Neutrino Model Building

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Abstract. We review various models of flavour symmetries that have been proposed in order to explain the solar and atmospheric neutrino deficits. Although there is a wide range of solutions within different frameworks, it is possible to identify some common characteristics which, in view of the expected neutrino data, may be used to constraint or even exclude certain classes of models.

1 Phenomenology of neutrino mass textures

The SuperKamiokande data [1] clearly indicate a ν_μ/ν_e ratio in the atmosphere that is significantly smaller than the Standard Model expectations. The most natural way to explain this deviation is by introducing ν_μ–ν_τ oscillations, with $\delta m^2_{\nu_\mu \nu_\tau} \approx (10^{-2} \text{ to } 10^{-3})$ eV^2 and $\sin^2 2\theta_{\mu\tau} \geq 0.85$. Similarly, the solar neutrino deficit, confirmed by SNO, can be resolved by vacuum or matter-enhanced (MSW) oscillations [3]. It turns out that both deficits can be accommodated in minimal schemes with

(a) three light neutrinos with hierarchical masses, of the order of the required mass differences for the atmospheric and solar deficits. Then, the atmospheric neutrino data would require $m_3 \approx (10^{-1} \text{ to } 10^{-1.5})$ eV, while m_2 is significantly smaller and its magnitude depends on the particular solar neutrino solution.

(b) textures with three almost degenerate neutrinos; if the mass scale is $\mathcal{O}$(eV), neutrinos may also provide a component of hot dark matter.

(c) inverted hierarchy solutions, with $|m_1| \sim |m_2| \gg |m_3|$, where $m^2_{1,2} \sim \Delta m^2_{atm}$ and $\Delta m_{12} = \Delta m^2_{sun}$. Unlike case (b), in these schemes no significant hot dark matter component is obtained.

The minimal schemes with only three neutrino masses, allow only two independent mass differences and thus the LSND result indicating $\bar{\nu}_\mu$–$\bar{\nu}_e$ and ν_μ–ν_e oscillations [4], cannot be simultaneously explained unless a sterile light neutrino state is introduced. In models with light sterile neutrinos, one has to take into account the constraints from cosmological Big Bang Nucleosynthesis: a sterile neutrino that mixes with an active one, thus being in equilibrium at the time of nucleosynthesis, can change the abundance of primordially produced elements, such as ^4He and deuterium. The larger the mixing and the mass differences between the sterile and active neutrinos, the bigger the deviations from the observed light element abundances. This implies that models where the sterile component contributes to solar rather than atmospheric neutrino oscillations, are accommodated easier within the standard nucleosynthesis scenarios. Here, in order to retain a simple connection of the neutrino masses with the known charge lepton and quark hierarchies, we focus on models without sterile neutrinos.

Naturally light neutrinos can be obtained via the see-saw mechanism [5]; this relates the light neutrino mass matrix, m_{eff}, with the Dirac neutrino matrix, m_D, and the heavy right-handed Majorana mass matrix, M_R, via the formula

$$m_{eff} = m_D \cdot M_R^{-1} \cdot m_D^T \tag{1}$$

Finally, the leptonic mixing matrix is given by $V_{MNS} = V_\nu^\dagger V_\ell$, where V_ℓ diagonalizes the charged-lepton mass matrix, while V_ν diagonalizes the light neutrino mass matrix, m_{eff} [6].

The various phenomenological textures can be classified by their predictions for the neutrino mixing and mass hierarchies. In particular, we would like to know the answer to the following questions:

- Is the atmospheric neutrino mixing maximal, or close-to-maximal?
- Which solution do we have for the solar neutrino problem?
- Are the neutrino masses Dirac or Majorana [1]?
- Are the neutrinos degenerate or hierarchical?
- In a given model and basis, does the mixing dominantly arise from V_ν or V_ℓ?

Even before passing in detail to phenomenological textures, we can gain some insight by looking at the simple formulas for mass eigenvalues and mixing for a 2×2 matrix. ¿From

$$m_{eff} = \begin{pmatrix} m_{22} & m_{23} \\ m_{23} & m_{33} \end{pmatrix}, \quad \sin^2 2\theta = \frac{4m_{23}^2}{(m_{33} - m_{22})^2 + 4m_{23}^2}$$

$$m_2 + m_3 = \text{Trace}[m_{eff}], \quad m_2\, m_3 = \text{Det}[m_{eff}]$$

we see that:

(i) For large 23 mixing in m_{eff}, *large hierarchies require 0-determinant solutions.*

(ii) On the other hand, if the large mixing comes from the charged-lepton sector, we can have *large hierarchies without 0-determinant solutions.*

2 Introduction to Abelian flavour symmetries

The fact that the fermion mass matrices exhibit a hierarchical structure suggests that they are generated by an underlying family symmetry. Under such

[1] Some light on this may be shed by experiments on neutrinoless double beta decay $(d + d \to u + u + e + e)$, which may only occur for Majorana neutrinos. On the other hand, only Dirac masses give rise to diagonal neutrino magnetic moment contributions.

a symmetry different generations of fermions have different charges. Requiring that only invariant operators are allowed, will determine the magnitude of masses.

Let us start with the simplest possibility, which is this of an abelian flavour-symmetry. We denote the charges of the Standard Model fields under the symmetry as appears in Table 1.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c
$U(1)$	α_i	β_i	γ_i	b_i	c_i	d_i

Table 1. $U(1)$ *charges of the various fields, where i stands for a generation index.*

The Higgs charges are chosen so that the terms $f_3 f_3^c H$ (where f denotes a fermion and H denotes H_1 or H_2) have zero charge. Then, only the (3,3) element of the associated mass matrix will be non-zero. The remaining entries are generated when the $U(1)$ symmetry is spontaneously broken, via standard model singlet fields, θ, $\bar{\theta}$, with non-trivial opposite $U(1)$ charges, and equal vacuum expectation values. The suppression factor for each entry depends on the family charge: the higher the net $U(1)$ charge of a term $f_i f_j^c H$, the higher the power n in the term $f_i f_j^c H \left(\frac{\theta}{M}\right)^n$ that has zero charge. For example, if only the 2–3 and 3–2 elements of the matrix are allowed by the symmetry at order $\epsilon \equiv \theta/M$, one has the following hierarchy of masses:

$$\mathcal{M} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \tag{2}$$

where M is an intermediate mass scale, determined by the mechanism that generates the non-renormalisable terms. The symmetry breaking arises via an extension of the "see-saw" mechanism, mixing light to heavy states (known as the Froggatt–Nielsen mechanism [7]).

We can now see that the choice $\alpha_i = \beta_i = \gamma_i = (-4, 1, 0)$ and zero Higgs U(1) charges, leads to a solution which reproduces the known quark hierarchies [8]:

$$M_{up} \propto \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, M_{down} \propto \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix}, \quad \text{where } \bar{\epsilon} = \sqrt{\epsilon} \approx 0.22$$

Similarly, one may derive acceptable charged lepton and neutrino textures, that also reproduce the neutrino data.

3 Neutrino masses in Left-Right symmetric models

In Left-Right symmetric models, the $U(1)$ family charges are strongly constrained since the symmetry requires identical $U(1)$ charges of the left- and right-handed fields. In the previous section, we discussed the predictions for quark masses. For charged leptons, the choice of charges

$$\begin{aligned} b_i = c_i = d_i &= \left(-\frac{7}{2}, \frac{1}{2}, 0\right) \\ b_i = c_i = d_i &= \left(\frac{5}{2}, \frac{1}{2}, 0\right) \end{aligned} \tag{3}$$

leads to two possible charged-lepton matrices :

$$M_\ell \propto \begin{pmatrix} \bar\epsilon^7 & \bar\epsilon^3 & \bar\epsilon^{7/2} \\ \bar\epsilon^3 & \bar\epsilon & \bar\epsilon^{1/2} \\ \bar\epsilon^{7/2} & \bar\epsilon^{1/2} & 1 \end{pmatrix}, M_\ell \propto \begin{pmatrix} \bar\epsilon^5 & \bar\epsilon^3 & \bar\epsilon^{5/2} \\ \bar\epsilon^3 & \bar\epsilon & \bar\epsilon^{1/2} \\ \bar\epsilon^{5/2} & \bar\epsilon^{1/2} & 1 \end{pmatrix} \tag{4}$$

Both these matrices lead to natural lepton hierarchies for $\bar\epsilon \approx 0.22$ and imply large but non-maximal lepton mixing.

What about neutrino masses? The neutrino Dirac mass is specified to be of the same type as for the charged leptons, but with a different expansion parameter. Actually, since neutrinos (charged leptons) and up-type (down-type) quarks couple to the same Higgs, they should have the same expansion parameter $\epsilon(\bar\epsilon)$. Then,

$$m_D \propto \begin{pmatrix} \epsilon^7 & \epsilon^3 & \epsilon^{7/2} \\ \epsilon^3 & \epsilon & \epsilon^{1/2} \\ \epsilon^{7/2} & \epsilon^{1/2} & 1 \end{pmatrix}, \ m_D \propto \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon^{5/2} \\ \epsilon^3 & \epsilon & \epsilon^{1/2} \\ \epsilon^{5/2} & \epsilon^{1/2} & 1 \end{pmatrix}$$

for the two choices of charges in (3) respectively.

Of course the mass structure of neutrinos is more complicated, due to the heavy Majorana masses of the right-handed components. These arise from a term of the form $\nu_R \nu_R \Sigma$, where Σ is a $SU(3) \times SU(2) \times U(1)$ invariant Higgs scalar field with $I_W = 0$. The possible choices for the Σ charge will give a discrete spectrum of possible forms for the Majorana mass, M_R. For example, if Σ has the same charge with the Higgs doublets, the form of the heavy Majorana mass matrix will be similar to that of the charged leptons. For simplicity of presentation, here we isolate this choice of Σ charge and discuss the set of textures that result out of the solution with $b_i = c_i = d_i = \left(\frac{5}{2}, \frac{1}{2}, 0\right)$. However we stress that for this model based on an abelian $U(1)$, the form of M_R only affects the neutrino eigenvalues, *but not the mixing*. For the particular choice made, one finds that [9]:

$$m_{eff} = \begin{pmatrix} \bar\epsilon^{10} & \bar\epsilon^6 & \bar\epsilon^5 \\ \bar\epsilon^6 & \bar\epsilon^2 & \bar\epsilon \\ \bar\epsilon^5 & \bar\epsilon & 1 \end{pmatrix}, \ m_{eff}^{diag} = \begin{pmatrix} \bar\epsilon^{15} & & \\ & \bar\epsilon^3 & \\ & & 1 \end{pmatrix}$$

To explain the neutrino deficits by hierarchical masses, we need: m_{ν_μ}/m_{ν_τ} $\mathcal{O}(0.01-0.1)$, which is fulfilled. Moreover,

$$V_{MNS} = V_\nu^\dagger V_\ell = \begin{pmatrix} 1-... & \bar\epsilon^2 & \bar\epsilon^{5/2} \\ -\bar\epsilon^2 & 1-... & \sqrt{\bar\epsilon}+\bar\epsilon \\ -\bar\epsilon^{5/2} & -\sqrt{\bar\epsilon}-\bar\epsilon & 1-... \end{pmatrix}$$

Then, $sin^2 2\theta_{\mu\tau}$ is ≈ 1, as required. What is interesting to observe, is that the $(e-\mu)$ mixing is specified by the charged lepton masses to be

$$V_\ell^{e\mu} \approx \frac{M_\ell^{12}}{m_\mu} \approx \bar\epsilon^2 \approx 0.05$$

thus predicting the small angle MSW solution for the solar neutrino deficit. This is in fact a feature often encountered in models with Abelian flavour symmetries, where the lack of charge quantization implies that we may not specify the phases and therefore we may not require accurate cancellations between the various entries of the mass matrices. In this case, large neutrino mass hierarchies and small angle solar mixing determined by the charged lepton sector are naturally favoured. Bimaximal mixing can still arise, but has to be generated by the see-saw conditions *in a model-dependent way.*

4 Fermion masses from GUT symmetries

An interesting question that arises is whether realistic fermion mass structures are consistent with the constraints on an Abelian family symmetry in GUT-embedded solutions and, if yes, which GUT schemes would be favoured. Along these lines, a huge number of proposals have appeared in the literature [10].

Here, in order to have a minimal model dependence and increase predictivity, we will assume that the mass textures are entirely determined by the U(1) and the GUT-multiplet structure *without additional help from Higgs or heavy GUT fields.* This means that the GUT structure is only used in order to constrain the $U(1)$ charges. Moreover, since we have an abelian flavour symmetry, we will assume a large charged-lepton mixing, and no cancellations in m_{eff}.

4.1 $SO(10)$

In an $SO(10)$ GUT, all quarks and leptons are accommodated in the 16 representation of the group. This implies that the quark and lepton charges for the left- and right-handed fields of a given family are the same. Moreover, since both Higgs fields of the Minimal Supersymmetric Standard Model fit in a single 10-plet of $SO(10)$, in the simplest scheme one would predict left-right-symmetric mass matrices with similar structure for all fermions.

However, this implies the prediction $V_{\mu\tau} \approx V_{cb}$, which may only be reconciled with observations either with the help of coefficients (which is unnatural) or by considering the effects of the additional Higgs multiplets that are required for breaking $SO(10)$ down to $SU(3) \times SU(2) \times U(1)$. This way, one can generate several operators with rank ≥ 4 in the mass matrices, however the predictivity of the $U(1)$ symmetry is reduced since the choice of an operator at a given entry is only phenomenological.

4.2 $SU(5)$

The field structure of $SU(5)$ implies that

$$\begin{aligned} Q_{(q,u^c,e^c)_i} &= Q_i^{10} \\ Q_{(l,d^c)_i} &= Q_i^{\overline{5}} \\ Q_{(\nu_R)_i} &= Q_i^{\nu_R} \end{aligned}$$

and therefore: (i) the up-quark mass matrix is symmetric and (ii) the charged-lepton mass matrix is the transpose of the down-quark one, thus relating the left-lepton with the right-quark mixing. This explains how the large mixing angle that is observed in atmospheric neutrinos can be consistent with the small V_{CKM} mixing, without any tuning. A possible pattern for masses and mixings is:

$$\frac{M_{up}}{m_t} = \begin{pmatrix} \bar{\epsilon}^6 & \bar{\epsilon}^5 & \bar{\epsilon}^3 \\ \bar{\epsilon}^5 & \bar{\epsilon}^4 & \bar{\epsilon}^2 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}, \frac{M_{down}}{m_b} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon}^3 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon}^2 \\ \bar{\epsilon} & 1 & 1 \end{pmatrix}, \frac{M_\ell}{m_\tau} = \begin{pmatrix} \bar{\epsilon}^4 & \bar{\epsilon}^3 & \bar{\epsilon} \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & 1 \end{pmatrix}$$

On the other hand, obtaining the correct $V_{CKM}^{12,21}$, inevitably leads to a larger m_{up} than indicated by the data. The abelian symmetry alone may not guarantee the smallness of m_{up} without introducing large coefficients or cancellations. However such a small term may in principle be generated by alternative means.

4.3 Flipped-SU(5)

In the case of the flipped-$SU(5)$, the fields Q_i, d_i^c and ν_i^c belong to a 10 of $SU(5)$, while u_i^c and L_i belong to a $\overline{5}$. Finally, the e_i^c fields belong to singlet representations of $SU(5)$. This assignment implies symmetric down-quark mass matrices. The structure of the up-quark mass matrix will depend on the charges of the right-handed quarks. However, as these are the same with the charges of the left-handed leptons, the mass matrix will be constrained by the need to generate large mixing for atmospheric neutrinos. In this model, it turns out that the contribution from the up-quark sector to V_{cb} is negligible [9] and therefore $V_{cb} \simeq \sqrt{m_s/m_b}$. This is too large and requires a significant coefficient adjustment. However, it has been shown that the string-embedded flipped $SU(5)$ model, due to its additional (although highly constrained) structure, works in a nice way [11].

4.4 $SU(3)_c \times SU(3)_L \times SU(3)_R$

In $SU(3)_c \times SU(3)_L \times SU(3)_R$, the left- and right-handed quarks belong to a $(3,3,1)$ and $(\bar{3},1,\bar{3})$ respectively and thus their $U(1)$ charges are not related. On the other hand the left and right-handed leptons belong to the same $(1,3,\bar{3})$ representation and hence must have the same $U(1)$ charge. Thus, the lepton mass matrices have to be symmetric, and similar to those of a left-right symmetric model. Since the quark mass matrices are asymmetric (with different expansion parameters but similar structure for up- and down-quarks), it is straightforward to chose $U(1)$ charges, such that all quark hierarchies are fulfilled. This choice, does not impose any constraints on the lepton charges.

A summary of fermion mass parameters for the various GUT groups that we discussed is presented in Table 2.

	SU(5)	SO(10)	flip.SU(5)	$SU(3)^3$	L-R-sym.
M_{up}	Symmetric	Symmetric (similar structure for all fermions)	Asymmetric	Asymmetric	Symmetric
M_{down}	Asymmetric	Symmetric	Symmetric	Asymmetric	Symmetric
M_ℓ	$M_\ell = M_{down}^T$	Symmetric	Asymmetric correlated to up	Symmetric	Symmetric
m_D	uncorrelated (ν_R in singlet)	Symmetric	$m_D = M_{up}^T$	Symmetric	Symmetric
V_{cb}	$V_{\mu\tau} \gg V_{cb}$ $\surd$	$V_{\mu\tau} \approx V_{cb}$ $\times$	very large $V_{\mu\tau} \gg V_{cb}$ $\times$	$V_{\mu\tau} \gg V_{cb}$ $\surd$	large $V_{\mu\tau} \gg V_{cb}$?
M_{up}	a bit high ?			$\surd$	$\surd$

Table 2. *Table of fermion mass and mixing parameters for different GUT groups, in schemes where a single $U(1)$ symmetry and the minimal GUT fermion multiplet structure entirely determine the fermion mass matrices. A "$\times$" implies that this simple framework has to be extended in order to obtain acceptable fermion mass patterns.*

This table, by no means provides a non-go statement. It simply indicates that if we want to construct for instance an $SO(10)$ or a flipped-$SU(5)$ neutrino model, we should introduce additional degrees of freedom in our theory, thus departing from the simpler schemes where the flavour-symmetry and the GUT-structure alone can predict all fermion mass textures.

5 Abelian versus non-Abelian flavour symmetries

So far, we discussed models with abelian flavour symmetries and we saw that:

• Large splitting between masses is to be expected, naturally leading to large neutrino hierarchies.

• Due to the existence of unknown phases and thus order unity coefficients, it is more difficult to obtain degenerate neutrinos.

• In many models lepton hierarchies consistent with mostly small angle MSW solutions, but large angle solutions to the solar neutrino problem are possible, when generated by the see-saw conditions.

The situation changes when passing to non-Abelian flavour symmetries [12]. Let us for instance look at the simple case where the the lepton fields are $SO(3)$ triplets. Then, degenerate lepton textures are to be expected. Subsequently we break SO(3) so that large charged lepton but small neutrino mass splitting is generated. This means that in non-Abelian models we expect that:

• Solutions with almost-degenerate neutrinos can be naturally generated.

• Textures with (almost)-bimaximal mixing are mostly predicted.

Successful non-abelian models with bimaximal mixing and large neutrino hierarchies have also been proposed. For instance the $SU(3)$ flavour model of [13], gives rise to matrices of the form

$$M_{\nu_D} \propto \begin{pmatrix} 0 & \lambda\,\epsilon^3 & \lambda\,\epsilon^3 \\ -\lambda\,\epsilon^3 & \epsilon^2 & \epsilon^2 \\ -\lambda\,\epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \; M_{\nu_R} \propto \begin{pmatrix} \epsilon_\nu^4 & 0 & \epsilon_\nu^3 \\ 0 & \epsilon_\nu^3 & \epsilon_\nu^3 \\ \epsilon_\nu^2 & \epsilon_\nu^3 & 1 \end{pmatrix} \tag{5}$$

with approximate bimaximal mixing and neutrino hierarchies obeying $\frac{m_2}{m_3} \sim \epsilon_\nu < \epsilon$, thus consistent with the LOW or quasi-vacuum solar solutions.

¿From this discussion, we conclude that different types of theoretical models "prefer" different solutions of the solar and atmospheric neutrino deficits. There is a large number of proposals in the literature, however the new neutrino data can help us to constrain or even exclude many of the existing models.

6 Stability of neutrino textures under quantum corrections

So far, we discussed neutrino mass textures at the unification scale. However, as has been discussed in [14], in the presence of neutrino masses, the running of the various couplings from the unification scale down to low energies is modified. For the neutrino sector, the Dirac neutrino Yukawa coupling, λ_N, runs until the scale M_N. Subsequently it decouples and the quantity that runs is the effective neutrino operator m_{eff}. For i, j, generation indices, in a supersymmetric model one finds that

$$\frac{1}{m_{eff}^{ij}} \frac{d}{dt} m_{eff}^{ij} = \frac{1}{8\pi^2} \left(-c_i g_i^2 + 3\lambda_t^2 + \frac{1}{2}(\lambda_i^2 + \lambda_j^2) \right)$$

$$16\pi^2 \frac{d}{dt}\sin^2 2\theta_{23} = 2\sin^2 2\theta_{23}(1-2\sin^2\theta_{23})\lambda_\tau^2 \frac{m_{eff}^{33}+m_{eff}^{22}}{m_{eff}^{33}-m_{eff}^{22}}$$

We see that $\sin^2 2\theta_{23}$ is affected by quantum corrections if λ_τ is large (large $\tan\beta$ of a supersymmetric model) and if $m_{eff}^{33}-m_{eff}^{22}$ is small. This formula implies that the mixing can be amplified or even destroyed, as we go down to low energies. Moreover, we observe that the neutrino masses will in fact vary non-trivially with the energy. It is convenient to define the integrals

$$I_g = \exp[\frac{1}{8\pi^2}\int_{t_0}^{t}(-c_i g_i^2 dt)] \tag{6}$$

$$I_t = \exp[\frac{3}{8\pi^2}\int_{t_0}^{t}\lambda_t^2 dt] \tag{7}$$

$$I_i = \exp[\frac{1}{8\pi^2}\int_{t_0}^{t}\lambda_i^2 dt], \quad i = e,\mu,\tau \tag{8}$$

Then [14],

$$m_{eff} \propto \begin{pmatrix} m_0^{11}\ I_e & m_0^{12}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_0^{13}\ \sqrt{I_e}\ \sqrt{I_\tau} \\ m_0^{21}\ \sqrt{I_\mu}\ \sqrt{I_e} & m_0^{22}\ I_\mu & m_0^{23}\ \sqrt{I_\mu}\ \sqrt{I_\tau} \\ m_0^{31}\ \sqrt{I_e}\ \sqrt{I_\tau} & m_0^{32}\ \sqrt{I_\mu}\ \sqrt{I_\tau} & m_0^{33}\ I_\tau \end{pmatrix}$$

where the initial conditions are denoted by m_0^{ij}. As we have already mentioned, these conditions are defined at M_N, the scale where the neutrino Dirac coupling λ_N decouples from the renormalization-group equations. ¿From (9), we see that the relative structure of m_{eff} is only modified by the charged-lepton Yukawa couplings. On the contrary, the top and gauge couplings give only an overall scaling factor. We then see that while these renormalization effects are not significant for schemes with hierarchical neutrino masses, in models with degenerate neutrinos at the GUT scale, they can have a dramatic effect. In particular, they can spoil the required neutrino degeneracy, even for small $\tan\beta$ [14].

7 Neutrino masses and lepton-flavour-violating processes

In the Standard Model with massive neutrinos, processes such as $\mu \to e\gamma$ or $\mu - e$ conversion on nuclei are extremely suppressed. This is not however the case in supersymmetric theories, due to the existence of heavy fermions in the loop-diagrams that mediate the above processes. The magnitude of the rates depends on the masses and mixings of superparticles and for non-diagonal sfermion masses at M_{GUT}, large rates are in general predicted [15].

However, even if the sfermion mass matrices at a high scale are diagonal (as in no-scale or gauge-mediated models), renormalization effects of the Minimal Supersymmetric Standard Model with right-handed neutrinos will spoil this diagonal form. Indeed, the Dirac neutrino and charged-lepton Yukawa couplings cannot, in general, be diagonalized simultaneously. Since both these sets of lepton Yukawa couplings appear in the renormalization-group equations, the slepton mass matrices receive off-diagonal contributions, which in the basis where m_ℓ is diagonal, are

$$\delta m_{\tilde{\ell}}^2 \propto \frac{1}{16\pi^2}(3+a^2)\ln\frac{M_{GUT}}{M_N}\lambda_N^\dagger\lambda_N m_{3/2}^2, \tag{9}$$

(where a is related to the trilinear mass parameter and $m_{3/2}^2$ is the common value of the scalar masses at the GUT scale).

This implies that different models predict in general different rates for lepton-flavour violation: the larger the mixing and the larger the neutrino mass scales that are required, the larger the rates. Consequently, for schemes with degenerate neutrinos and bimaximal mixing, we expect significantly larger effects than for hierarchical neutrinos with a small vacuum mixing angle. Note however that, for the just-so solutions to the solar neutrino problem (where a $\delta m^2 \approx 10^{-10}$ eV^2 is required), the predicted rates in the case of hierarchical neutrinos are small, even if the leptonic mixing in the $e-\mu$ flavours is large.

8 Summary

We discussed different models of neutrino mass textures, in the light of the atmospheric and solar neutrino data. There have been various proposals in the literature, on how the viable phenomenological textures may arise in models with flavour and GUT symmetries. Here, we focus in identifying common characteristics of the various theories, in a way that the new data can constrain or even exclude whole classes of models. For instance, a generic feature of models based on Abelian flavour symmetries is that, in the absence of see-saw cancellations, most models predict a small mixing angle for the solar neutrino deficit and hierarchical neutrino masses. On the other hand, non-Abelian flavour symmetries can generate more naturally degenerate neutrinos with bimaximal mixing. The future data will certainly give us some additional information on the neutrino parameters and thus on the flavour structure of the underlying fundamental theory.

Quantum corrections may drastically modify the neutrino masses and mixings from high to low scales. Finally, low energy lepton-flavour-violating processes, may in certain frameworks provide additional insight to neutrino mass textures.

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