Precise Kinetic Energy Measurement in Penning Trap

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Introduction

Neutrinos play a very crucial role in our understanding of β-decay and the universe and their masses are still unknown. Different groups around the world are trying to determine [1] the masses of different types of neutrinos. Several attempts are going on to determine the mass of an electron-neutrino from the end-point energies of β-decay spectra and KATRIN tritium experiment places a best limit on the electron-neutrino mass $m_\beta \leq 2.05$ eV/c^2 at 95% C.L[2]. One of the important ingredients in such measurements is high precision measurements of the kinetic energy of the electrons. We present a simulation work showing the feasibility of determining the kinetic energy of trapped energetic electrons or ions in a Penning trap with high precision from the measurement of the oscillation frequencies of the trapped particle. The simulation work has been done using SIMION 8.1 [3] for the mono-energetic 17830 eV conversion electrons from a ^{83m}Kr source.

In Penning Trap, the superposition of homogeneous axial magnetic field and weak quadrupole electric field results in three dimensional confinements of charged particles. The axial oscillation frequency of a trapped electron with electric charge q in the purely harmonic potential region of a trap is given by

$$\omega_z = \sqrt{\frac{qV_0C_2c^2}{(R.E.+K.E.)d^2}} \qquad (1)$$

where V_0 is the applied voltage between the ring and the end-cap electrode, C_2 is the quadrupolar coefficient of trap potential, d is the characteristic length of the trap, R.E. is the rest mass energy (0.511 MeV) of electron, K.E. is the kinetic energy of the electron and c is the velocity of light [4]. The image induced by axial motion of trapped electrons on the trap electrodes is picked up detection circuit and its FFT (Fast Fourier Transform) give the required frequency which is related to kinetic energy of trapped particle. In order to measure the electron energy to a precision ΔE, we need to measure the frequency to a relative precision of Δf/f =ΔE/R.E. For ΔE =100 eV this implies Δf/f = 2 x 10^{-4}. To achieve a precision of Δf in frequency, according to Nyquist's theorem we need to monitor the signal for t_{min} =2/Δf. To achieve this accuracy of Δf/f = 2 x 10^{-4} , we had to run the simulation for 2 consecutive days at each particle positions inside the trap. We have found through simulation that the frequency of the trapped high energy electron is different at different locations in the trap. However, it is highest when it is trapped centrally with minimum axial energy and this frequency could be used to extract the kinetic energy very accurately. This technique can be applied to even higher precision of frequency measurement as Penning trap is routinely used in various labs to measure frequency with an accuracy of 1x10^{-6} or better. This accuracy would imply measuring mass of neutrino with an accuracy of 1 eV/c^2 or better and it would make it competitive with the best technique available so far in the world. But, its simulation would require much longer computational time.
As it is required to trap mono-energetic electrons with energy of 17830 eV, we will catch them using magnetic mirroring effect where the entire energy of the electrons is transferred in the radial motion and hence it will be possible to trap them centrally with application of small voltage (~100V) on the trap electrodes.

Magnetic Mirror

In simulation, the magnetic field has been generated with a finite solenoid considering edge effects. When the electrons are released from the region of weak field, they experience the gradient of magnetic field while moving towards the centre of solenoid. Depending on the initial condition of electron emission from the source, they undergo reflection from the strong magnetic field. It is known as magnetic mirroring. Mirror effect can be shown mathematically by assuming particle's magnetic moment to be conserved. This is known as the adiabatic invariance. Let B_0, B_1 be the magnetic field strengths in weak and strong field regions. Considering the invariance of the magnetic dipole moment of the rotating charged particle in inhomogeneous magnetic field, we obtain [2]

$$\mu = \frac{m v_{0,\perp}^{2}}{2B_0} = \frac{m v_{1,\perp}^{2}}{2B_1} \qquad (2)$$

So, in the weak magnetic field region, most of the energy is parallel to the magnetic field i.e. the parallel component of velocity is high. Whereas, in the strong magnetic field region, the perpendicular velocity $v_{1,\perp}$ is higher i.e. the whole motion is perpendicular to the direction of magnetic field. Thus, the high energy electron slows down and reflects back from the strong field region without any loss of energy. The trap is placed inside the solenoid in the place where the electron with desired energy would be reflected. With proper switching of trapping potential and chopping the initial emission from the source, it would be possible to trap electrons almost centrally with minimum energy in axial direction.

Kinetic energy damping simulation

In the experiment, the image charge induced by a trapped electron on the electrodes would be picked up by a resonant tank circuit. It follows from the conservation of energy that the energy which is extracted in the process of detection of the trapped electron would be drawn out from the energy of the stored electron. As a result, resistive cooling of the trapped electron would take place and the kinetic energy of the trapped electron would be damped continuously. The simulations have been done for 17830 eV energy electron trapped centrally in 100 V trap with minimum energy in axial direction and the exchange of energy with the detection circuit has been considered. Fig. 1 shows the FFT of such detection signal. It is found that instead of a single narrow peak which is expected at theoretical value of axial frequency= 254.8MHz as obtained from eqn. 1, one obtains a broad peak as shown in Fig. 1 where the low frequency rising edge matches exactly with the theoretical value. Thus, in spite of the variation of KE of the trapped electron during the measurement process, it is possible to determine the value of initial KE from the FFT signal.

Conclusion

This work gives a completely new way of measuring high kinetic energy of electrons using Penning trap and magnetic mirror. An energy resolution of 105 eV at 17830 eV has been achieved through simulation. This technique has the potential ability to achieve energy resolution beyond the accuracy achieved so far. Our work opens up the possibility of measuring the kinetic energy of electrons with an accuracy of 1 eV/c^2 or better and could be useful in neutrino mass measurement experiments.

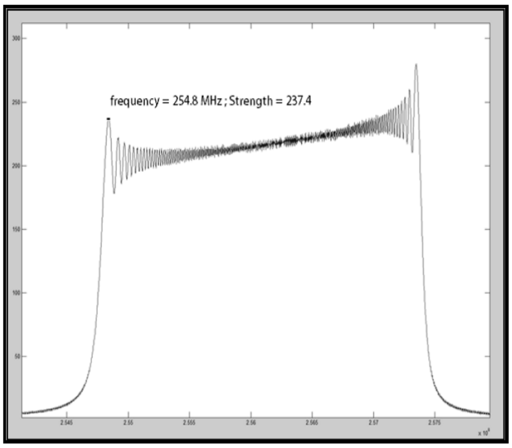

Fig. 1. Typical FFT of axial motion of trapped electron with continuous energy damping due to resonant circuit

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