Spherical Collapse in the Symmetron Model

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Abstract. The Symmetron model is a scalar field model for dark energy in which the scalar field is coupled to matter and is invariant under a reflection symmetry $\phi \to -\phi$. The coupling of the symmetron to matter is such that the symmetry is exact in high density environments and spontaneously broken when the density drops below a critical value. We study the nonlinear evolution of density perturbations using the spherical collapse model. We calculate the extrapolated linear density contrast δ_c which is a fundamental quantity to obtain the halo-mass function for this model.

1. Introduction

In recent years, there has been much interest in exploring the possibility that the cosmic acceleration of the universe could be caused by a scalar field rather than by a cosmological constant. In the first class of models (called *quintessence models*), the scalar fields generally mediate a long range force of gravitational strength, which would induce dramatic violations of the equivalence principle, strongly constrained by local tests of gravity. Over the last decade, however, it has been realized that these scalar fields can mediate a long range force (\sim Mpc) while satisfying local tests of gravity by imposing a *screening mechanism*, such as the Chameleon Mechanism [1], the Vainshtein Mechanism [2] and the Symmetron Mechanism, that we will discuss below.

2. The Symmetron model

Scalar-tensor theories are characterized by a conformal factor and by their interaction potential. In such a context, the symmetron model was proposed in [3, 4, 5] and is described by this action:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} \mathcal{R} - \frac{1}{2} \left(\partial\phi\right)^2 - V\left(\phi\right) \right\} + \int d^4x \sqrt{-\tilde{g}} \quad \mathcal{L}_m\left(\psi_i, \tilde{g}_{\mu\nu}\right) \tag{1}$$

where g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar, ψ_i are the different matter fields and $M_{Pl} \equiv \frac{1}{\sqrt{8\pi G}}$ where G is Newton's constant. Matter fields are minimally coupled to the Jordan frame metric $\tilde{g}_{\mu\nu}$, conformally related to the Einstein frame metric $g_{\mu\nu}$ by:

$$\tilde{g}_{\mu\nu} \equiv A^2\left(\phi\right) g_{\mu\nu} \tag{2}$$

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Varying the action with respect to the scalar field ϕ , we obtain the field equations for ϕ in the Einstein frame ¹

$$\Box \phi = \frac{\partial V(\phi)}{\partial \phi} + \rho \frac{\partial A(\phi)}{\partial \phi}$$
(3)

So the field evolves according to an effective potential

$$V_{eff}(\phi) = V(\phi) + \rho A(\phi)$$
(4)

For the symmetron model of interest, we choose:

$$V(\phi) = \bar{V} + V_0 e^{-\frac{\phi^2}{2M^2}}$$
(5)

and

$$A\left(\phi\right) = e^{\frac{\lambda\phi^2}{2M^2}} \tag{6}$$

in which \overline{V} is the cosmological constant energy density, V_0 is a small energy density fixed by the phase transition, λ is one dimensionless coupling constant and M is the mass scale. So we have:

$$V_{eff}(\phi) = \bar{V} + V_0 e^{-\frac{\phi^2}{2M^2}} + \rho e^{\frac{\lambda\phi^2}{2M^2}}$$
(7)

Since $\phi^2 \ll M^2$, we can expand $V_{eff}(\phi)$ in Taylor series so we obtain the effective potential described in [5]. With the above choice for the signs (we will assume $\lambda > 0$), there is a density-dependent phase transition. Indeed, looking at the second derivative of the potential in $\phi = 0$,

$$M^{2} \frac{d^{2} V_{eff}}{d\phi^{2}}|_{\phi=0} = -V_{0} + \lambda\rho$$
(8)

we see that it changes sign at

$$\rho\left(z_{t}\right) = \rho_{0}\left(1 + z_{t}\right)^{3} = \frac{V_{0}}{\lambda}$$

$$\tag{9}$$

where z_t is the redshift in which the phase transition occurs. As long as ρ is high enough $(z \ge z_t)$, the minimum of V_{eff} exhibits the \mathbb{Z}_2 symmetry and the vacuum expectation value (VEV) goes to zero $(\phi_{VEV} = 0)$. In low density regions $(z \le z_t)$, instead, the potential breaks the reflection symmetry spontaneously and the scalar acquires a VEV:

$$\phi_{VEV} = \phi_0\left(z\right) = \pm M \sqrt{\frac{6}{\lambda+1} \log\left(\frac{1+z_t}{1+z}\right)} \tag{10}$$

where we have chosen the minimum with the "+" sign. Fluctuations $\delta\phi$ around a local background value ϕ_{VEV} , as would be detected by local experiments, couple as $\approx \frac{\lambda}{M^2} \phi_{VEV} \delta\phi\rho$, that is, the coupling is proportional to the local VEV.

2.1. Scalar coupling

We define the coupling of the scalar field to matter $\beta(\phi)$ as in [6]:

$$\beta\left(\phi = \phi_0\right) = \beta_0\left(z\right) = M_{Pl} \frac{d\log A\left(\phi\right)}{d\phi} \mid_{\phi = \phi_0} = M_{Pl} \frac{\lambda\phi_0\left(z\right)}{M^2} \tag{11}$$

¹ where $\rho = A^3 \tilde{\rho}$ is the matter density, which is conserved in the Einstein frame and $\tilde{\rho}$ is the 00 component of the energy-momentum tensor $\tilde{T}_{\mu\nu} = -\left(2/\sqrt{-\tilde{g}}\right)\delta\mathcal{L}_m/\delta\tilde{g}^{\mu\nu}$ in the Jordan frame, which is covariantly conserved: $\tilde{\nabla}_{\mu}\tilde{T}^{\mu}_{\nu} = 0.$

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where $\phi_0 = 0$ before the phase transition, and $\phi_0 \neq 0$ after the phase transition. We take $\beta_0 (z = 0)$ as a parameter of our model:

$$\beta_0 \left(z = 0 \right) = \frac{\lambda M_{Pl}}{M} \sqrt{\frac{6}{\lambda + 1} \log \left(1 + z_t \right)} \cong \frac{\lambda M_{Pl}}{M} \sqrt{6 \log \left(1 + z_t \right)} \tag{12}$$

because $\lambda \ll 1$. We have fixed $\beta_0 \approx 0.75$ from the value of the error on the parameter σ_8^2 which is about 5% and the phase transition $z_t = 1$ near today.

2.2. Spherical solutions

To study the implication for tests of gravity, we follow the same steps of [5, 6]. In analogy with the Chameleon Mechanism (see [1]), we define a dimensionless parameter γ :

$$\gamma \equiv \frac{\lambda}{M^2} \left(\rho - \bar{\rho}\right) R^2 = 6 \frac{M_{Pl}^2}{M^2} \Phi \tag{13}$$

Physically, this ratio measures the surface Newtonian potential Φ relative to M/M_{Pl} . We define *screened objects* the case in which the scalar force is confined only within a thin-shell beneath the surface and *unscreened objects* the case in which the scalar force is non zero inside the object. To satisfy experimental constraints we want the Milky Way to be screened so we set:

$$\frac{M}{M_{Pl}} = \mu \cong 10^{-3} \tag{14}$$

following [5].

3. Spherical collapse

A popular tool to study the non-linear growth in cold dark matter is the spherical collapse [7, 8]. This non-linear approximation was first used in the standard cold dark matter scenario, but later also in the cold dark matter model with a cosmological constant (ΛCDM) [9].

3.1. Application to Symmetron model

Now we study how to apply the spherical collapse to symmetron model.

At the beginning, the initial density contrast of the bubble is very small ($\delta_{m,i} = 0.0003$) so we start by $\gamma \ll 1$: initially the sphere is in the *thick-shell* regime and the scalar field has this value both inside and outside the sphere. Then, the transition from *thick-shell* to *thin-shell* regime depends on the initial radius of the bubble (R_i). For small values of the initial radius, the sphere remains in the *thick-shell* regime for all the time that it takes to collapse and we have that the scalar field is inside the bubble and in the background; in this case, the γ parameter is always less than one. For large initial radii, the sphere is initially in the *thick-shell* regime; at some point, the γ parameter becomes larger than one and we turn off the scalar field inside the sphere. In this case we observe the transition from *thick* to *thin-shell* regime. We denote by ϕ the scalar field inside the bubble and by $\overline{\phi}$ the scalar field in the background.

The flat background Universe is described by Friedmann equations:

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{1}{6M_{Pl}^2} \left(\bar{\rho}_m A\left(\bar{\phi}\right) + \bar{\rho}_\phi + 3\bar{p}_\phi\right) \tag{15}$$

where:

$$\bar{\rho}_{\phi} = \frac{1}{2}\dot{\bar{\phi}}^2 + V\left(\bar{\phi}\right) \tag{16}$$

² where σ_8 parameter is defined as: $\sigma_8^2 (r = 8Mpc/h) = \int d^3k/(2\pi)^3 |\tilde{W}(kr)|^2 P_L(k)$.

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$$\bar{p}_{\phi} = \frac{1}{2}\dot{\bar{\phi}}^2 - V\left(\bar{\phi}\right) \tag{17}$$

are respectively the density and the pressure of the scalar field and

$$\bar{H}^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2} \left(\bar{\rho}_m A\left(\bar{\phi}\right) + \bar{\rho}_\phi\right) \tag{18}$$

We can derive the evolution equation for the bubble radius R by following the same steps of [10]. So we have:

$$\left(\frac{\ddot{R}}{R}\right) = -\beta\left(\phi\right)\dot{\bar{\phi}}\left(\bar{H} - \frac{\dot{R}}{R}\right) + \left(\frac{\ddot{a}}{a}\right) - \frac{1}{6M_{Pl}^2}\bar{\rho}_m\delta_m\left(1 + 2\beta\left(\phi\right)^2\right)$$
(19)

Since the scalar field is rolling slowly along the minimum, we can neglect all terms proportional to $\dot{\phi}$. In the *thick-shell* regime the scalar field is inside the sphere and in the background: $\phi = \phi = \phi_0$. So we use equations 15-19 in which we put $\phi = \phi_0$.

In the transition regime from *thick* to *thin-shell*, at the beginning we use the equations of the previous case. When γ parameter becomes bigger than one, we put $\phi_0 = 0$. The equations are the same for the background, while the evolution equations for the radius R become:

$$\left(\frac{\ddot{R}}{R}\right) = \left(\frac{\ddot{a}}{a}\right) - \frac{1}{6M_{Pl}^2}\bar{\rho}_m\delta_m \tag{20}$$

when we turn-off the scalar field inside the sphere. The important thing is that the dynamics



Figure 1. Evolution of radius $\frac{R}{R_i}$ vs (z + 1) for $\delta_{m,i} = 0.0003$. The blue curve represents ΛCDM model, the red curve represents the transition regime from *thick* to *thin-shell* obtained by fixing $R_iH_0 = 0.45$ and the yellow curve represents the *thick-shell* regime, obtained by fixing $R_iH_0 = 0.1$.

of collapse and the type of regime depends on the initial radius of the sphere (R_i) . For small initial radius $(R_iH_0 \leq 0.1 \text{ means } R_i \leq 300 \text{ Mpc})$, we have that $\gamma < 1$ for all the time. If we integrate numerically equations 15-19 from $z_i \sim 7000$ to $z_f = 0$, with an initial density contrast $\delta_{m,i} = 0.0003$, we observe that the radius collapses at $z \approx 0.2$, as we can see from figure 1. For large initial radius $(0.1 < R_iH_0 \leq 0.45 \text{ means } R_i \text{ between about 300 and 1400 Mpc})$ we have the transition regime: we start by integrating numerically the same equations of the previous case from $z_i \sim 7000$ to the moment in which $\gamma \geq 1$ ($z \sim 0.7$). Then we use equation 20 for radius evolution when we pass to *thin-shell* regime until the end $z_f = 0$. In this case, the scalar force is confined only within a *thin-shell* beneath the surface so the sphere collapses later respect to the case in which the object is unscreened.

3.2. Determination of δ_c

We follow the same steps of [10] to obtain the linear evolution of density contrast:

$$\ddot{\delta}_{m,L} = \left(\beta\dot{\bar{\phi}} - 2\bar{H}\right)\dot{\delta}_{m,L} + \frac{1}{a^2}\nabla^2\Phi_{eff}$$
(21)

where Φ_{eff} is the effective gravitational potential given by

$$\Phi_{eff} \equiv \Phi + \beta \delta \phi \tag{22}$$

which follows the modified Poisson equation

$$\nabla^2 \Phi_{eff} = -\frac{a^2}{2M_{Pl}^2} \bar{\rho}_m \delta_m \left(1 + 2\beta^2\right) \tag{23}$$

If we assume the adiabatic approximation, we can neglect all terms proportional to $\dot{\phi}$ so equation 21 becomes:

$$\ddot{\delta}_{m,L} \cong -2\bar{H}\dot{\delta}_{m,L} + \frac{1}{a^2}\nabla^2\Phi_{eff} \tag{24}$$

where \bar{H} is defined in equation 18. We numerically solve equation 24 from $z \sim 7000$ to z = 0and we calculate the value of δ at redshift of collapse z_c by varying the value of $\delta_{m,i}$. From figure 2, the two models (*thick-shell* regime and transition from *thick* to *thin-shell* regime) approach ΛCDM at high redshift after the phase transition. At z = 0, the difference in δ_c with respect to ΛCDM is about 2% for *thin-shell* regime, while is about 4% for *thick-shell* regime.



Figure 2. Extrapolated linear density contrast at collapse δ_c vs. $z_c + 1$ for Einstein de-Sitter EdS (green curve), ΛCDM (blue curve), transition from *thick* to *thin-shell* regime (red curve) and *thick-shell* regime (yellow curve) cases.

4. Halo-mass function

We use a model of halo-mass function based on spherical collapse and on the Sheth-Tormen (ST) prescription, by following the procedure of [11]. We calculate the halo-mass function by fixing the value of δ_c at z = 0, as we can see from figure 3.



Figure 3. Left panel: Halo mass function as a function of the virialization mass M_v for thick-shell regime (red dashed line) by fixing $\delta_c (z = 0) \simeq 1.64$ and $\beta_0 = 0.75$, for the transition regime (blue dotdashed line) by fixing $\delta_c (z = 0) \simeq 1.70$ and $\beta_0 = 0.75$. The green band shows the region between the two values of δ_c . The solid black line corresponds to the ΛCDM case, i.e. $\beta_0 = 0$. Right panel: Relative deviations of the halo mass function from ΛCDM model in function of M_v .

5. Conclusions

Dark matter haloes are the building blocks of cosmological observables associated with structures in the universe. In this model, the scalar coupling which occurs after the phase transition leads to an enhanced abundance of massive halos with respect to the Standard Model of Cosmology. In figure 3 (right panel) we observe that the number of haloes increases significantly, especially at the high mass end, by up to 50-100% for cluster-sized haloes. The transition regime (blue curve) suppresses the abundance of haloes in the high mass end, as expected. A similar behavior was found in [11] for f(R) models.

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7. References

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