Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 2

NATURALNESS OF ELECTROWEAK SYMMETRY BREAKING*

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(Received November 15, 2006)

After revisiting the hierarchy problem of the Standard Model and its implications for the scale of New Physics, I consider the fine tuning problem of electroweak symmetry breaking in two main scenarios beyond the Standard Model: SUSY and Little Higgs models. The main conclusions are that New Physics should appear on the reach of the LHC; that some SUSY models can solve the hierarchy problem with acceptable residual fine tuning and, finally, that Little Higgs models generically suffer from large tunings, many times hidden.

PACS numbers: 11.15.Ex, 11.30.Qc, 12.60.Jv, 12.60.Cn

1. Hierarchy problem of the SM

It is well known that, although the Standard Model (SM) works extremely well (at the permille), it is probably not fundamental but rather an approximate description of particle physics valid up to some high energy scale Λ, where a more fundamental theory takes over. We do have some clues about the value of Λ coming from the SM electroweak symmetry breaking (EWSB) sector. This breaking is described in the SM by a fundamental doublet scalar with a mexican-hat potential

$$V = \tfrac{1}{2}m^2h^2 + \tfrac{1}{4}\lambda h^4\,. \tag{1}$$

The Higgs mass parameter m^2 is assumed to be negative so that h develops a non-zero vev that breaks the EW gauge symmetry spontaneously. As m^2 is the only mass scale that appears in the SM Lagrangian it sets the scale of EWSB, with $v^2 = -m^2/\lambda$ fixed to $\simeq (246~\mathrm{GeV})^2$ to get right the masses

* Presented at the "Physics at LHC" Conference, Kraków, Poland, July 3–8, 2006.

of Z^0 and $W^\pm$. The spectrum contains a Higgs boson with mass squared $m_{\rm h}^2 = 2\lambda v^2$ that is unknown but expected to be light from loop effects in fits to precision EW data.

There is a theoretical problem with this picture of EWSB: m^2 is sensitive to high energy scales through quantum corrections. Loops of $W^\pm$ and Z^0 gauge bosons, the top quark and the Higgs itself give a one-loop quadratically divergent correction to m^2. More precisely one gets [1]

$$\delta m^2 = \frac{3\Lambda^2}{64\pi^2}(3g^2 + g'^2 + 8\lambda - 8\lambda_t^2) + \dots \, . \tag{2}$$

If one believes that the SM is valid all the way up to the Planck mass, $\Lambda \sim M_{\rm Pl}$, δm^2 is huge and has to be balanced with extreme precision against the tree-level value of m^2. This is the Big Hierarchy Problem [2]. If, following indications from the absence of indirect effects of some non-renormalizable operators, one believes the SM is valid up to $\Lambda \sim 10$ TeV one still has a problem (the Little Hierarchy Problem [3]) albeit softer.

Turning the argument around, naturalness of EWSB requires $\delta m^2 \sim \lambda v^2 \sim m^2$ and an upper bound on the scale of New Physics follows. For instance,

$$\frac{\delta m^2}{m^2} < 10 \Longrightarrow \Lambda \lesssim 2 \text{ TeV}\, , \tag{3}$$

for $m_{\rm h} \sim 130$ GeV. The importance of this figure is obvious: it implies that New Physics beyond the SM should be on the reach of LHC.

This naive estimate has been refined [4, 5] taking into account higher order effects [for moderate values of Λ the dominant loop corrections are summed up to leading-log order [6] simply by Eq. (2), but with couplings evaluated at the high scale Λ] and the sensitivity of m^2 to other parameters besides Λ, like λ and λ_t. Before presenting the final result let me remind you how to estimate numerically the fine tuning associated to EWSB. Consider your favourite model for EWSB, which should give v as a function of some input parameters p_α (usually these are defined at some UV scale). Following Barbieri and Giudice [7], we adopt as a measure of the fine tuning associated to p_α the quantity Δ_{p_α} defined by $\delta m^2/m^2 = \Delta_{p_\alpha}(\delta p_\alpha/p_\alpha)$, where δm^2 is the change induced in m^2 by a change δp_α in p_α. For a given model, we can arrive at a global fine tuning figure by adding the different Δ_{p_α} in quadrature, $\Delta \equiv \sqrt{\sum_\alpha \Delta_{p_\alpha}^2}$ [8]. Absence of fine tuning requires $\Delta \lesssim \mathcal{O}(10)$, which corresponds to 10% tuning (roughly speaking Δ^{-1} measures the probability of a cancellation among terms of a given size to obtain a result which is Δ times smaller. For discussions and refinements see [9]).

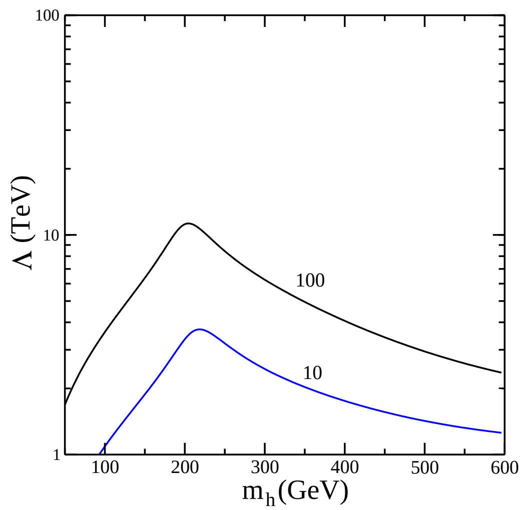

Fig. 1. Upper bound on the scale of New Physics (Λ) from the absence of 1/10 (1/100) tuning. (Improved calculation, including sensitivity to λ and λ_t, see [14])

Going back to the SM, Fig. 1 shows the fine tuning $\Delta = \{\Delta_\Lambda^2 + \Delta_{\lambda_t}^2 + \Delta_\lambda^2\}^{1/2}$. One obtains then

$$\Lambda \lesssim 3\text{–}4 \text{ TeV}\,, \tag{4}$$

(2.5 TeV on average) to avoid more than 10% fine tuning. So, one maintains the naive expectation that New Physics beyond the SM should be on the reach of the LHC. (In fact one can even argue that this bound is conservative, being based on an underestimate of the effect of New Physics [5]. For further qualifications on the meaning of this kind of analysis see [10].)

Besides the general bound of Eq. (4), more concrete (and solid) implications from naturalness can be deduced in particular scenarios for the New Physics beyond the SM. In what follows we consider two scenarios which are particularly well motivated: SUSY and Little Higgs models. Both of them try to reduce the sensitivity of m^2 to high energy scales Λ by introducing new particles (so that new loop corrections cancel the SM dangerous contributions) and new symmetries (so that the required cancellation is natural).

2. The SUSY fine tuning problem

In SUSY, the new particles introduced are the superpartners of SM particles and SUSY ensures the cancellation of quadratic divergences to all orders! The Higgs mass is protected because SUSY relates the Higgs to chiral fermions, whose mass is under control. However, SUSY must be broken, with the new superpartners having masses $\sim \tilde{m} \lesssim 1$ TeV. As a result, loops of particles and antiparticles do not cancel completely and quadratic divergences are replaced by corrections proportional to the soft SUSY breaking mass scale $\tilde{m}$. For instance, top-stop loops give

$$\delta m^2 \sim -\frac{\lambda_t^2 \tilde{m}^2}{16\pi^2} \log \frac{M_{\rm mes}^2}{\tilde{m}^2} \ , \tag{5}$$

where $M_{\rm mes}$ is the high energy scale at which SUSY is transmitted to the observable sector (this log can be interpreted as a RG effect). The improvement in naturalness with respect to the SM case (with $\Lambda \sim M_{\rm mes}$) is enormous. In addition, the negative contribution in (5) explains EWSB dynamically (*i.e.* gives a reason for $m^2 < 0$).

Focusing on the Minimal Supersymmetric Standard Model (MSSM), it is known that EWSB suffers from a residual fine tuning problem. In order for EWSB to be natural, $\tilde{m} \lesssim$ few hundred GeV is needed in (5), which implies that superpartners should be not too heavy. Experimental lower bounds already force the ordinary MSSM to be significantly fine tuned [7, 9, 11]. Consider for instance the upper bound on the lightest Higgs boson mass

$$m_{\rm h}^2 \leq M_Z^2 \cos^2 2\beta + \frac{3 m_t^4}{2\pi^2 v^2} \log \frac{M_{\rm SUSY}^2}{m_t^2} + \dots \ , \tag{6}$$

where m_t is the (running) top mass ($\simeq 166$ GeV for $M_t = 173$ GeV). Since the experimental lower bound, $(m_{\rm h})_{\rm exp} \geq 115$ GeV, exceeds the tree-level contribution, the radiative corrections must be responsible for the difference, and $M_{\rm SUSY} \gtrsim 3.6\, m_t$ is required. This implies sizable soft terms, $\tilde{m} \gtrsim 2 m_t$, and then large fine tunings.

The typical tuning in this model is shown in Fig. 2. There are three main reasons for such large values of fine tuning:

1. In the MSSM, λ is calculable and quite small: $\sim (1/8)(g^2 + g_Y^2) \cos^2 2\beta \simeq (1/15) \cos^2 2\beta$. This amplifies whatever cancellations are taking place inside m^2.

2. Although for a given size of $\tilde{m}$ the radiative corrections reduce the fine tuning, sizable radiative corrections require large $\tilde{m}$. A given increase in $\tilde{m}^2$ reflects linearly in m^2 but only logarithmically in λ, so the fine tuning usually gets worse.

3. Typically, the large logarithms in (5) and the numerical factors compensate the one-loop factor, so that the residual corrections to m^2 can be quite sizable.

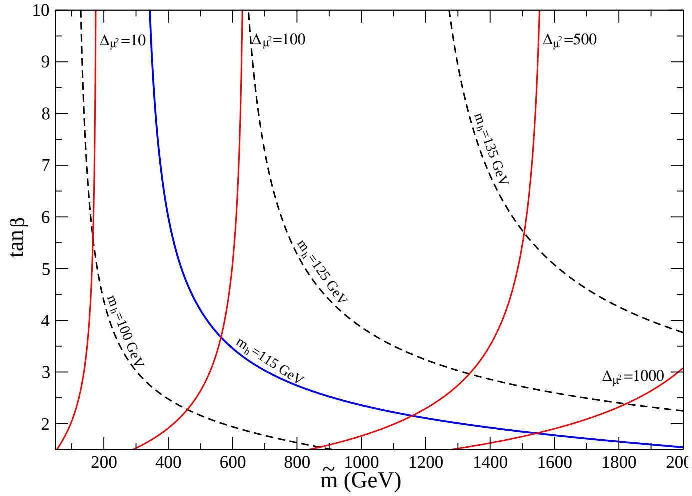

Fig. 2. Solid: Fine tuning (measured by Δ_{μ^2}) in the MSSM with universal soft masses, in the $(\tilde{m}, \tan\beta)$ plane. Dashed: contour lines of constant Higgs mass.

It should be kept in mind, however, that it is not too difficult to come up with alternative SUSY models which perform better than the MSSM concerning fine tuning of EWSB. In fact, it is fair to say that SUSY is the most powerful tool for controlling the naturalness of EWSB. One popular model is the Next-to-Minimal Supersymmetric SM (NMSSM) which adds a singlet chiral multiplet to the MSSM. In this model, λ gets larger thanks to additional F-term contributions from the singlet, improving point (1) of the list above [12]. One can also consider scenarios with low SUSY breaking scale (not far from the TeV) in which it is natural [13] to have tree-level contributions to λ that can make it larger [point (1)]. This helps in evading the LEP Higgs mass bound without the need of large radiative corrections [point (2)] and, moreover, in such models RG effects are expected to play no significant role since the cut-off scale is much closer to the EW scale [point (3)]. All these three improvements can cooperate to make EWSB much more natural than in the MSSM [13, 14].

3. Fine tuning in Little Higgs models

Little Higgs (LH) models try to solve the Little Hierarchy problem, that is, to explain the smallness of the Higgs mass compared with 10 TeV. There are many models in the market but they typically have the following structure: below $\Lambda \sim 10$ TeV (beyond which some UV completion takes over) there are new particles (new gauge bosons, fermions and scalars) that fit together with the SM particles into multiplets of some global symmetry G, spontaneously broken at the scale $f \sim \Lambda/(4\pi) \sim 1$ TeV. In this process the new particles gain masses of order f but the SM Higgs is special: besides being a (pseudo)-Goldstone boson, G is also explicitly broken in a "collective" way so that $m_{\rm h}$ is suppressed and under control. Diagrammatically, what happens is that loops of the new heavy particles cancel the quadratic divergences coming from SM loops (the equality of couplings necessary to render this cancellation natural is ensured by G). As a result, δm^2 is not of order $\Lambda^2/(16\pi^2)$ but rather $f^2/(16\pi^2)$ which is of electroweak size. Parametrically, this solves the Little Hierarchy problem.

However, a closer look reveals some difficulties. In contrast with the SUSY solution, the LH cancellation takes place only at one-loop. As an example, the top quadratic divergence is cancelled by some heavy fermion with the same quantum numbers and mass $M_{\rm T} \sim f$, giving

$$\delta m^2 \sim -\frac{\lambda_t^2 M_{\rm T}^2}{16\pi^2} \log \frac{\Lambda^2}{M_{\rm T}^2} \,, \tag{7}$$

where now $\Lambda \lesssim 10$ TeV [compare with (5)]. These models are also able to explain $m^2 < 0$ as the result of the large negative correction shown above (which is the dominant one). In fact this correction introduces the first problem: **(1)** Typically $M_{\rm T}^2 \geq \mathcal{O}(\lambda_t^2 f^2)$ and one gets $\delta_{\rm T} m^2/m^2 \geq \mathcal{O}(30)$. This number is quite large and points immediately to a fine tuning problem.

This large $\delta_{\rm T} m^2$ has to be compensated by some other correction, for instance from new heavy scalar degrees of freedom with a sufficiently large mass M_ϕ. In practice, **(2)** both λ and M_ϕ come from the same sector of the model and it is difficult to achieve a large value of M_ϕ while keeping λ small [8]. Some cancellation is required and this worsens the total fine tuning. Finally **(3)**, EWSB requires that some parameters, call them c generically, are numerically much smaller than its natural value (estimated by looking at the radiative corrections they receive). Schematically one has $c = c_0 + c_{\rm rad} \ll c_{\rm rad}$. This is a further fine tuning problem.

The problems just discussed seem to be generic and the most popular Little Higgs models suffer from fine tuning problems [8]. Fig. 3 compares the fine tuning performance of several Little Higgs models: the Littlest Higgs [15], a modified version of it [16] (curve labelled "Littlest 2"), a Littlest Higgs model with T-parity [17] and the so-called Simplest Little Higgs model [18]. Each curve gives the minimum value of Δ accessible by varying the parameters of the model. For comparison, the curve labelled "SM" represents the fine-tuning of the Little Hierarchy problem in the SM (*i.e.* with $\Lambda =$ 10 TeV) and the "MSSM" line shows the fine-tuning of the MSSM. This last curve has been obtained for large $\tan\beta$ (which minimises the fine-tuning), but disregarding stop-mixing effects, which can help in reducing the fine tuning. Usually, only in a marginal area of the parameter space of each LH model is the fine-tuning close to the lower bound shown, so the LH curves in Fig. 3 are very conservative estimates of the fine tuning.

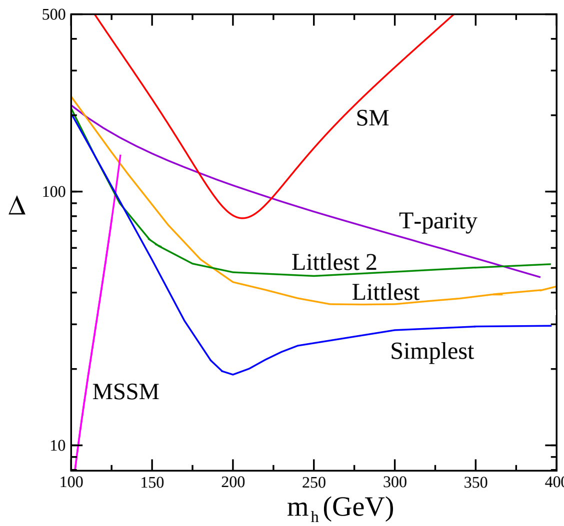

Fig. 3. Fine tuning performance of different Little Higgs models, compared with the SM with $\Lambda = 10$ TeV and the MSSM (with $m_{\tilde{t}} = 1$ TeV).

Generically, we see from Fig. 3 that the value of Δ for all these models is $\geq \mathcal{O}(100)$ in most of parameter space, and larger that 20–30 in all cases. Such fine tuning is larger than the MSSM one, at least for the especially interesting range $m_{\rm h} \lesssim 130$ GeV ($m_{\rm h} \gtrsim 135$ GeV is not available in the MSSM if the SUSY masses are not larger than ~ 1 TeV).

I should also emphasise here that in order to compare different LH models we chose $f = 1$ TeV (in the Simplest Model there are two breaking parameters and we chose $f_1 = f_2 = 1$ TeV). In some models such value is already too low and causes problems with precision EW data, which tend to favour larger values of f (that is, they prefer heavier extra particles) [19]. If one takes into account the constraints from precision EW data in the fine tuning analysis the results would be much worse: generically the fine tuning will grow with f as $\Delta \propto f^2$.

In conclusion, although LH models solve parametrically the Little Hierarchy problem these models generically have a substantial fine tuning built-in, usually much higher than suggested by the rough considerations commonly made and comparable to the little hierarchy tuning.

I would like to thank the organisers for their kind invitation and for achieving a very stimulating atmosphere. I also thank Alberto Casas and Irene Hidalgo for a very fruitful and enjoyable collaboration. This work is supported in part by the Spanish Ministry for Education and Science through a M.E.C. project (FPA2004-02015) and by a Comunidad de Madrid project (HEPHACOS; P-ESP-00346).

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