Pseudo-supersymmetry and a Tale of Alternate Realities

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Abstract

We discuss how all variant 10d maximal supergravities, including star supergravities and supergravities in different signatures, can be obtained as different real slices of two complex actions. As an application we study the domain-wall/cosmology correspondence in this approach. We give a 10d example where the domain-wall and corresponding cosmology can be viewed as different real slices of the same complex solution. In this case the pseudo-supersymmetry of the cosmological solutions can be understood as the invariance under supersymmetry of a variant supergravity.

1 Introduction

In this proceeding we will discuss how one can obtain supergravity actions for different signatures as different real slices of a single complex action. The strategy we will follow in obtaining actions and supersymmetry transformation rules for these supergravities, is based on the observations made in [1]. Sometimes these real slices lead to different supergravity theories with the same signature. For such cases we will find a connection between the star theories of [2] and the domain-wall/cosmology correspondence [3]. For related work see [4–7]. This proceeding is based on the work done in [11].

The starting point of our construction will be a complex action that then can be reduced to different real actions. In this proceeding we will not address the question of how one can in general construct sensible complex actions. The idea is to start from a known action in terms of some real fields that is invariant under some real symmetry group. The first step is to construct a complexified version of this action that is invariant under the complexified symmetry group. We require that the real action we started from can be obtained from this complexified action by imposing certain reality conditions and similarly for the symmetries. At this point one faces the natural question: are there different real slices leading to other theories? As it will turn out, theories in different signatures are found by taking different reality conditions for a single complex action. In the case one has *extended* supersymmetry it can even happen that one finds multiple real

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theories in one signature. It is these issues that we have worked out in detail for (massive) type IIA, type IIB and 11d supergravity in [11] and we refer to it for all the details that we omit in this proceeding.

This general scheme of finding different real actions as consistent real slices of a given complex action can be applied quite generally. One would expect the general procedure presented below to hold for all kinds of theories in various dimensions although subtleties can arise and some particular details might change from case to case.

2 Holomorphic complexification

To start we will deal with the first of the two questions posed above. We will show how one can find complex actions that can respectively be restricted to the known actions of (massive) IIA and IIB by reality conditions, and that are furthermore invariant under the complexified super Poincaré group. How the different formulations of the real 10d super Poincaré algebra can be found from the unique ten-dimensional complex OSp(1|32) algebra was described in detail in [1].

In complexifying an action it is crucial that all fields appear holomorphically in the complex action [8–10]. In other words we replace fields that take values in $\mathbb{R}$ by fields that take values in $\mathbb{C}$ in such a way that no complex conjugates appear. If one does the same complexification on the symmetry transformations, the complexified action is guaranteed to be invariant under these complex transformations as checking the invariance is a pure algebraic computation that nowhere assumes reality of the involved parameters[2].

This procedure of 'holomorphic complexification' is rather straightforward and only requires some more consideration in case of the spinors. Usually spinors appear in the action through bilinears written in terms of the Dirac conjugate $\bar{\chi} = \chi^\dagger A$, where A is the product of all time-like Gamma matrices. In this form there appears a complex conjugation and as such the action is not holomorphic in the spinor χ. There is an easy way around this as using the reality condition on the spinors the original real action can equivalently be written in terms of the Majorana conjugate $\bar{\chi} = \chi^T \mathcal{C}$, where $\mathcal{C}$ is the charge conjugation matrix. In this form spinors appear holomorphically and complexification now amounts to ignoring the reality condition on the spinors.

3 Back to reality

Starting from the complex action and supersymmetry transformations that we obtained with the method above we will now explain how one can construct different real actions by taking different real slices. In order to formulate reality conditions in 10 dimensions, we will work with a doublet notation, allowing us to treat type IIA and type IIB theories in a single framework. The 64-component doublets are the following

$$\chi = \begin{pmatrix} \chi^+ \\ \chi^- \end{pmatrix} \ (\text{type IIA}) , \qquad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \ (\text{type IIB}) . \tag{1}$$

Let us start by explaining what we mean by taking a real slice. A reality condition on the fields cannot be chosen at will, but has to satisfy certain consistency conditions.

[2] One might think that complexifying the supersymmetries in a maximal supergravity theory leads to a supergravity with 64 supercharges. This is however not the case. One should view the complexified action as a mathematical tool and not as a new theory describing new physical degrees of freedom.

First of all, one can only impose a limited number of reality conditions on the fermions, i.e. the dilatino λ, gravitino ψ_μ and ϵ. A general analysis shows that using this doublet notation, a general reality condition in $D = 10$ can be denoted as follows [11]:

$$\chi^* = -\alpha_\chi \mathcal{C} A \rho \chi \,, \tag{2}$$

where α_χ represents a phase factor. Note that the condition (2) now contains a 2×2-matrix ρ, that can mix the two components of the doublets (1). We will take the following possibilities for $\rho \in \{\mathbb{1}_2, \sigma_1, i\sigma_2, \sigma_3\}$. For the gauge potentials a general reality condition is given by[3]:

$$B^*_{\mu\nu} = \alpha_B B_{\mu\nu} \,, \quad C^{(2n-1)*}_{\mu_1\cdots\mu_{2n-1}} = \alpha_n C^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}} \,, \tag{3}$$

where again the α-factors represent phases. From the complex action it is easy to see that we can take the the dilaton ϕ and vielbein e_μ^a to be real.

The α-factors appearing in the reality conditions on the bosons and the fermions are not independent. Demanding a real action and consistency with supersymmetry relates them, which means that both sides of the supersymmetry rules should have the same behaviour under complex conjugation. In this way, the reality conditions on the fermions determine those of the bosons. The result is summarized in table 1. Note in particularly

	A				B		
t mod 4	0	1		2	1		3
type	*M$^+$	MW	*MW	M$^+$	MW	*MW	SMW
ρ	σ_3	$\mathbb{1}$	σ_3	$\mathbb{1}$	$\mathbb{1}$	σ_3	$i\sigma_2$
$\alpha_\epsilon = \alpha_\psi$	i	1	1	i	1	1	1
α_λ	i	1	-1	$-i$	1	1	1
α_B	$-$	$+$	$+$	$-$	$+$	$+$	$-$
$\alpha_0 = \alpha_2$, $\alpha_{1/2} = \alpha_{5/2}$	$+$	$+$	$-$	$-$	$+$	$-$	$+$
α_1, $\alpha_{3/2}$	$-$	$+$	$-$	$+$	$+$	$-$	$-$

Table 1: Possible reality conditions on the fields of type II supergravities. t is the number of time-like directions in space-time. We reserve the * when $\rho = \sigma_3$ in (2). M, MW and SMW then correspond to what is known in the literature as Majorana, Majorana-Weyl and symplectic Majorana-Weyl. Every set of reality conditions (column) corresponds to a different variant supergravity theory. From this table the actions and supersymmetry transformations of all 10d variant supergravities can be constructed.

that in (1,9) space-time dimensions we have *two* different real slices in the same signature. This is due to the fact that we only have a $\mathcal{N} = 2$ theory in this signature and we can use ρ to mix the component of the doublets (1). This (1,9) signature example turns out to be the link to the domain-wall/cosmology correspondence of [3].

4 Domain-wall/cosmology correspondence

Let us now turn to solutions of these complex theories and shown that one can obtain solutions of the different real theories by taking real slices. The most simple example of this is complex massive IIA. In particular, we show in [11] that in this way a supersymmetric domain-wall and a pseudo-supersymmetric [3] cosmology can arise as two different

[3]To have a uniform notation the reality condition for $G^{(0)}$ is given in terms of some formal $C^{(-1)}$.

real slices of one complex solution. The domain-walls are solutions in (1,9) massive IIA supergravity, while the cosmologies arise as solutions of the star version [2]. *In this sense the pseudo-supersymmetry of cosmologies corresponds to supersymmetry in the star theory.* This is a 10d example where the domain-wall/cosmology correspondence of [3] can be embedded into an extended supergravity context.

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