



A novel method for stellar electron-capture rates of excited nuclear states

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ABSTRACT

We propose a novel shell-model method to calculate stellar electron-capture (EC) rates of highly-excited nuclear states. This method is based on the Projected Shell Model that can incorporate high-order multi-quasiparticle configurations in a large model space. By taking the EC calculation from ^{59}Co to ^{59}Fe as the first example, we study the effects of nuclear excitations (including both single-particle and collective excitations) on the stellar EC rates. It is found that the predicted EC rates exhibit large variations as functions of excitations in both parent and daughter nuclei, which sensitively depend on the density and temperature in the stellar environment. We emphasize the importance of inclusion of multi-quasiparticle configurations constructed in a large model space for the calculation of EC rates used in astrophysical studies.

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Weak interaction processes play crucial roles in many astrophysical environments and conditions when reactions through the strong and electromagnetic forces are in chemical equilibrium [1–9]. In particular, electron captures (EC) on nuclei are important for the cooling of neutron star crust [10], the core-collapse and thermonuclear supernovae [5,11], the r -process nucleosynthesis, and the thermonuclear explosions of accreting white dwarfs [12] etc. On one hand, EC on nuclei reduces the pressure provided by the degenerate relativistic electron gas, and drives corresponding nuclei to the more neutron rich regions. On the other hand, the produced neutrinos in the EC processes can cool astrophysical environments by carrying away energy.

Following the pioneering works by Fuller, Fowler, and Newman (FFN) [1–4], stellar weak-interaction rates can be determined, provided that the corresponding Gamow-Teller (GT) transition (and Fermi transition, if applicable) strength distributions among nuclei are known with high accuracy. Experimentally, GT transitions can be studied by either the traditional β -decay experiments or the modern charge-exchange (CE) reactions [13,14]. The former has the advantage of high energy resolution while the excitation energy is limited; the latter allows access to higher excitation energy while the energy resolution is relatively low. Although the CE reactions have been well studied with great success over the decade, it is

still impossible to measure the GT strength of all states for most of the nuclei involved in astrophysical environments. Therefore, one has to rely heavily on theoretical calculations.

Motivated by the calculation of FFN with the independent particle model [1–4], over the past decades, stellar EC rates have been determined by the large-scale shell-model (SM) method for sd -shell (with mass numbers $A = 17 - 39$) [15–17] and some pf -shell ($A = 45 - 65$) nuclei [18–20,11]. The success is benefited from the fact that both experimental levels and GT strength distributions can be theoretically reproduced with considerable accuracy. However, as such shell-model dimensions grow roughly exponentially with the mass number A , EC rates for medium-heavy and heavy nuclei with $A > 65$, which are important for the Urca process that affects neutron star cooling [10] etc., must be determined by other approaches. For heavier nuclei, different versions of the quasiparticle random phase approximation (QRPA) [21–29] are often used. In these approaches, the calculation depends on effective truncations on model space (for single-particle motion in the mean field) and/or configuration space (for many-body correlations among nucleons beyond the mean field). On the other hand, only the ground state and some low-lying states are considered in many stellar EC rate calculations [15,11,30]. Presently, EC rates of highly-excited states from medium to heavy mass regions remain less unexplored. In this Letter, we introduce a new method to study stellar EC rates at highly-excited nuclear states, and discuss the effects of the truncations on the EC rates.

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Following the work of FFN [1–4], the rate (in s^{-1}) for stellar EC process, i.e., $(Z, A) + e^- \rightarrow (Z-1, A) + \nu_e$, reads as

$$\lambda^{\text{EC}} = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(k_B T)}}{G(Z, A, T)} \sum_j B_{ij} \Phi_{ij}^{\text{EC}}, \quad (1)$$

where the sums run over states (with angular momentums J_i and J_j , and excitation energies E_i and E_j) of the parent (i) and daughter (j) nuclei, respectively. In a thermal equilibrium with temperature T , the population probability of excited states for the parent nucleus follows a Boltzmann distribution. $G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/(k_B T))$ is the partition function with k_B being the Boltzmann constant. The constant K can be determined from superallowed Fermi transitions, and $K = 6146 \pm 6$ s [31] is adopted in the present work. Φ_{ij}^{EC} is the phase space integral given by,

$$\Phi_{ij}^{\text{EC}} = \int_{\omega_l}^{\infty} \omega p(Q_{ij} + \omega)^2 F(Z, \omega) S_e(\omega) d\omega, \quad (2)$$

where ω ($p = \sqrt{\omega^2 - 1}$) labels the total energy (the momentum) of the electron in unit of $m_e c^2$ ($m_e c$), and

$$Q_{ij} = \frac{1}{m_e c^2} (M_p - M_d + E_i - E_j), \quad (3)$$

is the available energy. ω_l is the capture threshold in EC process, with M_p (M_d) denoting the nuclear mass of the parent (daughter) nucleus. $\omega_l = 1$ if $Q_{ij} > -1$, or $\omega_l = |Q_{ij}|$ if $Q_{ij} < -1$, and S_e is the electron distribution function,

$$S_e(\omega) = \frac{1}{\exp[(\omega - \mu_e)/k_B T] + 1}, \quad (4)$$

with μ_e the chemical potential for electron. Correspondingly one can get the positron distribution $S_p(\omega)$ with $\mu_p = -\mu_e$ in Eq. (4). For astrophysical environment with density ρ and electron-to-baryon ratio Y_e , one has

$$\rho Y_e = \frac{1}{\pi^2 N_A} \left(\frac{m_e c}{\hbar} \right)^3 \int_0^{\infty} (S_e - S_p) p^2 dp, \quad (5)$$

where N_A is the Avogadro's number. The μ_e can be determined by Eqs. (4), (5) for given ρY_e and T . The expression of the Fermi function $F(Z, \omega)$ (in Eq. (2)) that reflects the Coulomb distortion of the electron wave function near the nucleus, can be found in Refs. [1,19] and checked numerically following Ref. [32].

The last term in Eq. (1), B_{ij} , is the reduced probability of nuclear transitions. Generally only the Fermi and GT contributions (which are shown to be quite sufficient) are considered. In this work we follow Refs. [11,30] and neglect the Fermi transition, i.e.,

$$B_{ij} = B_{ij}(\text{GT}^+) = \left(\frac{g_A}{g_V} \right)_{\text{eff}}^2 \frac{\langle \Psi_{J_j}^{n_j} | \sum_k \hat{\sigma}^k \hat{\tau}_+^k | \Psi_{J_i}^{n_i} \rangle^2}{2J_i + 1} \quad (6)$$

where $\hat{\sigma}$ ($\hat{\tau}$) is the Pauli spin operator (isospin operator). Ψ_j^n labels the nuclear many-body wave function of the n -th eigen-state with angular momentum J (i for the parent and j for the daughter nuclei). $(g_A/g_V)_{\text{eff}}$ is the effective ratio of axial and vector coupling constants with corresponding quenching of the GT strength [33,34]

$$\left(\frac{g_A}{g_V} \right)_{\text{eff}} = f_{\text{quench}} \left(\frac{g_A}{g_V} \right)_{\text{bare}}, \quad (7)$$

with $(g_A/g_V)_{\text{bare}} = -1.2599(25)$ [31] and f_{quench} is the quenching factor.

The key step is to calculate $B(\text{GT})$, which is entirely a nuclear structure problem. It is seen from Eq. (6) that the reliability of the calculation depends on the quality of nuclear many-body method and the good understanding of the quenching effect. These are crucial not only for β -decay and EC processes [35,29] but also for the neutrinoless double beta ($0\nu\beta\beta$) decay [36–38]. The quenching factor takes care of possible missing correlations among nucleons and non-nucleonic degrees of freedom [36,38,35]. Throughout the work we adopt $f_{\text{quench}} = 0.74$ which is the same as in the single-shell shell-model calculations of the sd - and some pf -shell nuclei. The effect of such f_{quench} value will be discussed later. From Eq. (1), both nuclear ground states and excited states must involve in stellar EC rate calculations, especially at high temperature. Nuclear excitations include the single-particle and collective excitations as well as the interplay between them. Therefore, novel nuclear structure theories that are able to go beyond the applicabilities of the above-mentioned traditional approaches must be developed.

In the present work, we apply the Projected Shell Model (PSM) [39–42] to calculate GT transitions among odd-mass nuclei. PSM is based on the shell model concept, but unlike the conventional spherical shell model, the PSM model space is defined in the deformed basis. In PSM, one first performs deformed Nilsson mean-field calculations [43] with pairing correlations included [44], to describe (deformed) nuclei in the intrinsic system. Single-particle excitations (different orders of quasiparticle (qp) states) are considered by building explicitly a large configuration space. As in Refs. [45,42], for example, up to 7-qp states are considered for odd-neutron nuclei

$$\left\{ \hat{a}_{\nu_i}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\nu_i}^\dagger \hat{a}_{\nu_j}^\dagger \hat{a}_{\nu_k}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\nu_i}^\dagger \hat{a}_{\pi_j}^\dagger \hat{a}_{\pi_k}^\dagger |\Phi(\varepsilon)\rangle, \right. \\ \left. \hat{a}_{\nu_i}^\dagger \hat{a}_{\nu_j}^\dagger \hat{a}_{\nu_k}^\dagger \hat{a}_{\pi_l}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\nu_i}^\dagger \hat{a}_{\nu_j}^\dagger \hat{a}_{\nu_k}^\dagger \hat{a}_{\pi_l}^\dagger \hat{a}_{\pi_m}^\dagger |\Phi(\varepsilon)\rangle, \right. \\ \left. \hat{a}_{\nu_i}^\dagger \hat{a}_{\nu_j}^\dagger \hat{a}_{\nu_k}^\dagger \hat{a}_{\pi_l}^\dagger \hat{a}_{\pi_m}^\dagger \hat{a}_{\pi_n}^\dagger |\Phi(\varepsilon)\rangle \right\} \quad (8)$$

and for odd-proton nuclei

$$\left\{ \hat{a}_{\pi_i}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\pi_i}^\dagger \hat{a}_{\pi_j}^\dagger \hat{a}_{\pi_k}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\pi_i}^\dagger \hat{a}_{\nu_j}^\dagger \hat{a}_{\nu_k}^\dagger |\Phi(\varepsilon)\rangle, \right. \\ \left. \hat{a}_{\pi_i}^\dagger \hat{a}_{\pi_j}^\dagger \hat{a}_{\pi_k}^\dagger \hat{a}_{\nu_l}^\dagger \hat{a}_{\nu_m}^\dagger |\Phi(\varepsilon)\rangle, \hat{a}_{\pi_i}^\dagger \hat{a}_{\pi_j}^\dagger \hat{a}_{\pi_k}^\dagger \hat{a}_{\nu_l}^\dagger \hat{a}_{\nu_m}^\dagger \hat{a}_{\nu_n}^\dagger |\Phi(\varepsilon)\rangle, \right. \\ \left. \hat{a}_{\pi_i}^\dagger \hat{a}_{\pi_j}^\dagger \hat{a}_{\pi_k}^\dagger \hat{a}_{\nu_l}^\dagger \hat{a}_{\nu_m}^\dagger \hat{a}_{\nu_n}^\dagger |\Phi(\varepsilon)\rangle \right\} \quad (9)$$

where $\hat{a}_\nu^\dagger$ ($\hat{a}_\pi^\dagger$) labels neutron (proton) qp creation operator associated with the deformed qp vacuum $|\Phi(\varepsilon)\rangle$ with collective intrinsic deformation ε . We take the EC process from ^{59}Co to ^{59}Fe as the first example of the PSM calculation. A large model space is adopted for neutrons and protons, i.e., the indices $i, j, k, \dots$ in Eqs. (8), (9) run over four major harmonic-oscillator shells with $N = 2, 3, 4, 5$.

Symmetries that are broken in the intrinsic system can generally be restored by the projection technique [46]. Projection is the transformation that brings the intrinsic wave functions back to the laboratory frame where physical quantities are defined [40]. For example, the broken rotational symmetry in deformed intrinsic mean fields can be restored by an exact angular-momentum projection operator

$$\hat{P}_{MK}^J = \frac{2J+1}{8\pi^2} \int d\Omega D_{MK}^J(\Omega) \hat{R}(\Omega), \quad (10)$$

where $\hat{R}$ and D_{MK}^J , respectively, are the rotation operator and Wigner D -function with Euler angle Ω [47]. Nuclear many-body wave functions in the laboratory system can then be expanded in the projected basis,

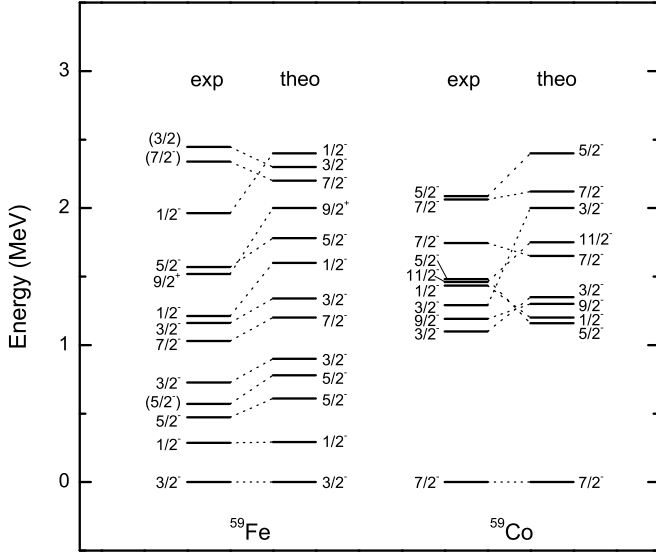


Fig. 1. Calculated energy levels for ^{59}Fe and ^{59}Co , as compared with experimental data [48].

$$|\Psi_{JM}^n\rangle = \sum_{KK'} F_{JKK'}^n \hat{P}_{MK}^J |\Phi_K\rangle, \quad (11)$$

with $|\Phi_K\rangle$ labels the qp states in Eqs. (8), (9). The expansion coefficients $F_{JKK'}^n$ can be obtained by solving the Hill-Wheeler-Griffin equation in PSM, where a separable two-body GT force is included in the original PSM Hamiltonian (see Ref. [42] for the details of the PSM model for GT transition we have developed). In this work all numerical details and parameters for medium-heavy nuclei are adopted exactly as in Ref. [42].

EC processes in the iron-nickel mass range with $A = 50 - 65$ are decisive in the initial stage of the collapse and the dynamics of the explosion for core-collapse supernovae [19,20,24,49]. Correct description of both GT strength distributions and spectra in the involved nuclei are important for corresponding EC rates calculations [20,11]. In Fig. 1 we show the calculated energy levels for the daughter nucleus ^{59}Fe and the parent ^{59}Co with the excitation energy $E_x < 2.5$ MeV, as compared with available data in Ref. [48]. In the calculation the intrinsic quadrupole and hexadecapole deformation in Eqs. (8), (9) are adopted as $\varepsilon_2 = 0.200$, $\varepsilon_4 = 0.053$ for ^{59}Fe and $\varepsilon_2 = 0.133$, $\varepsilon_4 = 0.013$ for ^{59}Co , respectively, taken from Ref. [50]. It is seen that the data are correctly described, especially for the spin and parity of the ground states for both odd-mass nuclei. By analyzing corresponding wave functions, the main configurations (labelled by the Nilsson notation) of the ground states are determined as $\nu 3/2^- [312]$ for ^{59}Fe and $\pi 7/2^- [303]$ for ^{59}Co . Besides, the gap between the ground state and the first excited state for ^{59}Co is well reproduced, indicating that the underlying deformed mean-field levels are described correctly by the Nilsson model. As typical *pf*-shell nuclei, most low-lying states of ^{59}Fe and ^{59}Co have negative parity states, and positive-parity states exist only at relatively high excitation regions.

In Fig. 2, we present the calculated GT strength distribution $B(\text{GT}^+)$ from (a) the ground state, (b) the first $3/2^-$ excited state (see Fig. 1) and (c) the second $3/2^-$ and 15th $5/2^-$ states of ^{59}Co to all states of ^{59}Fe as functions of their excitation energy with $E_x < 12$ MeV, and compare them with available data [51] as well as with the results from the shell model (with the GXPF1a interaction) [11] and the QRPA [30]. Calculations with different configuration spaces (i.e., qp states with higher order are included step by step) are shown to compare with each other. Due to the low excitation-energy resolution (about 0.9 MeV) of the ^{59}Co (*n, p*) reaction in Ref. [51], the data are plotted in 1-MeV-wide bins. The

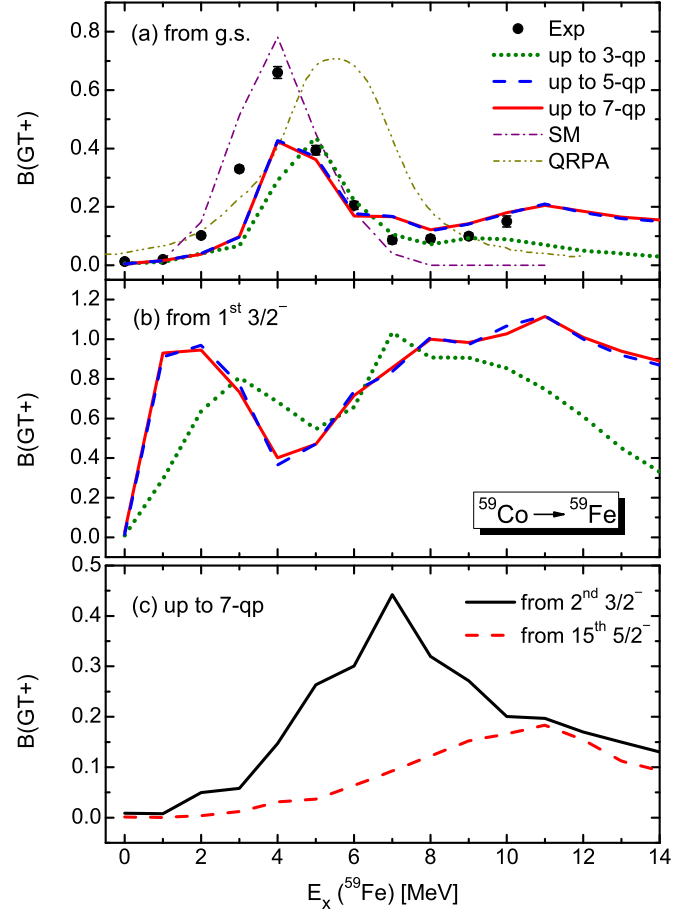


Fig. 2. Calculated GT strength distribution $B(\text{GT}^+)$ for the transition ^{59}Co to ^{59}Fe plotted vs the excitation energy of the daughter nucleus in different configuration spaces. (a) From the ground state of ^{59}Co , as compared with the data [51] and calculations from the SM [11] and QRPA [30]. (b) From the first $3/2^-$ excited state of ^{59}Co . (c) From the second $3/2^-$ and 15th $5/2^-$ states of ^{59}Co .

calculated results are also plotted in the same way to avoid overlaps. It is seen from Fig. 2(a) that when only 1- and 3-qp states are included in the configuration space, the calculated $B(\text{GT}^+)$ shows a peak at $E_x = 5$ MeV and a decreasing trend beyond $E_x \sim 9$ MeV, inconsistent with the data. When 5- and 7-qp states are considered additionally, the peak shifts correctly to 4 MeV and the calculated $B(\text{GT}^+)$ begins to show an increasing trend at $E_x \sim 8$ MeV. We note that for these two calculations, the 7-qp states do not bring in much contribution as in both plots, the curves “up to 5-qp” and “up to 7-qp” are nearly identical. On one hand, the 5- and 7-qp states contribute in wave functions of excited states with $E_x \gtrsim 3$ MeV, leading to the shift of the $B(\text{GT}^+)$ peak. On the other hand, the increasing trend of $B(\text{GT}^+)$ for ^{59}Co at $E_x > 9$ MeV is well reproduced, for the first time theoretically, benefited from the large model space and mixing with higher-order qp configurations in PSM. It is noticed however from Fig. 2(a) that different from the calculations of the SM and QRPA, our calculated $B(\text{GT}^+)$ underestimates the data at $E_x \sim 3 - 5$ MeV. However, in the high excitation region, our calculated $B(\text{GT}^+)$ starts showing an increasing trend at 8 MeV, in agreement with the data. For the other two $B(\text{GT}^+)$ calculations, the QRPA shows a completely opposite trend after 8 MeV, and the SM obtains no strength after that excitation, possibly due to the limited model space. We speculate that the possible reason for our calculated smaller peak than the data at $E_x \sim 3 - 5$ MeV may be that we adopted the same quenching factor $f_{\text{quench}} = 0.74$ in Eq. (7) as in the spherical shell-model calculations. As illustrated by *ab initio* studies, larger f_{quench} (≈ 0.9) [35]

is suggested for *sd*- and *pf*-shell nuclei when more correlations among nucleons are considered in the many-body wave functions. We would also expect the use of larger f_{quench} in our calculations because we have larger model and configuration spaces, so that the data in Fig. 2(a) can be better explained. The rest of the quenching effect might result from non-nucleonic degrees of freedom like the chiral two-body currents [36,35]. Following Ref. [38], corresponding study on open-shell heavy deformed nuclei is in progress.

If we compare the $B(\text{GT}^+)$ distribution from the ground state as shown in Fig. 2(a) with Fig. 2(b), we see that the $B(\text{GT}^+)$ from the first $3/2^-$ excited state exhibits a very different distribution, in both the distribution shape and strength. First of all, they have completely opposite trends at $E_x < 9$ MeV (note that in both plots, the inclusion of 5- and 7-qp states leads to the same first-peak shift and increase at $E_x > 9$ MeV). The average $B(\text{GT}^+)$ strength in Fig. 2(b) is about 0.75 for the excitation-energy region considered here, which is about four times larger than the one from the ground state (see Fig. 2(a)). It is noticed from Eq. (1) that although excited states have low population probabilities at specific temperature, they might also contribute largely to the stellar EC rate owing to possible large $B(\text{GT}^+)$ values.

In most $B(\text{GT})$ discussions in finite-temperature environments, one often adopts the Brink-Axel hypothesis [52,53] which supposes that the $B(\text{GT})$ of excited states has the same distributions as the ground state, with only a shift in excitation of the corresponding states [1–4,18–20,16]. While this hypothesis has been widely applied, there have been works pointing out violations of the hypothesis, with the suggested modifications [17,54]. When comparing Fig. 2(a), which illustrates the distribution of the $B(\text{GT}^+)$ from the ground state of the parent ^{59}Co , with Fig. 2(c) from the two excited states, we find that the $B(\text{GT}^+)$ from the second $3/2^-$ (the 15th $5/2^-$) state of ^{59}Co , with excitation energy $E_i \approx 2.1$ ($E_i \approx 3.8$) MeV, shows a similar GT peak centering around $E_x \approx 7$ ($E_x \approx 11$) MeV, as compared to the peak centering around 4 MeV in Fig. 2(a). However, the total $B(\text{GT}^+)$ strength of the three distributions has clearly a decreasing trend as the excitation of the initial parent state goes up. If one recalls that the GT Brink-Axel approach treats all excited states as having the same bulk strength distribution as the ground state, our results tend to warn that the Brink-Axel hypothesis should be applied with caution. In addition, the $B(\text{GT}^+)$ distribution of the first $3/2^-$ state of ^{59}Co in Fig. 2(b) is very different from that of the ground state. We can qualitatively understand these results by looking at the microscopic structure of the relevant states. The main configurations of the $7/2^-$ ground state and the first $3/2^-$ state in the parent ^{59}Co are rather pure, which are found to be $\pi 7/2^- [303]$ and $\pi 3/2^- [301]$ of the single-qp configuration, respectively. Thus the differences in the GT peaks in Fig. 2(a) and 2(b) are determined by the mixing of configurations in the daughter ^{59}Fe . For Fig. 2(c), the wave functions of both the second $3/2^-$ and the 15th $5/2^-$ have increasing degree of configuration mixing already in the parent states. The main configuration of the former is $\pi 7/2^- [303] \otimes \nu 1/2^- [310] 3/2^- [312]$ and that of the latter is $\pi 7/2^- [303] 7/2^- [303] 5/2^- [312] \otimes \nu 3/2^- [312] 5/2^- [303]$, both include the ground state configuration $\pi 7/2^- [303]$ in ^{59}Co . Thus in Fig. 2(c), the configuration mixing occurs in both parent and daughter nuclei. Comparing Fig. 2(c) with 2(a), the GT peak appears because the multi-qp configuration contains the one of the ground state, but the height and width of the distribution are determined by the degree of the configuration mixing.

Fig. 3 displays the calculated EC rates from ^{59}Co to ^{59}Fe in astrophysical environments with different temperature T_9 (GK) and density ρY_e (mol/cm³) conditions. Cases without (panels (a–e)) and with (panels (f–j)) the inclusion of excited states of the parent nucleus ^{59}Co are shown in different configuration spaces, where about 600 excited states of ^{59}Co with excitation energy $E_i \leq 7.0$

MeV are taken into account (which are expected to be sufficient for the temperatures considered in Figs. 3 and 4 as seen from Eq. (1)). The ones derived from the experimental $B(\text{GT}^+)$ distributions [51] are also plotted. It is noted that the comparison can only have a qualitative meaning because these experimental rates are not necessarily the actual rates in the corresponding astrophysical environments since population on excited states of the parent nucleus is neglected and integer values for the excitation energy of the daughter nucleus ^{59}Fe are adopted in the derivation due to the limited energy resolution (see Fig. 2(a)).

For the cases where only the ground state (g.s.) of ^{59}Co is considered, one can see from Fig. 3 (a–e) that the EC rates generally increase with temperature and density. This is because the electron chemical potential μ_e increases with the density, and the distribution of $S_e(\omega)$ becomes more diffused with the temperature so that more GT transitions to the states of the daughter nucleus with higher excitation energy would contribute to the rates (see Eqs. (4), (5)). When only 1- and 3-qp states are included in the configuration spaces for both nuclei, the experimental rates cannot be even qualitatively reproduced by the calculations, and the discrepancies are sizable particularly at lower temperatures and densities. This is because in these conditions the EC rates are mainly sensitive to the $B(\text{GT}^+)$ of low-lying states (in the daughter nucleus) for which the calculations in a small configuration space are not satisfactory (see Fig. 2(a)). The discrepancy is improving at higher temperature and ρY_e since the rates will be more dependent on the global structure of the GT distribution. When 5- and 7-qp states are considered additionally, the agreement between theory and data becomes much better, and the experimental rates can be well reproduced except for some minor systematical underestimates. One must evaluate these underestimates with caution because of the following two reasons. On one hand, in the calculations of EC rates λ^{EC} in Eq. (1) we used exactly the calculated levels (i.e., excitation energies E_j as shown in Fig. (1)) instead of the integer values for E_x (see Fig. 2(a)) in the derivation of the experimental rates, or the experimental spectra directly as in some other calculations. However, it is known that the EC rate is very sensitive to the excitation energy of the states in the daughter nuclei [11], and this would be a place where uncertainties are introduced. On the other hand, as mentioned above, we adopted a small quenching factor $f_{\text{quench}} = 0.74$ which should be larger for the present calculations. Thus we conclude that when only transitions from the ground state of ^{59}Co are considered (panels (a–e)) for which the 5- and 7-qp states are not important for the description of the ground state of ^{59}Co , the inclusion of higher order single-particle (qp) excitations in the daughter nucleus ^{59}Fe are necessary for a correct prediction of EC rates.

In stellar environments, nuclei have considerable probability to be thermally populated in excited states, which would enhance significantly their total decay rates [55]. Hence transitions from excited states of the parent nuclei must be considered for an accurate calculation and prediction of corresponding stellar weak-interaction rates, especially at higher temperature when more excited states are involved effectively as shown in Eq. (1). Both kinds of the well-known nuclear excitations, the single-particle and collective excitations as well as their interplay would play crucial roles. This is precisely where the detailed nuclear structure physics enter into the discussion. By comparing panels (f–j) with panels (a–e) in Fig. 3, we see that the EC rates are enhanced significantly when transitions from excited states of ^{59}Co are taken into account. The enhancement depends on different density, temperature and the configuration space. For smaller configuration spaces with only 1- and 3-qp states, the enhancement is dramatic at low density and temperature, seen from panels (d, i) and (e, j). For larger configuration spaces where up to 7-qp states are considered, the enhancement is sensitive mainly to temperature. For example, for

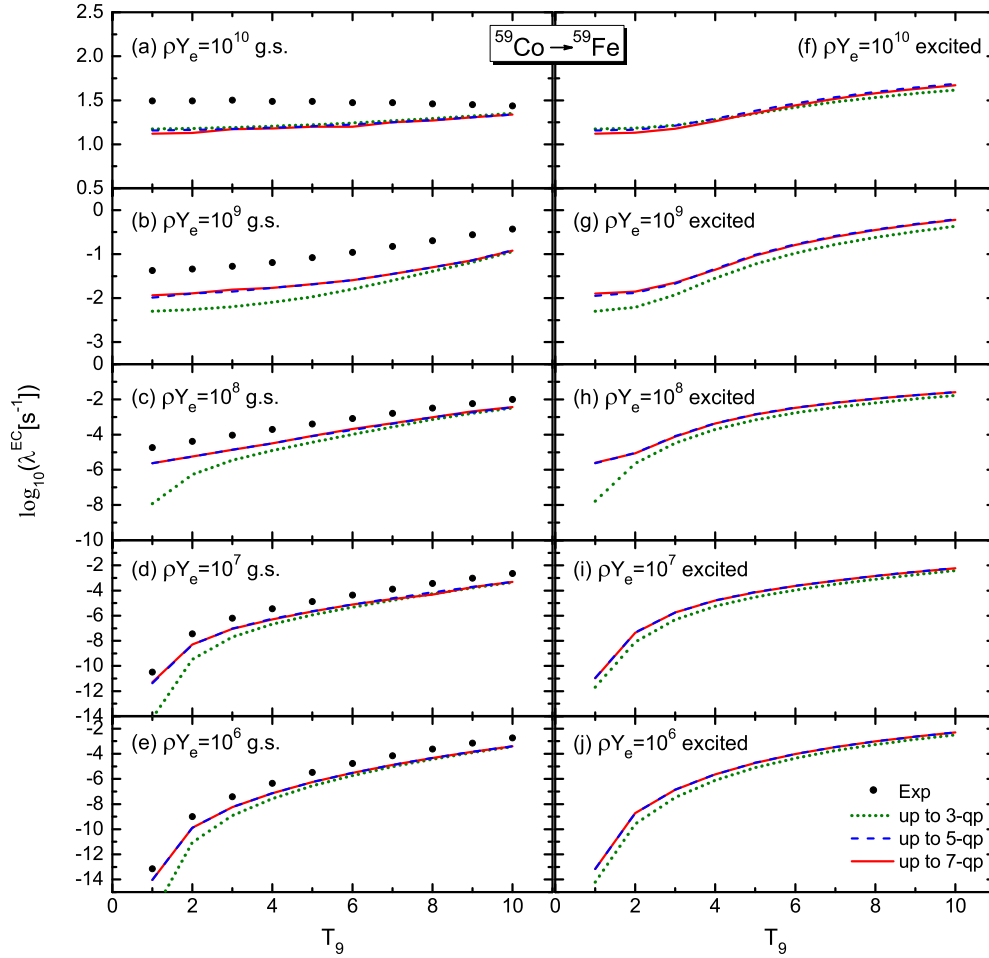


Fig. 3. Electron-capture rates for $^{59}\text{Co} \rightarrow ^{59}\text{Fe}$ as a function of the temperature T_9 (GK) for densities $\rho Y_e = 10^{10}, 10^9, 10^8, 10^7$, and 10^6 mol/cm 3 . Calculations without (panels (a-e)) and with (panels (f-j)) excited states in different configuration spaces are shown and compared with the data derived from the experimental $B(\text{GT}^+)$ distributions [51]. See text for details.

$\rho Y_e = 10^8$ mol/cm 3 , as comparing the red-solid lines in panels (c) and (h), the enhancement is negligible for $T_9 \leq 2$ GK and about one order of magnitude for $T_9 \geq 4$ GK. This is because that for low temperatures, the excited states of ^{59}Co cannot be involved at all due to the large gap above the ground state as shown in Fig. 1. As temperature rises, the first and many other excited states above the gap are suddenly involved at specific T_9 . This suggests that for some medium-heavy odd-mass nuclei where many low-lying states exist near the ground state, the EC rates would be enhanced significantly at all densities and temperatures, which will affect corresponding Urca process that cools the neutron star crust [10]. When comparing the results with and without the 5- and 7-qp states in panels (f-j), it is seen that the higher order qp (single-particle) excitations are important not only for low temperature and density (as in the cases in panels (a-e)), but also for high temperature and some high densities (see panel (g)). This indicates that single-particle excitations in both parent and daughter nuclei are crucial for prediction of stellar EC rates.

Finally we discuss effects from nuclear collective excitations. We vary the collective intrinsic deformation ε_2 in PSM in constructing the configuration spaces in Eqs. (9), (8) for both the parent and daughter nuclei. Note that ε_2 is commonly used to indicate nuclear shapes, with $\varepsilon_2 = 0$ for a nucleus in spherical shape, and $\varepsilon_2 > 0$ ($\varepsilon_2 < 0$) for prolate (oblate) shape. We study the effects on the EC rates in the two cases that are crucial for supernova explosion. One is with $\rho Y_e = 10^7$ mol/cm 3 and $T_9 = 3$ GK, which represents the conditions of the core during the silicon-burning

stage in a pre-supernova star [56]. The other is with $\rho Y_e = 10^9$ mol/cm 3 and $T_9 = 10$ GK, which represents the conditions for the pre-collapse of the core [57]. Up to 7-qp states are included in the configuration space for both cases. It is seen from Fig. 4(a) that for low temperature and density when GT transitions to low-lying states of the daughter nucleus ^{59}Fe mainly contribute to the EC rates, the EC rates are very sensitive to the ε_2 of ^{59}Fe . The rates are large when ^{59}Fe would be spherical for which the $B(\text{GT}^+)$ to low-lying states dominate. Such effects would definitely influence the stellar evolution, especially the initial conditions of the gravitational collapse of the core. For high temperature and density when many excited states of both the parent and daughter nuclei are involved and contribute, the EC rates are less sensitive and basically symmetric to the deformations of both nuclei, seen from Fig. 4(b). The rates turn out to be larger when nucleons prefer to perform smaller collective deformation ε_2 in both nuclei, and are less sensitive when $0.1 \leq |\varepsilon_2| \leq 0.3$ for both nuclei. Nuclei are expected to have smaller intrinsic deformation at higher temperature [58], which would definitely enhance the predicted rates, especially for odd-mass nuclei [42,59,60]. It is interesting that the situation is very different from the one of $0\nu\beta\beta$ decay where the rates are negligible except that both nuclei share the same deformation [61–63]. The reason is that in $0\nu\beta\beta$ decay only the transition from ground state to ground state is considered in the calculation as the closure approximation is usually employed and single-particle excitations are neglected, while all transitions between excited states for both nuclei contribute to the stellar EC

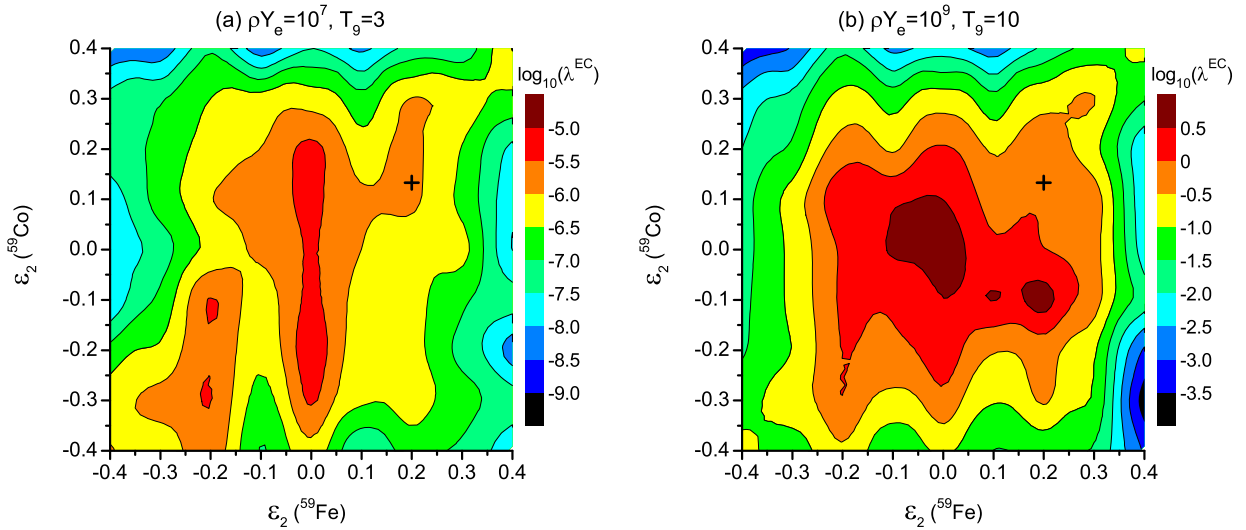


Fig. 4. Electron-capture rates for $^{59}\text{Co} \rightarrow ^{59}\text{Fe}$ at different densities and temperatures, i.e., (a) $\rho Y_e = 10^7$ mol/cm 3 , $T_9 = 3$ GK and (b) $\rho Y_e = 10^9$ mol/cm 3 , $T_9 = 10$ GK, as a function of the intrinsic deformation of the parent and daughter nuclei. The cross symbol marks the location for the physical values of deformations. See text for details.

rates. To reduce uncertainties and provide accurate predictions for stellar EC rates, a state-by-state evaluation by models with both a large model space and a large configuration space, as well as a correct handling of collective intrinsic deformation is required.

In summary, weak interaction processes are crucial for many stellar environments and astrophysical conditions, in which nuclei are exposed to high temperature and high density environments, and thus have considerable probability to be thermally populated in excited states. Transitions between excited states would then enhance significantly corresponding rates. Uncertainties in these calculations would finally affect the reliability of the description for stellar evolution and many astrophysical phenomena. In this work we studied the effects of nuclear excitations (including both single-particle and collective excitations) on the evaluation of stellar electron-capture (EC) rates by the Projected Shell Model with a large model space and a large configuration space. By taking the EC from ^{59}Co to ^{59}Fe as the example, we found that high-order single-particle excitations and their interplay with collective excitations for both the parent and daughter nuclei enhance significantly the stellar EC rates; the enhancement is dramatic especially for low temperatures and densities. The collective excitations would also affect the rates, the effects depends also on temperature and density, and the rates usually get larger with smaller collective deformation in both nuclei.

Potential applications of the method presented here in nuclear astrophysics are study of uncertainties and systematical calculations for all stellar weak-interaction rates (EC, β^+ decay, positron capture, β^- decay) of medium-heavy and heavy nuclei, and particularly when these nuclei are deformed. This is crucial for many astrophysical scenarios such as core-collapse supernova, nucleosynthesis, and cooling of neutron stars etc., in which an accurate database of stellar weak-interaction rates is much desired.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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