RESEARCH ARTICLE

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Matching WMAP 3-year results with the cosmological Slingshot primordial spectrum

Abstract We consider a recently proposed scenario for the generation of primordial cosmological perturbations, the so called Cosmological Slingshot scenario. We first obtain a general expression for the Slingshot primordial power spectrum which extends previous results by including a blue pre-bounce residual contribution at large scales. Starting from this expression we numerically compute the CMB temperature and polarization power spectra arising from the Slingshot scenario and show that they excellently match the standard WMAP 3-year best-fit results. In particular, if the residual blue spectrum is far above the largest WMAP observed scale, the Slingshot primordial spectrum fits the data well by only fixing its amplitude and spectral index at the pivot scale $k_{\mathrm{p}} = 10^{-3}\,h\,\mathrm{Mpc}^{-1}$. We finally show that all possible distinctive Slingshot signatures in the CMB power spectra are confined to very low multipoles and thus very hard to detect due to large cosmic variance dominated error bars at these scales.

Keywords String cosmology · CMP

1 Introduction

It is well known that Standard (non-inflationary) Cosmology is afflicted by three severe problems [1]: homogeneity, isotropy and flatness. Inflation is the standard accepted paradigm for the resolution of these problems. Nevertheless, as a fundamental origin of Inflation is as yet lacking, many attempts to alternatively solve the homogeneity, isotropy and flatness fine tunings have been recently put forward.

In this paper we consider one of these alternatives, namely the scenario developed in [2; 3] so called the "Cosmological Slingshot Scenario", or shortly the *Slingshot*.

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In the Slingshot, our Universe is a probe $D3$-brane "orbiting" with an open trajectory in a IIB supergravity background, namely the Klebanov–Tseytlin (KT) metric [4] (the bulk). If the probe brane approach of [5] used in [2; 3] can be used, the Slingshot trajectory results on an induced cosmological evolution on the brane. More precisely, a brane observer experiences a Friedman–Robertson–Walker non-singular bouncing universe. In the Slingshot, the problems that afflict standard cosmology are circumvented [2] by using similar mechanisms introduced in pre big-bang [6] and cyclic [7] scenarios. Besides, the Slingshot also predict a power spectrum of primordial perturbations. In [2; 3], and in this letter, the primordial spectrum of scalar perturbations due to the fluctuation of the Slingshot brane on the KT background is indeed calculated under the approximation that the backreaction of the Slingshot brane into the bulk is negligible. The validity of this approximation is supported by the fact that the KT background, in which the Slingshot brane is moving, is produced by a large number of D3-branes having all the *same* tension ("mass") as the Slingshot brane.

The plan of the paper is as follows. In Sect. 2 we will summarize the previous results of [2; 3]. We will then extend those results in two ways. First of all we will consider a new blue contribution to the primordial power spectrum that was previously not accounted for. This will produce the general parametrization of the Slingshot primordial spectrum shown at the end of Sect. 3.

As a second step, in Sect. 4, we will use this general parametrization in order to numerically compute the temperature and polarization CMB angular power spectra arising from the Slingshot and we will compare them to WMAP data [8]. In particular we will show that a suitable and natural choice of the Slingshot parameters *allow to reproduce the standard WMAP 3-year best-fit power spectra*. We will then try different choices for the Slingshot parameters and see if they can produce distinctive model-dependent signatures in the results. Finally we will draw our conclusions in Sect. 5.

We are now ready to conclude this section, but we would like to make a final remark first. In [3] an analytic expansion of the Slingshot spectrum for large multipoles ($\ell > 10$) had actually already been shown to match the WMAP best fit of a power law spectrum with spectral index $n_{\mathrm{s}} \simeq 0.95$. However, this result held *only* at a given *pivot scale* (chosen as $k_{\mathrm{p}} \sim 10^{-3}\,\mathrm{h}\,\mathrm{Mpc}^{-1}$). Our numerical approach in this paper shows instead that the spectrum found in [3], matches the WMAP experimental results at *all* scales, and not only around the pivot scale. This is not an obvious result as the Slingshot primordial spectrum presents a non-trivial running of the spectral index.

2 The original Slingshot power spectrum

The Slingshot power spectrum of scalar perturbations is related to the quantum fluctuation of the Slingshot $D3$-brane. The way of producing this perturbation is similar to the one introduced by [9] but without the drawbacks outlined by [10] (see [3]). The fluctuation of the brane is of quantum origin and it is in a pure state whenever the comoving wave length of the perturbation is below a fundamental quantum length l_{c}. This fundamental length can be consistently chosen to be the first massive mode of the fundamental String or the M-theory minimal length.

During the pre-bounce phase of the Slingshot, the perturbations created in the far past (in the Bunch-Davis vacuum), eventually come back to their vacuum state whenever their wavelength $\lambda = a/k < l_c$, where k is the wave number of the fourier mode associated with the wavelength λ and a is the scale factor of the induced cosmology on the brane. Viceversa, as the brane re-expands, the wavelength of a given perturbation will grow again. It is therefore clear that after some time, the perturbation wavelength will reach the scale l_c. At this point the perturbation collapses into its classical state becoming a coherent state as the perturbation is over-damped by the Universe expansion [11]. After waiting enough time, a stochastic background of primordial perturbations is therefore dynamically (and continuously) created. The distribution of these perturbations is gaussian with variance set by the quantum correlations during the quantum to classical transition. However, since the Slingshot Scenario represent a bouncing cosmology, not all the wavelength of primordial perturbations can be produced with this mechanism. In fact the maximal wavelength that can be produced is

$$\lambda_{\text{cut-off}} = \frac{a_b}{k_{\text{cut-off}}} = l_c, \tag{1}$$

where a_b is the size of the scale factor at the bouncing. This obviously creates a natural cut-off on the power spectrum.

A remark here is due. The physical process we have just discussed has been developed in the String frame. There, at zeroth order on the brane velocities [3], the gravitational coupling is running ($G_N \sim a^2$) and particle masses are fixed. The Einstein frame, in which the gravitational coupling is constant, can be then easily obtained by re-scaling the metric by a^{-2}. At the background level then, the spacetime in Einstein frame is Minkowski and all particle masses run (note that also the matter Lagrangian is re-scaled). It is therefore easy to convince ourselves that any physical quantity in the two frames is completely equivalent (see [12] for a general discussion and [3] for the Slingshot case). Let us, however, comment the special case of the perturbation spectrum. In String frame the perturbed metric is $ds_S^2 = a^2(1 + 2\Phi)dt^2 + \cdots$ where Φ is the Bardeen potential. By re-scaling to the Einstein frame such that $ds_E^2 = (1 + 2\Phi)dt^2 + \cdots$, we still obtain the same Bardeen potential Φ. The power spectrum of primordial perturbations (a physical quantity) is therefore unchanged by the change of conformal frames. However, one might still be puzzled whether the String frame cut-off on the perturbations spectrum is still there in the Einstein frame. In the Einstein frame the wavelength of a perturbation is constant in time, i.e. $\lambda = 1/k$. However, the quantum length in this frame is re-scaled, together with any other physical length, by a factor a^{-1}. In the Einstein frame the quantum length is therefore "bouncing". This produces, in the Einstein frame, the same cut-off as observed in the String frame.

In [3], the Slingshot primordial power spectrum is calculated to be

$$P_{\text{red}}(k) = \frac{A}{k} e^{\frac{1}{2}W_{-1}\left(-\frac{k_0^4}{k^4}\right)} \left[1 - e^{\frac{1}{2}\Delta(k_0/k; k_{\text{cut-off}})}\right], \tag{2}$$

where W_{-1} is the Lambert W function in the real branch -1 (see [13] for a description of the Lambert function properties), A is an overall normalization of the spectrum and finally k_0 is a parameter defining the spectral index at large k_s.

The cut-off function $\Delta(k_0/k; k_{\text{cut-off}})$ is defined as

$$\Delta(k_0/k; k_{\text{cut-off}}) = W_{-1}\left(-\frac{k_0^4}{k^4}\right) - W_{-1}\left(-\frac{k_0^4}{k_{\text{cut-off}}^4}\right), \tag{3}$$

where $k_{\text{cut-off}}$ fixes the cut off as P_k is positive definite. If the ratio k_0/k is small we have $W_{-1}(-k_0^4/k^4) \simeq 4\ln(k_0/k)$. Therefore for large wave number (i.e. small scales)

$$e^{\frac{1}{2}W_{-1}\left(-\frac{k_0^4}{k^4}\right)} \simeq \frac{k_0^2}{k^2} . \tag{4}$$

In this limit $e^{\frac{1}{2}\Delta} \ll 1$ and the spectrum looks scale invariant.

In order to match the best WMAP fit of a power law power spectrum, following [3], we fixed the spectral index of the Slingshot to be $n_s = d\ln k^3 P(k)/d\ln k + 1 = 0.95$ at the pivot scale $\ell = 100$. If we neglect the correction due to the cut-off scale, this fixes the parameter k_0 to be $k_0 = 8.86648 \times 10^{-7} \, h\,\text{Mpc}^{-1}$. This parameter is much smaller than any wave number we are going to consider. We can then use the analytical properties of the Lambert W function $W_{-1}(-x) \simeq \ln(x) - \ln(-\ln(x))$, to find the approximate spectrum

$$P_{\text{red}}(k) \simeq \frac{A}{k^3\sqrt{\ln\frac{k}{k_0}}} \left[1 - \frac{k_{\text{cut-off}}^2}{k^2}\sqrt{\frac{\ln\frac{k_{\text{cut-off}}}{k_0}}{\ln\frac{k}{k_0}}}\right] . \tag{5}$$

This completes the description of the previous results obtained in [2; 3]. In the following section we are going to consider an additional contribution to the Slingshot primordial power spectrum that was not kept into account in previous works.

3 A blue pre-bounce residual

As previously discussed, primordial perturbations are in general present even during the pre-bounce era. These perturbations, corresponds to the quantum fluctuation of the Slingshot brane during its motion down the throat of the CY. The induced Bardeen potential Φ evolves with the angular brane motion and its perturbations, as discussed in [3]. Nevertheless, we can use the approximations used in [2] where the Bardeen potential results decoupled from the angular brane motion.

In this case, Φ, follows schematically the Mukhanov equation [2]

$$\delta r'' + \left(k^2 - \frac{J^2}{r^4}\right)\delta r = 0, \tag{6}$$

where $r\Phi = \delta r$, J is the brane angular momentum, r parameterize the brane position in the CY throat and finally $'$ is the derivative with respect to the conformal time. J^2/r^4 corresponds to r''/r. Differently from the inflationary case, however, r''/r *is not* the Hubble horizon.

The induced scale factor of the Universe a is related to r as $a = \frac{r}{L\sqrt{\ln r/r_{\rm s}}}$. L is proportional to the number N of $D3$-branes in the stack and $r_{\rm s}$ is the radius of the blown up sphere at the tip of the CY (see [2] for more details). We then see that for $k \gg \frac{J}{r^2}$ the system oscillates, particles are not created and the Bardeen potential stays in its vacuum. In the opposite case, $k \ll \frac{J}{r^2}$, the system is instead over-damped and eventually Φ becomes constant. There, particles are created and the system evolves stochastically. In the large k region we therefore have $\langle \Phi(k)\Phi(k')\rangle \propto \frac{\delta(k,k')}{kr^2}$. At the matching point $k = \frac{J}{r^2}$, we then have a constant spectrum of perturbations, i.e. a power law spectrum with spectral index $n_{\rm s} = 4$. Note that if $J = 0$, i.e. for a brane with no-angular momentum, the perturbation is never over-damped and therefore the spectrum will be $P(k) \propto k^{-1}r^2$ at any times. In this case the resulting spectral index will be $n_{\rm s} = 3$ as found in [14].

These conclusions can also be drawn more precisely by following [3] and by considering the semi-classical to quantum matching point at $k = Jr^{-2}$.

We then conclude that a blue spectrum of primordial perturbations, coming from the pre-bounce, must be added to the $P_{\rm red}(k)$. However, this spectrum will survive from being destroyed by the quantum region only for perturbation scales $k < k_{\rm cut\text{-}off}$, as discussed before. Therefore the full spectrum of perturbation turn out to be

$$
P(k) = \begin{cases} P_{\rm red}(k), & \text{if} \quad k > k_{\rm cut\text{-}off} \\ P_{\rm blue}(k) \equiv \frac{B}{k_0^3} & \text{if} \quad k < k_{\rm cut\text{-}off}. \end{cases} , \tag{7}
$$

where the amplitude B is a completely free parameter. The blue part of the spectrum (7) did not appear previously in the literature. Thus, Eq. (7) constitutes the most general parameterizations of the Slingshot power spectrum and completes the previous results of [2; 3]. In the following section we will numerically compute the CMB angular power spectra arising from this primordial spectrum.

4 Matching the WMAP results

The CMB temperature and polarization angular power spectra are obtained from the primordial power spectrum of scalar perturbations $P(k)$ through the well-known formula (see, e.g. [15]):

$$
C_\ell^{XX} = (4\pi)^2 \int\limits_{k_{\rm min}}^{k_{\rm max}} dk k^2 P(k) \left(\Delta_\ell^X(k)\right)^2, \tag{8}
$$

where $\Delta_\ell^X(k)$ are the radiation transfer functions and $X = T, E$ defines temperature and polarization, respectively. The temperature and polarization transfer

Fig. 1 Temperature and polarization CMB power spectra in the slingshot scenario, compared to the standard WMAP 3-year best-fit cosmological model. The cosmological parameters $\Omega_b h^2 = 0.02218$, $\Omega_c h^2 = 0.1010$, $\Omega_\Lambda = 0.77$, $\tau = 0.09$, $h = 0.74$ are the same in the two model. The primordial spectral index in the standard scenario has been chosen as $n = 0.95$, while the slingshot primordial power spectrum is described in Sects. 2 and 3. The slingshot power spectrum normalization is chosen in order to match the amplitude of the CMB angular power spectrum at $\ell = 100$ obtaining $k_0 = 8.86648 \times 10^{-7} \, h \, \text{Mpc}^{-1}$. We consider several different cut-off scales and $B \ll \bar{B}$ (see text for further details)

functions can be extracted from a Boltzmann code like, e.g. CMBfast. The angular power spectrum predicted by the Slingshot scenario can then be calculated starting from the primordial power spectrum of formula (7) and numerically evaluating the Lambert W-functions. The Slingshot power spectrum expression contains two new free parameters: the cut-off scale $k_{\text{cut-off}}$ and the amplitude B of the large scale part $P_{\text{blue}}(k)$. We will now study the effects of varying these parameters on the final C_ℓs.

The first alternative we consider is to choose $k_{\text{cut-off}}$ such that $k_{\text{cut-off}} \ll k_{\text{min}}$ where k_{min} is the smallest wavenumber appearing in the integral defined by Eq. (8). A cut-off below k_{min} then clearly affects only scales that are unobservable. The choice of B is then not relevant in this case and we can replace $P(k)$ in Eq. (8) with $P_{\text{red}}(k)$ defined in Eq. (2). We calculated the CMB power spectrum using this power spectrum and transfer functions obtained from the WMAP 3-year best-fit cosmological parameters. The resultant Slingshot C_ℓs in this case are represented by the dot-dashed green line in Fig. 1. In the same figure, the solid black line represents the standard WMAP best-fit power spectrum. The two spectra present a very good match, thus showing that, just by fitting the two parameters A and k_0, *the Slingshot model allows to well reproduce the standard CMB angular power spectrum from WMAP*. More precisely, an explicit calculation of the likelihood for the Slingshot shows that the goodness- of-fit relative to the standard WMAP 3-year spectrum is $\Delta\chi^2_{\text{eff}} = 3$, which, being non statistically significant [8], makes the slingshot still a good fit of the data. As we were already stressing above, this result was not obvious due to the non-trivial running of the Slingshot primordial power spectrum.

Let us now consider a cut-off on scales that are relevant for the CMB, i.e. $k_{\text{cut-off}} \gtrsim k_{\text{min}}$ in Eq. 8.

In this case the wavenumber $k_{\text{cut-off}}$ will roughly define an angular cut-off $\ell_{\text{cut-off}}$ below which the angular power spectrum is basically obtained from a constant primordial power spectrum $P(k) = P_{\text{blue}} \equiv B/k_0^3$ (see Eq. (7)). At this point we found it useful for our analysis to determine a value of the normalization parameter B which makes the amplitudes of $P_{\text{red}}(k)$ and $P_{\text{blue}}(k)$ to roughly coincide at the pivot scale (that we chose to be $k \sim 10^{-3}$ in our analysis). In order to match WMAP data for $\ell > \ell_{\text{cut-off}}$ we need $|k^3 P_{\text{red}}(k)| \sim 10^{-10}$. Thus matching the two amplitudes yields:

$$k_p^3 \frac{B}{k_0^3} = k_p^3 P_{\text{red}}(k_p) \sim 10^{-10}. \tag{9}$$

Fig. 2 Comparison between WMAP data and low CMB multipoles for different values of $k_{\text{cut-off}}$ and fixed $B \ll \bar{B}$ in the Slingshot scenario

Fig. 3 We compare different values of the low-ℓ spectrum normalization B for a given value of $\ell_{\text{cut-off}} \sim 10$ and optical depth to last scattering $\tau = 0.09$. The *solid black line* represents the standard WMAP 3-year best-fit power spectrum while the *dashed blue* and *dotted red lines* are the Slingshot power spectra for small and large values of B, respectively

With our values of $k_0 \sim 10^{-6}$ and $k_{\text{p}} \sim 10^{-3}$, we get $B \equiv \bar{B} \sim 10^{-19}$. We can now distinguish between two cases: $B \ll \bar{B}$ and $B \gtrsim \bar{B}$.

Let us first take $B \ll \bar{B}$ and consider several different $k_{\text{cut-off}}$. In this case the amplitude of P_{blue} is much smaller than the amplitude of P_{red}. We then expect to see a suppression of power on scales $\ell < \ell_{\text{cut-off}}$. This is shown in Figs. 1 and 2. The same pictures also suggest that a cut-off scale $k_{\text{cut-off}} \lesssim 2 \times 10^{-4}$ is still a good-fit to the data: the goodness-of-fit relative to the WMAP best-fit spectrum is $\Delta \chi^2_{\text{eff}} \leq 3$ in this range (for a similar discussion applied to a different model see [16]).

Let's now move to the case $B \gtrsim \bar{B}$. We can now choose B large enough to eliminate the suppression of the larger angular scales that we have just described above. In Fig. 3 we consider an angular cut-off scale $l_{\text{cut-off}} \sim 10$ and we show that a choice of the normalization $B \sim \bar{B}$ can significantly improve the goodness-of-fit relative to the case $B \ll \bar{B}$ with the same cut-off ($\Delta \chi^2_{\text{eff}} = -112$). In other words this suggests that a full likelihood analysis of the slingshot parameters (which is beyond the purpose of this work) would show some degeneracy between $k_{\text{cut-off}}$ and B. Nevertheless it is important to note that this does not allow to arbitrarily increase the cut-off scale. The slope of the angular power spectrum for $\ell < \ell_{\text{cut-off}}$ is indeed completely different in the two regimes and this becomes rapidly evident for large $\ell_{\text{cut-off}}$. We then conclude that also in this case any possible Slingshot-related signature is unfortunately confined to the first few CMB multipoles, characterized by a large cosmic variance. For this reason it seems impossible to use the CMB TT, TE, and EE angular power spectra as a way to discriminate between Slingshot and standard inflationary cosmology. To this purpose further investigation in other directions might be interesting (e.g. non-Gaussian signatures, gravitational wave background).

Even if we give up the idea of finding specific observable signatures of the Slingshot model in the CMB temperature and polarization power spectra, we are still left with the interesting following question: if we *assume* the Slingshot as the scenario for the generation of primordial fluctuations, and we repeat the analysis of WMAP results in this framework, are we going to see any change in the final cosmological parameters? In the acoustic peaks region both the Slingshot and the inflationary power spectrum have the same slope, so we already know that the answer to the previous question is 'no' for most of the parameters. As the only differences can be at small ℓ, it seems that the only parameter that can in principle be affected is the optical depth at reionization τ. Let us elaborate on this. WMAP is known to predict a large optical depth to reionization $\tau \simeq 0.09$, or equivalently an early reionization at a redshift $z \sim 10$. The signature of this early reionization is mainly in the bump observed at low ℓ in the TE and EE angular power spectra: there wouldn't be any primordial polarization signal on large angular scales in absence of early reionization. This conclusion still holds in the Slingshot scenario

Fig. 4 We consider a relatively low value of the optical depth to reionization, $\tau = 0.06$, and study the effect of varying the normalization parameter B in order to look for possible degeneracies between τ and B. The *solid black line* is the standard inflationary power spectrum whereas the *dotted red line* is the Slingshot prediction for a suitably large value of B. Even if we can improve the fit of polarization data with respect to the standard case, we produce a bump in low-ℓ temperature spectrum which does not allow to fit the data well ($\Delta\chi^2 = 1{,}219$)

(it is only related to the physics of Compton scattering). However, the low-ℓ part of the polarization spectrum is described in the Slingshot by three parameters, B, $k_{\text{cut-off}}$ and τ. Possible degeneracies among these parameters could then eventually change the best-fit value of τ.

In particular it might now happen that a value of τ significantly smaller than in the standard scenario could still allow a good fit of the low-ℓ polarization bump if we compensate for it by *increasing the amplitude B*. This also works in the opposite direction: we can increase τ and reduce B accordingly. This mechanism is clearly very efficient if we limit ourselves to considering polarization data only. The situation, however, drastically changes when we account for temperature data. A large τ produces a low-ℓ bump in the TE and EE but it does not affect the TT power spectrum. A large B instead produces a large enhancement of the low-ℓ TT power spectrum as well. This effect is not compatible with the data if we have to compensate for a very small (large) τ with a very large (small) B. In other words temperature data contribute to largely breaking the degeneracy between τ and B that arises from polarization data alone. An example of this is in Fig. 4 where we try to fit data with an optical depth $\tau = 0.06$ and all the other WMAP parameters unchanged.

5 Conclusions

In this paper we have studied some phenomenological implications of the cosmological Slingshot scenario introduced in [2; 3]. In the first part of the paper we have derived an expression of the primordial power spectrum of cosmological perturbations arising from the Slingshot (formula 7). This expression generalizes previous results by [2; 3] in that it contains a blue contribution to the spectrum which had not been considered before. In the second part of the paper we have numerically computed the CMB temperature and polarization power spectra arising from the Slingshot primordial spectrum. First, we showed that a suitable choice of the Slingshot parameters allows to match Slingshot predictions with the WMAP 3-year best-fit power spectrum. More precisely we showed through a relative goodness-of-fit approach that the slingshot predictions are not, in a statistical sense, worse than the best WMAP fit of a power law primordial spectrum. This conclusion has been drown by fitting the slingshot power spectrum spectral index to be 0.95 (best WMAP fit) at some pivot scale and by keeping all the standard cosmological parameter unchanged. To gain a more precise insight it would be, however, very important to perform a Montecarlo analysis were all parameter, in particular the spectral index, can change. This might in principle find a better fit to the data. However, as the aim of the present paper was only to show that, with the same WMAP parameters, the slingshot power spectrum is a good fit of the data,

the above mentioned complete statistical analysis is left for future work. In the last part of the paper we finally looked for possible specific signatures of the Slingshot in CMB data that could allow to distinguish it from the standard scenario. Unfortunately all the distinctive Slingshot features turn out to be confined to the low-ℓ part of the spectrum, where large cosmic variance dominated error bars prevent from any significant discrimination between the two scenarios.

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