CURRENT ALGEBRA

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INTRODUCTION

I will report on developments in Current Algebra since the 1966 High-Energy Conference at Berkeley. During these two years I have received preprints of work on current algebra weighing altogether about 55 kilograms. In order to make some sense of all this activity in a one-hour talk, I will begin with a brief tour of the whole subject, which will incidentally give me a chance to mention some important developments that I cannot discuss in detail. After that, I will go into some selected topics as fully as time allows.

At the centre of our field is an approximate dynamic symmetry of the strong interactions, chiral SU(2) × SU(2), or chirality for short. By a dynamic symmetry I mean one like general covariance or local gauge invariance, which, although a symmetry of the Lagrangian, does not imply relations among S-matrix elements for fixed numbers of particles[1]. Instead, the approximate chiral symmetry of the Lagrangian is used to infer the existence of a set of vector and axial vector currents which are conserved or partially conserved and which obey the Gell-Mann commutation relations[2]. These properties of the currents are collectively called "current algebra" and if we like we may accept the current algebra and forget about Lagrangians. The direct physical consequences of current algebra, and hence of the underlying chiral symmetry, are contained in a set of soft-pion theorems, of the sort originally derived by Nambu[3], just as the older dynamic symmetries, general covariance and local gauge invariance, are directly manifested in low-energy theorems for gravitons and photons.

Unfortunately the pion is not massless, so chirality is only an approximate symmetry, and the derivation of soft pion theorems is not entirely straightforward. The calculation of the scattering lengths for pion collisions with arbitrary targets has recently been put on a firmer basis by Fubini and Furlan[4]; they estimate corrections to the old soft-pion formulae, and show that these are small for πN scattering but not for $\pi\Lambda$ or $\pi\Sigma$.

At this point I should mention that I am concentrating here on SU(2) × SU(2), but most of what I will say applies also to chiral SU(3) × SU(3), except of course that this symmetry is badly broken by the large K and η masses.

I should also explain that the weak and electromagnetic hadron currents are believed to be given by linear combinations of the vector and axial-vector currents arising from chiral symmetry, and for this reason current algebra has a lot to tell us about the weak and electromagnetic interactions of hadrons, but chirality would be a useful dynamical symmetry even if no photons or leptons were coupled to its currents. I will not be able to keep these different aspects of current algebra separate in my talk, but it should be kept in mind that a statement that "the strong interactions admit the construction of currents with such-and-such properties" is a different and a weaker statement than that "the weak and electromagnetic currents of the hadrons have such-and-such properties".

To return to our tour: most of the more useful pure soft-pion applications of current algebra (such as $\pi N \to \pi N$, $\pi N \to 2\pi N$, $\gamma N \to \pi N$, $Y \to N\pi$, $K \to \pi\ell\nu$, $K \to 2\pi\ell\nu$, $K \to 3\pi$, etc.) had already been worked out by the time of the Berkeley Conference, so I will merely provide a partial list of recent calculations:

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Electroproduction of pions: Fubini and Furlan[4].
Photoproduction of pion pairs: Carruthers and Huang[5]; Biswas, Smith and Campbell[6]; Narayanaswamy and Renner[7].
Radiative hyperon decay: Ahmed[8]; Gupta, Majumdar and Tripathy[9].
Pion production in nucleon-nucleon collisions: Beder[10]; Schillaci, Silbar and Young[11].
Nuclear forces: Brown, Green and Gerace[12].

In general I would say that the soft-pion theorems are well-understood and in good agreement with experiment, the single outstanding failure being for the process $\eta \to 3\pi$. [On this topic, I refer you to the excellent review by Bell and Sutherland[13].]

In order to go beyond the soft-pion results of chiral symmetry proper, it is necessary to add new information, either about more detailed properties of the currents, or about the energy and momentum dependence of their matrix elements, or both. At this point our itinerary becomes very complicated, so I have supplied a logical road map in Fig. 1. [For definiteness I have drawn this map for SU(2) × SU(2); the map for SU(3) × SU(3) is similar but even a little more complicated.]

The most conservative and well-verified assumption we can make here is that the amplitudes for forward pion (and/or photon scattering behave at high energy as required by Regge-pole theory. Together with the soft-pion theorems, this leads us to the familiar sum rules of Adler[14] and Weisberger[15], Furlan, Fubini and Rossetti[16], etc., which agree very well with experiment where they can be tested, i.e. when the target is a nucleon. In this case it appears experimentally that the sum rule is saturated to within 10% or so by a handful of π-N resonances[17], which suggests that we should try to saturate all forward pion-scattering amplitudes, elastic and inelastic, with a finite though perhaps large number of single-particle states. From this assumption flow the purely algebraic aspects of chiral symmetry, which I will discuss in Section 2.

Chirality suggests, but does not require, that the time-components of the vector and axial-vector currents should obey strictly local commutation relations. When combined with reasonable assumptions about the asymptotic behaviour of current correlation functions at fixed off-shell masses, this local algebra leads to the Fubini-Dashen-Gell-Mann sum rules[18] for the absorptive parts of these functions. It is not possible to express these sum rules in terms of purely hadronic observables, but in the special case of zero-momentum transfer they yield useful results, such as Adler's "β-sum rule" for neutrino cross-

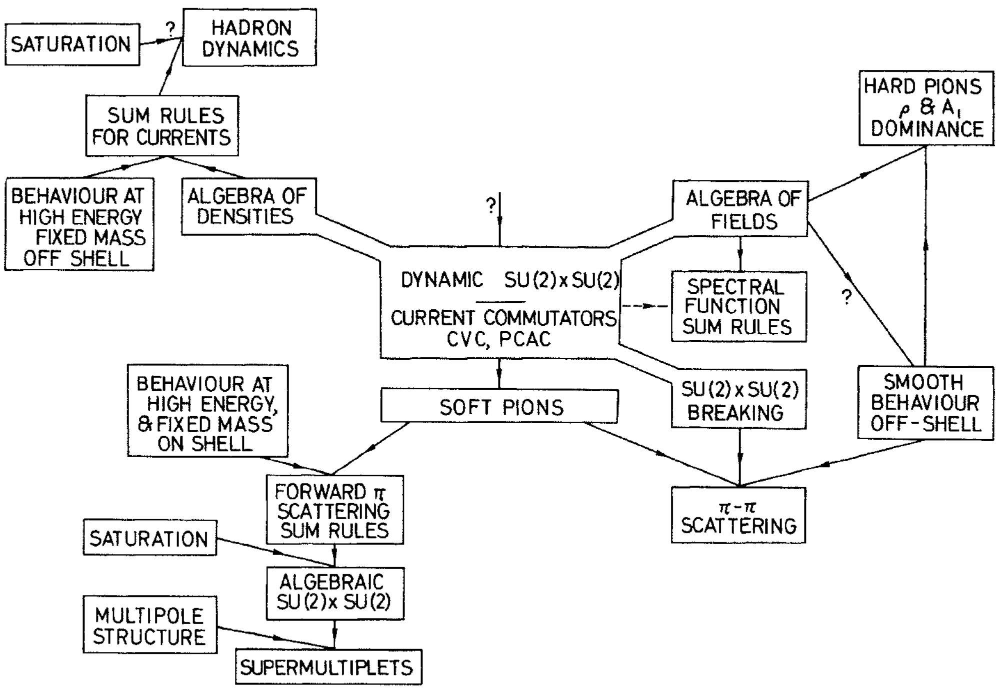

Fig. 1 A logical map of current algebra. (Not drawn to scale.)

sections[19] and the corresponding inequality of Bjorken[20] for electron cross-sections. These sum rules were discussed by Dashen (1966) and their present status has been reviewed in Kroll's report, so I will not go into details here. However, I should remark that the asymptotic assumptions on which they rest cannot be derived from ordinary Regge phenomenology but have been shown by Bronzan, Gerstein, Lee and Low[21], and by Singh[22] to require a fixed pole at $j = 1$. It may be that such asymptotic requirements can be replaced with assumptions about current commutation relations at light-like separations, as suggested by Okubo[23], Jersák and Stern[24], and Leutweyler[25]. [It may also be that they are wrong, as argued forcefully by Sakurai[26].] During the last two years, many authors[27] have been trying to learn something about hadron dynamics from saturated forms of the non-forward sum rules, with what I gather are discouraging results. I know very little about this program, and will not discuss it further here.

The assumptions I have discussed so far deal only with the behaviour of matrix elements of currents when the four-momenta carried by the currents are held at fixed mass. A different and apparently unrelated assumption is that the dependence on these four-momenta is as smooth as possible on and off the mass-shell, i.e., as smooth as allowed by current algebra constraints and by the known singularity structure of the matrix elements. "Smoothness" is useful because it allows us to squeeze physical information out of various hypotheses about the currents which go beyond chirality itself.

One such hypothesis deals with the mechanism which breaks chiral symmetry. Symmetry breaking is generally a mysterious phenomenon, but the success of the Gell-Mann-Okubo mass formula for broken SU(3) suggests that chiral SU(2) × SU(2) may be broken by a term whose chiral transformation character is as simple as possible. By invoking smoothness, we can then derive values for the π-π scattering lengths[28]. In Section 3 I will review the arguments about whether these values are actually correct.

Another hypothesis which supplements chirality is that it acts as a local gauge invariance, broken only by masses. The Lagrangian must then contain a number of vector and axial-vector Yang-Mills fields which obey a set of detailed commutation relations, known as the algebra of fields. I will describe this algebra in Section 4 and will use it there to derive the spectral function sum rules. This apparatus becomes particularly useful if we also assume that the weak and electromagnetic currents of the hadrons are linear combinations of gauge fields. I do not have any strong feeling that this is right; it may well be that the weak and electromagnetic currents are bilinear in quark fields, or what have you (in which case our map needs correction), but the charm of the algebra of fields is that it yields so many predictions that we may be able to prove it right or wrong experimentally.

By using the algebra of fields in conjunction with the assumption of smooth dependence on four-momenta, we can begin to calculate matrix elements for processes like ρ decay, in which the pions cannot be treated as soft. (See Section 5.) This really amounts to using Gell-Mann and Zachariasen's[29] version of ρ (and A_1) dominance together with soft-pion theorems, but with certain corrections which are required by off-shell Ward identities and crossing symmetry. This approach can also be extended to processes involving strange mesons, such as $K_{\ell 3}$ decay, under suitable assumptions about SU(3) × SU(3) breaking. (See Section 6.)

I will mention in passing that H. Sugawara[30] has invented an amusing theory (reviewed in the Discussion Session by Sommerfield), in which the energy-momentum tensor is constructed solely from gauge fields. Bardacki, Frishman and Halpern[31] have shown that this theory is obtained from a canonical theory of gauge fields if the bare mass and coupling constant are allowed to vanish together.

There is one other application of the algebra of fields, which we are just beginning to understand: it may be that the smoothness assumption itself can be derived from these detailed commutation relations. The idea here is that an amplitude which is free of strong singularities in a given variable will generally vary smoothly with that variable, provided that it obeys a dispersion relation with few enough subtractions, while Bjorken[32] has shown how to relate the number of subtractions in certain off-shell dispersion relations to the commutators of the fields and their derivatives. It is noteworthy that this is just the approach used by Fubini and Furlan[4] in their elucidation of the soft-pion theorems. Other steps in this direction have been taken by Perrin[33], Schnitzer and Wise[34], and Brown and West[35], but their results are not yet systematic enough for me to be able to describe them coherently here.

Before concluding this tour, I would like to return for a moment to our starting point, and ask where chiral symmetry comes from? It is well known that the more familiar dynamic symmetries, local gauge invariance and general covariance, are simply consequences of Lorentz invariance as applied to massless particles of spin one and spin two[36]. On the other hand, chirality can certainly not be derived in this way, because we can easily write down theories of massless pions, such as the old pseudoscalar coupling model, which violate chiral symmetry and yet are perfectly acceptable as relativistic quantum theories. Mandelstam[37] has recently suggested that a massless pion would have to obey chiral low-energy theorems, because, even though the pion has zero spin, it belongs to an M = 1 conspiracy. This implies the Adler condition[38] that soft-pion amplitudes vanish (except for poles) as the pion four-momentum vanishes, and from this condition Mandelstam is able to derive the universal formula for pion scattering at zero energy. His derivation heightens the analogy between chirality and the older dynamic symmetries, because it uses precisely the same method[36] by which one shows that Lorentz invariance requires the conservation of charge and the equality of gravitational to inertial mass. Mandelstam[39] has reported that if a massless pion has M = 1 then hard- as well as soft-pion amplitudes must vanish, unless the pion is on a daughter trajectory, which does not seem likely. However, Mandelstam's work leaves us with the conclusion that dynamic chiral symmetry may ultimately be explained if the Adler condition can be derived, either from conspiracy theory, or from something else.

Author's Note: I have tried to provide a not too incomplete guide to only that part of the literature from 1966 to 1968 that deals directly with the topics discussed in the text. Thus I have not included references to important work on generalized sum rules, η-decay, Compton scattering sum rules, electromagnetic mass differences, higher-order weak interactions, etc., some of which more properly falls in the province of other rapporteurs.

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1. CHIRAL DYNAMICS

To begin the more detailed part of this report, I will discuss the development of some new techniques for deriving results from chiral symmetry. The standard technique, described in Dashen's report at the 1966 Conference, is based on the use of current commutation relations and conservation or partial conservation equations to derive Ward-like identities for current correlation functions. These identities generally involve unknown terms, so we either pass to a suitable soft-pion limit in which these terms drop out, or, if results are needed for hard pions, we may use the "smoothness" hypothesis to parametrize our ignorance.

Since 1966 there have appeared a large number of papers in which current-algebra results are re-derived (or occasionally derived for the first time) from lowest-order Feynman perturbation theory. This may give an impression, either that perturbation theory has suddenly become generally applicable to the strong interactions, or that theorists have suddenly become soft headed. Both impressions are, I think, false. The "standard technique" of current algebra tells us that the matrix elements for soft-pion emission or scattering in a process $\alpha \to \beta$ is uniquely determined once we know the theory is chiral-invariant, and determined moreover to be of the form[1] symbolized in Fig. 2. (For simplicity I have chosen the process $N + N \to N + N + 23\pi$ to illustrate my remarks.) The important thing to note here is that this matrix element is of the lowest possible order in the "coupling constant" F_π^{-1} (where $F_\pi \simeq 190$ MeV is the pion-decay amplitude), except of course that we must use the true values for g_A/g_V and for the matrix element $M_{\beta\alpha}$ of the process sans soft pions. Hence, if we compute soft-pion matrix elements using any chiral-invariant Lagrangian in lowest order, and put in the true values of g_A/g_V and $M_{\beta\alpha}$, then we must necessarily get the right answer[2].

These observations gave a new relevance to the questions of how chiral symmetry is realized, and of how chiral-invariant Lagrangians are to be constructed. There are many possible realizations[3] of SU(2) × SU(2), but if we want the lowest-order matrix elements for soft-pion processes explicitly to

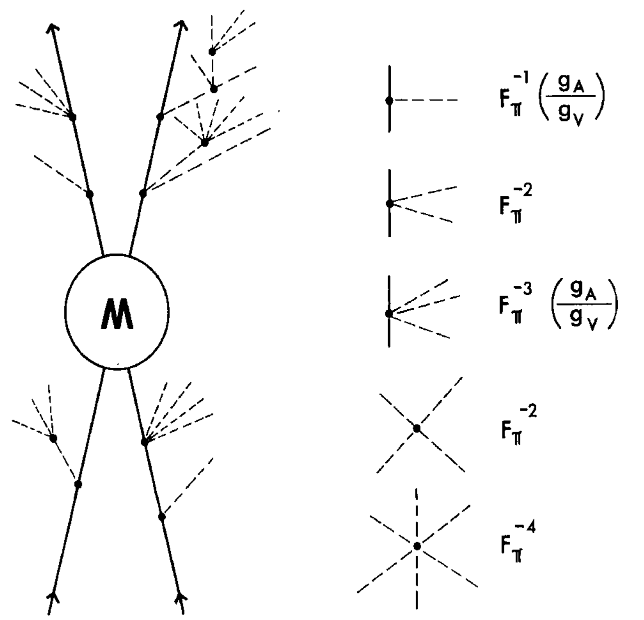

Fig. 2 Symbolic representation of current-algebra results for emission of 23 soft pions in nucleon-nucleon scattering. (Solid lines are nucleons, dashed lines pions.) The coupling constants associated with various vertices are shown on the right.

have the form shown in Fig. 2 (as is necessary to enable us to insert true values for g_A/g_V and $M_{\beta\alpha}$ in the right places) then we must realize chiral symmetry through non-linear transformations which carry the pion field into a function of itself. It has been shown by Brown[4], by Bardeen and Lee[5], and by myself[6] that these transformations are unique, up to possible re-definitions of the pion field $\vec{\pi}$. It proves convenient to define $\vec{\pi}$ so that its transformation rule for an infinitesimal boost $\vec{\varepsilon}$ is that given by Schwinger[7], Wess and Zumino[8], Lee and Nieh[9], Chang and Gürsey[10], etc.:

$$\delta\vec{\pi} = \frac{F_\pi}{2}\left(1 - F_\pi^{-2}\vec{\pi}^2\right)\vec{\varepsilon} + F_\pi^{-1}\left(\vec{\pi}\cdot\vec{\varepsilon}\right)\vec{\pi}\,. \qquad (1.1)$$

The rule for any other field ψ is then

$$\delta\psi = i\,F_\pi^{-1}\left(\vec{t}\times\vec{\pi}\right)\cdot\vec{\varepsilon}\psi \qquad (1.2)$$

where $\vec{t}$ is the isospin matrix for ψ.

[Recall that SU(2) × SU(2) is algebraically the same as SO(4), so infinitesimal chiral transformations can be described in terms of isospin "rotations" and chiral "boosts".] The rules for constructing chiral-invariant Lagrangians are, simply, that they must conserve isotopic spin, and be constructed out of miscellaneous hadron fields ψ, their covariant derivatives[6)]

$$D_\mu \psi \equiv \partial_\mu \psi + 2i\left(F_\pi^2 + \vec{\pi}^2\right)^{-1} \vec{t} \cdot \left(\vec{\pi} \times \partial_\mu \vec{\pi}\right) \qquad (1.3)$$

and the covariant derivative of π:

$$D_\mu \vec{\pi} \equiv \left(1 + F_\pi^{-2} \vec{\pi}^2\right)^{-1} \partial_\mu \vec{\pi} \; , \qquad (1.4)$$

but not the pion field itself. For instance, the soft-pion limit of a process like N + N → N + N + 23π can be calculated by using the phenomenological Lagrangian

$$\mathcal{L} = -\bar{N}\gamma^\mu D_\mu N - \tfrac{1}{2} D^\mu\vec{\pi}\cdot D_\mu\vec{\pi} + i\,F_\pi^{-1}\left(\frac{g_A}{g_V}\right)\bar{N}\gamma_5\gamma^\mu N\, D_\mu\vec{\pi} -$$

$$-\tfrac{1}{2}\, m_\pi^2 \left(1 + F_\pi^{-2}\vec{\pi}^2\right)^{-1} \vec{\pi}^2 \qquad (1.5)$$

provided that chirality is broken by the pion mass in the simplest possible way, as discussed in Section 3. [These results have been extended to SU(3) × × SU(3) by Lévy[11)], Gasiorowicz and Geffen[12)], Mitter and Swank[13)], Majumdar[14)], Bardeen and Lee[15)], Volkov[16)], and Macfarlane, Sudberg and Weisz[17)], and to general compact Lie algebras by Callan, Coleman, Wess and Zumino[18)], and by Volkov[19)]]. The non-linearities in Eqs. (3) and (4) generate the trees of soft pions shown in Fig. 2, and arise simply because the pion carries isotopic spin, just as the non-linearities of General Relativity arise because the graviton carries energy and momentum.

Most of the additional hypotheses which supplement chiral symmetry proper can be expressed in terms of these chiral-invariant Lagrangians, as follows:

i) The results of saturating Adler-Weisberger sum rules with single-particle states can be obtained[20)] by requiring that the sum of all lowest order graphs for forward pion scattering behave no worse at high energy than the actual scattering amplitude. [Note that individual tree graphs must behave badly at high energy in a chiral-invariant theory because of the derivatives and non-linearities in Eqs. (3) and (4).]

ii) The "smoothness" assumption referred to in the Introduction is incorporated if we do not include terms in the Lagrangian which involve more derivatives than necessary.

iii) Assumptions about the chiral behaviour of the term in the Lagrangian which breaks chiral symmetry are incorporated[7,8)] by explicit construction of this term as a suitable function of $\vec{\pi}$.

iv) The algebra of fields is incorporated by giving the Lagrangian suitable gauge-invariance properties[7,8)].

The great advantage of the use of chiral-invariant Lagrangians is that it makes current algebra calculations easy for anyone who knows how to evaluate Feynman diagrams. Another advantage is that it makes a direct connection between chiral symmetry and soft-pion theorems, thus avoiding the peculiar mystique that has grown up about the currents and their algebra. In addition, some authors have tried to use these Lagrangians for more ambitious purposes. Fradkin[21)] and Fried[22)] have begun a study of higher-order corrections. Schwinger[23)] regards the Lagrangians as phenomenological, to be used only in lowest order, but imposes on them additional "partial" symmetries. Nambu[24)] has motivated the use of lowest order diagrams as the first term in an expansion in $\hbar$, while Biswas, Braun, Pandit and Sudarshan[25)] regard it as the first term in an expansion in G_0^{-1}, where G_0 is an unrenormalized strong-coupling constant. It is too early to know how these speculations will turn out.

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2. SATURATED SUM RULES FOR FORWARD SCATTERING OF PIONS

By the time of the Berkeley Conference it was known that a great many useful relations among coupling constants and masses can be derived by saturating the Adler-Weisberger sum rules with single-particle states. Nevertheless, the mood in Ben Lee's discussion section at Berkeley was gloomy, because every step toward a general saturation scheme seemed to lead to the introduction of more and more inexplicable mixing angles. Since then the gloom has not precisely lifted, but the systematic study of all current-algebra sum rules for forward pion scattering has allowed us to see the problem a bit more clearly.

Let us consider forward elastic or inelastic scattering of a pion on an arbitrary target, and for simplicity let us take the pion mass as zero. The scattering amplitude can be split into pieces with isospin T = 0, 1, or 2 in the t-channel, and each piece yields a different kind of sum rule.

For T = 1 the sum rule is the familiar one derived by Adler and Weisberger[1] under the assumption that all T = 1, G = +1 trajectories have $\alpha(0) < 1$.

When saturated with single-particle states it gives the chiral commutation relations

$$[X_a(\lambda), X_b(\lambda)] = i\,\varepsilon_{abc}\,T_c. \quad (a = 1,2,3) \tag{2.1}$$

Here $X_a(\lambda)$ is the axial-vector coupling matrix for helicity λ, with a an isospin index, and T_a is the isospin matrix, with

$$[T_a, T_b] = i\,\varepsilon_{abc}\,T_c. \tag{2.2}$$

The isovector character of the axial current gives

$$[T_a, X_b(\lambda)] = i\,\varepsilon_{abc}\,X_c(\lambda). \tag{2.3}$$

Taken together, Eqs. (2.1), (2.2) and (2.3) tell us that $X_a(\lambda)$ and T_a form for each helicity λ a representation of the algebra SU(2) × SU(2). Single-particle states of definite mass are generally mixtures of states which behave irreducibly under this algebra, and the elements of $X_a(\lambda)$ between physical particle states can easily be calculated in terms of the mixing angles. It is important to note that X is a numerical matrix (not an operator in Hilbert space) whose elements are directly measurable physical quantities; in particular, the rate for a transition process $\alpha \to \beta + \pi_a$ (with α at rest and unpolarized, β having helicity λ) is:

$$\Gamma(\alpha \to \beta + \pi_a) = \frac{(m_\alpha^2 - m_\beta^2)^3}{4\pi\, m_\alpha^3 (2J_\alpha + 1) F_\pi^2} \left| [X_a(\lambda)]_{\beta\alpha} \right|^2 . \tag{2.4}$$

(This is for $m_\pi = 0$; corrections for the finite pion mass are easily supplied.)

By itself, the chiral algebra (2.1) - (2.3) would give no clue to the values of the many mixing angles needed to construct physical states. We can find such a clue in the sum rules of d'Alfaro, Fubini, Furlan and Rossetti[2)], derived from the dispersion relations with T = 2 in the t-channel under the assumption that the T = 2, G = +1 Regge trajectories have $\alpha(0) < 0$. The saturation of these sum rules has been studied in a great many special cases by Gilman and Harari[3)] and discussed here by Gilman. The general result is that the mass-matrix $m^2_{\beta\alpha} \equiv m^2_\alpha\, \delta_{\beta\alpha}$ is for each helicity λ the sum of a chiral scalar and the fourth component of a chiral four-vector. That is,

$$m^2 = m_0^2(\lambda) + m_4^2(\lambda) \tag{2.5}$$

where

$$[X_a(\lambda), m_0^2(\lambda)] = 0 \tag{2.6}$$

and

$$[X_a(\lambda), m_4^2(\lambda)] = i\, m_a^2(\lambda) \tag{2.7}$$

$$[X_a(\lambda), m_b^2(\lambda)] = -i\, \delta_{ab}\, m_4^2(\lambda) . \tag{2.8}$$

[Of course m^2 does not depend on λ, but its behaviour under commutation with $X_a(\lambda)$ does.] The term $m_4^2(\lambda)$ is not smaller than $m_0^2(\lambda)$, and governs the reducibility of the states of definite mass.

So much for the T = 1 and T = 2 sum rules; now what about T = 0? If all Regge trajectories with the quantum numbers of the vacuum had $\alpha(0) < 0$, then the T = 0 amplitude would also obey a sum rule like that for T = 2, and by saturating this sum rule with single-particle states we would find that $m_4^2(\lambda) = 0$, so states of definite mass could be put in irreducible representations of SU(2) × SU(2), and chirality would be a good symmetry of the ordinary algebraic kind, like isospin. Of course, this is not the case; there are vacuum trajectories with $\alpha(0) > 0$, and therefore chirality is not even approximately valid as an algebraic symmetry. However, there is some evidence[4)] that the leading vacuum trajectories have much smaller residues for inelastic than for elastic reactions. If we idealize this observation by supposing that the residues vanish in inelastic amplitudes then these amplitudes obey T = 0 sum rules, which when saturated with single-particle states yield the result that[5)]

$$[m_4^2(\lambda)]_{\beta\alpha} = 0 \quad \text{for} \quad \beta \neq \alpha . \tag{2.9}$$

Of the many examples discussed by Gilman and Harari I will mention only one, which deals with the $\lambda = 0$ states of non-strange mesons with $G\Pi(-)^J = +1$. They assumed in effect that $\vec{\rho}$ and $\vec{\pi}\cos\psi - \vec{A}_1 \sin\psi$ form an antisymmetric chiral tensor while σ and $\vec{\pi}\sin\psi + \vec{A}_1\cos\psi$ form a chiral four-vector, so that the matrix elements of $\vec{X}(0)$ can be calculated by ordinary SO(4) tensor analysis. They found that

$$\Gamma_\rho = \frac{m_\rho^3}{12\pi F_\pi^2}\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{\frac{3}{2}} \cos^2\psi \tag{2.10}$$

$$\Gamma_\sigma = \frac{3m_\sigma^3}{8\pi F_\pi^2}\left(1 - \frac{4m_\pi^2}{m_\sigma^2}\right)^{\frac{1}{2}} \sin^2\psi \tag{2.11}$$

$$m_{A_1}^2 = m_\rho^2 + m_\sigma^2 \tag{2.12}$$

$$m_\rho^2 / m_{A_1}^2 = \sin^2\psi \tag{2.13}$$

plus formulae for the $\lambda = 0$ A_1-decay amplitudes. Here ψ is the π-A_1 mixing angle, but what is its numerical value? If we impose the demand that $m_4^2(0)$ be diagonal we find that $\psi = 45°$, which gives

$$\Gamma_\rho = 135 \text{ MeV}, \quad \Gamma_\sigma = 610 \text{ MeV}, \tag{2.14}$$

$$m_\rho = m_\sigma = m_{A_1}/\sqrt{2} \ . \tag{2.15}$$

This is most encouraging, for reasons which will become clearer as we go along.

If all pion transitions involved p-wave pions, then $[X_a(\lambda)]_{\beta\alpha}$ would be proportional to a Clebsch-Gordan coefficient $C_{J_\beta 1}(J_\alpha\lambda;\ \lambda 0)$. It can then be shown[5] that X_a belongs to a larger algebra, that of SU(4). This is highly academic, because pion transitions are certainly not all p-wave, but it does suggest that the origin of possible supermultiplet symmetries like $SU(6)_W$ is to be sought in chiral sum rules plus limitations on the partial-wave structure of pion transition amplitudes.

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3. WHAT ARE THE π-π SCATTERING LENGTHS?

If chiral symmetry is broken in the simplest possible way (that is, by a term in the Lagrangian which behaves as the fourth-component of a chiral four-vector) and if the Feynman amplitude for low-energy π-π scattering is a smooth function of s, t, and u, then the π-π scattering lengths are[1] $a_0 \simeq 0.2\ m_\pi^{-1}$, $a_1 \simeq 0.35\ m_\pi^{-3}$, $a_2 \simeq -0.06\ m_\pi^{-1}$. This result has raised a good deal of controversy which continues to the present, so I will take a few minutes to review some of these arguments.

On the theoretical side, we can ask whether the extrapolation technique used to compute the scattering lengths is valid. This has been studied during the last two years by Khuri[2], Fujii[3], Yabuki[4], Sucher and Woo[5], Meiere[6], Iliopoulos[7], Akiba and Kang[8], Donnachie[9], Levers[10], Amatya, Pagnamenta and Renner[11], Franklin[12], and Arnowitt, Friedman, Nath and Suitor[13]. I will not be able to go into details here; suffice it to say that the consensus of these authors is that the extrapolation technique is at least self-consistent, i.e., that with π-π scattering lengths this small, the unitarity cut is not a strong enough singularity to invalidate the assumed smoothness of the scattering matrix element.

On the experimental side, it is clear that we are not going to have definitive values of the π-π scattering lengths until the analysis of $K_{\ell 4}$ decay outlined by Pais and Treiman[14] is carried out. However, it has at least been possible to use current algebra, in conjunction with the measured cross-section[15] for the process $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ near threshold, to provide a semi-empirical estimate of the $\pi^+\pi^-$ scattering length. If the π-π scattering lengths are as small as indicated by current algebra, we would in general expect the peripheral and non-peripheral contributions to the matrix element for pion production to be of about the same magnitude, which would prelude any conclusions about the π-π interaction. However, current algebra determines the non-peripheral part of this matrix element uniquely (and independently of our choice of the symmetry-breaking mechanism), so that the low-energy cross-section can be calculated as a function of the linear combination of scattering lengths, $2a_0 + a_2$, which determines $\pi^+\pi^-$ scattering. This calculation was first done by Chang[16], who found agreement with experiment for the expected value $2a_0 + a_2 \simeq 0.34\ m_\pi^{-1}$. Recently Olsson and Turner[17] have re-done the calculation with $2a_0 + a_2$ left arbitrary. [See Fig. 3; the lower curve corresponds to a model of chiral symmetry breaking due to Schwinger[18].] Their results indicate that $|2a_0 + a_2| = (0.4 \pm 0.1)m_\pi^{-1}$ in very satisfactory agreement with the expected value $0.34\ m_\pi^{-1}$.

At higher energies the pion production process is dominated for small momentum transfer by the peripheral contribution, and hence provides a direct estimate of the π-π phase shifts. The difficulty now is of course that current algebra does not by itself determine the phase shifts at these energies. This difficulty can to some extent be circumvented by using dynamical theory to calculate a unitary π-π S-matrix, taking various assumed scattering lengths as an input to the calculation. Such calculations have been carried out by Arnowitt, Friedman, Nath and Suitor[13], Tryon[19], Brown and Goble[20], Franklin[12], and Amatya, Pagnamenta and Renner[11]. To go into details would take us out of our subject; I will summarize these calculations by saying that the current algebra π-π scattering lengths lead to s-wave solutions in which the T = 0 phase shift is positive and rises to about 90° somewhere between 0.7 and 1.0 GeV; and the T = 2 phase shift remains small and negative, in agreement with some of the analyses of pion production data[21], and also with one of Lovelace's solutions[22] of the backward πN dispersion relations. Allow me to remind you that the saturation of current algebra sum rules also requires the presence of a broad T = 0 s-wave resonance near the ρ-meson mass.

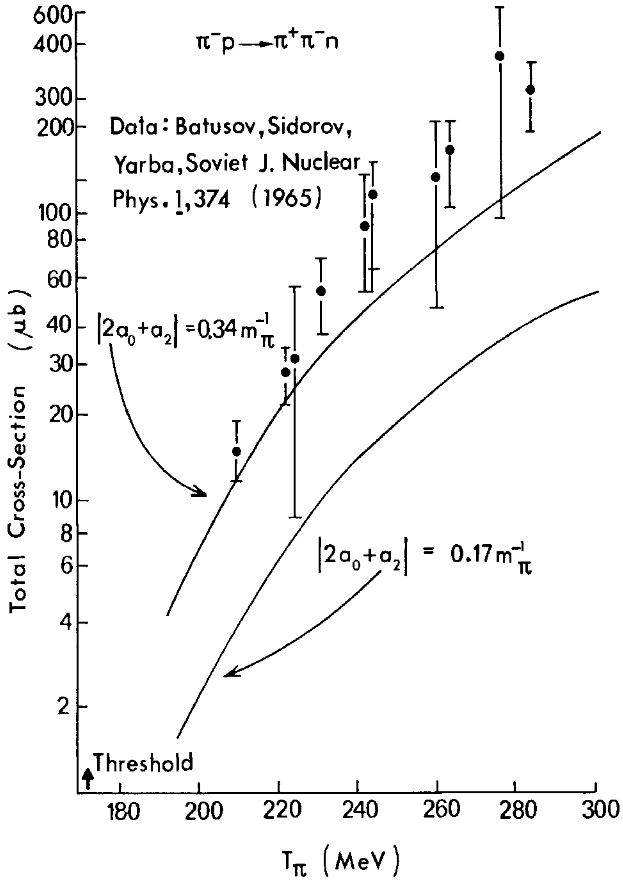

Fig. 3 Theoretical and experimental results for the process $\pi + N \rightarrow 2\pi + N$.

I should emphasize here that the values I have quoted for the scattering lengths would not be expected to be correct if there were a broad π-π resonance at low energy, say below 500 MeV. The effect of such a resonance has been carefully studied by Carbone, Donini, Furlan and Sciuto[23]. They find that an s-wave T = 0 σ-resonance would nearly double a_0 if $m_\sigma = 3m_\pi$ but would increase a_0 by only 20% if $m_\sigma = 5m_\pi$. (In the preprint of this paper a "σ term" is arbitrarily thrown away, but this error has since been corrected.)

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4. ALGEBRA OF FIELDS AND SPECTRAL-FUNCTION SUM RULES

The idea that strong interactions may be partially invariant under some sort of local gauge group goes back to the 1954 paper of Yang and Mills[1]). In such a theory the hadron Lagrangian would be of the form[2])

$$\mathcal{L} = -\tfrac{1}{4} F_{\alpha\mu\nu}F^{\alpha\mu\nu} - \tfrac{1}{2} m_0^2 \Phi_{\alpha\mu} \Phi_\alpha^\mu + \mathcal{L}_m(\psi, D_\mu \psi) \quad (4.1)$$

where

$$F_{\alpha\mu\nu} \equiv \partial_\mu \Phi_{\alpha\nu} - \partial_\nu \Phi_{\alpha\mu} + g_0\, C_{\alpha\beta\gamma}\, \Phi_{\beta\mu}\, \Phi_{\gamma\mu} \quad (4.2)$$

$$D_\mu \psi \equiv \partial_\mu \psi + g_0\, T_\alpha\, \Phi_{\alpha\mu}\, \psi \,. \quad (4.3)$$

Here $\Phi_{\alpha\mu}$ are the gauge fields, m_0 and g_0 are their bare mass and coupling constant, $C_{\alpha\beta\gamma}$ is the structure constant of some compact simple Lie algebra, ψ is a generic hadron field other than $\Phi_{\alpha\mu}$, and T_α is the representation on ψ of the Lie algebra. The field equations for Φ are then of the form

$$m_0^2 \Phi_\alpha^\nu - \partial_\mu F_\alpha^{\mu\nu} = g_0 J_\alpha^\mu \;, \quad (4.4)$$

where J_α^ν is the current as usually defined

$$J_\alpha^\nu = F_\beta^{\nu\mu}\, C_{\alpha\beta\gamma}\, \Phi_{\gamma\mu} + \sum_\psi \frac{\partial \mathcal{L}_m}{\partial D_\nu \psi}\, T_\alpha\, \psi \,. \quad (4.5)$$

Despite the prophetic attempts by Sakurai[3]) to identify the gauge fields $\Phi_{\alpha\mu}$ with pionic resonances of unit spin, it proved quite difficult to derive any predictions which could definitely test such ideas.

In the last two years this body of speculation has been put on a firmer basis by two related developments. First, Kroll, Lee and Zumino[4]) suggested that it is the gauge fields $\Phi_{\alpha\mu}$, rather than the "currents" $J_{\alpha\mu}$, which serve as the source of the electromagnetic field. [The difference is just the conserved term $\partial_\mu F_\alpha^{\mu\nu}$ in Eq. (4.4).] Second, Lee, Zumino and I[5] noticed that the gauge fields $\Phi_{\alpha\mu}$ obey an algebra, which agrees in its general outlines with the algebra of currents proposed by Gell-Mann[6], but is in detail far simpler. We found that

$$[\Phi'_{\alpha 0}(\vec{x},t), \Phi'_{\beta 0}(\vec{y},t)] = i\, C_{\alpha\beta\gamma}\, \Phi'_{\gamma 0}(\vec{x},t)\, \delta^3(\vec{x}-\vec{y}) \tag{4.6}$$

$$[\Phi'_{\alpha 0}(\vec{x},t), \Phi'_{\beta i}(\vec{y},t)] = i\, C_{\alpha\beta\gamma}\, \Phi'_{\gamma i}(\vec{x},t)\, \delta^3(\vec{x}-\vec{y}) + i(m_0^2/g_0^2)\, \delta_{\alpha\beta}\, \partial_i \delta^3(\vec{x}-\vec{y}) \tag{4.7}$$

$$[\Phi'_{\alpha i}(\vec{x},t), \Phi'_{\beta j}(\vec{y},t)] = 0 \tag{4.8}$$

$$[\partial_0 \Phi'_{\alpha i}(\vec{x},t) - \partial_i \Phi'_{\alpha 0}(\vec{x},t), \Phi'_{\beta j}(\vec{y},t)] =$$

$$= -i(m_0^4/g_0^2)\, \delta_{\alpha\beta}\, \delta_{ij}\, \delta^3(\vec{x}-\vec{y}) + i\, C_{\alpha\beta\gamma}\, \Phi'_{\gamma i}(\vec{x},t)\, \partial_j \delta_3(\vec{x}-\vec{y}) - i(g_0^2/m_0^2)\, C_{\alpha\delta\gamma}\, C_{\beta\delta\epsilon}\, \Phi'_{\gamma i}(\vec{x},t)\, \Phi'_{\epsilon j}(\vec{x},t)\, \delta^3(\vec{x}-\vec{y}) \tag{4.9}$$

where $\Phi'_{\alpha\mu}$ is a renormalized field, defined as

$$\Phi'_{\alpha\mu} \equiv (m_0^2/g_0)\, \Phi_{\alpha\mu}\ . \tag{4.10}$$

Eqs. (4.6) and (4.7) are the same as the commutation relations of Gell-Mann[6], except that the Schwinger term in Eq. (4.7) is a c-number which can be shown to be finite; Eq. (4.8) differs from what would be expected for currents constructed from quark fields; and Eq. (4.9) represents new information. Also, the field equation (4.4) gives

$$\partial_\mu \Phi'^{\mu}_{\alpha} = \partial_\mu J^{\mu}_{\alpha} \tag{4.11}$$

so Φ'^{μ}_{α} satisfies whatever conservation or partial conservation conditions are satisfied by J^{μ}_{α}, and therefore qualifies as a candidate for the weak as well as the electromagnetic currents. [If Schwinger terms appear in the commutators of the $J_{\alpha 0}$ then Eq. (4.6) must be suitably modified.]

The most immediate consequences of these commutation relations are obtained by taking their vacuum expectation values. From Eqs. (4.7) and (4.9) we obtain the first and second "spectral-function sum rules"

$$\int \rho^{(1)}_{\alpha\beta}(\mu^2)\, \mu^{-2}\, d\mu^2 + \int \rho^{(0)}_{\alpha\beta}(\mu^2)\, d\mu^2 = \frac{m_0^2}{g_0^2}\, \delta_{\alpha\beta} \tag{4.12}$$

$$\int [\rho^{(1)}_{\alpha\beta,V}(\mu^2) - \rho^{(1)}_{\alpha\beta,A}(\mu^2)]\, d\mu^2 = 0 \tag{4.13}$$

where $\rho^{(j)}_{\alpha\beta}$ are the spin-j parts of the spectral function, i.e.

$$\langle \Phi'_{\alpha\mu}(x)\, \Phi'_{\beta\nu}(0)\rangle_0 = (2\pi)^{-3} \int d^4p\, \theta(p^0)\, e^{ip\cdot x} \left\{ \rho^{(1)}_{\alpha\beta}(-p^2)\, [g_{\mu\nu} - p_\mu p_\nu/p^2] + \rho^{(0)}_{\alpha\beta}(-p^2)\, p_\mu p_\nu \right\} . \tag{4.14}$$

I should add by way of explanation that these sum rules can be applied to chiral groups, which are not simple, because parity conservation relates the currents $V^\mu - A^\mu$ and $V^\mu + A^\mu$. Also the second sum rule (4.13) can only be used to relate corresponding vector and axial-vector currents, because it is only in this case that the quadratic term in the commutator (4.9) drops out.

This is not the only way of deriving the spectral-function sum rules. In the earliest derivation[7], limited to SU(2) × SU(2), the first rule was obtained by assuming that Schwinger terms are c-numbers, and the second rule was obtained from an asymptotic assumption suggested by the field current identity. Related derivations and extensions to other groups were subsequently given by Glashow, Schnitzer and myself[8], Nieh[9], Acharya[10], Lu[11], Perrin[12], Kowalski[13], and Gross and Jackiw[14]. A quite different approach was also suggested by Das, Mathur and Okubo[15], who obtained the sum rules as superconvergence relations, by requiring that leading terms in the asymptotic expansion of the current propagator become SU(2) × × SU(2) or SU(3) × SU(3)-invariant at high momenta. The nice thing about their approach is that it leaves scope for playing around with various hypotheses about the transformation character of different terms in the propagator at high momenta, as done later by Das, Mathur and Okubo[16], Kimel[17], and Akiba and Kang[18]. There is no time to go through all the possibilities here, so I will stay on the safer ground of working only with the sum rules (4.12) and (4.13).

Most of the applications of spectral function sum rules have been obtained by saturating the integrals with narrow single-particle states. [Finite-width corrections have been studied by Geffen and Walsh[19], Gounaris and Sakurai[20], Vaughn, Blackman and Wali[21], Lam and Raman[22], and Acharya[10].] For SU(2) × SU(2), the sum rules give

$$\frac{g_\rho^2}{m_\rho^2} = \frac{g_{A_1}^2}{m_{A_1}^2} + F_\pi^2 \qquad (4.15)$$

$$g_\rho^2 = g_{A_1}^2. \qquad (4.16)$$

The g's are the coupling constants of the spin-one mesons to their respective currents. In particular, g_ρ determines the rate for $\rho^0 \to e^+ + e^-$; from this source, or alternatively from the still mysterious KSFR relation[23], we also have

$$g_\rho^2 \simeq 2m_\rho^2 F_\pi^2 \qquad (4.17)$$

so Eqs. (4.12) and (4.13) give

$$m_{A_1}/m_\rho \simeq \sqrt{2} \qquad (4.18)$$

in agreement with experiment (the A_1 appears to be getting realer) and also in agreement with the saturated pion-scattering sum rule discussed in Section 2.

The first spectral-function sum rule was applied to the neutral currents of SU(3) by Das, Mathur and Okubo[16], Oakes and Sakurai[24], Divakaran and Pandit[25] and others[26]. They found

$$\frac{g_\rho^2}{m_\rho^2} = \frac{3}{4}\left(\frac{g_\phi^2}{m_\phi^2} + \frac{g_\omega^2}{m_\omega^2}\right) \qquad (4.19)$$

where g_ϕ and g_ω are the couplings of ϕ and ω to the hypercharge current. It follows directly from Eq. (4.19) that

$$\frac{1}{3}\, m_\rho \Gamma(\rho \to e^+e^-) = m_\omega \Gamma(\omega \to e^+e^-) + m_\phi \Gamma(\phi \to e^+e^-). \qquad (4.20)$$

As discussed by Ting in his report, it now appears that this relation is verified by experiment. [Incidentally De Alwis[27],Das, Mathur and Okubo[16], and Sakurai[24] have pointed out that Eq. (4.19) and the observed facts that $g_\phi \neq 0$ and $m_\phi >> m_\omega > m_\rho$ show that the second spectral-function sum rule does not work when used to compare the vector currents with each other, as has been suggested by Glashow, Schnitzer and myself[8]. In this context it also cannot be derived in a straightforward way from the algebra of fields. It is also possible that the first spectral function sum rule is inexact here because the bare vector meson masses break SU(3), as suggested by Okubo[28] and Sugawara[29]. This would invalidate Eqs. (4.19) and (4.20), but not of course Eqs. (4.15) to (4.18).]

Finally, we may attempt the saturation of the SU(3) × SU(3) spectral-function sum rules by vector mesons ρ, ω, ϕ, K^*; axial mesons A_1, D, E, K_A; pseudoscalar mesons π, η, X, K; and a scalar κ. This gives[30]

$$\frac{g_{KA}^2}{m_{KA}^2} + F_\kappa^2 \simeq \frac{3}{4}\left(\frac{g_D^2}{m_D^2} + \frac{g_E^2}{m_E^2} + F_\eta^2 + F_X^2\right) \simeq \frac{g_{A_1}^2}{m_{A_1}^2} + F_\pi^2 \simeq$$
$$\simeq \frac{g_{K^*}^2}{m_{K^*}^2} + F_\kappa^2 \simeq \frac{3}{4}\left(\frac{g_\phi^2}{m_\phi^2} + \frac{g_\omega^2}{m_\omega^2}\right) \simeq \frac{g_\rho^2}{m_\rho^2} \simeq F_\pi^2 \qquad (4.21)$$

$$g_{KA}^2 \simeq g_{K^*}^2; \quad g_D^2 + g_E^2 \simeq g_\phi^2 + g_\omega^2; \quad g_\rho^2 \simeq g_{A_1}^2 \,. \qquad (4.22)$$

(The F's and g's are the couplings of spin-zero and spin-one mesons to their respective currents.) Dar and Weisskopf[31] have pointed out that these relations explain certain otherwise mysterious regularities found in fits of the quark model to experiment. In the discussion below they will be used to supply values of the coupling constants needed in "hard π or K" calculations.

The algebra of fields and spectral-function sum rules also have quite a different kind of application;

they determine the asymptotic behaviour at high energy of electromagnetic and weak interaction processes. For instance:

i) The spectral functions $\rho^{(1)}_{\alpha\beta}(\mu^2)$ are directly related to the cross-section for hadron production in e^+e^- and $e^-\bar{\nu}$ scattering at $s = \mu^2$. Dooher[32] has noted that because the integrals in the first sum rule converge, these cross-sections must vanish as $s \to \infty$ faster than $1/s^2$. This is in sharp contrast with the result of Bjorken[33], and Gribov, Ioffe and Pomeranchuk[34], that in the quark model of currents the hadron-production cross-sections go like $1/s$.

ii) Bjorken[35] has derived a lower bound for backward scattering of electrons from nucleons at large fixed momentum-transfer, which depends on the commutators of the spatial components of the electromagnetic current. In the quark model this lower bound is just what would be expected for Compton scattering from a point Dirac particle, but in the algebra of fields it is zero.

iii) The spectral-function sum rules ensure the convergence of higher-order electromagnetic or weak corrections in a few soft-pion or kaon calculations, such as that of the pion electromagnetic mass difference by Das, Guralnik, Mathur, Low and Young[36]. However, in most cases the radiative corrections remain divergent[37]. I gladly leave this topic to be discussed by Kroll and Treiman.

I am not sure whether or not we should take these high-energy predictions very seriously. The original motivation of the algebra of fields and the spectral-function sum rules was to give concrete expression to the ρ- and A_1-dominance of matrix elements at moderate energies. Also, the fundamental role played here by elementary spin-one fields should make us uneasy, especially because the Adler-Weisberger sum rules depend crucially on the assumption that the ρ meson is on a Regge trajectory. (However, Gell-Mann informs me that Sugawara has constructed currents, obeying the algebra of fields, as non-linear functions of spin-zero fields.) And of course the divergence of higher-order weak and electromagnetic corrections is not very reassuring. However, none of these qualms apply to applications of the algebra of fields at moderate energies, such as the saturated spectral-function sum rules discussed above, or the "hard-pion" results, to which we now turn.

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5. HARD-PION CALCULATIONS

The information we have assembled concerning the currents does not by itself enable us to calculate matrix elements for processes like ρ decay in which none of the participating particles is particularly soft. To fill this need, a "hard-pion" method has been developed by a number of authors including Gerstein, Schnitzer and myself[1], Arnowitt and his co-workers at Northeastern[2], Brown and West[3], and Das, Mathur and Okubo[4]. The several versions differ in detail, but their results are the same, which, if it does not show that they are right, does at least show that they are based on coherent and reasonable assumptions.

As an illustration of this method let us consider the three-point functions

$$\langle T\{A_a^\mu(x), A_b^\nu(y), V_c^\lambda(z)\}\rangle_0$$
$$\langle T\{\partial_\mu A_a^\mu(x), A_b^\nu(y), V_c^\lambda(z)\}\rangle_0$$
$$\text{and}\ \langle T\{\partial_\mu A_a^\mu(x), \partial_\nu A_b^\nu(y), V_c^\lambda(z)\}\rangle_0$$

where V and A are the vector and axial-vector currents of SU(2) × SU(2). By taking divergences on the various free indices μ,ν,λ, and using the chiral commutation relations (with c-number Schwinger terms, as in the algebra of fields), we may easily derive five independent Ward identities which relate these functions to each other and to the V and A propagators. However, these Ward identities determine the three-point functions directly only at unphysical points in momentum space, where four-momenta vanish. In order to derive useful information we have to supplement current algebra with assumptions about the smoothness of the three-point functions as functions of the four-momenta. But these three-point functions have poles in momentum space, corresponding to the physical $A_1A_1\rho$, $\pi A_1\rho$, and $\pi\pi\rho$ couplings, so before we invoke a smoothness assumption we must first separate out these various poles. To do this, we first define a "proper" $\pi\pi\rho$ vertex Γ_λ by

$$\int d^4x\, d^4y\, d^4z\, e^{-iqx} e^{ipy} e^{-ikz} \langle T\{\partial_\mu A_a^\mu(x), \partial_\nu A_b^\nu(y), V_c^\lambda(z)\}\rangle_0 = i\,\varepsilon_{abc}(2\pi)^4\, \delta^4(p-q-k)\, m_\pi^4\, F_\pi^2\, g_\rho\, \Delta_\pi(q)\, \Delta_\pi(p)\, \Delta_\rho^{\lambda\lambda'}(k)\, \Gamma_{\lambda'}(q,p,k)$$

then define the proper $\pi A_1\rho$ vertex $\Gamma_{\nu\lambda}$ by

$$\int d^4x\, d^4y\, d^4z\, e^{-iqx} e^{ipy} e^{-ikz} \langle T\{\partial_\mu A_a^\mu(x), A_b^\nu(y), V_c^\lambda(z)\}\rangle_0 = \varepsilon_{abc}(2\pi)^4\, \delta^4(p-q-k)\big[m_\pi^2\, F_\pi^2\, g_\rho\, \Delta_\pi(q)\, p^\nu\, \Delta_\pi(p)\, \Delta_\rho^{\lambda\lambda'}(k)\, \Gamma_{\lambda'}(q,p,k) + $$
$$+ m_\pi^2\, F_\pi\, g_\rho\, g_{A_1}\, \Delta_\pi(q)\, \Delta_{A_1}^{\nu\nu'}(p)\, \Delta_\rho^{\lambda\lambda'}(k)\, \Gamma_{\nu'\lambda'}(q,p,k)\big]$$

and finally define the proper $A_1A_1\rho$ vertex $\Gamma_{\mu\nu\lambda}$ by a similar formula which separates it from the $\pi A_1\rho$ and $\pi\pi\rho$ contributions to $\langle T\{A_a^\mu, A_b^\nu, V_c^\lambda\}\rangle_0$. (Here Δ_π, Δ_ρ, and Δ_{A_1} are just the usual free-particle propagators.) The five Ward identities for the three-point functions may be re-written in terms of the proper vertices; when this is done we find that <u>these five relations completely determine the $\pi\pi\rho$ and $\pi A_1\rho$ vertices in terms of the $A_1A_1\rho$ vertex</u>, and provide a single constraint on the $A_1A_1\rho$ vertex itself to the effect that

$$\Gamma_{\mu\nu\lambda}(q,p,k) = \frac{2m_\rho^2}{g_\rho}\,\{g_{\mu\nu}(p+q)_\lambda - g_{\mu\lambda}p_\nu - g_{\nu\lambda}q_\mu + \gamma_{\mu\nu\lambda}(q,p,k)\}\ ,$$

where $\gamma_{\mu\nu\lambda}$ is an unknown quantity obeying the constraint

$$k^\lambda\, \gamma_{\mu\nu\lambda} = 0\ .$$

So far there have been no approximations except for single-particle saturation of the V and A propagators. Now we are ready to invoke smoothness. Since all poles have been removed from the Γ's, we are free to suppose that $\Gamma_{\mu\nu\lambda}$ is as smooth as possible, which in this case means that it is at most <u>linear</u> in four-momenta. It follows then that

$$\gamma_{\mu\nu\lambda} \simeq (2+\delta)\,(g_{\mu\lambda}\,k_\nu - g_{\nu\lambda}\,k_\mu)$$

where δ is an unknown parameter, related to the anomalous magnetic moment of the A_1. All proper vertices are now completely determined [and agree precisely with the couplings appearing in the phenomenological Lagrangian of Wess and Zumino[5], which is why we believe that such Lagrangians can be used in the tree approximation]. If we adopt the formulae for g_ρ, g_{A_1}, and m_{A_1} given in the last section, we find the decay rates:

$$\Gamma_\rho = \frac{m_\rho^3}{24\,\pi\,F_\pi^2}\left(\frac{3-\delta}{4}\right)^2\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{\frac{3}{2}} \simeq \left(\frac{3-\delta}{4}\right)^2 \times 135\ \text{MeV} \qquad (5.1)$$

$$\Gamma_{A_1}(\lambda=0) = \frac{m_\rho^3}{96\sqrt{2}\,\pi\,F_\pi^2}\left(\frac{3+\delta}{2}\right)^2 \simeq \left(\frac{3+\delta}{2}\right)^2 \times 30\ \text{MeV} \qquad (5.2)$$

$$\Gamma_{A_1}(\lambda=+1) = \frac{m_\rho^3}{192\sqrt{2}\,\pi\,F_\pi^2}(2+\delta)^2 \simeq (2+\delta)^2 \times 15\ \text{MeV}\,. \qquad (5.3)$$

The value $\delta = -1$ is particularly attractive, because then formula (5.1) for the ρ width is precisely the same as that derived from Adler-Weisberger sum rules (with π-A_1 mixing angle $\psi = 45°$) and also because the pion electromagnetic form-factor then has the simple unsubtracted form $(1 - t/m_\rho^2)^{-1}$. However, if the ρ width is as small (and the e^+e^- branching ratio as large) as was indicated by the Novosibirsk colliding-beam experiment[6] then δ will have to be reduced in absolute value to about $-\frac{1}{2}$, as advocated particularly by Geffen and Walsh[7]. (Ting's report indicates that the ρ width is increasing again.) The Lagrangian model of Schwinger[8] has $\delta = 0$. All of these δ values give total widths for $A_1 \to \rho\pi$ which are consistent with the little that is known about A_1 decay.

The hard-pion results for the A-A-V vertex function has also been applied by Brown and West[9], and Riazuddin and Fayyazuddin[10] to the process $\pi \to e + \nu + \gamma$ and by Arnowitt et al.[2] to the process $A_1 \to \pi + \gamma$ (and A_1 photoproduction); these matrix elements depend on just the one unknown parameter δ. A similar treatment of the A-V-V vertex by Brown and West[9] leads to a single relation among the rates for $\pi^0 \to 2\gamma$, $\omega \to \pi^0\gamma$, and $\rho^0 \to \pi^0\gamma$, from which one obtains a width of 0.03 MeV for $\rho^0 \to \pi^0\gamma$. This analysis also suggests large corrections to the old vector-meson-dominance calculations of Gell-Mann, Sharp and Wagner[11], particularly for the ρ-pole contribution to $\omega \to 3\pi$. [A related calculation, which assumes weak SU(3)-breaking, has been recently carried out by Brown, Munczek and Singer[12].] This approach can also be applied to four-point functions, with the disadvantage that there are now poles for virtual particles of unlimitedly high spin. With reasonable saturation assumptions, Gerstein and Schnitzer[1] find that the direct decay $A_1 \to 3\pi$ is somewhat slower than the mode $A_1 \to \rho\pi$ considered above, and they also evaluate corrections to the soft-pion formulae for ρ-π scattering.

These hard-pion results are generally in satisfactory agreement with experiment, but a critical test has yet to be made, and probably must wait until the properties of the A_1 can be definitely established. However, there is one theoretical problem that will have to be settled before we can really begin to use these methods with confidence. How can we be sure of avoiding conflicts between the on-shell approach, particularly the saturation of forward pion-scattering sum rules, and the off-shell methods, which use "smoothness" and the algebra of fields. In some cases, the two approaches are in satisfactory agreement, as for the A_1 mass and the ρ width. But in other cases they definitely disagree, as for the A_1 decay amplitudes. [Gilman and Harari[13] find from the forward-scattering sum rules that the rate for $A_1 \to \rho\pi$ should essentially vanish for ρ-helicity $\lambda = \pm 1$, and should be about 110 MeV for $\lambda = 0$, both results in contradiction to the hard-pion results (5.2) and (5.3).] The on-shell methods are better a priori, in that they rely on verifiable assumptions about behaviour at high energy, while the off-shell methods are better a priori in that the higher resonances and multi-particle states that we neglect are all of known spin. [So far, experiment favours the on-shell results[14].] It is still possible that the left- and right-hand sides of Fig. 1 can be brought into agreement by including a few more resonances in one or another sum rule, but an attempt in this direction by Chandra, Mohapatra and Okubo[15] gave discouraging results.

It is more likely that we should revise the hard-pion approach by basing our smoothness assumptions, not on pure intuition, but on physical criteria, such as the requirement of consistency with dispersive sum rules, or by using more of the detailed information provided by the algebra of fields, as discussed in the Introduction to this report[16].

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6. SU(3) × SU(3) BREAKING: κ MESONS AND $K_{\ell 3}$ DECAY

The "hard-pion" technique can be extended to strange-particle processes, but here it is essential to know the chiral transformation properties of the term in the Lagrangian which breaks chiral symmetry. Gell-Mann[1] has suggested that this term belongs to the simplest possible symmetry breaking representation of SU(3) × SU(3), which is $(3,\bar{3}) + (\bar{3},3)$. The Lagrangian would then be of the form

$$\mathcal{L} = \mathcal{L}_0 + \varepsilon_i \phi_i \tag{6.1}$$

where $\mathcal{L}_0$ is the chiral-invariant, the ϕ_i are 18 scalar and pseudoscalar objects which make up the representation $(3,\bar{3}) + (\bar{3},3)$, and the ε_i are symmetry-breaking parameters (of which only two are actually allowed not to vanish by isospin, hypercharge, and parity conservation). The divergences of the 16 currents of SU(3) × SU(3) are then given by

$$\partial^{\mu} J_{\mu}^{\alpha} = \varepsilon_i T_{ij}^{\alpha} \phi_j \ , \tag{6.2}$$

where T^{α} form the $(3,\bar{3}) + (\bar{3},3)$ representation of the algebra of SU(3) × SU(3). By taking the matrix elements of this equation between the vacuum and single-meson states we immediately obtain the couplings F_{π}, F_K, etc., of the mesons to their currents, in terms of masses, the ε_i, and the usual renormalization constants Z of the ϕ_i. But the quantities $\varepsilon_i(T^{\alpha})_{ij}$ are not linearly independent, so we can eliminate the unknown ε_i and find relations such as[2]

$$m_{\pi}^2 F_{\pi} Z_{\pi}^{-\frac{1}{2}} = m_K^2 F_K Z_K^{-\frac{1}{2}} + m_{\kappa}^2 F_{\kappa} Z_{\kappa}^{-\frac{1}{2}} \ . \tag{6.3}$$

[The κ meson is an hypothetical 0^+ strange meson coupled to the strangeness-changing vector current, as suggested by Nambu[3].] We also have at our disposal a sort of broken Goldstone theorem[2], which is derived[4] by using Eq. (6.2) to evaluate the divergence of the two-point function $\langle T\{J_{\mu}^{\alpha}, \phi_i\}\rangle_0$, and which expresses the inverse-square meson masses (times Z factors) as ratios of appropriate components of $T^{\alpha}\langle\phi\rangle_0$ and $T^{\alpha}\varepsilon$. The ε can be expressed in terms of the quantities $m^2 F$, and the $T^{\alpha}\langle\phi\rangle_0$ are linearly dependent, so we obtain additional linear relations, such as[2]

$$F_{\pi} Z_{\pi}^{\frac{1}{2}} = F_K Z_K^{\frac{1}{2}} + F_{\kappa} Z_{\kappa}^{\frac{1}{2}} \ . \tag{6.4}$$

[There are further relations similar to (6.3) and (6.4) which involve the η and η'.] By eliminating Z_{π} from relations (6.3) and (6.4) we obtain a formula for the κ mass

$$m_{\kappa}^2 = \frac{F_K F_{\kappa} m_{\kappa}^2 - y\left[F_K^2 m_K^2 - F_{\pi}^2 m_{\pi}^2\right]}{y\ F_{\kappa}\left[F_{\kappa} - F_K y\right]} \ , \tag{6.5}$$

where

$$y = \sqrt{Z_K/Z_{\kappa}} \ . \tag{6.6}$$

It follows then that m_{κ} is subject to an upper bound

$$m_{\kappa}^2 \le \left(F_K m_K - F_{\pi} m_{\pi}\right)^2 / F_{\kappa}^2 \tag{6.7}$$

provided F_K and F_{π} are of the same sign, which must be the case unless SU(3) is very badly broken. [If F_K and F_{π} are of opposite sign then the right-hand side of formula (6.7) becomes a lower bound for m_K^2.]

In order to make contact with experimental reality we must turn to $K_{\ell 3}$ decay. The hard-pion technique described in the last section has been used to calculate the $K_{\ell 3}$ form-factors, by Gerstein and Schnitzer[5], Arnowitt, Friedman and Nath[6], B.W. Lee[7], Chang and Leung[8], Glashow and myself[2], Ueda[9], and Riazzuddin and Sarker[10]. All but the last two calculations are in essential agreement, but the papers by Gerstein and Schnitzer, and Arnowitt, Friedman and Nath are somewhat more comprehensive than the others, so I will use their results as the basis for my discussion.

The $K_{\ell 3}$ form-factors are defined in terms of the K-π matrix element of the weak current:

$$\langle\pi|J_{\lambda}|K\rangle \equiv \left[(P_K + P_{\pi})_{\lambda}\ f_+(t) + (P_K - P_{\pi})_{\lambda}\ f_-(t)\right] \sin\theta, \tag{6.8}$$

where θ is Cabibbo's angle. The assumed smoothness of the vertex functions sans poles implies that $f_{\pm}(t)$ are dominated by the 1^- and 0^+ K^* and κ poles, with one subtraction in $f_+(t)$ and none in $f_-(t)$:

$$f_+(t) = f_+(0) + \frac{g_{K^*K\pi}\ g_{K^*}\ t}{m_{K^*}^2\left(m_{K^*}^2 - t\right)} \tag{6.9}$$

$$f_-(t) = \frac{-\ g_{K^*K\pi}\ g_{K^*}\left(m_K^2 - m_{\pi}^2\right)}{m_{K^*}^2\left(m_{K^*}^2 - t\right)} + \frac{g_{\kappa K\pi} F_{\kappa}}{m_{\kappa}^2 - t} \ . \tag{6.10}$$

The extra information provided here by broken chiral symmetry consists of formulae for $f_+(0)$ and $g_{K\kappa\pi}$:

$$f_+(0) = \frac{F_K^2 + F_{\pi}^2 - F_{\kappa}^2}{2F_{\pi}F_K} \tag{6.11}$$

$$g_{\kappa K\pi} = \frac{1}{2F_{\pi}F_{\kappa}F_K}\left[\left(m_K^2 - m_{\pi}^2\right)\left(F_{\pi}^2 + F_K^2 - F_{\kappa}^2\right) + \right.$$
$$\left. + \ m_{\kappa}^2\left(F_K^2 - F_{\pi}^2 + F_{\kappa}^2 - 2\ F_K F_{\kappa}\ y\right)\right] \ , \tag{6.12}$$

where $y \equiv \sqrt{Z_K/Z_{\kappa}}$ is the same quantity that appears in Eq. (6.7) for the κ mass.

In order to complete the calculation of the $K_{\ell 3}$ form-factors, it is necessary to use some information from other experiments:

a) The rate for $K^* \to K + \pi$ determines $g_{K^*K\pi}$. (It is not possible to calculate this coupling constant *a priori*, because it turns out to depend upon an unknown parameter δ' analogous to the δ entering in ρ decay. The experimental K* width indicates that δ' is about -1.)

b) The Cabibbo theory together with the ratio $\Gamma(K_{\mu 2})\Gamma(\pi_{e3})/\Gamma(\pi_{\mu 2})\Gamma(K_{e3})$ of observed leptonic decay rates tells us that

$$\frac{F_K}{F_\pi\, f_+(0)} = \frac{2F_K^2}{F_\pi^2 + F_K^2 - F_\kappa^2} = 1.28 \ . \qquad (6.13)$$

c) The first spectral-function (4.21) sum rule yields a relation between g_{K*}, F_κ^2, and F_π^2:

$$\frac{g_{K*}^2}{m_{K*}^2} + F_\kappa^2 = \frac{g_\rho^2}{m_\rho^2} \simeq 2\,F_\pi^2 \ . \qquad (6.14)$$

So far the cited papers agree [except that Arnowitt et al.[6] do not use the relation between $g_{\kappa K\pi}$ and m_κ^2], but we still need a little more information, which we can take from a variety of <u>incompatible</u> assumptions:

(1) (References 2, 5, 6). By using formula (4.21) together with the second spectral-function sum rule (4.22) relating the vector and axial-vector strangeness-changing currents, we obtain a relation among the F's:

$$\frac{2F_\pi^2 - F_\kappa^2}{2F_\pi^2 - F_K^2} = \frac{m_{KA}^2}{m_{K*}^2} = 1.98 \ .$$

With Eq. (6.13), this gives

$$|F_K/F_\pi| = 1.08, \quad |F_\kappa/F_\pi| = 0.58, \quad f_+(0) = 0.85 \ .$$

Since $F_\kappa \neq 0$ there must be a 0^+ contribution to the propagator of the $\Delta S = 1$ current, and Eq. (6.8) sets an upper limit $m_\kappa \leq 670$ MeV on its mass. (This use of the second spectral-function sum rule is dangerous, even if we do not question its validity, because it certainly converges less rapidly than the first sum rule, and hence is less well saturated by low-lying states.)

(2) (Reference 7). If there are no important 0^+ strange-particle contributions to the spectral function then $F_\kappa = 0$, and Eq. (6.13) gives

$$|F_K/F_\pi| = 1.34, \quad f_+(0) = 1.05 \ .$$

The κ term in $f_-(t)$ now becomes a subtraction constant, for Eq. (6.5) shows that $m_\kappa^2 = \infty$ (with y finite), and Eq. (6.12) then gives

$$\frac{g_{\kappa K\pi} F_\kappa}{m_\kappa^2} \to \frac{F_K^2 - F_\pi^2}{2\,F_\pi\, F_K} \ .$$

(3) (Reference 8). If the divergence of the strangeness-changing vector current has a form-factor which vanishes at infinity then Eqs. (6.9) and (6.10) give

$$g_{\kappa K\pi}\, F_\kappa = (m_K^2 - m_\pi^2) f_+(0)$$

so Eqs. (6.11) and (6.12) determine y as

$$y = \frac{F_K^2 - F_\pi^2 + F_\kappa^2}{2\,F_K\, F_\kappa}$$

and Eq. (6.5) determines the κ mass as

$$m_\kappa^2 = \frac{2m_K^2 F_K^2}{F_\kappa^2 + F_K^2 - F_\pi^2} + \frac{2m_\pi^2 F_\pi^2}{F_\kappa^2 - F_K^2 + F_\pi^2}$$

or, using Eq. (6.13) and neglecting the m_π^2 term,

$$m_\kappa \simeq m_K \left(1 - \frac{1}{1.28}\right)^{-\frac{1}{2}} = 1050 \text{ MeV} \ ,$$

which would put the κ in an octet with the $\pi_V(1016)$ and $\eta_V(1070)$. The inequality (6.7) now provides a lower bound for F_K:

$$F_K \geq F_\pi \frac{m_\pi}{m_K} + \sqrt{\frac{2\,F_\kappa^2 F_K^2}{F_\kappa^2 + F_K^2 - F_\pi^2}} \ ,$$

which with Eq. (6.13) yields

$$|F_K/F_\pi| \geq 1.21, \quad |F_\kappa/F_\pi| \leq 0.44 \ .$$

The maximum value of F_K/F_π is set by the requirement that F_κ^2 in Eq. (6.10) is positive; at this maximum $F_\kappa = 0$, and F_K has the same value as in Case (2), but $g_{\kappa K\pi}\, F_\kappa$ and m_κ^2 now are finite (and y is infinite).

The coupling constants and $K_{\ell 3}$ decay parameters arising from these three different assumptions are shown in Table 1. We note the following:

i) The calculated values of $f_+(0)$ and F_K/F_π do not differ very much from the value unity they would have if SU(3) were a good algebraic symmetry like isospin. This is not a trivial remark, because we have <u>not</u> used SU(3) as an algebraic symmetry, but only as part of the dynamical group SU(3) × SU(3). The reason we have found SU(3) to be more or less successful here is that the input values for decay

TABLE 1

Values of coupling constants and $K_{\ell 3}$ decay parameters derived using ratios of observed K_{e3}, $K_{\mu 2}$, π_{e3}, $\pi_{\mu 2}$ decay rates, first spectral-function sum rule, observed K* decay rate, and either (1) second spectral-function sum rule, or (2) neglect of κ, or (3) unsubtracted κ-K-π form factor. Here $\lambda_\pm \equiv f'_\pm(0)m_\pi^2/f_\pm(0)$.

	(1)			(2)	(3)	
$\lvert F_K/F_\pi\rvert$	1.08			1.34	1.21 (min)	1.34 (max)
$\lvert F_\kappa/F_\pi\rvert$	0.58			0	0.44	0
$g_{K^*K\pi}\, g_{K^*}/m_{K^*}^2$	0.85			0.93	0.88	0.93
$f_+(0)$	0.85			1.05	0.94	1.05
$f_+(t_{max})$	1.01			1.22	1.10	1.22
λ_+	0.024			0.021	0.022	0.021
m_κ(MeV)	490	620	670	∞	1050	1050
$g_{\kappa K\pi}\, F_\kappa/m_\kappa^2$	0.03	0.15	0.23	0.30	0.19	0.21
$f_-(0)$	-0.21	-0.09	-0.01	+0.04	-0.06	-0.05
$f_-(t_{max})$	-0.23	-0.06	0.04	0.00	-0.08	-0.07
$\lambda_- f_-(0)$	-0.003	0.002	0.004	-0.006	-0.003	-0.003

amplitudes and j = 1 meson masses we have used are more or less SU(3) invariant. What we really need is some general and systematic method of deciding where in these calculations we should impose algebraic SU(3) a priori. Some interesting suggestions along these lines were presented in the Discussion Section by Gell-Mann[1] and Oneda[11], but the role of algebraic SU(3) remains somewhat obscure. [The Ademollo-Gatto theorem[12] tells us that $f_+(0)$ should differ from unity by terms of second order in SU(3) symmetry breaking, a requirement automatically met by Eq. (6.4). Bjorken and Quinn[13] have further shown that $f_+(0)$ should be less than unity if SU(3) is weakly broken; this is satisfied by Eq.(6.11) if and only if $F_\kappa^2 > (F_K - F_\pi)^2$, which would rule out Case (2). There are models[14] in which F_K/F_π also differs from unity by amounts of second-order in symmetry breaking, while Gell-Mann, Oakes and Renner[1] have shown that F_K/F_π should be unity in an approximation which treats π's and K's at rest as "soft".]

ii) Everyone predicts that the K_{e3} slope parameter $\lambda_+ \equiv f'_+(0)m_\pi^2/f_+(0)$ is in the range 0.021 - 0.024. This should probably be scored as a success, both because it appears to be in agreement with experiment and also because $\lambda_+ = 0.024 = m_\pi^2/m_{K^*}^2$ is precisely the value required if $f_+(t)$ is to vanish at infinity.

iii) Everyone also agrees that $f_-(t)$ is very small over the whole range of t, unless there exists a κ meson of mass close to the K-meson mass. [However, the precise values of $f_-(t)$ differ very much from case to case, and should not really be taken too seriously.] If ξ is really of order -1 then the smoothness assumptions made here will probably have to be revised, probably by including a large subtraction constant in f_-.

iv) The soft-pion theorem of Callan and Treiman[15], and Mathur, Okubo and Pandit[16], is satisfied exactly here, in the sense that $f_+ + f_- = F_K/F_\pi$ if we set $m_\pi = 0$ and $t = m_K^2$. When m_π is given its actual value and t is set equal to its maximum value $(m_K - m_\pi)^2$ then the CTMOP relation is still satisfied to within 20%, except where there is a κ meson of mass below 600 MeV. However, the CTMOP relation is badly violated if used at t = 0.

v) The crucial question which will have to be answered by experiment is, whether there exists a κ meson, and if so, what is its mass? Our three theoretical cases require (1) a κ below 670 MeV; (2) no κ at all, and (3) a κ at about 1050 MeV. The Goldhaber-Trilling group[17] and Krisch[18] have looked for the κ of Case (1), and not found any, but Krisch[19] and Schnitzer[20] estimate that neither experiment was sensitive enough to detect a κ with the properties required by Case (1). [Incidentally, such a κ might appear as a low-energy K-π resonance, or as a virtual state, or as a K-π bound state which decays into K + 2γ, with a width estimated by Hertel[21] to be comparable to the π^0 width.] The κ of Case (3) may have been seen in a recent experiment by the UCLA group[22]. The Adler-Weisberger sum rule for K-π scattering requires[23] the existence of normal-parity strange-particle states in addition to the K*(890), but these could have spin and parity 1^-, 2^+, 3^-,... instead of 0^+.

There have been a great many calculations of the $K_{\ell 3}$ form-factors which use other methods, particularly those of dispersion theory. Time will allow me to do no more here than to give a list of references[24]. In general, their results are not in good agreement with those I have described, which may perhaps be because they do not use the full off-shell content of the current-algebra identities, and because they impose strong restrictions on the number of subtractions allowed.

Sarker[25] has used hard-pion and kaon methods to recalculate the $K_{\ell 4}$ form-factors, and has found improved agreement with experiment. However, it is perhaps premature to worry about $K_{\ell 4}$ decay while $K_{\ell 3}$ decay is so problematical; in particular the possible effect of a κ meson of low mass has yet to be taken into account.

These methods (or equivalent methods using phenomenological Lagrangians) have also been used to attack other processes where SU(3) × SU(3) breaking plays an important role, such as[26] $X^0 \to \eta + 2\pi$, $K_A \to K + \rho$, $K_A \to K^* + \pi$, $K_A \to \omega + K$, $\phi \to K + \bar{K}$, $K \to \mu + \nu + \gamma$, etc. It appears that, lacking any reliable rules for imposing SU(3), these matrix elements involve so many unknown parameters of the type of δ and δ' that any decisive confrontation of theory and experiment will be difficult.

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7. CONCLUSION

In closing, let us turn away from these details, and ask how far we have been advanced by current algebra towards understanding the elementary particles? I confess to extreme optimism. First, current algebra has given us confidence that we can now cope with a wide variety of problems, involving soft or hard pions, kaons, photons, etc. This feeling of confidence is much like that which has long been enjoyed in electrodynamics or General Relativity, which should not be surprising, because in all these cases it stems from a dynamic symmetry which governs the interactions of a massless (or nearly massless) boson. Second, current algebra has brought the strong, weak, and electromagnetic interactions into closer connection than ever before. Even gravitation has been brought into the picture, if only on the level of analogy. At the same time, current algebra is itself divided by its own internal contradictions, between algebraic SU(3) and dynamic

SU(2) × SU(2), between Regge on the mass-shell and Bjorken off it. Let us hope for a further synthesis by the time of the Fifteenth High-Energy Conference.

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DISCUSSION

WILSON: Prof. Weinberg suggested in the beginning of his talk that the use of Feynman graphs to compute soft-pion amplitudes was based on a formal argument. In particular the formal argument suggests that one uses only the tree graphs and not closed loop graphs. But in fact one learns from the current algebra analysis (of π-π scattering lengths especially) that soft pions are weakly coupled to themselves and to heavy particles, which suggests the Feynman graph approach is justified as a perturbation approach: in particular it would be reasonable to compute unitarity corrections to soft-pion calculations using closed loop Feynman graphs with all necessary renormalizations to make these finite.

WEINBERG: A great many people over the last two years have looked at the unitarity corrections to soft-pion calculations. (See Refs. 2-13 and 19, 20 of Sect. 3.) Generally speaking, it makes no difference whether one uses a phenomenological Lagrangian or the standard technique of current algebra. I agree that it would be very desirable to have a real Lagrangian perturbation theory based on the comparative weakness of soft-pion interactions. (This idea has also been suggested to me by M. Lévy.)

SCHLEIN: I have an experimental comment. I would like to point out that we have presented evidence to this Conference that the T = 1/2 s-wave $\pi\pi$ phase shift passes through 90° in the neighbourhood of 1100-1200 MeV. The evidence comes from an extrapolation to the pion exchange pole of the experimental intensity distribution for the reaction $K^+p \to K\pi\Delta^{++}$ at 7.3 GeV/c. For low momentum transfer to the Δ^{++}, the intensity distribution between the K*(890) and K*(1900) has all the characteristics of arising from a broad (∿ 400 ± 100 MeV) T = 1/2 s-wave resonance. We also obtain an s-wave scattering length $a_0^T = 1/2 =$ = 0.34 ± 0.09 fermi.

WEINBERG: Just to underline what Schlein said, the 1100-1200 MeV κ meson fits in very nicely with the calculation of Chang and Leung, who find that the κ meson must be at 1050 MeV. I think the width of 400 MeV is rather larger than we would have expected from their calculation, but if there really is a κ meson at around 1100 MeV (and if the experimenters can make up their minds that ξ is near zero rather than -1) then I think that the theory of $K_{\ell 3}$ decay will be in good shape. Incidentally, the κ-K scattering length reported by Schlein is in rough agreement (within $1\frac{1}{2}$ standard deviations) with that predicted by current algebra.

PANDIT: Rajasekharan and I have done a calculation of the first order SU(3) breaking to semi-leptonic decays by techniques of equal time commutators. We find $f_K = f_\pi$. Thus just like the case of vector decay (according to Ademollo-Gatto theorem) Cabibbo's angle θ_A is determined from $K_{\ell 2}$, $K_{\pi 2}$ decay rates correct to first-order breaking. Thus θ_A and θ_V appear different to the first order in SU(3) breaking!

WEINBERG: There have been a number of specific models of SU(3) symmetry breaking suggested in which F_K equals F_π in first order (see Ref. 14 of Sect. 6). I think this is still an open question.

BREIT: In connection with the pion trees and soft pions coming out of nucleon lines, which of the applications to nucleon-nucleon interactions that you were referring to do you consider to be the most successful?

WEINBERG: The example of $N + N \to N + N + 23\pi$ was of course academic. However single-pion production in nucleon-nucleon scattering has been calculated by

Beder and Schillaci, Silbar, and Young, and their results agree well with experiment near threshold. It would also be very interesting to calculate the total emission rate of arbitrary numbers of soft pions in high-energy nucleon-nucleon collisions and then use some sort of diffraction theory to compute the elastic amplitude, thus getting a closed system of equations for high-energy nucleon-nucleon scattering. Attempts to sum up the soft-pion emission rates have been made by Perrin and others, but it is a very difficult combinatoric problem. Apart from nucleon-nucleon collisions, there are a great many successful soft-pion calculations, notably $\pi N \to \pi N$, $\pi N \to 2\pi N$ (see Sect. 3), $\gamma N \to \pi N$, $K \to 3\pi$, and so on. It would take more than another hour to go through all them in detail.

GOLDBERGER: What is the possibility for making a dynamical theory based on using currents as the fundamental element: theories of the variety of Sugawara and Sommerfield?

WEINBERG: I will answer unwillingly. I am not sympathetic to these theories. I think they are very imaginative and very interesting mathematically, but I think there has been too much emphasis on the currents in current algebra. I tried in my talk to emphasize the idea that what has been happening in current algebra is that we are learning how to use a dynamic symmetry of the strong interactions, and that the currents themselves merely provide a means to that end. I would hope that models like that of Sugawara do not have anything to do with reality because they do not agree with my intuition of the way physics should be; what physical principle leads us to expect that the energy momentum tensor should be composed only of currents? Fortunately, there is some nice work by Callan and Gross (Ref. 30 of Introduction) which shows how experiment may be able to test these models, but again, with all due respect to their imaginativeness, I hope that they are not right.