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TESTS OF SCALING OF THE PROTON ELECTROMAGNETIC STRUCTURE FUNCTIONS*

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ABSTRACT

Deep-inelastic structure functions W_1 and W_2 have been extracted from electron-proton scattering cross sections that were measured in recent experiments at SLAC. The structure functions display deviations from scaling in the variable ω in the kinematic range $1.5 \leq \omega \leq 3.0$ and $2 \leq Q^2 \leq 15$ GeV^2.

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We have extracted the structure functions W_1 and W_2 from deep-inelastic electron-proton scattering cross sections that were measured in two experiments[1,2] at the Stanford Linear Accelerator Center (SLAC). In the first Born approximation, the differential cross section for the scattering of electrons of energy E to a final energy E' through an angle θ is related to W_1 and W_2 by

$$d^2\sigma/d\Omega dE' = \sigma_M\left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2\theta/2\right] ,$$

where σ_M is the Mott cross section, $\nu = E - E'$, and $Q^2 = 4EE' \sin^2\theta/2$. The differential cross section is also related to the longitudinal and transverse virtual photoabsorption cross sections σ_L and σ_T by

$$d^2\sigma/d\Omega dE' = \Gamma\left[\sigma_T(\nu, Q^2) + \epsilon\sigma_L(\nu, Q^2)\right] ,$$

where Γ is the flux of transverse virtual photons and $\epsilon = \left[1 + 2(1+\nu^2/Q^2)\tan^2\theta/2\right]^{-1}$. Extraction of W_1 and W_2 at some (ν, Q^2), which requires the differential cross sections for at least two values of θ, is equivalent to the extraction of σ_T and $R=\sigma_L/\sigma_T$. Here $W_1 = (K/4\pi^2\alpha)\sigma_T$ and $W_2 = (K/4\pi^2\alpha)\sigma_T(1 + R)/(1 + \nu^2/Q^2)$ where $K = (W^2 - M^2)/2M$, $W = (M^2 + 2M\nu - Q^2)^{1/2}$, and M is the proton mass.

Bjorken[3] originally conjectured that the two functions $2MW_1(\nu, Q^2)$ and $\nu W_2(\nu, Q^2)$ should scale in the variable $\omega = 2M\nu/Q^2$ (i.e., become functions only of ω) in the limit $\nu \to \infty$, $Q^2 \to \infty$, with ν/Q^2 fixed. Previous scaling tests[4] used values of νW_2 which had been extracted from measured cross sections by assuming the average value $R = 0.18 \pm 0.10$ to be valid throughout the kinematic range of the data. Within experimental errors, those tests revealed that νW_2 was consistent with scaling in ω for $Q^2 \geq 1$ GeV^2 and $W \geq 2.6$ GeV. When data for $W \geq 1.8$ GeV were included, then νW_2 scaled better in the variable $\omega' = \omega + M^2/Q^2$. Such tests of scaling at finite ν and Q^2 are dependent on the choice of the scaling variable. Other scaling variables have been proposed,[5,6] all of which approach ω as $Q^2 \rightarrow \infty$. In the following tests, we restrict ourselves to the scaling variables ω, ω', and[6] $\omega_L = M/\left[(Q^2+\nu^2)^{1/2} - \nu\right]$.

The cross sections[1,2] employed in the present scaling tests were measured with the same spectrometers as those used in the previous tests,[4] but have smaller statistical errors. The bulk of the cross section data used in these extractions of W_1 and W_2 was measured[1] at 18^o, 26^o, and 34^o with the SLAC 8-GeV spectrometer. Additional cross sections used in this analysis were measured[2] at 6^o and 10^o with the SLAC 20-GeV spectrometer. The analyses[1,2,7] of the raw experimental data from these two experiments were similar and the radiative correction procedures[2,7] were identical. A normalization factor[7] of 1.02 ± 0.02 was applied to the 6^o and 10^o data prior to the extraction of W_1 and W_2.

In the present analysis, unlike previous analyses, the two structure functions have been extracted from the cross section data without any assumptions about R. An array of kinematic points (ν, Q^2) (with $W \geq 2$ GeV and $Q^2 \geq 1$ GeV^2) lying on constant-ω contours was chosen[7] to reflect the distribution of measured cross sections. At each angle values of $\Sigma(\nu, Q^2, \theta) = (1/\Gamma) d^2\sigma/d\Omega\, dE'$ were obtained by interpolation of the measured differential cross sections. At every (ν, Q^2) point for which there were interpolated data from two or more angles, σ_L and σ_T were available as the slope and intercept of a linear fit to Σ versus ϵ. Values of W_1 and W_2 and their errors were obtained from the extracted σ_L and σ_T and their errors. Plots of νW_2 and $2MW_1$ versus Q^2 for values of ω between 1.5 and 5.0 are shown in Fig. 1. The total systematic uncertainties are estimated to be 6-8% in νW_2 and 7-10% in $2MW_1$.

We have tested scaling in $\xi = \omega, \omega_L$, or ω' by fitting functions of the form $F_i = g_i(\xi)\left[1 - 2Q^2/\Lambda_i^2\right]$, to this data for $F_1 = 2MW_1$ and $F_2 = \nu W_2$ in two regions of ω and Q^2. Here $g_i = \Sigma p_{in}(1-1/\xi)^n$ where n ranges from 3 to 7. Best fit values for Λ_i^2 and for the polynomial coefficients p_{in} were obtained simultaneously. Our studies indicate that the results for Λ_1^2 and Λ_2^2 are insensitive to the choice of the functional forms for g_1 and g_2. In the region $1.5 \leq \omega \leq 3.0$ ($2 \leq Q^2 \leq 15$ GeV^2) F_1 and F_2 show deviations from scaling in ω which are characterized by $2/\Lambda_1^2 = 0.0324 \pm 0.0048$ GeV^{-2} ($\Lambda_1^2 = 62 \pm 9$ GeV^2) and $2/\Lambda_2^2 = 0.0268 \pm 0.0026$ GeV^{-2} ($\Lambda_2^2 = 75 \pm 7$ GeV^2). The data show smaller, but still significant, deviations from scaling in ω_L. These are characterized by $2/\Lambda_1^2 = 0.0225 \pm 0.0058$ GeV^{-2} ($\Lambda_1^2 = 89 \pm 23$ GeV^2) and $2/\Lambda_2^2 = 0.0167 \pm 0.0030$ GeV^{-2} ($\Lambda_2^2 = 120 \pm 21$ GeV^2). In this same region, possible deviations from scaling in ω' are small, as indicated by the best fit values $2/\Lambda_1^2 = 0.0098 \pm 0.0070$ GeV^{-2} and $2/\Lambda_2^2 = 0.0040 \pm 0.0036$ GeV^{-2}. As the data in the region $4 \leq \omega \leq 10$ ($2 \leq Q^2 \leq 5$ GeV^2) are less precise and the Q^2 range is smaller, we report only lower limits (95% confidence) for Λ_1^2 and Λ_2^2 of 40 GeV^2 (using either ω, ω_L, or ω' as the scaling variable). We have no accurate data for $\omega > 10$ and little can be said about scaling in that region.

Systematic uncertainties in Λ_1^2 and Λ_2^2 arise from uncertainties in the relative normalization[7] of the two experiments, in the experimental parameters (e.g., fluctuations in the energy and direction of the incident beam), and in the radiative corrections. The total systematic uncertainty in Λ_1^2 or Λ_2^2 (which is comparable to the statistical error) is computed from the quadratic sum of these three uncertainties and is included in the errors and limits quoted above.

Deviations from scaling, suggested by a number of theoretical models,[8-13] were further examined by fitting functions with explicit Q^2-dependent terms to F_1 and F_2 for fixed ω in the range $1.5 \leq \omega \leq 3.0$.

Within parton models,[3] it has been suggested[8,9] that deviations from scaling in ω arise because the partons themselves have structure. Chanowitz and Drell[8] have suggested a fall-off of the form $F_i = a_i\left[1 - 2Q^2/\Lambda_i^2\right]$. The quantities Λ_1 and Λ_2 are expected to be equal and independent of ω and to represent the effective mass of vector gluons associated with a parton form factor. In a generalized version[9] of this model, in which the partons have an additional anomalous magnetic moment, Λ_2 could be greater than Λ_1. Best fit values of Λ_1^2 and Λ_2^2 from fits of the above form are given in Table I, along with their statistical errors. All data with $W \geq 2$ GeV and $Q^2 \geq 2$ GeV^2 were used in these fits. Similar values of Λ_i^2 are obtained if only data with $W \geq 2.6$ GeV are used (see Table I).

Deviations from scaling of the form $F_i = a_i/\left[1+Q^2/\bar{\Lambda}^2\right]^2$ would result from the exchange of a heavy photon[10] of mass $\bar{\Lambda}$, or from a parton-structure model in which a simple pole is assumed for the parton form factor. The quantities $\bar{\Lambda}_1^2$ and $\bar{\Lambda}_2^2$ from fits of this form to F_1 and F_2 (given in Table I) are expected to be equal and independent of ω.

Logarithmic deviations from scaling arise in field theoretic[10,11,12] models. The results of fits of the form $F_i = a_i\left[1 - b_i\,\ell n(Q^2/M^2)\right]$ are given in Table I. Perturbation theory calculations from renormalizable field theories[10,11] suggest large values for b_1 and b_2 (on the order of strong interaction coupling constants), in disagreement with the data. The small values of b_1 and b_2 obtained over the Q^2 range of this experiment can be accommodated within super renormalizable field theories.[10] The observed deviations from scaling in ω are also compatible with asymptotically free field theories.[12] Deviations from scaling of the form $F_i = a_i\left[M^2/Q^2\right]^{c_i}$, where the c_i could vary with ω, are suggested[13] by theories of anomalous dimensions. The results of such fits are given in Table I.

Another interpretation of the observed deviations from scaling in ω is that they reflect a low Q^2 (or low W) approach to scaling. Fits of the form $F_i = a_i\left[1+d_iM^2/Q^2\right]$ provide a comparison of the data with a $1/Q^2$ approach to scaling. The best fit coefficients (given in Table I) vary rapidly with ω and are close to what would be expected if F_i scaled in ω'.

From an analysis of the data without any assumptions about R we conclude that νW_2 and $2MW_1$ show significant fall-offs with Q^2 for fixed $1.5 \leq \omega \leq 3.0$ and $2 \leq Q^2 \leq 15\ \mathrm{GeV}^2$. Due to the limited Q^2 range, it is not clear whether the data favor models predicting deviations from scaling in ω with increasing Q^2, or models in which the asymptotic scaling values are gradually approached from above. The data, however, provide experimental limits on the magnitudes of Q^2-dependent scale breaking terms suggested by both types of models.

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TABLE CAPTION

I. Coefficients of scale breaking terms and associated statistical errors from fits to $F_1 = 2MW_1$ and $F_2 = \nu W_2$ versus Q^2 at fixed ω. All data with $W \geq 2$ GeV and $Q^2 \geq 2$ GeV^2 were used unless otherwise noted. Λ^2 and $\bar{\Lambda}^2$ are in units of GeV^2; $\Delta\Lambda_1^2$ and $\Delta\Lambda_2^2$ are the systematic uncertainties in Λ_1^2 and Λ_2^2 which arise from the three effects quoted in the text. Systematic uncertainties for the other fits are comparable to the statistical errors.

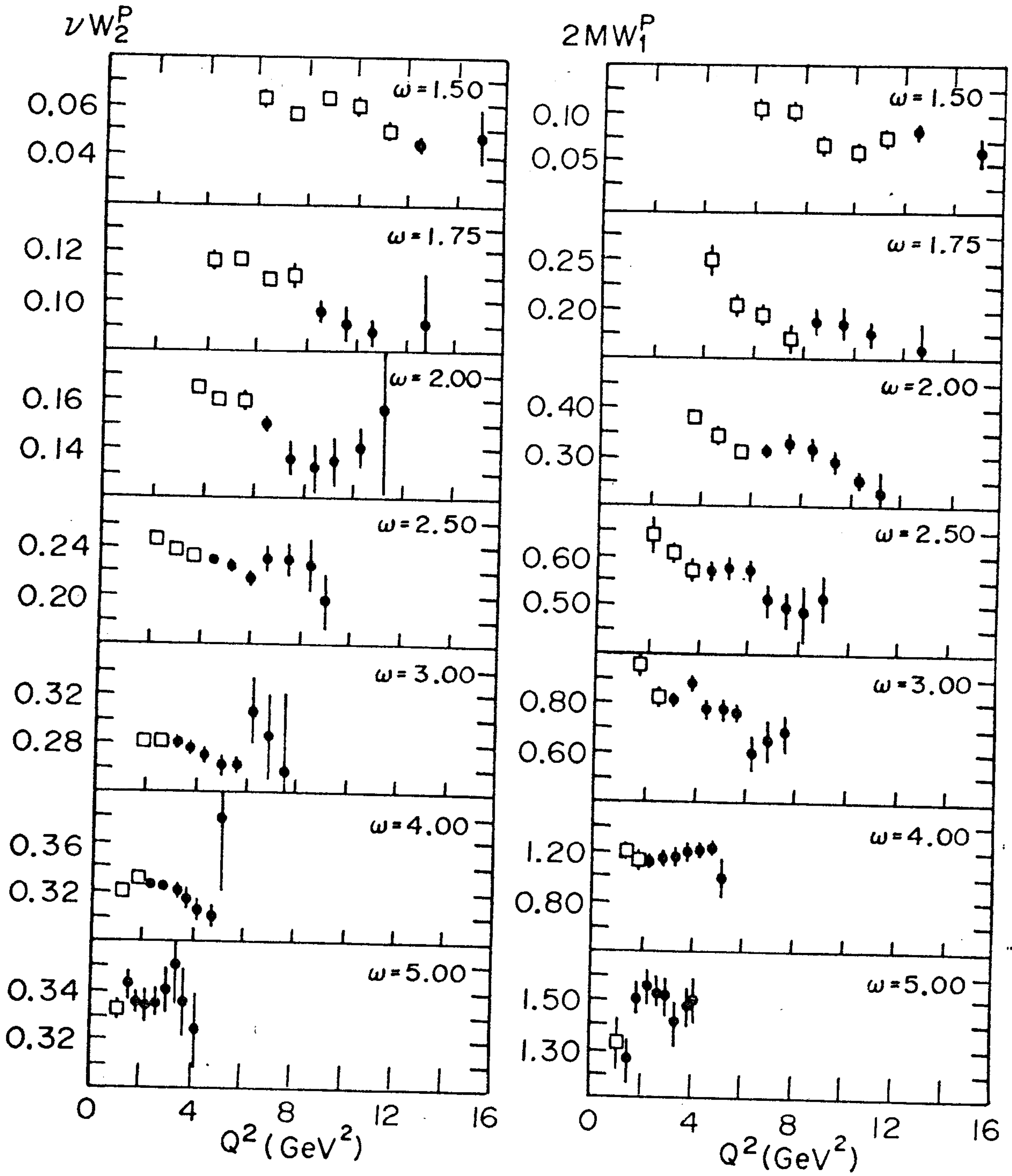
FIGURE CAPTION

1. Proton structure functions versus Q^2 for fixed ω. The symbols ● and ○ represent $W \geq 2.6$ GeV and $2.0 \leq W < 2.6$ GeV data respectively. Only statistical errors are shown. Note that the scales have suppressed zeros.

TABLE I

$F_i = a_i\left[1 - 2Q^2/\Lambda_i^2\right]$ (W $\geq$ 2.0 GeV)					$F_i = a_i/\left[1 + Q^2/\bar{\Lambda}_i^2\right]^2$		$F_i = a_i\left[1 - b_i \ln(Q^2/M^2)\right]$	
ω	Λ_1^2	$\Delta\Lambda_1^2$	Λ_2^2	$\Delta\Lambda_2^2$	$\bar{\Lambda}_1^2$	$\bar{\Lambda}_2^2$	b_1	b_2
1.50	59 ± 13	7	71 ± 13	10	24 ± 10	45 ± 14	0.235 ± 0.031	0.189 ± 0.027
1.75	68 ± 16	11	59 ± 7	5	40 ± 14	39 ± 8	0.187 ± 0.032	0.182 ± 0.019
2.00	58 ± 8	6	74 ± 10	10	36 ± 8	55 ± 10	0.189 ± 0.023	0.137 ± 0.017
2.50	67 ± 15	15	89 ± 11	15	52 ± 15	66 ± 11	0.130 ± 0.026	0.095 ± 0.013
3.00	59 ± 16	10	120 ± 43	29	47 ± 17	107 ± 42	0.125 ± 0.035	0.064 ± 0.021

$F_i = a_i\left[1 - 2Q^2/\Lambda_i^2\right]$ (W $\geq$ 2.6 GeV)					$F_i = a_i\left[M^2/Q^2\right]^{c_i}$		$F_i = a_i\left[1 + d_i M^2/Q^2\right]$	
ω	Λ_1^2	$\Delta\Lambda_1^2$	Λ_2^2	$\Delta\Lambda_2^2$	c_1	c_2	d_1	d_2
1.50	---	--	---	--	0.57 ± 0.16	0.33 ± 0.09	13.73 ± 9.00	4.84 ± 1.88
1.75	69 ± 43	22	74 ± 51	24	0.32 ± 0.09	0.28 ± 0.05	3.83 ± 1.51	2.59 ± 0.61
2.00	54 ± 12	7	106 ± 54	36	0.30 ± 0.06	0.18 ± 0.03	2.96 ± 0.81	1.29 ± 0.26
2.50	65 ± 22	15	103 ± 53	29	0.16 ± 0.04	0.11 ± 0.02	0.90 ± 0.28	0.48 ± 0.08
3.00	49 ± 12	7	157 ± 102	73	0.15 ± 0.05	0.07 ± 0.01	0.70 ± 0.31	0.31 ± 0.12



νW_2^P
$2MW_1^P$
ω = 1.50
ω = 1.75
ω = 2.00
ω = 2.50
ω = 3.00
ω = 4.00
ω = 5.00
0.06
0.04
0.12
0.10
0.16
0.14
0.24
0.20
0.32
0.28
0.36
0.32
0.34
0.32
0.10
0.05
0.25
0.20
0.40
0.30
0.60
0.50
0.80
0.60
1.20
0.80
1.50
1.30
0
4
8
12
16
Q^2 (GeV^2)

Fig. 1