Compilation of coupling constants and low-energy parameters

(selected from recent publications, June 1969)

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1. πN-scattering

(units $\hbar = c = m_{\pi^+} = 1$, $M \equiv m_p = 6.722$)
Review: Hamilton and Woolcock, RMP *35*, 737 (63)

Partial waves, $S^I_{l\pm} = \eta^I_{l\pm} e^{2i\delta^I_{l\pm}} = 1 + 2iq\, f^I_{l\pm}$ for $J = l \pm \frac{1}{2}$.

a) S-wave scattering lengths

$a_{2I} = f^I_{0\pm}(q=0)$; coupling constant, $\frac{G^2}{4\pi} = 4M^2 f^2$.

$a_1 - a_3$	$a_1 + 2a_3$	$f^2 \times 10^3$	$\frac{G^2}{4\pi}$	Reference
0.271	−0.002			Hamilton, PL *20*, 687 (66)
±0.007	±0.008			
0.263	0.069			Lovelace, Heidelberg Conf. (Sept. 67)
0.266	0.056			Lovelace, Irvine Conf. (Dec. 67)
0.288	0.039			Lovelace, Heidelberg Conf., extrap. by Steiner (method of Hamilton, PL 20)
0.297		for 81	14.64	Höhler, Baacke & Steiner, Z. Phys. *214*, 381 (68)
0.271		for 76	13.74	
	0.045			Zovko, NP B 10, 231 (69)
0.288	−0.021	80±5	14.46	Höhler, Schlaile & Strauß, Lund Conf. 69
±0.010	±0.010			
0.277	−0.026	81.5±1.6	14.73	Samaranayake & Woolcock, Lund Conf. 69
±0.009	±0.008			

Attention: for charge exchange, 0.5% isoscalar admixture to π° coupling changes the Born term in the forward dispersion relation at threshold by 6 % ($f^2_{\text{ch. ex.}} = 0.076$).

b) P-wave scattering lengths

$a_{2I,2J} = \lim_{q^2 \to 0} f^I_{1\pm}/q^2$.

$a_{11} - a_{31}$	$a_{13} - a_{33}$	$a_{11} + 2a_{31}$	$a_{13} + 2a_{33}$	Reference
−0.045	−0.245	for	$f^2 = 0.081$	Baacke, Höhler & Steiner
−0.039	−0.237	for	$f^2 = 0.077$	Z. Phys. *221*, 134 (69)
−0.051	−0.243	−0.168	0.396	Höhler et al., Lund Conf. 69
−0.049	−0.250	−0.160	0.431	Collins, Samaranayake & Woolcock, Lund Conf. 69

c) ρ-couplings

("universality" normalization: $G_\rho \tau/2$ between nucleons)

	2.8 ±0.2	Sakurai, PRL *17*, 1021 (66)
$\frac{G_{\rho\pi\pi} G_{\rho V}}{4\pi}$	= 2.85 ± 0.3	Hamilton, in "High Energy Physics I", p. 282 (Acad. Press 1967, ed. Burhop)
	(2.19)	Höhler et al., Z. Phys. *214*, 381 (68), criticize the method.

2. $K^{\pm}N$, $\pi\Sigma$ and $\pi\Lambda$ channels

(units $\hbar = c = fm = 1$)

a) K^+p scattering

$$q^{2l+1} \cot \delta_l = \frac{1}{a} + \frac{r}{2} q^2$$

$a_s = -.32 \pm .01$ $2a_{p3} + a_{p1} = -.01 \pm .01$	$r_s = .31 \pm .15$	A. D. Martin & Perrin, NP *B10*, 125 (69)
$a_s = -.28 \pm .01$		Lea, B. R. Martin & Oades, Lund Conf. 69

b) K^-p scattering

Review: Pilkuhn, The Interactions of Hadrons, North-Holland 1967.

Partial waves, $S^I_{l\pm} = 1 + 2i\sqrt{q}\, T^I_{l\pm} \sqrt{q}$, q = diagonal matrix of cms momenta.

s-waves, K-matrix, $T^I - i\, T^I q\, K^I = K^I$.

K^1	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$
$\bar{K}N$	0.222	0.781	0.381
$\pi\Sigma$	0.781	0.919	−0.174
$\pi\Lambda$	0.381	−0.174	0.463

B. R. Martin & Sakitt, Lund Conf. 69

K^0	$\bar{K}N$	$\pi\Sigma$
$\bar{K}N$	−1.879	0.919
$\pi\Sigma$	0.919	−0.315

Kim, PRL *19*, 1074 (67).

Inverse K-matrix, $(K^{-1})^I_{if} = M^I_{if} + \frac{r^I_i}{2}\, \delta_{if}\, (q_i^2 - q_i^2(0))$

$q^2_{\bar{K}N}(0) \equiv 0$, $q^2_{\pi^0\Sigma^0}(0) = .0328\ \mathrm{GeV}^2 = .843/fm^2$, $q^2_{\pi^0\Lambda}(0) = .0647\ \mathrm{GeV}^2 = 1.661/fm^2$.

M^1	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$
$\bar{K}N$	−3.60 ± .02	−2.86 ± .03	2.08 ± .07
$\pi\Sigma$	−2.86 ± .03	−1.40 ± .06	1.81 ± .04
$\pi\Lambda$	2.08 ± .07	1.81 ± .04	−2.31 ± .11
r^1	−0.13 ± .07	− .78 ± .23	−1.22 ± .45

M^0	$\bar{K}N$	$\pi\Sigma$
$\bar{K}N$	0 ± .02	−1.11 ± .04
$\pi\Sigma$	−1.11 ± .04	2.04 ± .10
r^0	.54 ± .08	− .89 ± .31

Kim, PRL *19*, 1074 (67).

Scattering lengths, $T^I = A^I(1 - iq\, A^I)^{-1}$, $A^I = a^I + i\, b^I$.

$A^0_{\bar{K}N}$	$A^1_{\bar{K}N}$	$A_{\pi\Lambda}$	$A^0_{\pi\Sigma}$	$A^1_{\pi\Sigma}$	Reference
−1.57 ± .04 +i(.54 ± .06)	−.24 ± .05 +i(.43 ± .05)				Kittel, Otter & Wacek, PL *21*, 349 (66)
−1.65 ± .04 +i(.73 ± .02)	−.13 ± .02 +i(.51 ± .03)	~1.7	1.09 ± .23	.39 ± .07 +i(.14 ± .03)	von Hippel & Kim, PRL *20*, 1303 (68)
−1.66 ± .02 +i(.69 ± .02	−.09 ± .03 +i(.54 ± .02)	.13 ± .07	.42 ± .03	.28 ± .03 +i(.16 ± .02)	B. R. Martin & Sakitt, Lund Conf. 69

c) Coupling constants, $G^2/4\pi$

Review and conventions: Queen, Restignoli & Violini, Fortschr. Phys. *17*, 467 (69).

$\Lambda\Sigma\pi$	$\Sigma\Sigma\pi$	Reference
21.5 ± 7.0	11.4 ± 5.0	Chan & Meiere, PL *28B*, 125 (68)
10.7 ± 0.9*	4.0 ± 2.0	for $a_{ps} = D_{ps}/(D_{ps} + F_{ps}) = 0.74$ (ps SU_3)
	10.0 ± 2.0	for $a_{pv} = D_{pv}/(D_{pv} + F_{pv}) = 0.60$ (pv SU_3)

* From Goldb.-Treiman rel. for $\Sigma \to \Lambda l\bar{\nu}$ decays, see sect. 6.

$p\Lambda K$	$p\Sigma K$	Reference	KN parametriz.
4.9 ± 0.9	< 3.0	Carter, PRL *18*, 801 (67)	CSL
5.0 ± 1.7	< 3.0	Davis et al., NP *B3*, 616 (67)	CSL
4.0 ± 0.9	< 2.4	Kim, PRL *19*, 1079 (67)	CSL
13.5 ± 2.1	0.2 ± 0.4		KM
~ 4.1	–	A. D. Martin & Poole, PL *25B*, 343 (67)	CSL
6.2 ± 1.0	–	Rood, NC *50A*, 493 (67)	KM
3.9 ± 0.6	< 3.1	Queen et al., NP *B* (69)	CSL
11.9 ± 2.7	0.4 ± 0.6		KM
5.9 ± 1.3	< 1.3	Granovskii & Starikov, SJNP *6*, 444 (68)	CSL
13 ± 3	0 ± 1	Chan & Meiere, PRL *20*, 568 (68)	KM
5.0 ± 1.9	< 2	B. R. Martin & Sakitt, Lund Conf. 69	EFR

$(p\Lambda K) + .84(p\Sigma K) = 5.1 \pm 4.0$	A. D. Martin & Ross, PL *26B*, 527 (68) (from K^+p)
$(p\Lambda K) + .847(p\Sigma K) = 12.35 \pm 2.8$	Perrin & Woolcook, NP *B4*, 671 (68)
$(p\Lambda K) + (p\Sigma K) = 15^{+6}_{-4}$	Cutkosky & Deo, PRL *20*, 1272 (68) (from K^+p)

CSL = constant scattering length, KM = K-Matrix, EFR = effective-range (consistency tests: A. D. Martin et al., NP. *B10*, 481 and PL *29B*, 311 (69)).

3. *NN* scattering

a) Scattering lengths and effective ranges

(units $fm = 1$), $k\cot\delta = +\frac{1}{a} + \frac{1}{2}rk^2$

pp	a = $7.786 \pm .008$	Slobodrian, PRL *21*, 438 (68)
	r = $2.840 \pm .009$	
nn	a = 18.42 ± 1.53	Haddock et al., PRL *14*, 318 (65), corr. by Nygren
	a = $13.1 \; ^{+3.4}_{-2.4}$	Butler et al., PRL *21*, 470 (68)
	a = 16.1 ± 1.0	Baumgartner et al., PL *16*, 105 (66)
	r = 3.2 ± 1.6	
np	$a_t = -5.425 \pm .004$	Houk & Wilson, RMP *40*, 672 (68), Err.
	$a_s = 23.714 \pm .013$	
	$r_s = 2.704 \pm .087$	
	$r_t = 1.749 \pm .008$	Wilson, Comments Nucl. Part. Phys. *2*, 141 (68) (quotes Houk)

Deuteron binding energy: 2.224644 ± 0.000034 MeV.

b) Meson-nucleon coupling constants

$\frac{G_\pi^2}{4\pi}$	$\frac{G_{\rho V}^2}{4\pi}$	$\frac{G_{\rho T}}{G_{\rho V}}$	$\frac{G_{\omega V}^2}{4\pi}$	Reference
14.4	4.8 ± 0.8	1.68	4.7 ± 2.1	Köpp & Söding, PL *23*, 494 (66)
14.01	3.12	2.38	8.02	Ueda & Green, PR *174*, 1304 (68)
14.02	3.12	2.39	7.97	Ueda & Green, NP *B10*, 289 (69)
$4.1^{+1.4}_{-0.8}$	–	–	–	Cutkosky & Deo, PRL *20*, 1272 (68)
$14.72 \pm .83$	–	–	–	Mac Gregor et al., PR *169*, 1128 (68)
$14.73 \pm .3$	$7.53 \pm 4.$	2.5 ± 1	13.5 ± 1.7	Bugg, NP B 5, 29 (68)

Note: Köpp & Söding take $G_{\rho T}/G_{\rho V}$ from a nucleon form factor fit. $G_{\omega T}$ and G_η are not well determined.

4. Photon couplings

a) Vector meson-photon couplings

$(\alpha^{-1}=137.036)$

Review: Ting, Proc. Vienna Conf. 1968 ($f_V=2\gamma_V$ of Ting).

Vector meson dominance:

$$J_\mu^{em}(x)=-\left\{\frac{m_\rho^2}{f_\rho}\rho_\mu(x)+\frac{m_\omega^2}{f_\omega}\omega_\mu(x)+\frac{m_\varphi^2}{f_\varphi}\varphi_\mu(x)\right\}$$

Leptonic decay width:

$$\Gamma(V\to e^+e^-)=\frac{1}{3}\alpha^2 m_V\left(\frac{f_V^2}{4\pi}\right)^{-1};\quad V=\rho,\omega,\varphi.$$

$f_\rho^2/4\pi$	$f_\omega^2/4\pi$	$f_\varphi^2/4\pi$	Reference
$2.08^{+0.28}_{-0.24}$	$18.76^{+4.96}_{-3.24}$	$12.16^{+4.28}_{-2.64}$	Ting, Vienna Conf. 68
1.86 ± 0.18	14.8 ± 2.8	11.0 ± 1.6	Augustin et al, PL *28B*, 503 (69)
2.10 ± 0.11	(from $\rho\to2\pi$ width)		Augustin et al, PL *28B*, 503 (69)
2.00 ± 0.28	16.0 ± 3.6	12.4 ± 2.8	Lohrmann, Lund Conf. 69

Note: All values refer to vector meson V on the mass shell; for the photon on the mass shell Ting gives $f_\rho^2/4\pi=2.12\pm0.16$.

b) Pion form factor

From π^+ electroproduction [Mistretta et al., PRL *20*, 1523(68)]:

$$F_\pi(t)=\left[1-\frac{t}{(0.56\pm.08)^2\,\text{GeV}^2}\right]^{-1}\quad (t<0\text{ only})$$

From e^+e^- colliding beams [Roos and Pisút, NP *B10*, 563 (69)]:

$$F_\pi(t)=\frac{t_1+t}{t_1}\,\frac{\omega_r^2-\omega_r\left(\frac{q}{q_r}\right)^3\Gamma_0}{\omega_r^2-t-i\omega_r\left(\frac{q}{q_r}\right)^3\Gamma_0};$$

$$t_1=4\{|\omega_r^2-\omega^2-i\omega_r\left(\frac{q}{q_r}\right)^3\Gamma_0|+\mu^2\}$$

$$\omega_r=770\pm4\text{ MeV};\quad \Gamma_0=122^{+7}_{-6}\text{ MeV};\quad \omega^2\equiv t=4(q^2+\mu^2).$$

c) Nucleon form factors

Weber, Proc. Stanford Conf. 1967:

Scaling law: $G_{E_p}(t)=\frac{G_{M_p}(t)}{\mu_p}=G(t)$ for $-t\leq4$ (GeV/c)^2

$\frac{G_{M_n}(t)}{\mu_n}=G(t)$ for $-t<1.2$ GeV^2, $G_{E_n}(t)\leq0.2$ but $\neq0$.

Dipole fit: $G(t)=\left(1-\frac{t}{M_D^2}\right)^{-2};\quad M_D^2=0.71$ GeV^2.

Deviations at larger t values:

from dipole fit for $2<-t<25.0$ (GeV/c)^2, Coward et al. PRL *20*, 292 (68); from scaling law for $0.6<-t<2$ (GeV/c)^2: $G_{E_p}(t)=\frac{G_{M_p}(t)}{\mu_p}[1+(0.066\pm0.023)t]$, Berger et al., PL *28B*, 276 (68).

d) Radiative decays $\pi^0\to\gamma\gamma$ and $\eta\to\gamma\gamma$

$$\Gamma_{\gamma\gamma}=\frac{1}{16}\alpha^2m^3\frac{g^2}{4\pi}$$

	π^0	η
$\Gamma_{\gamma\gamma}$ [eV]	8.0	1000
$g^2/4\pi$ [GeV^{-2}]	976	1820

We use $\tau(\pi^0)=0.82\times10^{-16}$sec and take $\Gamma_{\gamma\gamma}(\eta)$ from Particle Data Group, RMP *41*, 109 (69).

5. Pion photoproduction

a) Multipoles at threshold

$(\hbar=c=m_\pi=1)$.

Notation: CGLN, PR *106*, 1345 (57)

$100\,E_{0+}(\pi^+)=+2.853\pm0.016$	Adamovich et al., SJNP *7*, 643 (68)
$100\,E_{0+}(\pi^-)=-3.15\pm0.06$	Adamovich et al., SJNP *2*, 95 (66)
$100\,E_{0+}(\pi^0)=-0.18\pm0.09$	Govorkov et al., SJNP *4*, 265 (67)

b) γNN^* magnetic dipole moment

Review: Rollnik, Hercegnovi lectures 1967 (to be published).

μ^* [in units of μ_{proton}]	Reference
1.21 ± 0.02	Dalitz and Sutherland, PR *146*, 1180 (66)
1.17 ± 0.04	Donnachie and Shaw, NP *87*, 556 (67)
1.16 ± 0.03	Grilli et al., NC *49*, 326 (67)
1.07 ± 0.01	Ash et al., PL *24 B*, 165 (67)
1.18 ± 0.11	Pfeil, Thesis, Bonn Univ. (68)
1.23	Schwela and Weizel, Z. Phys. *221*, 71 (69)

6. Weak interactions

(units $c=\hbar=1=6.58218\times10^{-25}$ GeV sec). See Parker et al., RMP *41*, 375 (69).)

Review: Brene, Roos & Sirlin, NP *B6*, 255 (68).

a) μ-decay

$\Gamma_\mu = (2.9944 \pm .0011)\ 10^{-10}$ eV.

$g_\mu = (192\pi^3\ \Gamma_\mu\ m_\mu^{-5})^{1/2} = (1.1635 \pm .0002)\ 10^{-5}/\mathrm{GeV}^2$.

rad. correction, $g_{\mu,\ \mathrm{cor.}} = g_\mu \left(1 + \frac{\alpha}{4\pi}\left(\pi^2 - \frac{25}{4}\right)\right) =$

$= (1.1659 \pm .0002)\, 10^{-5}/\mathrm{GeV}^2$.

b) β-decay

$g_V = (1.140 \pm .002)10^{-5}/\mathrm{GeV}^2$	Brene et al., NP *B 6*, 255 (68)
$(1.144 \pm .002)10^{-5}/\mathrm{GeV}^2$	Freeman et al., PL *27 B*, 156 (68) (rad. corr. included, but model-dependent)
$g_A = (1.23 \pm .01)g_V$	
$= (1.40 \pm .01)10^{-5}/\mathrm{GeV}^2$	Christensen et al., PL *26 B*, 11 (67)

c) π-decay

$\Gamma(\pi \to \mu\nu) = (2.527 \pm .007)\, 10^{-8}$ eV.

$|g_\pi| = (4\pi\Gamma(\pi \to \mu\nu)\, m_\pi)^{1/2}\ m_\pi/m_\mu(m_\pi^2 - m_\mu^2) =$
$= (1.057 \pm .002)\, 10^{-6}/\mathrm{GeV}$

$f_\pi \equiv \sqrt{2}\ g_\pi/g_V = 130.9\ \mathrm{MeV} = 0.938\ m_{\pi+}$.

d) Goldberger-Treiman relation

$-g_\pi(t=0)\ G(t=0) = \frac{1}{2}(m_p + m_n)g_A$

$g_\pi(t=0)\ G(t=0)/g_\pi G = 0.917 \pm .03$ for $G = \sqrt{4\pi \times 14.73}$

$\Sigma \to \Lambda$ decays, $-g_\pi(t_{\Sigma\Lambda}) \cdot G_{\Lambda\Sigma\pi}(t_{\Sigma\Lambda}) = \frac{1}{2}(m_\Lambda + m_\Sigma)g_{A,\ \Lambda\Sigma\pi}(t)$

pv-coupling, $f_{\Lambda\Sigma\pi} = -\sqrt{\tfrac{4}{3}}\ \alpha_{pv} f$, $\alpha \equiv D/(D+F)$

$\alpha_{pv} = .613 \pm .023$ Brene, Roos & Sirlin, NP *B6*, 255 (68), and pr. comm.

ps-coupling, $G_{\Lambda\Sigma\pi} = -\sqrt{\tfrac{4}{3}}\ \alpha_{ps}\ G$, $\alpha_{ps} = \alpha_{pv}\frac{m_\Lambda + m_\Sigma}{m_p + m_n} = 1.23\ \alpha_{pv}$

$\alpha_{ps} = .75 \pm .03$ Pilkuhn & Swoboda, Lett. *NC.* 1, 854 (69)

e) Cabibbo angles from leptonic hyperon decays

$\theta_V = .233 \pm .012$, $\theta_A = .238 \pm .018$ Filthuth, CERN 69—7 (Proc. Top. Conf. on Weak Int.) $\alpha_{pv} = .60 \pm .04$.

f) Pionic hyperon decays

$$\Gamma = p\frac{(M+M')^2 - m_{\pi+}^2}{8\pi M^2}\left(|A|^2 + \frac{(M-M')^2 - m_{\pi+}^2}{(M+M')^2 - m_{\pi+}^2}|B|^2\right)$$

	Λ_-	Σ_-^-	Σ_+^+	Σ_0^+ $\gamma>0$	Σ_0^+ $\gamma<0$	Ξ_-^-	Ξ_0^0
$A \times 10^7$	3.30 ± .04	4.06 ± .07	0.04 ± .09	3.32 ± .30	2.50 ± .39	4.05 ± .07	3.32 ± .11
$B \times 10^7$	22.67 ± .71	−1.19 ± .93	41.43 ± .76	−25.02 ± 4.02	−33.35 ± 3.04	−16.11 ± 1.41	−9.86 ±2.21

Filthuth, CERN 69—7. *Note*: our A and B are dimensionless. For dimension $10^5/\sqrt{\mathrm{sec}}$, Γ contains an additional $1/m_{\pi+}$. The conversion factor to dimensionless A and B is 2.1715×10^{-7}.