16390

STUDY OF ULTRA-LOW EMITTANCE DESIGN FOR SPEAR3 USING LONGITUDINAL GRADIENT DIPOLE

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Abstract

Since its 2003 construction, the SPEAR3 synchrotron light source at SLAC has continuously improved its performance by raising beam current, top-off injection, and smaller emittance. This makes SPEAR3 one of the most productive light sources in the world. Now to further enhance the performance of SPEAR3, we are looking into the possibility of converting SPEAR3 to an ultra-low emittance storage ring within its site constraint.

INTRODUCTION

Introducing dipoles with a longitudinal field variation may provide an emittance significantly below the minimum emittance of a homogeneous TME bending magnet [1]. The emittance can be reduced further without increasing the number of dipoles. The minimization is limited by the ratio of maximum and average field in the dipole [2]. The equilibrium emittance in a storage ring is given by

$$\varepsilon = k\frac{I_5}{I_2} \tag{1}$$

with the synchrotron integrals

$$I_2 = \int h(s)^2 ds, \quad I_5 = \int h(s)^3 H(s) ds, \tag{2}$$

where *h(s)* is the local curvature in the magnet and *H(s)*:

$$H(s) = \beta_x(s)\eta'_x(s)^2 + 2\alpha_x(s)\eta_x(s)\eta'_x(s) + \frac{1+\alpha_x(s)}{\beta_x(s)}\eta_x(s)^2 \ . \tag{3}$$

Consequently, increasing the curvature *h(s) =B(s)(Tesla)/(3.335E(GeV))* in the dipole center where $H(s)$ is small and lowering it in the outer regions where $H(s)$ becomes larger while keeping the integral constant,

$$\int_L h(s)ds = \theta_B \tag{4}$$

will compensate for the variation of *H* and lead to a lower emittance.

Finding a function *h(s)* to minimize Eq. (1) and keeping Eq. (4) is not an easy task and the construction of the magnet that fulfills the *h(s)* function may not be feasible. An alternative approach that includes realistic boundary conditions and an analytic solution is given by Andreas Streun [3]. A superbend can be simplified as consisting of two components: a central high field part with given curvature *h*, corresponding to the highest possible field, and an outer low field part with lower curvature h_1 as shown in figure 1. With $l=L/2$ the half-length and θ_B the full angle, μl the measure of how far the high field region extends:

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-76SF00515.

$$h\mu l + h_1(1-\mu)l = \frac{\theta_B}{2}, \quad \mu_{\max} = \frac{\theta_B}{2hl}. \tag{5}$$

The emittance of a TME dipole can be expressed as:

$$\varepsilon(\beta_0,\eta_0,\mu) = k\frac{h^3\int_0^{\mu l} H_0(s)ds + h_1^3\int_{\mu l}^{l} H_1(s)ds}{h^2\mu l + h_1^2(1-\mu)l}, \tag{6}$$

where β_0 and η_0 are the beta and dispersion functions at the magnet center. Due to symmetry α_0 and η_0' are set to zero. The matching conditions for the initial parameters are obtained by [3]:

$$\frac{\partial\varepsilon(\beta_0,\eta_0,\mu)}{\partial\eta_0} = 0 \rightarrow \eta_0(\beta_0,\mu) \Rightarrow \frac{\partial\varepsilon(\beta_0,\mu)}{\partial\beta_0} = 0 \rightarrow \beta_0(\mu) \Rightarrow \varepsilon(\mu). \tag{7}$$

The results are back substituted to Eq. (6) to get the minimum emittance as a function of the μ-parameter. A similar result can be obtained for DBA dipole here the dipole is only the right half part in Fig. 1. The initial $\eta_{s,} \eta_s'$ are zero at the high field end.

$$\varepsilon(\beta_s,\alpha_s,\mu) = k\frac{h^3\int_0^{\mu L} H_0(s)ds + h_1^3\int_{\mu L}^{L} H_1(s)ds}{h^2\mu L + h_1^2(1-\mu)L} \tag{8}$$

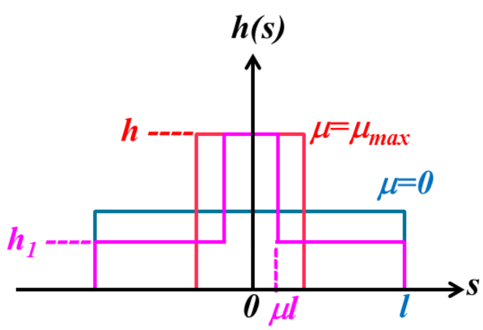

Figure 1 A simple model of longitudinal gradient dipole. The length of the gradient dipole is same as the original dipole. The lower field is smaller than the original field. The blue block is the homogenous dipole. The magenta is the gradient dipole and the red is the high field homogenous dipole. The parameter μ varies from 0 to $\mu_{\max}$.

OPTIMIZATION OF LONGITUDINAL STEP GRADIENT DIPOLE OF SPEAR3

We use the dipoles of a QBA cell that is designed to replace the SPEAR3 DBA cell as an example. The length of the middle dipole is 1.3525 m, θ_B is 0.10937, B_0=0.81 T. The length of the end dipole is 0.924 m and the angle is 0.075428. The maximum B field is 1.8 T. The β_0, η_0 and minimized emittance (I_5/I_2) as function of μ-parameter of the TME dipole are shown in figure 2 and the initial β_s, α_s parameters of the DBA dipole are shown in figure 3. For each μ-parameter there is a corresponding set of β_0 /β_s (blue) and η_0 /α_s (green) to reach the minimum emittance (red). Taking Fig. 2 the TME dipole for example, the μ-parameter range from 0 to $\mu_{\max}$ 0.4496, the minimum emittances of homogenous dipoles μ=0, and $\mu=\mu_{\max}$ are the same because bending angles θ_B is the same. The minimum emittance derived from Eq. (7) with μ=0, and $\mu=\mu_{\max}$ is same as is in Eq. (7) of reference [4] for the homogenous dipole. However the required β_0 and η_0 of these two dipoles for minimization of emittance are different. For stronger dipole $\mu=\mu_{\max}$ the required β_0 and η_0 are smaller and are harder to fulfil. There is a minimum emittance of the step gradient dipole when μ= 0.255. The emittance is about one fourth of the homogenous TME dipole. Table 1 shows a summary of the optimization parameters for the TME and DBA dipoles.

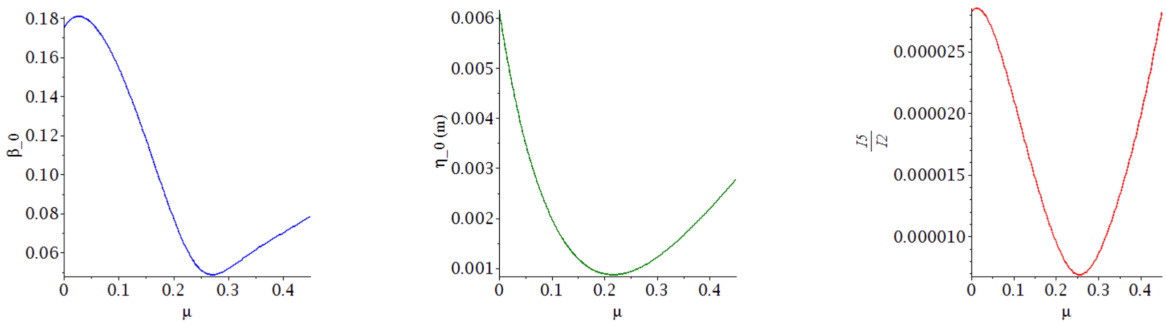

Figure 2: TME longitudinal step gradient dipole, the length of the gradient dipole is same as the original dipole. For every μ-parameter there is a corresponding set of β_0 (blue) and η_0 (green) to reach the minimum emittance (red).

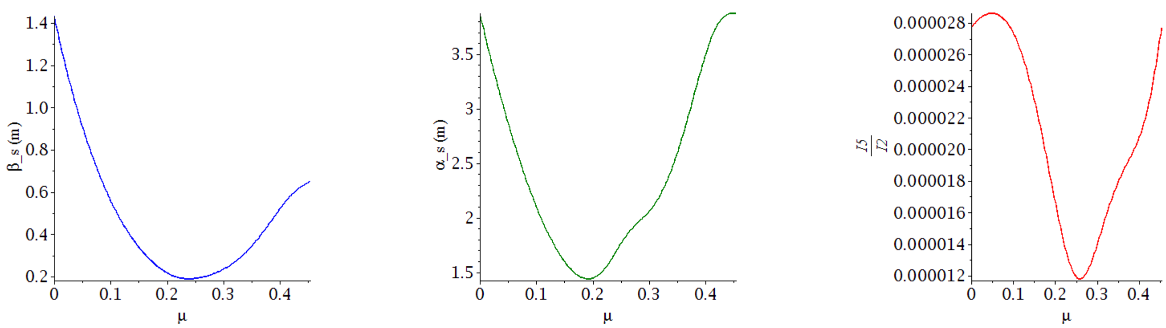

Figure 3: DBA longitudinal step gradient dipole, the length of the gradient dipole is same as the original dipole. For every μ-parameter there is a corresponding set of β_s (blue) and α_s (green) to reach the minimum emittance (red).

Table 1: Optimization parameters of gradient dipole. The length of the gradient dipole is same as the lower field homogenous dipole.

	TME*1 L=1.3525 m, θ_B=0.10937				DBA*2 L=0.924 m, θ_B=0.075428		
μ	0	0.255	0.4496		0	0.262	0.4538
B (T)	0.81	0.467/1.8	1.8		0.81	0.467/1.8	1.8
ε*3 (pm.rad)	330	80	330		330	138	330
β_0 (m)	0.1746	0.05	0.0785	β_s (m)	1.43	0.196	0.6496
α_0	0	0	0	α_s	3.87	1.8553	3.87
η_0 (m)	0.0062	0.00095	0.0028	η_s (m)	0	0	0

1. The β_0, α_0, η_0 shown are at middle of the magnet.
2. The β_s, α_s, η_s shown are at start of the magnet
3. Assuming J_x=1.

For a short cell like SPEAR3 there is a benefit to shorten the length of dipole to release space. An alternative method to apply the longitudinal step gradient dipole is to set the lower field to certain value like the original field 0.81 T. The variable now is the total length of the gradient dipole and the parameter μ is function of the dipole length as:

$$\mu = \frac{(\theta_B - h_1 l)}{(h - h_1) l} \tag{9}$$

The range of the dipole length is from μ =1 to μ =0. The schematic model of this is shown in figure 4. Eq. 6 and Eq. 8 are still valid. The β_0, η_0 and minimized emittance (I_5/I_2) as function of dipole length of TME are shown in figure 5 and of DBA dipole are shown in figure 6. Table 2 shows the summary of optimization parameters for the TME and DBA dipoles for fixed lower field.

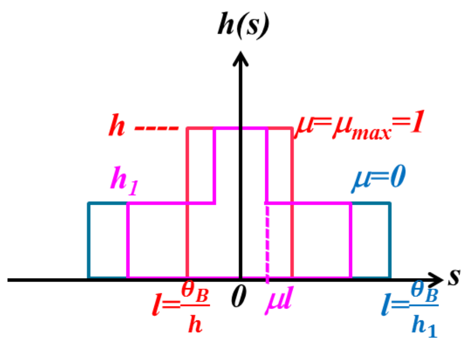

Figure 4 A schematic model of longitudinal gradient dipole. The lower field is same as the original dipole. The length of the gradient dipole is short than the original dipole. The blue block is the homogenous dipole. The magenta is the gradient dipole and the red is the high field homogenous dipole. The parameter μ varies from 0 to μ_{max}=1.

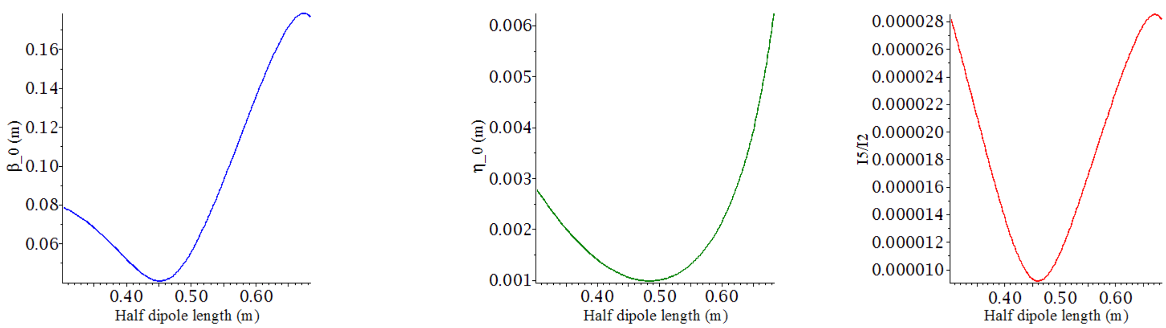

Figure 5: TME longitudinal gradient dipole, the lower field is same as the original dipole. For different dipole length there is a corresponding set of β_0 (blue) and η_0 (green) to reach the minimum emittance (red).

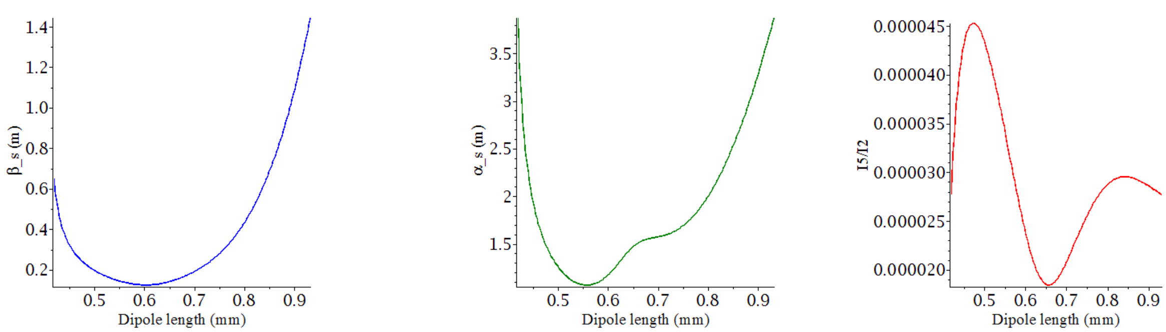

Figure 6: DBA longitudinal gradient dipole, the lower field is same as the original dipole. For different dipole length there is a corresponding set of β_s (blue) and α_s (green) to reach the minimum emittance (red).

Table 2: Optimization parameters of gradient dipole. The lower magnet field of the gradient dipole is same as the lower field homogenous dipole.							
	TME[*1] θ_B=0.10937				DBA[*2] θ_B=0.075428		
μ	0	0.39	1		0	0.345	1
L (m)	1.3525	0.92	0.608		0.924	0.655	0.4538
B (T)	0.81	0.81/1.8	1.8		0.81	0.81/1.8	1.8
ε^{*3} (p m.rad)	330	107	330		330	214	330
β_0 (m)	0.1746	0.041	0.0785	β_s (m)	1.43	0.144	0.6496
α_0	0	0	0	α_s	3.87	1.49	3.87
η_0 (m)	0.0062	0.001	0.0028	η_s (m)	0	0	0

1. The β_0, α_0, η_0 shown are at middle of the magnet.
2. The β_s, α_s, η_s shown are at start of the magnet.
3. Assuming J_x=1

Longitudinal step gradient dipole of QBA cell

With the optimization parameters in Tab. 2 we investigate the possibility of employing gradient dipoles in a SPEAR3 QBA cell. The cell length is 11.691 m. The lengths and angles of middle (TME) and end (DBA) dipoles are shown in Table 2. The optimal Courant-Snyder parameters at the middle of the TME dipole are β_0 /η_0 0.1746 m/ 0.0062 m. The optimal Courant-Snyder parameters at the entrance of the DBA dipole are β_s /α_s 1.43 m/ 3.87. Due to the constraints of the unit cell length and the strength limitations of magnets, it's not easy to obtain the optimal Courant-Snyder parameters in the design. The emittance degradation of QBA cell due to non-optimal Courant-Snyder parameters is shown in figure 7.

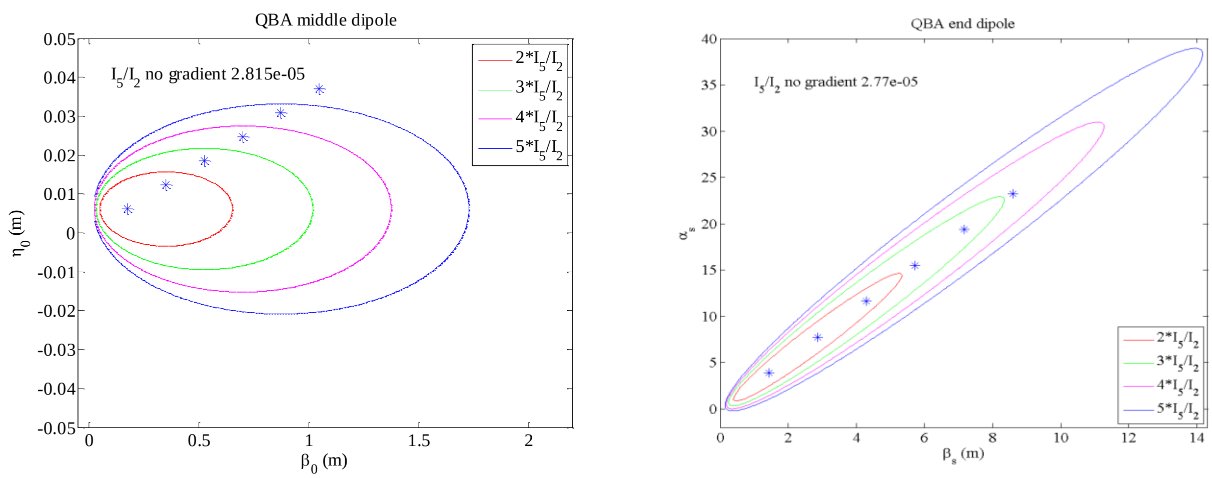

Figure 7 Contours of emittance degradation of QBA cell due to non-optimal Courant-Snyder parameters at the middle of dipole (left, TME dipole) and dipole entrance (right, DBA dipole). The blue stars represent different multipliers of the optimal Courant-Snyder parameters.

The emittance of the QBA cell for SPEAR3 is 675 pm.rad. The Courant-Snyder parameters at the middle of the TME dipole are β_0 /η_0 0.225 m/ 0.025 m. The Courant-Snyder parameters at the entrance of the DBA dipole are β_s /α_s 2.388 m/ 5.438. The tunes per QBA cell are $\Delta\nu_x/\Delta\nu_y$ 1.998/1.2498. The β_x/β_y at two ID straights is: 0.85 m/2.4

m. The horizontal damping partition J_x is 2.35. The optics functions of the QBA cell is shown in figure 8. The emittance is about 4.8 times larger than the theoretic minimum emittancewith $J_x = 1$.

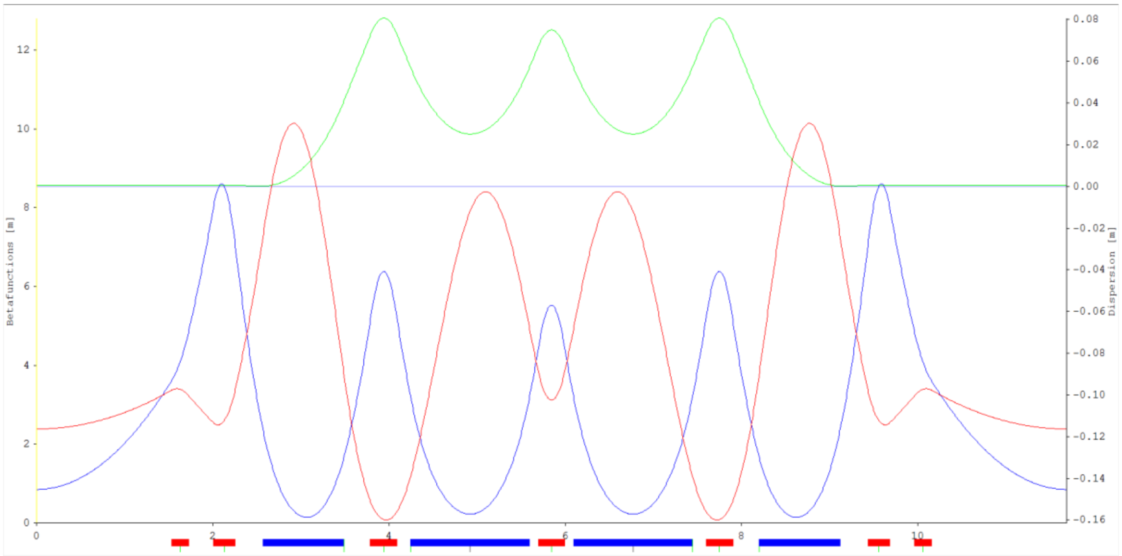

Figure 8: Optics functions of QBA cell.

The similar situation happens to the design of longitudinal gradient dipole. The emittance degradation of longitudinal gradient dipole due to non-optimal Courant-Snyder parameters is shown in figure 9.

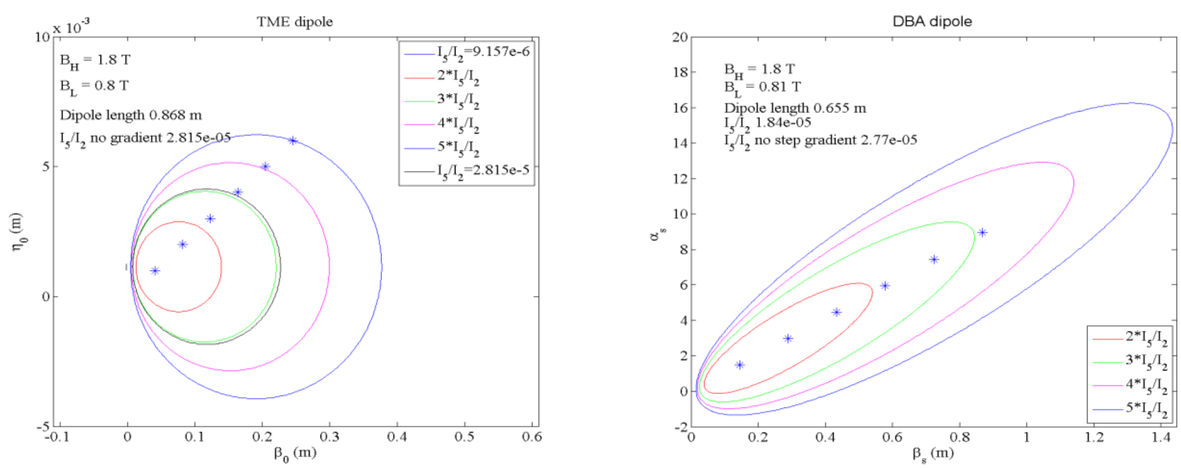

Figure 9 Contours of emittance degradation of longitudinal gradient dipole due to non-optimal Courant-Snyder parameters at the middle of dipole (left, TME dipole) and dipole entrance (right, DBA dipole). The blue stars represent different multipliers of the optimal Courant-Snyder parameters.

We first try to replace the middle QBA dipoles with longitudinal gradient dipoles. Six lattices named LQBA_LME1 to LQBA_LME6 are tried. The lattices represent six sets of Courant-Snyder parameters at the middle of gradient dipole corresponding to the blue stars in the left plot of Fig. 9. The Courant-Snyder parameters at the middle of the TME dipoles of the lattice LQBA_LME1 are matched to the requirements of minimum emittance of the longitudinal gradient dipole using the three quadrupoles (Q5 and QBL1) between the two TME dipoles as shown in figure 10. In fig. 10 the starting point is the middle of one of the longitudinal gradient dipoles. The sequence of the quadrupoles starts from left is: QBL1, Q5, QBL1, QBL2, Q4, Q2, Q1 and then in the reverse order. After the gradient dipoles the quadrupoles QBL2 and Q4 are used to match the achromatic conditions and the Q2 and Q1 to make the lattice symmetry at the middle of the straight. As a consequence there is little control of the Courant-Snyder parameters at the end dipoles. The same method is used to design the lattice LQBA_LME2 to LQBA_LME6 with multiplier from two to six to the required Courant-Snyder parameters of the minimum emittance at the middle of the TME dipoles. The emittance and the Courant-Snyder parameters achieved in these designs are summarized in table 3. The values shown in the first row of Tab. 3 are the values required to achieve the minimum emittance. The emittance is decreased as we release the required Courant-Snyder parameters of the minimum emittance at the TME dipole. The reason is the closer the matching values of the middle TME dipole gets the more the end dipole is mismatched, as shown in Tab. 3. The smallest emittance is when the Courant-Snyder parameters of middle dipole are five times the minimum values. Then the larger emittance from the middle dipoles starts to dominate the overall emittance. The emittance is close to the homogenous QBA cell. The strengths (k value in MAD8) of the quadrupoles are summarized in table 4. The strengths of Q5 and QBL1 are too strong due to the matching of small Courant-Snyder parameters at the TME dipoles.

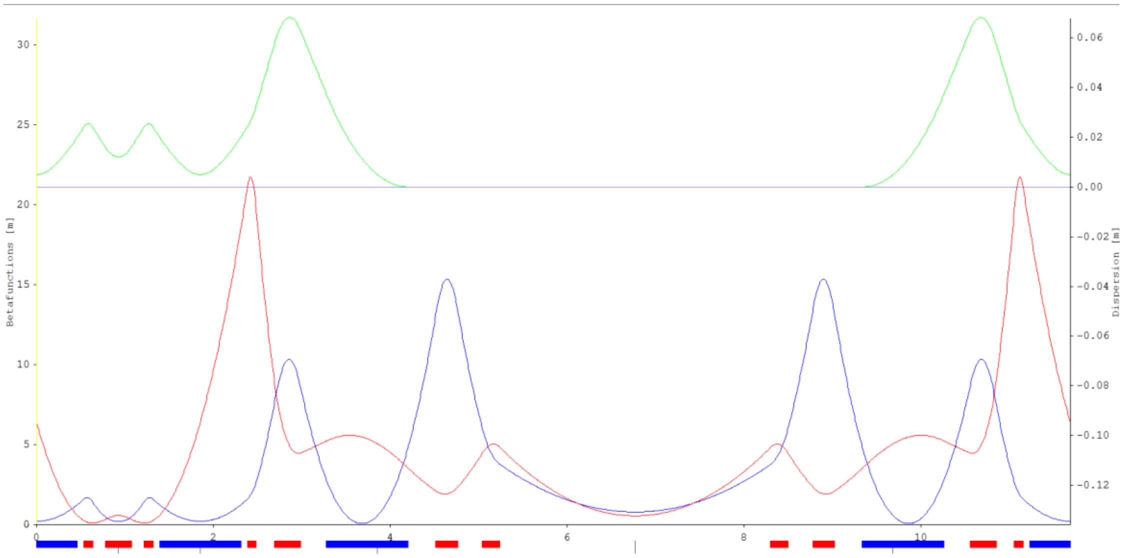

Figure 10: Optics functions of QBA cell with the replacement of the middle dipoles to longitudinal gradient dipoles. The start point is the middle of one of the longitudinal gradient dipoles. The sequence of the quadrupoles (red blocks) starts from left is: QBL1, Q5, QBL1, QBL2, Q4, Q2, Q1 and the reverse order.

Table 3: The emittance of QBA cell with replacement of middle dipole to longitudinal gradient dipole.

		EMIT	Jx		Matched middle dipole				End dipole	
		nm			BETX	ALFX	DX		BETX	ALFX
Matching value		~0.215	1		0.041	0.0	0.001		0.0895	0
LQBA_LME1		2.30	0.88077		0.041	0.001	0.001		2.883859	-1.63E+01
LQBA_LME2		1.10	0.92355		0.082	0	0.002		1.403758	-8.19969

LQBA_LME3	0.796	1.0089		0.123	0	0.003		9.12E-01	-5.26E+00
LQBA_LME4	0.677	1.0892		0.164	0	0.004		6.93E-01	-3.87E+00
LQBA_LME5	0.645	1.0847		0.205	0	0.005		5.60E-01	-2.84698
LQBA_LME6	0.656	1.0854		0.246	0	0.006		5.08E-01	-2.421491

Table 4: The strengths of quadrupoles of longitudinal gradient dipole.

	length(m)	LME1	LME2	LME3	LME4	LME5	LME6
Q5 (m^{-2})	0.3	-44.1039	-44.3922	-40.4576	-34.2808	-28.0423	-22.864
QBL1(m^{-2})	0.1	54.08222	52.97937	50.77415	47.76027	44.39379	41.0968
QBL2(m^{-2})	0.1	-31.9551	-28.0352	-26.6679	-24.8109	-23.6498	-22.8642
Q4(m^{-2})	0.3	12.95835	11.36055	11.40257	11.36888	10.6092	10.54414
Q2 (m^{-2})	0.25	7.815756	8.624916	8.566598	8.549702	9.296165	9.363056
Q1 (m^{-2})	0.2	-5.83151	-6.82594	-6.25488	-5.91207	-7.68929	-8.59566

The similar process is used to replace the end QBA dipoles with longitudinal gradient dipole. Five lattices named LQBA_LEV1 to LQBA_LEV5 are designed. The lattices represent five sets of Courant-Snyder parameters at the end of gradient dipole corresponding to the blue stars in the right plot of Fig. 9. The Courant-Snyder parameters at the end of the DBA dipoles of the lattice LQBA_LEV1 are matched to the requirements of minimum emittance of the longitudinal gradient dipole as shown in figure 11. The emittance and the Courant-Snyder parameters achieved in these designs are summarized in table 5. The emittance is increased as we release the required Courant-Snyder parameters of the minimum emittance at the DBA dipole. The emittance from the TME dipole after matching is also increased as the Courant-Snyder parameters at the DBA dipole are increased. This makes the emittances of five designs all larger than 10 nmrad. The strengths (k value in MAD8) of the quadrupoles are summarized in table 6.

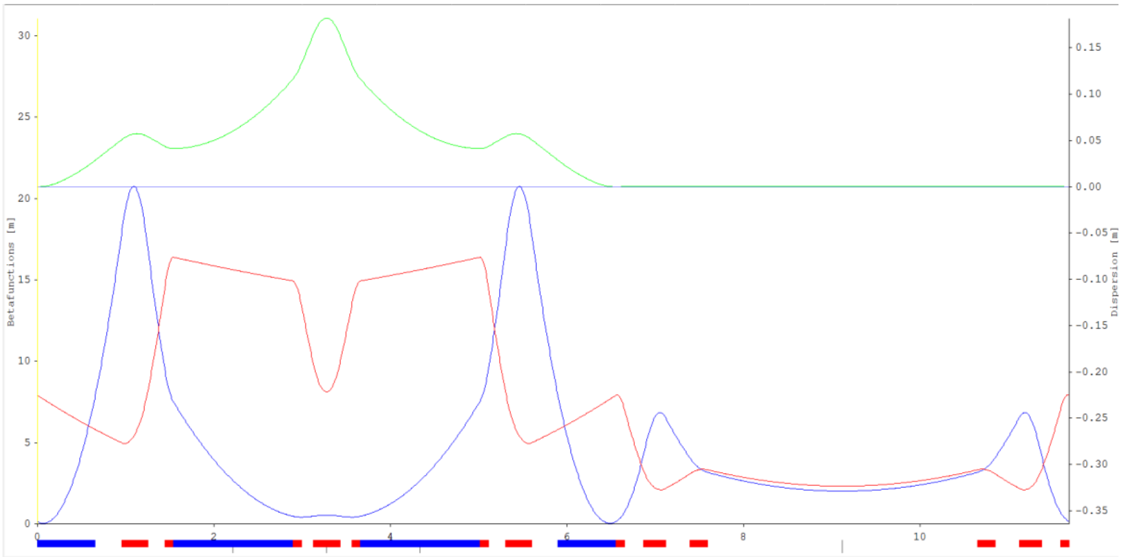

Figure 11: Optics functions of QBA cell with the replacement of the end dipoles to longitudinal gradient dipoles. The start point is the middle of one of the end longitudinal gradient dipoles. The sequence of the quadrupoles (red blocks) starts from left is: Q4, QBL2, QBL1, Q5, QBL1, QBL2, Q4, QBS, Q2, Q1, Q1, Q2 and QBS.

Table 5: The emittance of QBA cell with longitudinal gradient end dipole.

	EMIT	Jx	Matched end dipole			Middle dipole		
	nm		BETX	ALFX	DX	BETX	ALFX	DX
Matching value	~0.272		0.144	1.49	0	0.1746	0.0	0.0062
LQBA_LEV1	17.9	0.99721	0.144	1.50	0.0	2.66	2.61	6.03E-02
LQBA_LEV2	21.0	0.99724	0.288	2.98	0.0	3.17	2.89	5.94E-02
LQBA_LEV3	28.9	0.99725	0.432	4.47	0.0	4.43	4.10	5.89E-02
LQBA_LEV4	37.0	0.99727	0.576	5.96	0.0	5.66	5.37	5.76E-02
LQBA_LEV5	45.6	0.99727	0.720	7.45	0.0	6.98	6.66	5.85E-02

Table 6: The strengths of quadrupoles of longitudinal gradient dipole

	Length(m)	LEV1	LEV2	LEV3	LEV4	LEV5
Q1 (m^{-2})	0.2	-3.97356	-4.1697	-4.12376	-4.10641	-4.09941

Q2 (m^{-2})	0.25	12.40092	11.7359	11.57977	11.52679	11.49812
QBS(m^{-2})	0.1	-16.3364	-15.2797	-15.1816	-15.2583	-15.133
Q4(m^{-2})	0.3	7.653507	8.495893	8.517286	8.527028	8.530886
QBL2(m^{-2})	0.1	-9.9428	-0.0083	-0.0395	-2.223	-2.4562
QBL1(m^{-2})	0.1	-12.8822	-19.1178	-19.1401	-19.097	-19.0949
Q5(m^{-2})	0.3	8.830624	5.076596	5.100894	6.054101	6.152999

CONCLUSION

In this report we study the possibility to reduce the emittance further without increasing the number of dipoles by introducing dipoles with a longitudinal field variation. In general the emittance can be significantly below the minimum emittance of a homogeneous bending magnet. The minimization is limited by the ratio of maximum and average field in the dipole. A QBA cell that is designed to replace the SPEAR3 DBA cell is used in this study by replacing the homogeneous bending magnets with longitudinal step gradient dipoles. Due to the short cell length it is not possible to match the Courant-Snyder parameters at both the middle and the end dipoles to the desired values for the longitudinal step gradient dipole even with unlimited quadrupole strength, so it is not possible to take full advantage of longitudinal gradient in the SPEAR3 lattice. The best emittance achieved is to replace the middle dipoles with longitudinal step gradient dipoles. The maximum dipole field is 1.8 T and the lower field is 0.81 T. The emittance is 645 pm.rad which is close to the homogenous dipole QBA cell.

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