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UPPER AND LOWER BOUNDS ON WEAK INTERMEDIATE BOSON MASSES*

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ABSTRACT

Under a rather general class of assumptions, the most important being that weak and electromagnetic interactions are based upon a spontaneously broken gauge theory with an underlying simple gauge group, we estimate bounds on the least massive charged gauge boson. Typical values lie between 55 and 75 GeV, in agreement with those estimated from the Weinberg-Salam $SU(2) \otimes U(1)$ model. Less restrictive bounds are obtained for neutral bosons Z^0.

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I. INTRODUCTION

The apparent success of the Weinberg-Salam $SU(2)\otimes U(1)$ weak-electromagnetic gauge theory[1,2] in quantitatively accounting for the observed neutral-current cross sections increases one's confidence in the existence of charged ($W^{\pm}$) and neutral (Z^{0}) spin one intermediate bosons which mediate weak processes. In the simplest version of the $SU(2)\otimes U(1)$ model, the masses of W and Z are already well constrained by experiment. If it is true that the data[3] provide the limit

$$.25 \leq \sin^2\theta_W \leq .45 \tag{1.1}$$

it follows that

$$\begin{aligned} 56\text{ GeV} &\leq m_W \leq 75\text{ GeV} \\ 76\text{ GeV} &\leq m_Z \leq 87\text{ GeV} \end{aligned} \tag{1.2}$$

This result utilizes a mass formula ($m_W = m_Z \cos\theta_W$) which depends upon the details of the spontaneous breakdown mechanism. However, even if one ignores that relation and uses a two-parameter theory ($\sin^2\theta_W$, m_Z as parameters), essentially the same limits are obtained. More important is the assumption that the right-handed components of the nonstrange quarks transform as singlets under the weak group; if this is relaxed the experimental constraints on m_W are loosened.[4]

However, whatever the degree of success of the $SU(2)\otimes U(1)$ model in accounting for presently existing observations, it is widely felt that this model is only a small portion of a larger structure within which the totality of weak interaction physics resides.[5] In particular the $SU(2)\otimes U(1)$ model is not truly a unified theory of weak and electromagnetic interactions because of the presence of <u>two</u> coupling constants g and g'. The weak and electromagnetic parameters G_F and α are traded in for g and g'. An example of a truly unified theory is one based upon a simple gauge group G with a single coupling constant g, and with $SU(2) \otimes U(1) \subset G$. In such a theory there must be additional gauge bosons and quite possibly more quarks and leptons. The question naturally arises of whether at least some of the gauge boson masses will still lie in the range given by the $SU(2) \otimes U(1)$ model; Eq. (1.2). This is the problem posed in this paper. We endeavor to find a general set of assumptions with which it is possible to put bounds on the mass of charged and neutral bosons W and Z. While the limits are only as credible as the input assumptions, we believe it is still of use to carry out a study of the issue, if only to exhibit to what length one must go to avoid the conclusions reached in the context of the $SU(2) \otimes U(1)$ model. Indeed we believe our assumptions are in fact not very restrictive.

We now state the input assumptions we use in Sections II and III to obtain our results. As indicated below, these can be considerably relaxed (as we do in Section IV) without affecting significantly the conclusions:

Assumptions:

1. The weak interaction gauge group is contained in a simple group G: there is only one intrinsic gauge coupling constant g.

2. Gauge-bosons carrying lepton and/or baryon number contribute negligibly to the existing weak interaction phenomenology.

3. Only two (2-component) neutrinos ν_e and ν_μ contribute significantly to the existing weak interaction phenomenology. [By existing phenomenology we do not include the strong, but not yet conclusive, evidence[6] from SPEAR for a sequential heavy lepton $U^{\pm}$ and an associated neutrino.] This assumption actually is not vital and will be disposed of in Section IV.

4. For the predominant low energy phenomenology of charged and neutral weak currents (i.e., ignoring effects proportional to $\sin^2 \theta_C$, charm, other new

flavors, etc.) we may use the conventional 3-color, 3-quark model for fermion hadronic degrees of freedom.

5. Occasionally we shall also assume the Glashow-Iliopoulos-Maiani (GIM) mechanism;[7] i.e., there exists a fourth (colored) quark of c of charge $+\frac{2}{3}$; and (c, s_c) have the same weak couplings as (u, d_c) quarks.

6. As a consequence of these assumptions we may write for the phenomenological Lagrangian

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}}\left[\mathscr{J}_{\alpha}^{(cc)}\,\mathscr{J}_{(cc)}^{\dagger\alpha} + \bar{\nu}_{\mu}\gamma_{\alpha}(1-\gamma_5)\,\nu_{\mu}\mathscr{J}_{(NC)}^{\alpha} + \ldots\right] \tag{1.3}$$

where

$$\begin{aligned}\mathscr{J}_{cc}^{\alpha} &= \bar{e}\gamma^{\alpha}(1-\gamma_5)\,\nu_e + \bar{\mu}\gamma^{\alpha}(1-\gamma_5)\,\nu_{\mu} + \sum_{i=1}^{3}\bar{d}_i^{c}\gamma^{\alpha}(1-\gamma_5)\,u_i \\ &\quad + \sum_{i=1}^{3}\bar{s}_i^{c}\gamma^{\alpha}(1-\gamma_5)\,c_i ?\,? \;+ \ldots \\ &\equiv \sum_{j=1}^{M}\mathscr{J}_{(j)}^{\alpha}\end{aligned} \tag{1.4}$$

7. Higher-order radiative corrections may be neglected.

With these assumptions, we may now state our main results:

Let $W^{\pm}$ denote the charged intermediate boson of smallest mass which is coupled to $e\bar{\nu}_e$. Then

$$(75\ \text{GeV})\sqrt{B_{e\bar{\nu}_e}R_0} \leq m_W \leq (75\ \text{GeV})\sqrt{\frac{R_0}{M}} \tag{1.5}$$

Here

$$R_0 = \sum_{i\,\in\mathscr{R}} Q_i^2 \tag{1.6}$$

where the sum goes over all (4-component) fermions contained in an N-dimensional basis for some representation $\mathscr{R}$ of G, and where Q_i is the charge (in units of e; i.e., $Q_{e^\pm} = \pm 1$) of the ith fermion in the representation. Also

$$M = \text{number of independent terms in } \mathscr{J}^{\alpha}_{(cc)} \text{ in Eq. (1.4)} . \tag{1.7}$$

Notice that the $\Delta S=0$ hadron current associated with u and d_c quarks contributes 3 to M [A GIM charm-current contributes another 3 units to M]. Finally, to define the quantity $B_{e\bar{\nu}_e}$, first let $\langle j | c(W) | i \rangle$ denote the coupling constant (in terms of some basic coupling g) of the intermediate boson W to left-handed fermions j and i contained in the representation $\mathscr{R}$. That is the effective Lagrangian is to be written

$$\mathscr{L}_{eff} = g \sum_{j,i} \bar{u}_j \gamma^{\mu} \left(\frac{1-\gamma_5}{2}\right) u_i W^{\mu} \langle j | c(W) | i \rangle + \ldots \tag{1.8}$$

Then, define

$$B_{j\bar{i}} = \frac{|\langle j | c(W) | i \rangle|^2}{\sum\limits_{\substack{a,b \\ \text{in } \mathscr{R}}} |\langle b | c(W) | a \rangle|^2} \tag{1.9}$$

If the mass of all fermion degrees of freedom in the representation $\mathscr{R}$ is much less than $m_W/2$, then

$$B_{e\bar{\nu}_e} \cong \text{branching ratio of W into } e\bar{\nu}_e \tag{1.10}$$

We now give some examples, in order to exhibit the content of Eq. (1.5). Suppose

1. All leptons form a basis for a representation $\mathscr{R}$ of G. Then

$$\begin{aligned} R_0 &= 2 + ? \\ B^{-1}_{e\bar{\nu}_e} &= 2 + ? \\ M &= 2 + ? \end{aligned} \tag{1.11}$$

where the ? denotes contributions from all additional lepton degrees of freedom and from couplings not yet known or fully established. This gives

$$(75 \text{ GeV})\sqrt{\frac{2+?}{2+?}} \le m_W \le (75 \text{ GeV})\sqrt{\frac{2+?}{2+?}} \tag{1.12}$$

2. <u>All quarks form a basis for a representation $\mathscr{R}$ of G.</u> This option divides into two suboptions, according to whether or not the GIM mechanism is accepted:

a. Only the (u, $d_c = d \cos\theta_c + s \sin\theta_c$) current is considered an established contributor to the weak current and u, d, s to the electromagnetic current. Then

$$\begin{aligned} R_0 &= 2 + ? \\ B^{-1}_{e\bar{\nu}_e} &= M = 3 + ? \end{aligned} \tag{1.13}$$

and

$$61 \text{ GeV}\sqrt{\frac{3}{2}\cdot\frac{2+?}{3+?}} \le m_W \le 61 \text{ GeV}\sqrt{\frac{3}{2}\cdot\frac{2+?}{3+?}} \tag{1.14}$$

b. Colored u, d, s, c quarks are considered as established contributors to weak and electromagnetic currents (utilizing the GIM mechanism). Then

$$\begin{aligned} R_0 &= \frac{10}{3} + ? \\ B^{-1}_{e\bar{\nu}_e} &= M = 6 + ? \end{aligned} \tag{1.15}$$

and

$$56 \text{ GeV}\sqrt{\frac{9}{5}\cdot\frac{10/3+?}{6+?}} \le m_W \le 56 \text{ GeV}\sqrt{\frac{9}{5}\cdot\frac{10/3+?}{6+?}} \tag{1.16}$$

3. <u>Both leptons and quarks must be included in order to obtain a representation $\mathscr{R}$ of G.</u> This occurs, for example, in fully unified theories which include the strong force, such as the SU(5) theory of Georgi and Glashow.[8]

Again there are two suboptions:

a. Only u, d, s colored quarks and the $u\bar{d}_c$ weak current weak current are considered as established. Then

$$R_0 = 4 + ?$$
$$B^{-1}_{e\bar{\nu}_e} = M = 5 + ? \tag{1.17}$$

$$(67\ \text{GeV})\sqrt{\frac{5}{4}\cdot\frac{4+?}{5+?}} \leq m_W \leq (67\ \text{GeV})\sqrt{\frac{5}{4}\cdot\frac{4+?}{5+?}} \tag{1.18}$$

b. Colored quarks u, d, s, c are accepted with (GIM) $u\bar{d}_c$ and $c\bar{s}_c$ weak currents. Then

$$R_0 = \frac{16}{3}$$
$$B^{-1}_{e\bar{\nu}_e} = M = 8 \tag{1.19}$$

and

$$(61\ \text{GeV})\sqrt{\frac{3}{2}\cdot\frac{16/3+?}{8+?}} \leq m_W \leq (61\ \text{GeV})\sqrt{\frac{3}{2}\cdot\frac{16/3+?}{8+?}} \tag{1.20}$$

Hence without a major proliferation of degrees of freedom, we have

$$55\ \text{GeV} \lesssim m_W \lesssim 75\ \text{GeV} \tag{1.21}$$

just as in the case of the $SU(2)\otimes U(1)$ model. We emphasize, however, that this conclusion is obtained by a quite independent (but compatible) line of argument.

If leptons and hadrons form bases for independent representations $\mathscr{R}_\ell$ and $\mathscr{R}_h$ of G, the limits we have obtained in Eqs. (1.12) and Eqs. (1.14) or (1.16) are mutually exclusive. More degrees of freedom or couplings must be included in order to maintain compatibility. This situation is very similar to that discussed by others in the context of the $SU(2)\otimes U(1)$ model. In particular, the expression for $\sin^2\theta_W$ obtained by Georgi, Quinn, and Weinberg[9]

$$\sin^2\theta_W = \sum_{i\in\mathscr{R}} T_{3i}^2 \Big/ \sum_{i\in\mathscr{R}} Q_i^2 \tag{1.22}$$

(with the sum going over <u>two-component</u> fermion degrees of freedom) is closely connected to the bounds we obtain. In the limit of a single charged W coupled universally to fermion doublets, we evidently have $B^{-1}_{e\bar{\nu}_e} = M$. The inequality in Eq. (1.5) becomes an equality, with the same content as Eq. (1.22) above. Using the known leptons (μ, e, ν_μ, ν_e), one obtains

$$\sin^2 \theta_W = .25 \qquad (m_W = 75 \text{ GeV}) \tag{1.23}$$

Using only colored u, d, s, c quarks

$$\sin^2 \theta_W = .45 \qquad (m_W = 56 \text{ GeV}) \tag{1.24}$$

Combining both, as in the SU(5) model[8]

$$\sin^2 \theta_W = .375 \qquad (m_W = 61 \text{ GeV}) \tag{1.25}$$

An easy way to reconcile the lepton value with the hadron value without complete unification or a major proliferation is to introduce some neutral leptons possessing V+A couplings to electron and muon. Then we can immediately increase B^{-1} and M by two units without increasing R_0. We would then obtain, instead of Eq. (1.12),

$$(53 \text{ GeV})\sqrt{2 \cdot \frac{2+?}{4+?}} \leq m_W \leq (53 \text{ GeV})\sqrt{2 \cdot \frac{2+?}{4+?}} \tag{1.26}$$

Given the probable existence of a charged heavy lepton $U^{\pm}$ (which, by the way, would replace 53 GeV by 58 GeV in Eq. (1.26)) this might be considered an argument for the existence of neutral heavy leptons. In any case experimental searches for such objects are clearly of importance.[10]

The methods we use generally do not allow very stringent bounds or estimates for the mass of the lightest intermediate neutral boson Z^o. This is regrettable in the light of the importance[11] of the Z^o in contemplating future e^+e^- storage ring facilities. The useful information for Z^o bounds comes from the data on the semileptonic neutrino reactions. We find in general that the Z^o of lightest mass is bounded from above as follows:

$$m_{Z^o} \leq (75\ \text{GeV})\,(R_0)^{1/2}\left[\frac{2\left(1-R_{cc}^2\right)}{M'\left(R-R_{cc}^2\bar{R}\right)}\right]^{1/4} \tag{1.27}$$

where R_0 is defined in Eq. (1.6), and[12]

$$M' = \begin{cases} \geq 3 & \text{if only } Z^o \text{ couplings to u and d quarks are considered.} \\ \geq 6 & \text{if the GIM mechanism is accepted, and } Z^o \text{ couplings to c and s quarks are equal to the u and d couplings respectively.} \end{cases}$$

$$R = \frac{\sigma_{tot}(\bar{\nu}_\mu N \to \nu_\mu \text{ hadrons})}{\sigma_{tot}(\bar{\nu}_\mu N \to \mu^- \text{ hadrons})} = 0.28 \pm .04\ , \qquad R_{cc} = \frac{\sigma_{tot}(\bar{\nu}_\mu N \to \mu^+ \text{ hadrons})}{\sigma_{tot}(\nu_\mu N \to \mu^- \text{ hadrons})} = .38 \pm .02$$

$$\bar{R} = \frac{\sigma_{tot}(\bar{\nu}_\mu N \to \bar{\nu}_\mu \text{ hadrons})}{\sigma_{tot}(\bar{\nu}_\mu N \to \mu^+ \text{ hadrons})} = 0.39 \pm .06 \tag{1.28}$$

In deriving this bound, we have necessarily put both ν_μ and quarks in the same representation $\mathscr{R}$; hence R_0 must be summed over <u>both</u> quarks and leptons. However, putting numbers into Eq. (1.27) yields

$$m_{Z^o} \leq \begin{cases} 189\ \text{GeV} & \text{ordinary quarks and leptons;} \\ & R_0 = 4,\ M' = 3 \\ 184\ \text{GeV} & \text{GIM and ordinary leptons;} \\ & R_0 = 16/3,\ M' = 6 \end{cases} \tag{1.29}$$

Furthermore, it is not possible in general to obtain a lower bound on m_{Z^o} using only semileptonic neutral current data.

Stronger bounds <u>can</u> be obtained with stronger assumptions. For example, let us assume that only one Z^o contributes to the extant semileptonic neutral current phenomenology. In such a case we can replace Eq. (1.27) with the bound

$$m_{Z^o} \leq (75 \text{ GeV})\, R_0^{1/2} \left[\frac{(1+R_{cc})}{4M'(R+R_{cc}\bar{R})}\right]^{1/4} \tag{1.30}$$

where the quantities are defined as in Eq. (1.27). Putting numbers into Eq. (1.30) yields

$$m_{Z^o} \leq \begin{cases} 107 \text{ GeV} & \text{ordinary quarks and leptons;} \\ & R_0=4,\ M'=3 \\ 104 \text{ GeV} & \text{GIM and ordinary leptons;} \\ & R_0=16/3,\ M'=6 \end{cases} \tag{1.31}$$

A final improvement of a factor $2^{-1/4}$ can be obtained upon assuming ν_μ-ν_e universality in semileptonic neutral current phenomenology (in addition to the single-Z^o assumption). The factor 3 in Eq. (1.30) is replaced by 6, and the bounds become

$$m_{Z^o} \leq \begin{cases} 90 \text{ GeV} & \text{ordinary quarks and leptons;} \\ & R_0=4,\ M'=3;\ \nu_\mu\text{-}\nu_e \text{ universality} \\ 88 \text{ GeV} & \text{GIM and ordinary leptons;} \\ & R_0=16/3,\ M'=6;\ \nu_\mu\text{-}\nu_e \text{ universality} \end{cases} \tag{1.31}$$

If only one Z^o mediates semileptonic neutral current processes, a direct estimate of its mass can also be made:

$$m_{Z^o} \approx (75 \text{ GeV})\, R_0^{1/2} \left[\frac{B_{\nu_\mu\bar{\nu}_\mu} B_{had}(1+R_{cc})}{M'(R+R_{cc}\bar{R})}\right]^{1/4} \tag{1.32}$$

where $B_{\nu_\mu\bar{\nu}_\mu}$ is defined analogously to the B_i for W's; c.f. Eq. (1.9). B_{had} is the sum over u, d, s, (c) quarks of $B_{q\bar{q}}$:

$$B_{had} = \sum_{\substack{q=u_i,d_i,\\(s_i,c_i)}} B_{q\bar{q}} \tag{1.33}$$

The most naive guess for their values might be

$$B_{\nu_\mu \bar{\nu}_\mu} \sim B_{\nu_e \bar{\nu}_e} \sim B_{\mu\bar{\mu}} \sim B_{e\bar{e}} \sim \frac{1}{3} B_{u\bar{u}} \sim \frac{1}{3} B_{d\bar{d}} \sim \frac{1}{3} B_{c\bar{c}} \sim \frac{1}{3} B_{s\bar{s}} \tag{1.34}$$

yielding

$$B_{\nu_\mu \bar{\nu}_\mu} \sim \frac{1}{16} \qquad B_{had} \sim \frac{3}{4} \tag{1.35}$$

Inserting these numbers into Eq. (1.32), along with $R_0 = 16/3$ and $M' = 6$ gives

$$m_{Z^o} \sim 69 \text{ GeV} \tag{1.36}$$

[This is lower than the SU(2) ⊗ U(1) estimate because in that model $B_{\nu_\mu \bar{\nu}_\mu}$ is ~30% higher than the guess in Eq. (1.35).] The details of all these Z^o bounds are found in Section III.

We conclude this section by warning the reader not to believe that all these bounds, and in particular the conclusions summarized in Eq. (1.21), are completely general. They depend upon assumptions not fully based on experiment. The resourceful theorist can break the bounds by ingeniously violating the assumptions. One vulnerable assumption is that of the current-current structure of the charged-current effective Lagrangian. Only the <u>nondiagonal</u> terms are well-measured. At best, diagonal contributions are known to exist and to have the correct order of magnitude.[13,14] However, assumptions on the nature of the diagonal contributions <u>are</u> used in Section II in obtaining the bound in Eq. (1.5). This question is studied in Section IV for typical cases. It turns out that the upper bound is only increased by a factor $\lesssim (4/3)^{1/4} \approx 1.08$, if one only uses the existing data.

Another assumption we have made is that the neutrinos emitted in muon decay are identical to those emitted in semileptonic processes.[15] However, it again turns out that nothing changes significantly if this assumption is not made. For the case analyzed in Section IV, the bound is independent of information obtained from muon decay. And in general, it should be clear from the preceding discussion that omission of the information coming from leptonic decays would not modify the bounds in Eqs. (1.14) and (1.16), or change very much the upper bounds in Eq. (1.18) and (1.20).

A more difficult assumption to assess is our acceptance of the conventional fractionally charged color quark theory. Ideas along the lines pursued by Pati and Salam[16] (including the concept of pre-quarks)[17] or the "Berkeley" mixing models[18] might conceivably lead to different bounds. Consideration of these cases is beyond the scope of this paper.

An additional question has to do with the assumed neglect of radiative corrections. If leptons and hadrons must be unified within a single representation before reaching a simple group G, the mass-scale of leptoquark intermediate bosons could well be very large ($\sim 10^{15}$ GeV), and renormalization effects large. We have not succeeded in estimating this in a general way. However in the specific example studied by Georgi, Quinn, and Weinberg[9] with $SU(2) \times U(1) \subset SU(5)$, the factor $\sin^2 \theta_W$ was renormalized downward by about a factor 2; implying an upward renormalization of the intermediate-boson mass by $\sim$40%.

II. BOUNDS ON THE MASS OF THE W

To obtain these bounds on the W mass, we begin by embedding the group G into a larger unitary group. Let N denote the total number of two-component left-handed fermion degrees of freedom which form the basis for some (possibly reducible) representation $\mathscr{R}$ of G. These fermions might include some or all of the following: $e^-, e^+, \nu_e, \nu_\mu, \mu^+, \mu^-, u_i, d_i, s_i, \bar{u}_i, \bar{d}_i, \bar{s}_i$ (i=1, 2, 3); quarks possessing new flavors, more neutrinos, or heavy leptons. Any gauge transformation U in G, when applied to any one of these fermion degrees of freedom contained in $\mathscr{R}$, can only yield a linear combination of such fermion degrees of freedom. This is just a consequence of Poincare invariance. Invariance of the kinetic energy term in the fermion Hamiltonian

$$H_{kE} = -i \int d^3x \sum_{i \in \mathscr{R}} \psi_i^+(x)\, \sigma\cdot\nabla\, \psi_i(x) \tag{2.1}$$

under gauge transformations U requires that U, when considered in this N dimensional space of chiral fermions, be unitary. Hence $G \subset U(N)$, and[19] in fact, since G is simple, $G \subset SU(N)$.

We therefore, without loss of generality, restrict our attention to SU(N). We consider hypothetical bosons W_i^j (i, j=1, ... N) corresponding to the generators T_i^j of this SU(N) algebra. Only a subset of these W_i^j need be physical particles; the remainder we may without loss of generality presume to be physical but with an unobservably large mass. The physical bosons W_α of definite mass m_α may be mixtures of the W_i^j. We write, for convenience, the inverse relation

$$\langle j|W|i\rangle \equiv W_i^j = \sum_{\alpha=1}^{N^2-1} \langle j|c(\alpha)|i\rangle\, W_\alpha \tag{2.2}$$

where we include the possibly unphysical W_α of very large or infinite mass. In order that this mixing be unitary we have

$$\sum_{i,j=1}^{N} \langle j\,|c(\alpha)\,|i\rangle \langle j\,|c(\beta)\,|i\rangle^* = \delta_{\alpha\beta} \tag{2.3}$$

In addition, the linear independence of the W_i^j and their normalization condition demands [20]

$$\sum_{\alpha} |\langle j\,|c(\alpha)\,|i\rangle|^2 = \begin{cases} 1 & i\neq j \\ 1-\frac{1}{N} & i=j \end{cases} \tag{2.4}$$

We are now in a position to study limits on the mass of the charged W. Let the SU(N) gauge coupling be g, and let the indices i, j run over chiral fermion types, i.e., $i=e^-, \nu_e, e^+, \mu^-, \nu_\mu, \mu^+, u_1, u_2, \ldots$. Then by our assumptions (in particular the assumption that exchange of gauge bosons carrying nonvanishing lepton number contribute negligibly), the amplitude for muon decay is given by

$$\begin{aligned} \mathscr{M} &= \frac{G}{\sqrt{2}}\left[\bar{u}_e\gamma_\lambda(1-\gamma_5)\, v_{\nu_e}\right]\left[\bar{u}_{\nu_\mu}\gamma^\lambda(1-\gamma_5)u_\mu\right] \\ &\cong g^2 \sum_\alpha \left[\bar{u}_e\gamma_\lambda\left(\frac{1-\gamma_5}{2}\right)v_{\nu_e}\right]\frac{\langle e^-\,|c(\alpha)\,|\nu_e\rangle\langle\nu_\mu\,|c(\alpha)\,|\mu^-\rangle}{m_\alpha^2}\left[\bar{u}_{\nu_\mu}\gamma^\lambda\left(\frac{1-\gamma_5}{2}\right)u_\mu\right] \end{aligned} \tag{2.5}$$

Therefore

$$\frac{4G}{\sqrt{2}} = g^2 \sum_\alpha \frac{\langle e^-\,|c(\alpha)\,|\nu_e\rangle\langle\mu^-\,|c(\alpha)\,|\nu_\mu\rangle^*}{m_\alpha^2} \tag{2.6}$$

A simple, rather nonrestrictive limit follows rather directly from Eq. (2.6). Let $\alpha \equiv W$ denote the charged intermediate boson of smallest mass which contributes to the μ-decay process. Then, using Eq. (2.4)

$$\left|\frac{4G}{\sqrt{2}}\right| \leq \frac{g^2 \left| \sum_\alpha <e|c(\alpha)|\nu_e><\mu|c(\alpha)|\nu_\mu>^* \right|}{m_W^2}$$

$$\leq \frac{g^2 \sqrt{\left\{ \sum_\alpha |<e|c(\alpha)|\nu_e>|^2 \right\} \left\{ \sum_\beta |<\mu|c(\beta)|\nu_\beta>|^2 \right\}}}{m_W^2} = \frac{g^2}{m_W^2} \tag{2.7}$$

Hence

$$m_W^2 \leq \frac{\sqrt{2}\, g^2}{4G} \tag{2.8}$$

To translate this into a limit, we must know the value of g^2. This can be obtained from the coupling of the photon, which must be one of the gauge bosons W_α. The photon A^μ evidently has only diagonal couplings to the fermion degrees of freedom; thus only $<i|c(A)|i> \neq 0$. Indeed $<i|c(A)|i>$ must be proportional to the charge Q_i of the ith fermion. Then the normalization condition, Eq. (2.3), determines the coupling:

$$<i|c(A)|i> = \frac{Q_i}{\sqrt{\sum_{i=1}^{N} Q_i^2}} \tag{2.9}$$

Because the full electromagnetic coupling is

$$g\left[\bar{u}_i\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) u_i\right] <i|c(A)|i> \equiv eQ_i\left[\bar{u}_i\gamma_\mu u_i\right]$$

$$= eQ_i\left[\bar{u}_i\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) u_i\right] \tag{2.10}$$

it follows that

$$\frac{g}{\sqrt{\sum_{i=1}^{N} Q_i^2}} = \frac{g}{\sqrt{2R_0}} = e \tag{2.11}$$

where R_0 is defined in Eq. (1.6). There the definition was, as is usually the convention in such matters, a sum over four-component fermion degrees of freedom, while in Eqs. (2.9) and (2.11) the sum is over two-component degrees of freedom; hence the factor 2 in Eq. (2.11). Thus

$$g^2 = 8\pi\alpha R_0 \tag{2.12}$$

and from Eq. (2.8) we get the bound

$$m_W^2 \leq \frac{2\pi\alpha\sqrt{2}}{G} R_0 \tag{2.13}$$

or, putting in numbers

$$m_W \leq (75\ \mathrm{GeV})\sqrt{R_0} \tag{2.14}$$

This is not a very restrictive bound, inasmuch as experimentally $R_0 \gtrsim 7$. However m_W may be bounded more stringently upon assuming that the conventional current-current structure of weak interactions, when supplemented with sundry neutral current contributions in diagonal channels, is a good approximation to low-energy weak amplitudes. This implies that Eq. (2.6) generalizes:

$$\frac{4G}{g^2\sqrt{2}} \cong \sum_\alpha \frac{\langle j|c(\alpha)|i\rangle\langle \ell|c(\alpha)|k\rangle^*}{m_\alpha^2} \tag{2.15}$$

whenever (j, i) or (k, ℓ) are (e, ν_e), (μ, ν_μ), (d_i, u_i), or (s_i, c_i) [with the latter to be included if charm and the GIM mechanism is accepted].

To continue, define

$$<j\,|C(\alpha)\,|i> \equiv \frac{<j\,|c(\alpha)i>}{|m_\alpha|} \equiv <j\,|\vec{C}\,|i> \tag{2.16}$$

which is to be considered a vector in the N^2-1 dimensional complex space labeled by index α. Then Eqs. (1.3) and (1.4) imply

$$<j\,|\vec{C}\,|i> \cdot <k\,|\vec{C}\,|\ell> = \text{constant} = |<j\,|\vec{C}\,|i>|^2 \tag{2.17}$$

with

$$(j,i) \text{ or } (k,\ell) = (e^-, \nu_e),\ (\mu^-, \nu_\mu),\ (d_i, u_i)\ \left[\text{or } (s_i, c_i)\,?\right] \tag{2.18}$$

Therefore, for these pairs

$$<j\,|\vec{C}\,|i> \approx <k\,|\vec{C}\,|\ell> \tag{2.19}$$

For the components $\tilde{\alpha}$ which contribute significantly to the vector $<j\,|\vec{C}\,|i>$ in Eq. (2.19) we have not only

$$<j\,|C(\tilde{\alpha})\,|i> = <k\,|C(\tilde{\alpha})\,|\ell> \tag{2.20}$$

but also

$$<j\,|\,c(\tilde{\alpha})\,|i> = <k\,|\,c(\tilde{\alpha})\,|\ell> \tag{2.21}$$

with, as always, the pairs (j, i) going over at least the range given in Eq. (2.18). Thus one unique combination of intermediate bosons

$$\tilde{W} = \frac{\sum_{\tilde{\alpha}} <j\,|\,c(\tilde{\alpha})\,|i>\, W_{\tilde{\alpha}}}{\sqrt{\sum_{\tilde{\alpha}} |<j\,|\,c(\tilde{\alpha})\,|i>|^2}} \qquad (j,i) = (e, \nu_e), (\mu^-, \nu_\mu), (d_i, u_i)\ \ldots \tag{2.22}$$

necessarily is what is coupled to the pairs (j, i). At this point in the argument it is most convenient to choose an orthonormal basis for the W_α's which includes $\tilde{W}$. The orthogonality and normalization conditions, Eqs. (2.3) and (2.4), can still be applied in this basis to limit the coupling of $\tilde{W}$ to any one of the pairs (i, j).

Write, for this expansion, for general (j, i)

$$<j\,|W|\,i> \equiv <j\,|b(\widetilde{W})\,|i>\,\widetilde{W} + \sum_{\alpha'\neq\widetilde{W}} <j\,|b(\alpha')\,|i>\,W_{\alpha'} \tag{2.23}$$

From Eq. (2.3) rotated into this new basis

$$\sum_{j,i} |<j\,|b(\widetilde{W})\,|i>|^2 = 1 \tag{2.24}$$

Then, given M pairs (j, i) as enumerated in Eq. (2.18), all coupling in the same way to $\widetilde{W}$, it follows that

$$M\,|<e^-\,|b(\widetilde{W})|\nu_e|^2 \leq 1 \tag{2.25}$$

But the projection of $<j\,|W|\,i>$ onto $\widetilde{W}$ is also implied by Eqs. (2.2) and (2.22)

$$\begin{aligned} <j\,|W|\,i> &= \sum_{\widetilde{\alpha}} <j\,|\,c(\widetilde{\alpha})\,|i>\,W_{\widetilde{\alpha}} + \ldots \\ &= \sqrt{\sum_{\widetilde{\alpha}} |<j\,|\,c(\widetilde{\alpha})\,|i>|^2}\;\widetilde{W} + \ldots \end{aligned} \tag{2.26}$$

Hence

$$|<j\,|b(\widetilde{W})\,|i>|^2 = \sum_{\widetilde{\alpha}} |<j\,|\,c(\widetilde{\alpha})\,|i>|^2 \tag{2.27}$$

and

$$\sum_{\widetilde{\alpha}} |<e^-\,|\,c(\widetilde{\alpha})\,|\nu_e>|^2 = |<e^-\,|b(\widetilde{W})\,|\nu_e|^2 \leq \frac{1}{M} \tag{2.28}$$

Returning to Eq. (2.7), this now implies

$$\frac{4G}{g^2\sqrt{2}} \cong \sum_{\widetilde{\alpha}} \frac{|<e^-\,|\,c(\widetilde{\alpha})\,|\nu_e|^2}{m_{\widetilde{\alpha}}^2} \leq \frac{1}{m_W^2} \sum_{\widetilde{\alpha}} |<e^-\,|\,c(\widetilde{\alpha})\,|\nu_e>|^2 \leq \frac{1}{Mm_W^2} \tag{2.29}$$

instead of the bound of Eq. (2.8). The remainder of the argument leading up to Eq. (2.14) proceeds as before, but with the factor M improvement;

$$m_W \le (75\ \mathrm{GeV})\sqrt{\frac{R_0}{M}} \tag{2.30}$$

Implications of various choices for R_0 and M have already been discussed in Section I.

A lower bound on m_W can also be easily obtained. From Eq. (2.29) it is clear that

$$\frac{4G}{g^2\sqrt{2}} \ge \frac{|\langle e^- | c(W) | \nu_e \rangle|^2}{m_W^2} \tag{2.31}$$

Define

$$B_{e\bar{\nu}_e} = \langle e^- | c(W) | \nu_e \rangle^2 \quad \frac{|\langle e^- | c(W) | \nu_e \rangle|^2}{\sum_{i,j} |\langle j | c(W) | i \rangle|^2} \tag{2.32}$$

Were all fermion masses (for the fermions in $\mathscr{R}$) small compared to m_W, $B_{e\bar{\nu}_e}$ is just the branching ratio of W into $e^-\bar{\nu}_e$. From the above definition and the connection, Eq. (2.12) of g^2 with α and R_0, it follows that

$$m_W^2 \ge \left(\frac{2\pi\alpha\sqrt{2}}{G}\right) BR_0 = (75\ \mathrm{GeV})^2\, BR_0 \tag{2.33}$$

Again the implications of this result are discussed in Section I.

III. BOUNDS ON THE MASS OF THE Z^0

We may proceed in a similar way with the neutral-current contributions, although the bounds will be much less stringent, inasmuch as there are many forms which may be assumed for the structure of the neutral-current effective Lagrangian. As input data, we assume an effective neutral current Lagrangian as follows:

$$\mathscr{L}_{NC}^{eff} = \frac{G}{\sqrt{2}} \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\nu_\mu \left[\epsilon_L(e)\, \bar{e}\gamma_\lambda(1-\gamma_5)e + \epsilon_R(e)\, \bar{e}\gamma_\lambda(1+\gamma_5)e + \ldots \right.$$
$$+ \epsilon_L(u) \sum_{i=1}^{3} \bar{u}_i\gamma_\lambda(1-\gamma_5)u_i + \epsilon_R(u) \sum_{i=1}^{3} \bar{u}_i\gamma_\lambda(1+\gamma_5)u_i$$
$$\left. + \epsilon_L(d) \sum_{i=1}^{3} \bar{d}_i\gamma_\lambda(1-\gamma_5)d_i + \epsilon_R(d) \sum_{i=1}^{3} \bar{d}_i\gamma_\lambda(1+\gamma_5)d_i \right]$$
$$+ \ldots \tag{3.1}$$

The best bounds will come from exploiting the assumed color degeneracy of the quarks, and hereafter we shall disregard the pure leptonic terms involving $\epsilon_{L,R}(e)$. Data on ν_μ-hadron interactions[3] provide an estimate for the combinations $|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2$ and $|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2$. The commonly used ratios $R = \sigma_{nc}^{tot,\nu}(E)/\sigma_{cc}^{tot,\nu}(E)$, $R_{cc} = \sigma_{cc}^{tot,\bar{\nu}}(E)/\sigma_{cc}^{tot,\nu}(E)$, and $\bar{R} = \sigma_{nc}^{tot,\bar{\nu}}(E)/\sigma_{cc}^{tot,\bar{\nu}}(E)$ are related to the ϵ's in simple models[21] as follows:

$$R = \left\{|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2\right\} + R_{cc}\left\{|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right\}$$
$$\bar{R} = R_{cc}^{-1}\left\{|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right\} + \left\{|\epsilon_L(u)|^2 + \epsilon_L(d)|^2\right\} \tag{3.2}$$

We shall need the combinations

$$|\epsilon_L|^2 \equiv |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 = \frac{\left(R - R_{cc}^2\bar{R}\right)}{\left(1 - R_{cc}^2\right)} \approx .25 \pm .10$$

$$|\epsilon|^2 \equiv |\epsilon_L|^2 + |\epsilon_R|^2 \equiv |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 + |\epsilon_R(u)|^2 + |\epsilon_R(d)|^2$$
$$= \frac{(R + R_{cc}\bar{R})}{(1 + R_{cc})} \approx .30 \pm .10 \tag{3.3}$$

which are accessible to experiment as well as to the theoretical bounds. We are deliberately liberal with the error estimate, because our final result depends only on the fourth root of $|\epsilon|^2$; even such a large error assignment won't matter much.

Our problem is to use this information to bound the mass of Z^0's. The best strategy is to repeat the line of argument used for the charged W, and which led to Eq. (2.30). Thus the linear combination $\tilde{Z}$ of neutral bosons Z_α which couples to a given flavor and helicity of quark (say u_i)

$$\tilde{Z}(u) = \frac{\sum_\alpha \langle u_i | c(\alpha) | u_i \rangle Z_\alpha}{\sqrt{\sum_\alpha |\langle u_i | c(\alpha) | u_i \rangle|^2}} \tag{3.4}$$

must couple equally to each color of u quark. Since

$$\begin{aligned} \langle u_i | W | u_i \rangle &= \sum_\alpha \langle u_i | c(\alpha) | u_i \rangle Z_\alpha \\ &= \sqrt{\sum_\alpha |\langle u_i | c(\alpha) | u_i \rangle|^2}\ \tilde{Z}(u) \end{aligned} \tag{3.5}$$

this line of argument, using the normalization condition in Eq. (2.4), gives

$$3 \sum_\alpha |\langle u_1 | c(\alpha) | u_1 \rangle|^2 \le 1 - \frac{1}{N} \tag{3.6}$$

If the GIM mechanism is correct, we expect $\tilde{Z}(u)$ must also coupled in the same way to the c quarks. If this is presumed, then the factor 3 in Eq. (3.6) may be replaced by 6. Inclusion of the factor 1/N in that equation is a waste of ink; hereafter we drop it and write

$$\sum_\alpha \langle u_1 | c(\alpha) | u_1 \rangle|^2 \le \frac{1}{M'} \tag{3.7}$$

with

$$M' = \begin{cases} \ge 3 & \text{up and down quarks only} \\ \ge 6 & \text{GIM and charm presumed} \end{cases} \tag{3.8}$$

Then, in analogy to Eqs. (2.7) and (2.29),

$$\frac{4G\,|\epsilon_L(u)|}{g^2\sqrt{2}} = \left|\sum_\alpha \frac{\langle\nu_\mu|c(\alpha)|\nu_\mu\rangle\langle u_1|c(\alpha)|u_1\rangle}{m_\alpha^2}\right|$$

$$\leq \frac{1}{m_Z^2}\sqrt{\sum_\alpha |\langle\nu_\mu|c(\alpha)|\nu_\mu\rangle|^2 \sum_{\alpha'} |\langle u_1|c(\alpha')|u_1\rangle|^2}$$

$$\leq \frac{1}{\sqrt{M'}\,m_Z^2} \tag{3.9}$$

where again m_Z is defined as the contributing gauge boson of smallest mass. Hence, again using Eq. (2.12)

$$m_Z^2\,|\epsilon_L(u)| \leq \frac{2\pi\sqrt{2}\,\alpha}{G}\,\frac{R_0}{\sqrt{M'}} = (75\ \text{GeV})^2\,\frac{R_0}{\sqrt{M'}} \tag{3.10}$$

The same result evidently follows for $\epsilon_L(u)$ replaced by any other ϵ_s, e.g., $\epsilon_L(d)$, $\epsilon_R(u)$, $\epsilon_R(d)$, ... The best bound involving measured numbers is

$$m_Z^4\left\{|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2\right\} \leq \frac{2}{M'}\,(75\ \text{GeV})^4\,R_0^2 \tag{3.11}$$

or

$$m_Z \leq (75\ \text{GeV})\sqrt{R_0}\left(\frac{2}{M'|\epsilon_L|^2}\right)^{1/4} \tag{3.12}$$

inasmuch as $|\epsilon_L|^2 \geq |\epsilon_R|^2$ experimentally. As we discussed in Section I, this bound is not very stringent, as compared with the Z^0 mass estimate in the $SU(2)\times U(1)$ model.

The only really strong bound we have been able to find follows from the assumption that the observed neutral-current semileptonic processes are dominated (at moderate energies, $E_\nu \lesssim 20$ GeV) by the exchange of a single intermediate boson Z^o. If this is the case, we may use the inequality, Eq. (2.3)

$$\sum_{(i,j)} |\langle j | c(Z) | i \rangle|^2 \leq 1 \tag{3.13}$$

to good advantage. From Eq. (3.9), we now have

$$\frac{4G}{\sqrt{2}} |\epsilon_L(u)| \approx \frac{g^2}{m_Z^2} |\langle \nu_\mu | c(Z) | \nu_\mu \rangle \langle u_i | c(Z) | u_i \rangle| \tag{3.14}$$

with similar expressions for the other ϵ_s. Adding the four such equations together in quadrature and taking into account color (and possibly charm) as we did above Eq. (3.7) gives

$$\frac{4G}{\sqrt{2}} \sqrt{|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 + |\epsilon_R(u)|^2 + |\epsilon_R(d)|^2} \equiv \frac{4G}{\sqrt{2}} \sqrt{|\epsilon|^2}$$

$$\cong \frac{g^2}{m_Z^2} |\langle \nu_\mu | c(Z) | \nu_\mu \rangle| \sqrt{\sum_{\substack{q=u_i,\, d_i \\ (s_i,\, c_i)}} \frac{|\langle q | c(Z) | q \rangle|^2}{M'}} \tag{3.15}$$

The sum over q goes over both left-handed q and left-handed $\bar{q}$ (or right-handed q). Upon using Eq. (3.13), we obtain

$$\frac{4G}{\sqrt{2}} \sqrt{|\epsilon|^2} \leq \frac{g^2}{m_Z^2} \cdot \frac{1}{\sqrt{M'}} \cdot \langle \nu_\mu | c(Z) | \nu_\mu \rangle \sqrt{1 - |\langle \nu_\mu | c(Z) | \nu_\mu \rangle|^2}$$

$$\leq \frac{1}{2} \frac{g^2}{m_Z^2} \frac{1}{\sqrt{M'}} \tag{3.16}$$

This leads to the bound

$$m_Z^2 \leq \frac{2\pi\alpha\sqrt{2}}{G} \frac{R_0}{\sqrt{4M' |\epsilon|^2}} \tag{3.17}$$

or

$$m_Z \leq \frac{(75\ \mathrm{GeV})\ R_0^{1/2}}{(4M'\ |\epsilon|^2)^{1/4}} \tag{3.18}$$

As discussed in Section I, this reduces the upper bound on m_Z to ~110 GeV. A further improvement ensues if one assumes ν_μ-ν_e universality in the coupling of this Z to the neutrinos. If

$$<\nu_\mu\,|\,c(Z)\,|\nu_\mu> = <\nu_e\,|\,c(Z)\,|\nu_e> \tag{3.19}$$

then Eq. (3.16) may be written as

$$\frac{4G}{\sqrt{2}}/\sqrt{|\epsilon|^2} \leq \frac{g^2}{m_Z^2}\frac{1}{\sqrt{M'}} <\nu_\mu\,|\,c(Z)\,|\nu_\mu>\sqrt{1-2\,|<\nu_\mu\,|\,c(Z)\,|\nu_\mu>|^2}$$

$$\leq \frac{1}{2\sqrt{2}}\ \frac{g^2}{m_Z^2}\frac{1}{\sqrt{M'}} \tag{3.20}$$

and we gain a factor $2^{-1/4}$ in the upper bound for m_Z.

With the above assumptions we can find an estimate for m_Z as well, just as we found for the W. Write

$$|<\nu_\mu\,|\,c(Z)\,|\nu_\mu>| \equiv \sqrt{B_{\nu_\mu\bar{\nu}_\mu}}$$

$$\sum_{\substack{q=u_i,\,d_i\\ c_i,\,s_i\\ \text{also } L,R}} |<q\,|\,c(Z)\,|q>|^2 = B_{had} \tag{3.21}$$

where, as for the $W^\pm$, the B_s are the branching ratios of the Z^0 into modes s, provided all fermions in the representation $\mathscr{R}$ which are coupled to Z^0 have mass small compared to the Z^0 mass. In Eq. (3.21), B_{had} includes only those hadron states formed by u, d, s (c?). Inserting Eq. (3.21) into (3.15) yields

$$\frac{4G}{\sqrt{2}} \sqrt{|\epsilon|^2} \approx \frac{g^2}{m_Z^2} \cdot \frac{1}{\sqrt{M'}} \sqrt{B_{\nu_\mu \bar{\nu}_\mu} \cdot B_{had}} \tag{3.22}$$

where M' is defined in Eq. (3.8). Therefore

$$m_Z \approx \left(\frac{2\pi\alpha\sqrt{2}}{G}\right)^{1/2} R_0^{1/2} \left(\frac{B_{\nu_\mu \bar{\nu}_\mu} B_{had}}{M' |\epsilon|^2}\right)^{1/4} \tag{3.23}$$

With "reasonable" values of the parameters, this yields a value somewhat lower than that estimated in the SU(2) ⊗ U(1) model. This is not to be taken too seriously. We emphasize that the assumption that only one Z mediates the semileptonic neutral-current processes is crucial in this argument. A counterexample is two degenerate light Z's with equal couplings to quarks but with couplings to ν_μ which differ in sign. Such Z's might exist but would not contribute at all to semileptonic neutral current processes.

IV. EVASIONS OF THE ASSUMPTIONS

As we mentioned in the Introduction, there is little direct evidence for diagonal current-current interactions. It is therefore of interest to study what is lost if the conventional assumption for the magnitude of the diagonal current-current coupling is abandoned. To do this we return to the same line of argument used in Section II, starting at Eq. (2.17). We have

$$|\langle j|\vec{C}|i\rangle^* \cdot \langle k|\vec{C}|\ell\rangle| = \text{constant} = \frac{4G}{g^2\sqrt{2}} \tag{4.1}$$

only for the pair $(j,i) \neq (k,\ell)$. This means the (e, ν_e), (μ, ν_μ), and (d, u) vectors may point in different directions. We continue by following the line of argument leading to Eq. (2.29) from Eq. (2.17). We again introduce a new basis for the W_α, unitarily related to the original basis; for any such basis we retain the normalization condition, Eq. (2.3). In the vector notation of Eq. (2.16), applied to $\langle j|c(\alpha)|i\rangle$, this means that for any projection of $\langle j|\vec{c}\,|i\rangle$ on a unit vector $\hat{u}$, $(\hat{u}^*\hat{u}=1)$ we must have

$$\sum_{j,i} |\langle j|\vec{c}\,|i\rangle \cdot \hat{u}\,|^2 = 1 \tag{4.2}$$

There follows from this the inequality

$$|\langle e|\vec{c}\cdot \hat{u}\,|\nu_e\rangle|^2 + |\langle \mu|\vec{c}\cdot \hat{u}\,|\nu_\mu\rangle|^2 + M'\,|\langle d_i|\vec{c}\cdot \hat{u}\,|u_i\rangle|^2 \leq 1 \tag{4.3}$$

where M', defined in Eq. (3.8), is ≥ 3 or 6 depending upon whether charm and the GIM GIM mechanism are accepted and included. Evidently

$$m_W^2|\langle e|(\vec{C}\cdot\hat{u})\hat{u}\,|\nu_e\rangle|^2 \leq |\langle e|(c\cdot\hat{u})\hat{u}\,|\nu_e\rangle|^2 \tag{4.4}$$

as follows by expansion of the above expression in the original basis of W_α's of definite mass. Thus

$$m_W^2\left\{|\langle e|\vec{C}\cdot\hat{u}\,|\nu_e\rangle|^2 + |\langle \mu|\vec{C}\cdot\hat{u}\,|\nu_\mu\rangle|^2 + M'\,|\langle d_i|\vec{C}\cdot\hat{u}\,|u_i\rangle|^2\right\} \leq 1 \tag{4.5}$$

Define the quantities

$$\vec{E} = \left(\frac{g^2\sqrt{2}}{4G}\right)^{1/2} \langle e|\vec{C}\,|\nu_e\rangle$$

$$\vec{M} = \left(\frac{g^2\sqrt{2}}{4G}\right)^{1/2} \langle \mu|\vec{C}\,|\nu_\mu\rangle \tag{4.6}$$

$$\vec{H} = \left(\frac{g^2\sqrt{2}}{4G}\right)^{1/2} \langle d_i|\vec{C}\,|u_i\rangle$$

Insertion of these definitions into Eq. (4.5) gives the bound, for arbitrary unit vector $\hat{u}$,

$$m_W^2 \leq \left(\frac{g^2\sqrt{2}}{4G}\right)\cdot\left[|\vec{E}\cdot\hat{u}|^2 + |\vec{M}\cdot\hat{u}|^2 + M'|\vec{H}\cdot\hat{u}|^2\right]^{-1} \tag{4.7}$$

but with the additional constraints

$$|\vec{E}^*\cdot\vec{M}\,| = |\vec{E}^*\cdot\vec{H}\,| = |\vec{M}^*\cdot\vec{H}\,| = 1 \tag{4.8}$$

This defines the mathematical problem. Recall that for the simpler case treated in Section II we had $\vec{E}=\vec{M}=\vec{H}$, and the minimum of the right-hand side of Eq. (4.7) was obtained for $\hat{u}=\vec{H}$, giving

$$m_W^2 \leq \left(\frac{2\pi\alpha\sqrt{2}}{G}\right)\frac{R_0}{2+M} \equiv m_0^2 \tag{4.9}$$

To minimize the right-hand side of Eq. (4.7) in the general ease (with respect to variation of $\hat{u}$), one diagonalizes the matrix $E_i^* E_j + M_i^* M_j + M' H_i^* H_j$ and chooses the largest eigenvalue. Because of the large factor M' we expect the eigenvector to lie near the $\vec{H}$ direction. For simplicity, we here take $\hat{u}$ to lie along $\vec{H}$, namely $\hat{u}=\hat{H}^*$. Then

$$|\vec{E}\cdot\hat{u}|^2 = \frac{|\vec{E}\cdot\vec{H}^*|^2}{H^2} \geq \frac{1}{H^2} \tag{4.10}$$

Similarly

$$|\vec{M}\cdot\hat{u}|^2 \geq \frac{1}{H^2} \tag{4.11}$$

and

$$\left[|\vec{E}\cdot\hat{u}|^2 + |\vec{M}\cdot\hat{u}|^2 + M'|\vec{H}^*\cdot\hat{u}|^2\right] \geq \left(\frac{2}{H^2} + M'H^2\right) \geq 2\sqrt{2M'} \tag{4.12}$$

Hence

$$m_W^2 \leq m_0^2\left(\frac{2+M'}{2\sqrt{2M'}}\right) \tag{4.13}$$

Therefore the bound on m_W^2 is weaker than the previous case by a factor

$$\frac{2+M'}{2\sqrt{2M'}} = \begin{cases} \dfrac{5}{2\sqrt{6}} = 1.02 & M'=3 \\[2ex] \dfrac{2}{\sqrt{3}} = 1.16 & M'=6 \end{cases} \tag{4.14}$$

We note that it is no longer possible to obtain a <u>lower</u> bound for m_W in this more general case; the previous estimate, Eqs. (2.31) and (2.33), used in an essential way the positivity present in diagonal amplitudes.

Finally, as noted in Section I, even if we do not accept that the ν_e and ν_μ appearing in muon decay are the same as in semileptonic hadron processes (for example, perhaps muon decay is mediated by intermediate bosons distinct from those in semileptonic processes, with distinct neutrinos as well), we still obtain the upper bound expressed in Eq. (4.13). The only information from muon decay which was recorded in the above argument was the equation $|\vec{E}^{*}\cdot\vec{M}|\geq 1$, which in fact was never used. And information from the neutrino-experiments is persuasive that at least the predominant portion of semileptonic decay and reaction processes proceed[22] via a unique ν_μ (and ν_e as well[23]). Thus relaxation of the assumptions on neutrino identity lead to no essential changes in the results.

V. CONCLUSIONS

We have found that under a broad range of assumptions the estimated mass of the lightest charged intermediate boson $W^\pm$ lies in the range of 55 to 75 GeV. Similar attempts to bound the Z^o mass led to limits which are not very restrictive, unless it is assumed that only one Z^o contributes to the present neutral current phenomenology. This is, in the general context we have attempted here, probably too strong an assumption. It is regrettable this is the case, because resonant production of Z^o in e^+e^- colliding beams should be an extremely powerful way of studying the selection rules and dynamics of weak interactions in their natural energy regime.

But aside from the attempt in this paper to be general, we recognize that success of the $SU(2)\times U(1)$ model, if accurate experiments continue to agree with its predictions, will by itself make very credible the existence of $W^\pm$ in the mass region 65 ± 10 GeV and Z^o in the 80 ± 6 GeV region.

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