

DISCUSSION

PENTZ: You have explained the effect described in terms of the picture of two buckets, corresponding to two harmonic members of the radio-frequency acceleration differing by unity ($h, h \pm 1$), which may overlap. Is it correct, then, that the effect will occur when the bucket height is large enough for such overlap to occur?

SYMON: Yes.

PENTZ: Could one then study the effect experimentally by using two accelerating gaps separately programmed so as to locate two buckets at energy separations comparable to the buckets heights?

SYMON: Yes, this would be approximately equivalent situation to that which would exit in the case of acceleration at high harmonic number.

KOLOMENSKY: In your paper you have considered one-dimensional (longitudinal) motion. I think that the two-dimensional character of motion (i.e. coupling with the radial betatron oscillation) would be taken into account particularly in the case of large number of accelerating stations?

SYMON: Yes, I agree.

BEAM CURRENT LIMITATIONS IN THE AGS WITH MULTIPLE-TURN INJECTION RELATED TO ION SOURCE EMITTANCE CHARACTERISTICS

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As a consequence of the experimental observed behaviour that the injector (ion source) emittance increases approximately linearly with output current, the maximum number of particles stacked in transverse phase space with multiturn injection in the synchrotron is not necessarily obtained with maximum injector beam intensity.

Present high-intensity ion sources, such as the duoplasmatron source, r.f. source and P.I.G. source, show the behaviour that the emittance (two-dimensional phase space) increases linearly with beam output current (1), (2). This particular behaviour has some consequences related to optimum AGS intensity if multiple-turn injection is considered. To show this, it is useful to define the following quantities:

$$\epsilon_2 = \beta\gamma E_2$$

two dimensional momentum normalized emittance and

$$B_4 = \frac{I_{inj.}}{(\pi^{1/2}) (\epsilon_2)^2},$$

the source brightness, or momentum normalized density in four dimensional phase space. The last expression makes use of (3): $\epsilon_4 = (\epsilon_2)^2$ where ϵ_4 is the four dimensional normalized emittance.

For the high intensity ion sources mentioned above the experimentally observed current dependence of ϵ_2 and B_4 is illustrated in Fig. 1. * The observed current dependence is expressed as:

$$\epsilon_2 = \frac{I_{inj.}}{\pi \delta_2} \quad \text{and} \quad B_4 = \frac{2 (\delta_2)^2}{I_{inj.}}$$

with δ_2 a constant. $\delta_2 = d_2/\beta\gamma$ with d_2 the particle "density" in two dimensional phase space. It would be an oversimplification to assume a homogeneous density distribution. Nevertheless, also with stacking of high-density regions only, the present arguments are valid except for the fact that the constant δ_2 , would be larger. Because the brightness as defined here refers to four-dimensional phase space only, the time structure of the beam may change the δ_2 and B_4 values. The solid lines in Fig. 1 apply to the preinjector output current. Here the present experimental δ_2 value is approximately $250/\pi$ mA/cm-mrad.

After the Linac (50 MeV) the time average

* The B_4 values for the modified duoplasmatron source given in Fig. 1 refer to 90% of the total beam intensity and were obtained by interpolating the density distribution results of (5).

value of δ_2 becomes approximately $50/\pi$ mA/cm-mrad. This is because of longitudinal capture efficiency of the Linac, proton percentage of the preinjector beam, and effective dilution because of transverse-longitudinal coupling in the Linac. These factors can be taken into account and it has been shown (2) for the 50 MeV Linac that these δ_2 values are consistent.

With multiple-turn injection in the synchrotron the simplest model of horizontal phase-space stacking, disregarding other limiting factors, indicates that the maximum number of injectable turns is given by:

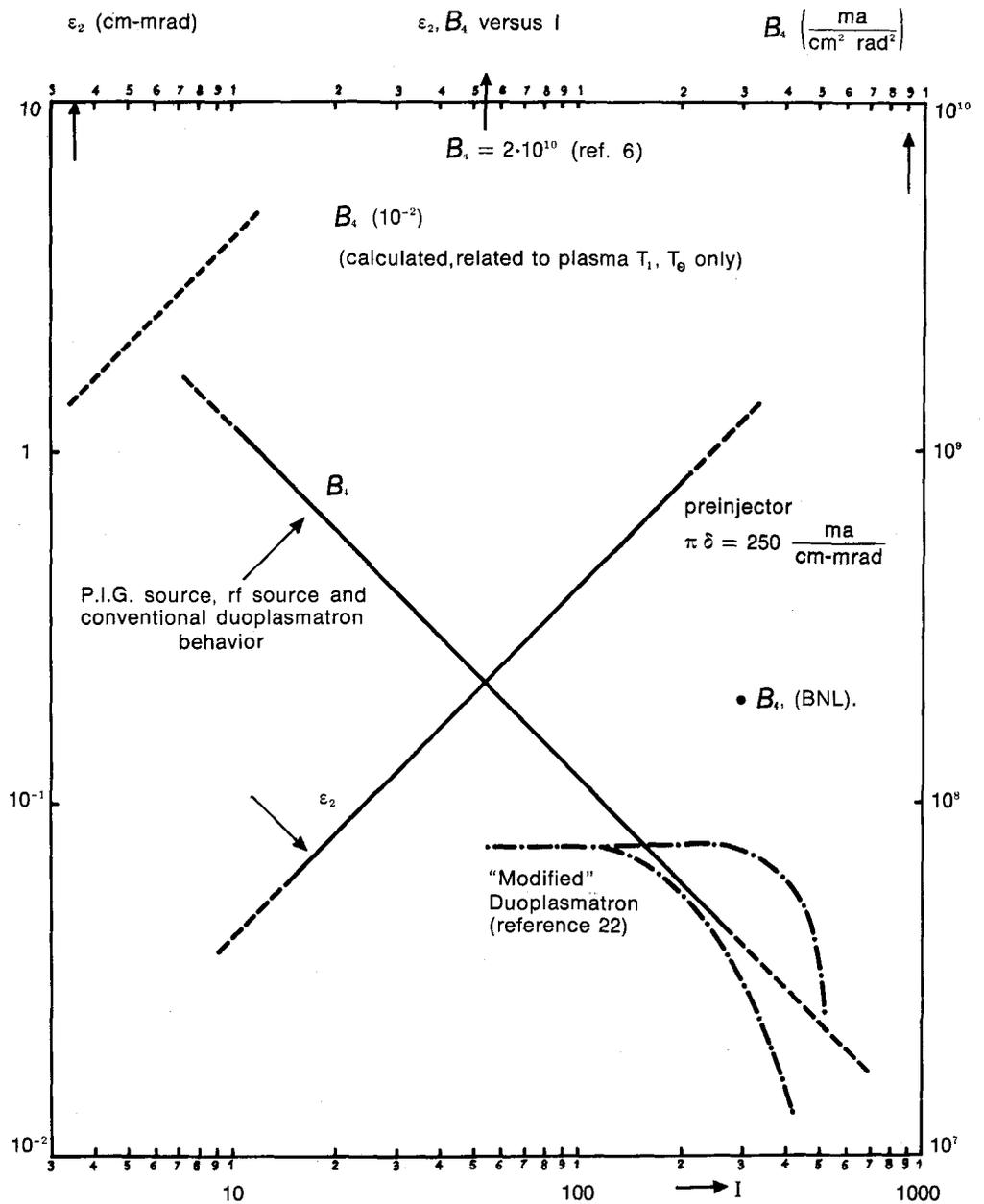
$$\eta_{max} = \frac{W_2}{E_2}$$

where W_2 is the synchrotron acceptance.

Using now the foregoing expressions and the experimentally observed emittance current behavior, one finds in a first approach for the maximum number of protons which can be stacked in the synchrotron:

$$N_H = (2\pi^2 \eta_{rf} \eta_{st} \cdot H W_{2,H} \cdot R_{eff}) \cdot \left(\frac{\delta_2 \gamma}{ec} \right)$$

Fig. 1



with η_{rf} = longitudinal capture efficiency; $\eta_{\text{st.H}}$ = horizontal stacking efficiency, and R_{eff} = synchrotron equilibrium orbit length/ 2π .

The significant conclusion is that, in a first approach, N_{H} can only be increased with an improvement of the δ_2 value and not with an increase of the Linac intensity. Actually, presently an attempt will be made to inject a larger number of turns into the ASG with a longer Linac pulse, but with reduced intensity.

Using the same simple model with vertical stacking also, one obtains similarly:

$$N_{\text{H,V}} = 2\pi^3 \eta_{\text{rf}} \eta_{\text{st.H}} \eta_{\text{st.V}} W_{2,\text{H}} \cdot W_{2,\text{V}} R_{\text{eff}} \left(\frac{\beta\gamma^2}{ec} \right) \frac{\delta_2^2}{I_{\text{inj}}}$$

or

$$N_{\text{H,V}} = N_{\text{H}} \left[(\pi \eta_{\text{st.V}} W_{2,\text{V}}) \beta\gamma \frac{\delta_2^2}{I_{\text{inj}}} \right]$$

The last expression indicates the improvement factor by also stacking in vertical phase space. The paradoxical point here is that with both horizontal and vertical stacking the maximum obtainable number of protons increases with decreasing Linac intensity. It is clear that this conclusion would be valid over a limited range of n_{max} values only. For a small beam intensity and consequently a smaller emittance value and larger n_{max} value it is to be expected that the over-all stacking efficiency decreases because of the minimum injection septum size. It is indicated, however, that with the present emittance-current characteristic an optimum value for injected beam intensity exists, if both horizontal and vertical stacking would be used. This value is not necessarily the maximum beam current obtainable from the injector.

It should also be noted that with both horizontal and vertical stacking $N_{\text{H,V}}$ increases with $\beta\gamma^2$ instead of being proportional to γ in the case of horizontal stacking only. This would be a consi-

deration when taking into account optimum injection energy.

An analysis (4) for the source brightness of the ion source indicates that ideally

$$B_s = B_0 \text{ and } \epsilon_2 = \left(\frac{1}{\pi B_0^{1/2}} \right)^{1/2} I_{\text{inj}}$$

In this case the qualitative expressions for the maximum number of stacked protons in the synchrotron with horizontal stacking only and with horizontal and vertical stacking combined become:

$$N_{\text{H}} = \left(2\pi^2 \eta_{\text{rf}} \eta_{\text{st.H}} W_{2,\text{H}} R_{\text{eff}} \right) \left(\frac{\gamma B_0}{ec} \right)^{1/2} I_{\text{inj}}^{1/2}$$

$$N_{\text{H,V}} = \left(2\pi^3 \eta_{\text{rf}} \eta_{\text{st.H}} \eta_{\text{st.V}} W_{2,\text{H}} \cdot W_{2,\text{V}} \cdot R_{\text{eff}} \right) \frac{\beta\gamma^2}{ec} B_0 =$$

$$= N_{\text{H}} \left[\pi \eta_{\text{st.V}} W_{2,\text{V}} \beta\gamma \frac{B_0^{1/2}}{I_{\text{inj}}} \right]$$

This is to be compared with the similar expression given in the foregoing. With horizontal stacking only and optimum source performance N_{H} would be directly proportional with $(B_0 I_{\text{inj}})^{1/2}$ indicating in this case not only the desirability of high source brightness but also high injector intensity.

Recent ion source emittance results, obtained both at BNL and elsewhere (5) indicate improved source brightness values, however, so far there does not seem to be any evidence that $B_s = B_0$, i. e. that the source brightness is independent of output current.

Actually, recent results obtained at Saclay (6) indicate an appreciable improvement in source brightness values obtained at lower beam intensities, however, again the source brightness varies approximately direct proportional to the inverse of the source output current.

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