Ordinary matter, dark matter, and dark energy on normal Zeeman space-times

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Abstract. Zeeman space-times are new, relativistic, and operator based Hamiltonian models representing multi-particle systems. They are established on Lorentzian pseudo Riemannian manifolds whose Laplacian immediately appears in the form of original quantum physical wave operators. In classical quantum theory they emerge, differently, from the Hamilton formalism and the correspondence principle. Nonetheless, this new model does not just reiterate the well known conceptions but holds the key to solving open problems of quantum theory. Most remarkably, it represents the dark matter, dark energy, and ordinary matter by the same ratios how they show up in experiments. Another remarkable agreement with reality is that the ordinary matter appears to be non-expanding and is described in consent with observations. The theory also explains gravitation, moreover, the Hamilton operators of all energy and matter formations, together with their physical properties, are solely derived from the Laplacian of the Zeeman space-time. By this reason, it is called Monistic Wave Laplacian which symbolizes an all-comprehensive unification of all matter and energy formations. This paper only outlines the normal case where the particles do not have proper spin but just angular momentum. The complete anomalous theory is detailed in [Sz2, Sz3, Sz4, Sz5, Sz6, Sz7].

1. Introduction

Dirac adamantly believed that the great challenges of particle physics should have been attacked by quantum wave operators which were explorable by the Hamilton formalism and the correspondence principle. Nonetheless, almost all modern theories such as the Standard Model or String Theory have been established by Lagrangian methods, where, as opposed to the waves, the particles are conceived as tiny billiard balls moving according to Yang-Mills equations established on suitable $SU(n)$ models by the principle of least action.

The Lagrangian method has become the major tool ever since Schwinger recognized the importance of gauge invariance in solving the infinity puzzle in QED and proposed to apply this principle to describe the weak interactions, as well. Dirac, however, most impatiently urged to return back to the complete Hamiltonian formalism and describe the quantum world in terms of natural Hamilton operators

and the associated matter waves. He himself implemented this tool in a circuitous way, which started out with a relativistic Lagrange function and the desired Hamilton operator was found after associating Hamilton functions to the Lagrangian considered in the first place [D]. By doing so, he wanted to see quantum theory as a new discipline but which is emerging in strong relations with the classical theories. Since this idea was not particularly helpful in solving the infinity puzzle, it has largely been ignored, since the mid 1940's.

The Zeeman manifold theory is a new operator based Hamiltonian model of a completely unified quantum theory which breaks away not just from the traditional Lagrangian but Dirac's circuitous Hamilton formalism as well. Namely, the desired Hamilton and the corresponding wave operators appear in the very first place as Laplace operators on particular, so called Zeeman manifolds and the corresponding Zeeman space-times. The main Lagrangian objects are introduced only in the second place which also include a natural compound scalar boson associated with the Hamilton and wave operators introduced in the first place. The wave operators are Laplacians on particular space-time models consistent with Einstein's general relativity. In order to distinguish this from the Hamilton formalism, it is called a "direct Einstein-Hamiltonian method" which could also be called "reversed Hamiltonian method" or operator based Hamiltonian model.

A clincher for this model passes the physical reality test is that all Hamilton operators established by Dirac's Hamilton formalism - which mostly arise from electromagnetic field theory - appear on Zeeman manifolds in the same classical form and one obtains relevant Hamilton operators even for weak, strong, and gravitational interactions, which have been explored, sofar, in SM and String Theory, just by Lagrangian methods. Although there are differences between them, the Zeeman manifold and space-times theories do not contradict SM but rather describes the electromagnetic, weak, and strong interactions in consent with this classical theory which has constantly been tested by experiments.

The metric on Zeeman manifolds - where the Hamilton operators are defined as Laplace operators - is still positive definite (Riemannian) and the relativistic Zeeman space-time having indefinite metric of Lorentz signature is introduced by appropriate extensions of Zeeman manifolds into the time direction. Then the indefinite (hyperbolic) Laplace operator appears as a natural relativistic quantum wave operator there. The quantum phenomena are described in terms of eigenwaves of this wave Laplacian. But it must be emphasized that the Zeeman space-time is only a stage; the true carrier of quantum physical contents are the waves and the probabilistic densities defined there.

The theory also implies a new cosmological model, expanding at an exponentially accelerating rate. It breaks away from the Big Bang theory in the sense that it describes the evolution of the universe as a dynamical "Eternal Whizz", started out infinitely long time ago and lasting forever. However, this model does not contradicts the Big Bang scenario. Namely, it is the Zeeman space-time, the stage of the quantum events, which has been existing forever. Even this stage is dynamical, going through several natural periods in which the quantum phenomena are described by the type of eigenfunctions characteristic for the period. There is also described a time-point, separating two periods, which could very well be the same as the time of the Big Bang. The reality of this model will be demonstrated by pointing out that the ordinary matter, dark matter, and dark energy appear according to the same ratios how they show up in Nature.

The prototype Zeeman manifolds are introduced on 2-step nilpotent Lie groups whose static and solvable extensions are the prototype Zeeman space-times. The metric on a two-step nilpotent Lie group is still Riemannian – namely, the positive definite left-invariant metric naturally present there – and the relativistic metric of Lorentz signature appears on the static and solvable extensions, respectively. The latter gives rise to the accelerating model where the exponentially accelerating expansion can be observed relative to the Euclidean metrics induced on the horospheres, defined as level sets by fixed time-values, such that the length of the horocycles' segments changes according to an exponentially accelerating expansion.

The main idea for realizing the unification is that the Hamilton and wave operators of all fundamental forces, as well as all matter-energy formations, arise from this single Laplace operator. More precisely, the wave functions associated with distinct matter-formations form distinct invariant subspaces and the corresponding Hamilton operators emerge as restrictions of the same Laplacian onto these invariant subspaces. The Zeeman Hamilton operator of charged point-particles orbiting in constant magnetic field emerges on torus bundles, defined by factoring the center of two step nilpotent Lie algebra by a lattice considered in the Z-space. The strong, weak, and gravitational Hamilton operators are associated with extended particles, whose wave functions are defined not on torus but Z-ball-bundles, where the balls are considered in the center.

Since this Laplacian gives rise to all Hamiltonians of particle theory, it is called Monistic Hamiltonian. The Laplacian on the Zeeman space-time is called Monistic Wave Operator. They work together with the Monistic Scalar Boson, comprising all bosons needed to carry out the actions of the Monistic Operator. The constituent bosons are the carriers of the corresponding fundamental forces. Their appearance as components of the Monistic Boson symbolizes the unification of fundamental forces.

This paper is focusing on the Hamilton and wave operators defined for the normal setting, where the particles can only have angular momentum. This is in contrast with those having proper spin where the corresponding operators act not on functions but spinors. The anomalous operators together with the Monistic Boson are outlined in [Sz2]. The full mathematical foundation of the theory is evolved in [Sz3]-[Sz7].

The paper consists of two major sections. The first one describes the Hamilton operators on Zeeman manifolds, including the explanation as to how does Schrödinger's classical Hamilton operator emerge from the Laplacian acting on functions periodic regarding a Z-lattice. These are the waves of point particles. By contrast, the waves of extended particles are defined on Z-ball bundles. The Monistic Hamilton Operator restricted onto this invariant function space defines the Hamiltonians by which the strong, weak, and gravitational interactions can be described. The second section is devoted for describing the expanding Zeeman space-time. The most important problem to be solved is to find appropriate decomposition of the Laplacian which respectively correspond to the wave operators of dark matter dark energy and ordinary matter. The investigations are carried out in relation to a new spectral mass assignment procedure which is analogous to the Higgs mechanism. One of its distinguishing feature is that it is not based on symmetry breaking but is purely established by the Monistic Wave Operator. Nonetheless, it relates to symmetry breaking.

2. Hamilton operators on Zeeman manifolds.

Zeeman space-times are established by relativistic extensions of Zeeman manifolds into the time direction. The latters are still endowed with positive definite Riemannian metrics whose Laplace-Beltrami operators serve as Monistic Hamilton Operators from which the Hamilton operators of the represented particle systems are derived. Because of the positiveness of the energy levels, the Hamilton operators must be elliptic, this is why the metric on a Zeeman manifold should be positive definite. The hyperbolic wave operator, which also includes the Monistic Hamilton Operator, will appear as Laplace operator on the Lorentzian Zeeman space-times, obtained by extension.

2.1. Mathematical construction of Zeeman manifolds.

In order to introduce the theory in a relatively easy way, this paper focuses mostly on prototype Zeeman manifolds, which, however, do not oversimplify the presentation. Quite to the contrary, these particular examples form a rather large subclass of the general category exhibiting all important features there. They are defined on two step nilpotent Lie groups, particularly on Heisenberg type groups endowed with natural left-invariant metrics. The prototype space-time indicated above will be introduced by the static resp. solvable (expanding) extensions of two step nilpotent Lie groups. The rudiments about H-type groups are as follows [K].

   The Lie algebra of these groups are defined on the Cartesian product $\mathcal{X} \times \mathcal{Z} = \mathbb{R}^k \times \mathbb{R}^l$ of Euclidean spaces, where the X-space $\mathcal{X}$ is of even dimensional, by a one-to-one linear map $J : \mathcal{Z} \to \mathrm{SkewEnd}(\mathcal{X})$ associating with any Z-vector $Z \in \mathcal{Z}$ a skew endomorphisms, J_Z, acting on the X-space. Then, for any two vectors $X, Y \in \mathcal{X}$, the Lie bracket $[X, Y] \in \mathcal{Z}$ is defined by $\langle [X, Y], Z \rangle = \langle J_Z(X), Y \rangle$, for any $Z \in \mathcal{Z}$. This, together with $[\mathcal{X} \oplus \mathcal{Z}, \mathcal{Z}] = 0$, completely determines a two step nilpotent Lie algebra. The particular H-type Lie algebras are introduced by the Clifford condition $J_Z^2 = -|Z|^2 \mathrm{Id}$ which identifies them with Clifford modules whose well known classification also gives rise to classifying all possible H-type algebras.

   The Lie group, N, determined by this Lie algebra is a Hadamard manifold, also defined on $\mathbb{R}^k \times \mathbb{R}^l$, where the group product is given by:

$$(X, Z)(X', Z') = (X + \frac{1}{2}X', Z + Z' + \frac{1}{2}[X, X']). \tag{1}$$

The Lie algebra is identified with the tangent space at 0, which carries the natural Euclidean inner product $\langle ., . \rangle = g_0(., .)$ whose left-invariant extension is the Riemannian metric $g_p(., .)$ considered on the metric Heisenberg type Lie group.

   Above, the left-invariant metric is defined by the left-invariant extension of the natural inner product given on the tangent space at the origin, nonetheless, it can also be described in terms of the unique connection, whose horizontal subspaces are spanned by the left-invariant extensions of the X-vectors in the Lie algebra. The connection form over an X-vector is defined in terms of an orthonormal basis $\{e_\alpha\}$ of the Z-space by $\omega_X(Y) = -\frac{1}{2} \sum_\alpha \langle J_\alpha(X), Y \rangle e_\alpha$. Then the invariant metric on the total space can also be described such that the metric of the X-space is uplifted to the horizontal subspaces, the horizontal and vertical subspaces are perpendicular, moreover, on the vertical subspaces, it agrees with the Euclidean metric of the Z-space. Since the connection is of non-zero curvature, this metric is of non-Euclidean. According to these, a H-type group appears on the total space of a trivial vector bundle $\mathcal{X} \times \mathcal{Z}$, where $\mathcal{X}$ is the base and $\mathcal{Z}$ is the fiber, which is endowed with a Yang-Mills

type metric. Yet, it is not a particular Yang-Mills model. One of the distinguishing features is that the main physical object – the Monistic Hamilton Operator – appears on the total space, as opposed to the Yang-Mills models where the gauge invariant objects are defined on the base.

The theory extends to certain trivial vector bundles defined over certain Kähler manifolds. There is only a limited collection of manifolds where the Laplace operator appears as a realistic physical Hamilton operator. It turns out that the HyperKähler-Zeeman manifolds, which also include the Calabi-Yau-Zeeman manifolds, are the only Riemannian manifolds where the Laplace-Beltrami operator meets the criteria of physical reality. Later arguments show that the weak interactions can only be modelled by such manifolds on which the same Riemannian metric is Kähler regarding not just one but more independent complex structures. Such are exactly the hyperKähler manifolds where these complex structures define higher dimensional Z-spaces and hereby giving rise to appropriate HyperKähler-Zeeman manifolds exhibiting all aspects needed for a full-fledged physical model. An other point showing that the prototype models are not generic is that they describe just magnetic aspects but the electric ones are beyond their scope. The detailed general theory, which reflects all possible aspects of physics, is described in [Sz6]. Except for the remarks in Section 2.4, this paper does not pursue this topic further.

2.2. The Monistic Hamilton Operator: $\mathcal{MH}$.

The unification is established such that the Hamilton operators of distinct matter formations are derived from the very same Laplace operator existing on Zeeman manifolds. By this reason it is called Monistic Hamiltonian and it is denoted by $\mathcal{MH} = -\Delta$. Its negative is the Beltrami-Laplace operator Δ, which appears on prototype manifolds in the form:

$$\Delta = \Delta_X + (1 + \frac{1}{4}|X|^2)\Delta_Z + \sum_{\alpha=1}^{l} \partial_\alpha D_\alpha \bullet . \tag{2}$$

Symbol $D_\alpha\bullet$ denotes directional derivative on the X-space by the Hopf vector fields, defined at X by $J_\alpha(X)$. The physically most important part of Δ is the last one that involves $D_\alpha\bullet$. It is a compound angular momentum operator defining orbiting spin for all particles appearing on Zeeman manifolds.

Formula (2) can be established by the identity $\Delta = \sum_i \mathbf{X}_i^2 + \sum_\alpha \mathbf{Z}_\alpha^2$, valid for left-invariant metrics defined on a Lie group. Here $\mathbf{X}_i$ and $\mathbf{Z}_\alpha$ denote orthonormal left-invariant vector fields, respectively. The negative sign before Δ insures the positiveness of the eigenvalues of $\mathcal{MH}$.

This is the Laplacian which directly emerges on particular Riemann manifolds, without using any kind of Hamilton formalism. Since this operator is defined on the total space, it can actually not be established by the traditional Hamilton formalism. In Yang-Mills and the related Hamilton formalism, the fundamental objects - such as gauge field - are defined on the base and must be gauge invariant, meaning that they do not change if they are defined regarding a new connection form $\tilde{\omega}$, obtained by adding a closed form to the original connection form ω.

This change is called gauge transformation of ω. Gauge transformation of the metric means defining the metric regarding the new connection. That is, by uplifting the metric from the base to the new horizontal subspaces, which stand perpendicular to the vertical subspaces, furthermore, one keeps the original inner product on the

vertical subspaces. It is rather remarkable that operator (2) is gauge invariant in the following sense: Metric $\tilde{g}$, defined from g by gauge transformation, is isometric to g, thus Laplacians Δ and $\tilde{\Delta}$ are isometrically equivalent.[Sz6]

2.3. Particle interpretations; Quarks vs. Splinters.

The model represents particles in two different ways. Primarily, it depicts them as "whole particles" but which naturally decompose into fractions - called splinters - which actually are the correspondents of quarks on Zeeman manifolds. The objects are preferably pictured by waves, defined on the total (X, Z)-space. Nevertheless, the wave-packets, visualizing the particles as tiny billiard balls, also are legitimate concepts, which can be included by the duality principle. On Zeeman manifolds, the laws of the quantum world are explored by the action of the Hamilton operator on the whole and splinter waves. This is opposed to the Lagrangian theories where the laws are explored by Lagrange equations describing the motions of the tiny billiard balls according to the principle of least action. The technical description of waves is provided in Sections 2.5 and 2.6.1. For better understanding, these particle interpretations are described in this section by less technical terms.

The whole particle waves are defined by functions which are invariant under the action of the compound angular momentum operator $\sum_{\alpha=1}^{l} \partial_\alpha D_\alpha \bullet$. A given whole particle system is defined by a system $\mathbf{Q} = \{Q_1, \ldots, Q_{k/2}\}$ of orthonormal X-vectors. For a unit Z-vector $Z_u = Z/|Z|$, the J_{Z_u} is a complex structure in terms of which the $\mathbf{Q}$ becomes a complex vector system. The required functions are introduced in terms of holomorphic and antiholomorphic polynomials defined for $\mathbf{Q}$. They portray the physical situation in $\mathcal{X} \times \mathcal{Z}$ so that the X-space, whose real dimension is $k = \dim_{\mathbb{R}}(\mathcal{X})$, appears as the exterior space for $\kappa = k/2$ number of "whole" particles which live in complex planes defined by the complex structures J_{Z_u} and system $\mathbf{Q} = \{Q_1, \ldots, Q_{k/2}\}$. Whereas, the Z-space is the common inner space shared by all of these particles. The particular form of the whole particle waves also allows to associate these particles with positive and negative charges as well.

The matter-particles appear in two forms, called point and extended particles, respectively. The waves of point particles are defined on torus bundles obtained by factoring the center by a lattice $\{Z_\gamma\}$ of the Z-space. Whereas, the waves of extended particles live on Z-ball bundles, defined by smooth fields of balls, $B_{\delta_Z(|X|)}(X)$, of radius $\delta_Z(|X|)$, considered - over each X-vector - about the origin in the Z-space.

The waves of point particles will be described in terms of the discrete Z-Fourier transform, defined for the lattice $\{Z_\gamma\}$ in the Z-space. They form an invariant subspace regarding the action of the Monistic Hamiltonian, where it appears as the Landau-Zeeman operator of charged particles orbiting in constant magnetic fields. The waves in this function space are "whole", consistent with the above description, which clearly exhibit the indivisibility of point particles.

The extended matter waves, however, will be introduced in terms of twisted Z-Fourier transforms, where an appropriate L^2-version of the Z-Fourier transforms is applied over each X-vector in the Z-space. They are appropriate adoptions of de Broglie's matter waves to the Zeeman setting. A particular feature of these waves is that they must satisfy prescribed boundary conditions, but which can not be enforced by "whole functions". The extended whole particles, also called Rutherford particles, form a special class including the neutrons, protons, and their combinations. In order to describe all extended particle waves satisfying the boundary conditions, the

whole waves must be decomposed into fractions, called splinter waves, which provide a profound insight into the inner structure of extended particles. They actually are waves of quarks, but, to avoid confusion, the particles arising from them are called splinter-particles, or splinters. A particularly important property is that all splinter waves are needed to determine a complete invariant subspace for the Monistic Hamilton operator, where its actions on the waves reproduce the electromagnetic, weak, strong, and gravitational interactions.

Summing up, the prototype Zeeman manifold models represent multi particle systems not by the traditional Cartesian product but by a certain modification where only the exterior world appears as the Cartesian product of 2-dimensional exterior worlds associated with individual whole particles. Their interior world, however, is not added to the X-space by the Cartesian product of the individual interior worlds but by a single Z-space which serves as a common interior world for all of the particles present on the manifold. It should strongly be emphasized that the Cartesian product strictly refers to whole-particles. This also means that the proper splinter-particles are added to the system not by Cartesian product but they live together with their whole parent particles and sibling splinters on the same Zeeman manifold. An other new feature is that not just the X- but the Z-space as well is considered to be space-like. Furthermore, the basic objects such as the Hamilton operators are considered on the total space and not just on the X-space. This is in sharp contrast with the Yang-Mills theory, where the basic objects appear on the base, and also contrasts the traditional view of Heisenberg groups where the Z-space (center) is the time axis.

2.4. Schwinger-type gauge invariance on Zeeman manifolds.

In QED, the importance of gauge invariance was recognized by Schwinger, in the late 40's. The presence of this property in a theory means that the results do not depend on the particular accountancy scheme used for their establishments. Schwinger exploited this principle in QED for maintaining the invariance of electric charge. His theory also implied the existence of a massless agent - the photon - that is carrying the information about the invariant charge from one electron to the other. He also suggested to work out these ideas for the much more complicated weak and strong forces and foresaw the possibility for the electroweak unification. His ideas inspired the Yang-Mills models of general gauge theories, as well as the Glashow-Salam-Weinberg realization of the electroweak unification. In Yang-Mills theory, the gauge bosons - carrying the informations about invariants - are of zero masses. This implication caused very serious problems in building up the weak interaction theory, where the gauge bosons must carry charges, thus their rest masses must be of non-zero. The difficulties have been resolved by theorizing the Higgs boson, providing mass to each particle of non-zero rest mass.

By an earlier remark, the Monistic Hamiltonian is gauge invariant, thus there is no fundamental contradiction with the Yang-Mills models. But there is an other important question to be answered, namely, how does Schwinger-type gauge invariance manifest itself, in other words, how is the information about invariant quantities distributed on Zeeman manifolds or Zeeman space-times? The answer is as follows.

As it is pointed out later, the charge can be introduced on Zeeman manifolds by the complex structures J_{Z_u}, defined for the unit Z-vectors Z_u, and the holomorphic and antiholomorphic linear functions appearing in whole particle waves. These complex structures actually give rise to a natural tool for delivering all invariant quantities

to each point of the X-space. They define the very same identification of Z-spaces over two distinct X-vectors than what is defined by adding the Z-space to the X-space by Cartesian product. Thus the distribution of invariant quantities by the complex structures or by the Z-spaces are equivalent procedures. Schwinger's gauge-invariance can completely be established on Zeeman manifolds by the Z-space, which, speaking topologically, is added to the X-space by Cartesian product. In other words, the common interior space gives rise to a natural tool delivering the very same information right into the insides of particles, over each point X of the exterior space. This information-distribution does not contradicts relativity, due to that that the matter waves described in terms of these invariants can not travel with speed greater than c. It neither implies that the bosons emerging in the theory move with speed c, therefore, their resting masses must a priori be zero. Contrary to this, there is a natural Hamiltonian mass-assignment process on the Zeeman space-time which attributes masses to charge carrying bosons. This process is analogous to the Higgs mechanism.

Schwinger's gauge invariance is furnished on general Zeeman manifolds, very similarly, by considering such Riemannian metrics on the X-space which are Kähler regarding not just one but number of complex structures. They determine a trivial Z-space over the X-space in the same way as on prototype manifolds. In the compact case, exactly the Calabi-Yau manifolds are those which allow such complex structures defining multidimensional trivial Z-spaces needed in the theory. The most generic versions are introduced in [Sz6], whose discussion is beyond the scope of this paper. Anyhow, the HyperKähler Zeeman manifolds, which include the Calabi-Yau-Zeeman manifolds, are the only general Zeeman manifolds which allow a completely unified relativistic quantum theory, similarly to that yielded by prototype manifolds.

The theory is explained in this paper on prototype manifolds only. In the next sections, the Hamilton operators of distinct particles will be established from the very same Monistic Laplacian (2). The main idea in carrying out this scheme is that the Hamilton waves of distinct particle systems define distinct invariant subspaces and the specific Hamilton operators are obtained by restricting the Monistic Hamiltonian onto the invariant subspace associated with the given particle system. One of the strongest indications for these investigations are heading into the right direction is that the Hamilton operator derived for a point particle system from the Monistic Hamilton Operator is equal to the Zeeman operator that has been established in classical quantum theory quite differently, by the Hamilton formalism and the correspondence principle.

2.5. Hamilton operators of point particles.

The Hamilton waves of a point particle system associated with a torus bundle $\mathcal{X} \times (\Gamma_Z \backslash \mathcal{Z})$ are defined by the discrete Z-Fourier transform: $\Psi = \sum_\gamma \psi_\gamma(X) e^{2\pi \mathbf{i} \langle Z_\gamma, Z \rangle}$, where $\Gamma_Z = \{Z_\gamma\}$ is a lattice in the Z-space defining the Z-torus bundle over the X-space. This formula gives representation for all Γ_Z-periodic functions. For any fixed lattice point Z_γ, functions of the form $\psi(X) e^{2\pi \mathbf{i} \langle Z_\gamma, Z \rangle}$ span a sub-space denoted by W_γ. They define the Fourier-Weierstrass decomposition, $\sum_\gamma W_\gamma$, of Γ_Z-periodic functions.

The Laplacian acts on W_γ according to the formula: $\Delta(\psi(X) e^{2\pi \mathbf{i} \langle Z_\gamma, Z \rangle}) = \Diamond_\gamma(\psi(X)) e^{2\pi \mathbf{i} \langle Z_\gamma, Z \rangle}$, where:

$$\Diamond_\gamma = \Delta_X + 2\pi \mathbf{i} D_\gamma \bullet -4\pi^2 |Z_\gamma|^2 (1 + \frac{1}{4}|X|^2), \tag{3}$$

It shows that the Laplacian leaves each of the subspaces W_γ invariant. Operator $\diamondsuit_\gamma$ is known in quantum physics as the Landau-Zeeman operator of charged particles orbiting in constant magnetic fields. The second term is the angular momentum operator associated with the magnetic dipole momentum.

To see this matching, recall that the 2D-Landau-Zeeman operator of a particle of charge C, orbiting in the (x, y)-plane in constant magnetic field directed toward the z-axis, is of the form [B, LL, P]:

$$-\frac{\hbar^2}{2m}\Delta_{(x,y)} - \mathbf{i}\frac{\hbar CB}{2mc}D_z \bullet + \frac{C^2B^2}{8mc^2}(x^2 + y^2), \qquad (4)$$

where $D_z\bullet = x\partial_y - y\partial_x$. Thus, by choosing $\pi|Z_\gamma| = \mu = CB/2\hbar c$ and multiplying the whole operator with $-\hbar^2/2m$, the $\lhd_\gamma$ (also denoted by $\lhd_\mu$) is transformed to the Landau-Zeeman operator.

In quantum theory this operator is established by Maxwell equations, Hamilton formalism, and correspondence principle. Whereas on Zeeman manifolds, its action is equal to that of the Monistic Hamilton Operator on Γ_Z-periodic functions - the Hamilton waves of point particles - which constitute an invariant subspace regarding the Monistic Hamiltonian. This coincidence with realistic quantum Hamilton operators clearly reveals that the mathematical Zeeman manifolds should be considered as profound quantum physical models. It immediately raises the question as to which quantum physical contents are revealed by the action of the Monistic Hamilton on the waves of extended particles. The answer is explored in the following section where it turns out that that action corresponds to the electroweak and strong interactions and also exhibits gravitation, while the action on the waves of point particle systems relates Δ to the electromagnetic phenomena.

By closing this section, let it be pointed out again that all Hamilton and wave operators established sofar act on functions thus the waves of particles can not carry mutual spin but just angular momentum. The anomalous theory is established in [Sz2]-[Sz6] under most general circumstances where Coulomb and other generic electric potential functions also appear in the Monistic Hamilton Operator. It is an important point that these potential functions not just simply added to the magnetic operator, as it is usually done in the literature today, but they appear as inherent parts in the Monistic Laplacian.

2.6. Extended particles on Zeeman manifolds.

The theory of extended particles are worked out on Z-ball bundles, defined by smooth fields, $B_{\delta(|X|)}(X)$, of Z-balls over the X-space where $\delta(|X|)$ denotes the radius of the ball over X. Since the Z-balls represent the common inside of extended particles, their size gives rise to a physical invariant. Thus, in physics, the $\delta(|X|)$ should be constant characteristic for the particle system. Nonetheless, the following considerations can also be carried out for varying radius-functions only depending on $|X|$.

2.6.1. Hamilton waves of extended particles. The Hamilton operators of extended particle systems are obtained by restricting the very same Monistic Hamilton Operator to an other invariant subspace, consisting of appropriate functions defined on the Z-ball bundles, which emerge then as the Hamilton waves of extended particle systems. Like for point particles, they also are represented by a sort of Z-Fourier transform operating on L_Z^2 functions over each X-vector in the Z-space, however, this version is

far from being just a simple generalization, passing from the discrete to the L_Z^2-case. Complications arise from the boundary conditions these waves must comply with which inquire the implementation of new operations – such as the below described projections $\Pi_{K_u}^{\mathbf{s}}(\psi(K_u, X)$ – which do not show up in discrete Z-Fourier transforms. To answer all these requirements, the Hamilton waves, $\Psi(X, Z)$, of extended particle systems are introduced by the so called twisted Z-Fourier transforms in the form:

$$\int_{\mathbb{R}^l} e^{\mathbf{i}\langle K, Z\rangle} \phi_{\mathbf{s}}(|X|, |K|) \Pi_{K_u}^{\mathbf{s}} \psi(K_u, X) dK = \tag{5}$$

$$= \Phi_{\mathbf{s}}(|X|, |Z|) \Pi_{Z_u}^{\mathbf{s}} \psi(Z_u, X), \quad \text{where} \tag{6}$$

$$\psi(K_u, X) = \varphi(K_u) \Pi_X^{(n)}(\prod z_i^{p_i} \overline{z}_i^{q_i})(K_u, X), \quad n = \sum_{i=1}^{k/2}(p_i + q_i). \tag{7}$$

The position of indices $\mathbf{s}$ in terms like $\phi_{\mathbf{s}} \Pi_{K_u}^{\mathbf{s}}$ indicates summation regarding $\mathbf{s}$. The other constituents are explained as follows.

The formula is a Fourier transform written up in the Z-space in terms of polynomials $\prod z_i^{p_i} \overline{z}_i^{q_i}$ defined by the complex structures J_{K_u}, associated with unit vectors $K_u = K/|K|$, and a fixed system $\mathbf{Q} = \{Q_1, \ldots, Q_{k/2}\}$ of orthonormal X-vectors representing $k/2$ number of whole particles. The holomorphic linear functions $z_i(K_u, X) = \langle Q_i + \mathbf{i} J_{K_u}(Q_i), X\rangle$ are defined regarding these vectors. The $\Pi_X^{(n)}$ projects the nth-order homogeneous polynomial of the X-variable standing in the argument to homogeneous harmonic polynomials of the same order. It can be expressed in the form $\sum_p a_p |X|^{2p} \Delta_X^p$, with recursively defined coefficients a_p [Sz4]. Function $\varphi(K_u)$ is the restriction of a homogeneous polynomial $\varphi(K)$, defined on the Z-space, onto the unit sphere of the Z-space.

Waves defined by the simpler formula:

$$\int_{\mathbb{R}^l} e^{\mathbf{i}\langle K, Z\rangle} \phi(|X|, |K|) \psi(K_u, X) dK \tag{8}$$

are the whole extended waves, which are also called Rutherford waves. They are eigenfunctions of the compound angular momentum operator, spanning a function space invariant under the action of the complete Monistic Hamilton Operator. The restriction defines now the Rutherford Hamilton Operator, which is very similar to that of Landau-Zeeman operator of point particles. But there arises a serious problem about these waves, namely, the boundary conditions on Z-ball bundles can not be enforced by them. This is the primary reason as to why the functions yielding the various boundary conditions should be sought by more sophisticated integral formulas like (5).

Such general waves are defined in terms of projections $\Pi_{K_u}^{\mathbf{s}}(\psi(K_u, X))$ in (5) which project ψ to spherical harmonics defined on the unit K-spheres, over each X-vector. Their explicit formulae, expressed in terms of the polynomials of the powered Laplacian $\Delta_{K_u}^r$, need numerous explanations. Namely, such formulae can primarily be established for projecting restrictions, $p(K_u)$, of homogeneous polynomials, $p(K)$, to spherical harmonics, and they depend both on the degrees of the function being projected and the function resulted by the projection. This is why index $\mathbf{s}$ must be compound, indicating both of these two degrees.

But this argument gives rise to an other complication, namely polynomials $z_i^{p_i} \overline{z}_i^{q_i}(K_u, X)$ are not restrictions of homogeneous polynomials onto the K_u-spheres but it is the sum of components having distinct degrees of homogeneity. The parts of

the same degree can be found by decomposing $z_i^{p_i} \overline{z}_i^{q_i}(K_u, X)$, where $z_i(K_u, X) = \langle Q_i + \mathbf{i}J_{K_u}(Q_i), X \rangle$, according to the powering and the distributivity laws into homogeneous components. Then $\Pi_{K_u}^{\mathbf{s}}$ acts on that component whose degree is indicated in $\mathbf{s}$. Projections $\Pi_K^{\mathbf{s}}$ acting on homogeneous polynomials can similarly be established as polynomials of the powered Laplacians Δ_K^r. Double radial function $\Phi_{\mathbf{s}}(|X|, |Z|)$ is established from $\phi_{\mathbf{s}}(|X|, |K|)$ by the Hankel transform, which describes the action of the Fourier integral standing on the left side of (5) in terms of radial functions [Sz2, Sz4, Sz6].

General waves (5) emerge from the Rutherford waves (8) by decomposing ψ by the projections $\Pi_{K_u}^{\mathbf{s}}$. Spherical harmonics $\Pi_{K_u}^{\mathbf{s}}(\psi(K_u, X))$ are called splinters of 0-generation, which are not "whole functions", like $\psi(K_u, X)$, but which describe the waves also in the insides of extended particles. Name "0-generation" indicates that the ψ is still a whole function. Higher generation functions $\psi^{(g)}$ are resulted below by successive application of the Monistic Hamiltonian on the above wave functions. The functions of higher order generations can not be considered whole anymore.

Projected functions $\Pi_{K_u}^{\mathbf{s}}(\psi^{(g)}(K_u, X))$ correspond to the Hamilton waves of quarks, the building blocks of the nucleus. For their deeper understanding, consider the homogeneous harmonic polynomials $\mathbf{P}^{\mathbf{s}}(K, X) = \Pi_K^{\mathbf{s}}(\psi^{(g)}(K, X))$ regarding the K-variable over each point X in the whole K-space, where the gradient vector field $\mathbf{B}^{\mathbf{s}}(K, X) = \mathrm{grad}_K(\Pi_K^{\mathbf{s}}(\psi^{(g)}(K, X))$ also is harmonic and corresponds to a closed 1-form in that Euclidean space. Thus we have: $\mathrm{div}_K(\mathbf{B}^{\mathbf{s}}) = 0$; $\mathrm{rot}_K(\mathbf{P}^{\mathbf{s}}) := *_K(d_K\mathbf{P}^{\mathbf{s}}) = 0$, that is, vector field $\mathbf{B}^{\mathbf{s}}$ satisfies the Maxwell equations of a pure magnetic field with which vanishing electric field, $\mathbf{E}^{\mathbf{s}} = 0$, is associated.

The whole system, $\{\mathbf{B}^{\mathbf{s}}\}$, defined for all compound index $\mathbf{s}$, is called magnetic labyrinth in the inside of extended particles. This version corresponds to the "little magnet" imagined by the physicists in the inside of point particles in order to visualize the spin-concept. Function $\mathbf{P}^{\mathbf{s}}$ represent the potential for the magnetic field $\mathbf{B}^{\mathbf{s}}$. The splinters - the wave functions of quarks - are the restrictions of individual magnetic potential fields onto the unit sphere of the K-space. Due to the Hankel transform, these objects appear on the Z-space in the very same form, where just K is substituted by Z.

Double radial functions $\Phi_{\mathbf{s}}(|X|, |Z|)$, defined from $\phi_{\mathbf{s}}(|X|, |K|)$ by the Hankel transform, are potential functions of the strong force keeping the quarks inside of particles. This errand is carried out by smooth functions defined inside of extended particles, whereas they vanish on the outside. If the degree of $\mathbf{P}^{\mathbf{s}}$ is s, then at the origin they behave asymptotically as $|Z|^s$. Thus the strong force is very weak inside - allowing the quarks moving almost freely there - whereas it gets infinitely strong at the boundary - not allowing to move the quarks to the outside. This paradoxical feature was recognized by Bjorken, in the 60's, and confirmed thereafter by numerous experiments.

2.6.2. Hamilton operators of extended particles. The very same Laplacian $\Delta = -\mathcal{MH}$ acting on extended matter waves (5) appears as the sum of operators:

$$(\Delta_X + (1 + \tfrac{1}{4}|X|^2)\Delta_Z)\Psi + (\mathbf{q} - \mathbf{p})\sqrt{-\Delta_Z}(\Phi_{\mathbf{v}} + \bigodot|_{\mathbf{v}}^{\mathbf{s}}\Phi_{\mathbf{s}})\Pi_{Z_u}^{(\mathbf{v})}\psi, \quad (9)$$

and $\sqrt{-\Delta_Z} \, \bigcirc|_{\mathbf{w}}^{\mathbf{s}}(\Phi_{\mathbf{s}}) \, \Pi_{Z_u}^{(\mathbf{w})}(\mathbf{W}^+ + \mathbf{W}^-)\psi,$ (10)

called Stabler and Decay Hamiltonian, respectively.

The proof of these formulas explores the action of Δ on wave functions (5). The most complicated computational details arise from the action of the compound angular momentum operator, which appears behind the integral sign as $\mathbf{i}|K|D_{K_u}\bullet$. Term $\sqrt{-\Delta_Z}$ origins from multiplication with $|K|$. The other terms are computed from the relation:

$$[D_{K_u}\bullet, \Pi_{Z_u}^{(\mathbf{s})}] = -\bigodot|_{\mathbf{v}}^{\mathbf{s}}\Pi_{K_u}^{(\mathbf{v})}D_{K_u}\bullet + \bigcirc|_{\mathbf{w}}^{\mathbf{s}}\Pi_{K_u}^{(\mathbf{w})}\sum_{\beta=1}^{l-1}\partial_\beta D_\beta\bullet, \qquad (11)$$

where $\{e_\beta(K_u)|\beta = 1, \ldots, l-1\}$ is an orthonormal basis in the tangent space of the unit sphere defined by unit vectors K_u.

The Stabler is associated with the term containing $D_{K_u}\bullet$, which results $(\mathbf{q} - \mathbf{p})$ in (9). Whereas, operator $(\mathbf{W}^+ + \mathbf{W}^-)$ in the Decay Hamiltonian is originated from $\sum_\beta \partial_\beta D_\beta\bullet$, which is involved to the second term standing on the right side of (11). It is a highly transmuting operator, indicating that it really is a decay-operator. It decreases the degrees of spherical harmonics and transmutes a holomorphic linear function to an antiholomorphic one and vice versa. Since these functions can be associated with positive and negative charges, the $D_\beta\bullet$ is considered to be acting as $\mathbf{W}^+$ on antiholomorphic functions and as $\mathbf{W}^-$ on holomorphic functions. When, due to the actions of these operators, the waves go through quantum leaps, then the change of energy levels is balanced by corresponding absorption or emission of bosons relating to $W^\pm$, introduced in the weak interaction theory.

Since the $\psi(K_u, X)$ is an eigenfunction of $-\mathbf{i}D_{K_u}\bullet$ with eigenvalue $\mathbf{q} - \mathbf{p}$, the Stabler does not transmutes ψ but it rather acts like a stabilizer, explaining its name. When the waves go through quantum leaps, due to the actions of this operator, then the change of energy levels is balanced by absorption resp. emission of Z-relating bosons, the other main objects in weak interaction theory. Explicit computations reveal that the spectrum is parity violating and the Stabler prefers to build up heavy stable protons accompanied with much lighter electrons. The Stabler actually emerges from this theory as the Hamilton operator of a new force whose agent is the Z-relating boson whose action results the stability of protons as well as their rather heavy weight, as compared to that of the electron. In the Standard Model, contrary to this, the stability is enforced by the barion numbers, which, however, do not yield any explanation neither for the heavy weight of protons nor for the parity violation. On Zeeman manifolds, the cooperative actions of the Stabler and the Decay operators govern all decays, such as alpha beta and gamma, and all above mentioned phenomena are spectrally explained there.

The Hamilton operator of the strong force is $\sqrt{-\Delta_Z}$, which appears in both Hamilton operators. Its action can completely be reduced to the Z-radial functions - the waves of the strong force. The strong force itself is defined by the gradient vector fields of these Z-radial functions. Since they vanish in the outside, the strong force can be infinitely strong at the boundary, thus it is appropriate to define the strong force Hamiltonian by $\sqrt{-\Delta_Z}$. In the inside of extended particles, it represents just a very weak force. At the boundary, however, it defines a distribution which corresponds to the infinitely large force acting at the boundary in order to keep the splinters in the inside of extended particles. When, due to the action of the strong force Hamiltonian, the waves go through quantum leaps, then the energy differences are balanced by corresponding emissions or absorptions of gluons.

It should be mentioned that Yukawa's strong force, whose agents are the pions, also appears in the theory. Namely, Yukawa's wave operator is inbuilt into the

Laplacian of the static Zeeman space-time (cf. Section 3.1). Its corresponding Hamilton operator is $-\Delta_Z$ and it appears together with $\partial^2/\partial t^2$. Due to its relation to the de Broglie waves, it becomes a major tool for establishing a new, spectral mass-assigning procedure which is analogous to the Higgs mechanism. On the accelerating Zeeman space-time, the $-\Delta_Z$ gets involved with the wave operator of dark energy and plays a major role to carry out the accelerating version of the new spectral mass assigning procedure. Term $\sqrt{-\Delta_Z}$, however, is engaged with the parabolic Schrödinger operator, expressed in terms of $\partial/\partial t$, which is the Wave operator of the non-expanding ordinary matter. According to these argument, the theory makes clear distinctions between the two forces and operator $\sqrt{-\Delta_Z}$ should really be considered as the Hamilton operator of Bjorken's strong force. It has an intimate relation to the graviton that is also described in the following discussions.

The arguments brought up sofar verify that the Stabler and Decay Hamiltonians describe nuclear processes in consent with the Standard Model. These Hamilton operators break down into part operators which not just reestablish the quantum world from Hamiltonian points of view but also imply such new features which actually are out of the scope of the Lagrangian theories. The most important among them is gravitation, appearing on the scene such that both the Stabler and Decay operators contain respective spool, $\odot|_{\mathbf{v}}^{\mathbf{s}}$, and winch, $\bigcirc|_{\mathbf{w}}^{\mathbf{s}}$ operators, binding any two splinter waves together produced by the repeated actions of the Monistic Hamilton Operator on a starting wave of 0 generation.

In order to understand this, imagine that one tries to determine eigenfunctions of Δ in a process, starting with the twisted Z-Fourier transform of a given function $\phi_{\mathbf{s}}(|X|,|K|)\Pi_{K_u}^{\mathbf{s}}(\psi^{(0)}(K_u,X))$. Index in $\psi^{(0)}$ indicate that it is a 0-generation function. If the Δ consisted only of the Stabler, then one would be able to find eigenfunctions in the form $\gamma_{\mathbf{s}}(|X|,|Z|)\Pi_{Z_u}^{\mathbf{s}}(\psi^{(0)}(Z_u,X))$, where double radial functions $\gamma_{\mathbf{s}}$ are derived from $\phi_{\mathbf{s}}$. Note that the generation index does not change in this case and the problem reduces to determining appropriate double radial functions $\gamma_{\mathbf{s}}(|X|,|Z|)$. However, due to the action of the Decay Hamiltonian, the problem can not be solved just by the 0-generation functions, but one also needs first generation function $\psi^{(1)}$ obtained from $\psi^{(0)}$ by the action of the Decay operator. Even these two generations will not be enough to determine eigenfunctions but one needs all generations, produced by repeated actions of the Decay operator. When all these splinter waves combined with double radial functions are produced, then all of them are needed to build up the eigenfunctions of Δ. Any two splinters of any two generations are connected by the spool and winch operators. They actually are Hamilton operators associated with very feeble pulling forces. The attraction is exerted in order to keep whole galaxies of particles together so as to form stable particle systems (galaxies) which appear to be as a single stable organization. If a particle in a stable galaxy is disturbed and tipped out of the equilibrium then the whole galaxy gets out of the eigen-state and this feeble force restores the balance by pulling the particle back to the stable position. This feeble force is nothing but the gravitation which manifests itself both on the microscopic and macroscopic levels. Its agents are the spool and winch gravitons which appear during quantum leaps caused by the actions of the operators.

Interactions due to spool and winch operators are just two out of three facets of gravitation. The third one emerges on most general Zeeman space-time models of the universe which are not just extensions of single H-type groups into the time direction but rather are Cartesian products of many of them in which a single H-type group is just an irreducible component. In such cases the spool and winch

operators are not universal in the sense that they act just within the components of the Cartesian product. The missing universality of gravitational interaction is supplied by the universal gravitation operator emerging from the action of the "mammoth" Schrödinger wave operator, which, as explained later, acts in strong cooperation with the spool and winch gravitations in order to establish an all-embracing gravitation also acting among the distinct components of the Cartesian product. An other important feature is that this gravitation directly relates to the mass generated by the spectral mass-assignment procedure.

By an informal description, the Laplace operator acts on Zeeman manifolds like an operator which decomposes the non-stable de Broglie waves into splinter waves by which systems of stable particle-galaxies corresponding to the eigenfunctions of the Laplace operator can be built up. The process breaks down into alpha beta and gamma decays, resulting heavy protons, light electrons, and all particles the microscopic and macroscopic universes consist of.

This theory modifies Einstein's general relativity from several points of view. The gravitation manifesting according to the Zeeman manifold model escapes the most serious contradictions of Einstein's theory such as the collapse of large universes due to the overwhelming gravitational attraction. The theory also gives explanation for other phenomena. Explicit spectral computations reveal, for instance, as to how are the stable protons built up and why is the proton much heavier then the electron. All this information is derived from the very same operator. This is how the Zeeman manifold model realizes unification on many levels of the microscopic and the macroscopic universes as well.

3. Static and accelerating Zeeman space-times.

3.1. Static Zeeman space-times.

3.1.1. Static Monistic Wave Operator   The static relativistic space-times are defined by metric Cartesian products, $N \times \mathbb{R}$, of Zeeman manifolds N with the time axis $\mathbb{R}$, where the latter is considered to be endowed with the indefinite inner product $< \partial_t, \partial_t > = -1$. Then, the tangent spaces $T(N)$ and $T(\mathbb{R})$ are perpendicular and, in the prototype case, the Laplacian Δ_{St} appears there in the form:

$$(\Delta_Z + \frac{2m\mathbf{i}}{\hbar}\partial_t - \partial_{tt}^2) + (\Delta_X + \frac{1}{4}|X|^2 \Delta_Z + \sum \partial_\alpha D_\alpha \bullet - \frac{2m\mathbf{i}}{\hbar}\partial_t). \quad (12)$$

This formula represents the Δ_{St} as the sum of two operators which decomposition can be established by introducing the trivial term $0 = \frac{2m\mathbf{i}}{\hbar}\partial_t - \frac{2m\mathbf{i}}{\hbar}\partial_t$ into the undecomposed operator. The reason for doing so is that this decomposition results two original wave operators - the Yukawa and the Schrödinger operators- of classical quantum theory. Also notice that energy operator $-\frac{2m\mathbf{i}}{\hbar}\partial_t = \frac{2m\mathbf{i}}{\hbar}\partial_{-t}$ in the Schrödinger operator represents positive energies if it is considered regarding time direction $-t$.

3.1.2. The Yukawa operator and the de Broglie waves   In order to properly exploit the apparent connections to the classical theory, one should recall some basic facts regarding Yukawa's strong force operator and its relation to de Broglie's waves:

$$\Psi_{deB}(Z, t) = \int \int \int A(K_1, K_2, K_3)e^{\mathbf{i}(\langle K, Z \rangle - \omega t)}dK_1 dK_2 dK_3, \quad (13)$$

$$\text{where} \qquad \sqrt{|K|^2 + \frac{m^2 c^2}{\hbar^2}} = \frac{\omega}{c}, \tag{14}$$

originally introduced for describing the ordinary matter having non-zero rest mass as matter-waves and the associated matter-particles as wave packets.

Waves $\Psi_{deB}(Z,t)$ satisfy the relativistic scalar wave equation:

$$(\Delta_Z - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\Psi_{deB}(Z,t) = \frac{m^2 c^2}{h^2}\Psi_{deB}(Z,t) \tag{15}$$

(cf. [2.2] in [P], Vol. 5, pages 3), which also serves as scalar wave equation for the strong force, used by H. Yukawa to explore the Pion - the agent of the strong force. Most remarkably, this operator appears in this pure form in the undecomposed Laplacian of the static Zeeman space-time, hereby relating the theory both to de Broglie's waves and Yukawa's scalar wave equation of the strong force. These relations also clarify as to why should operator $\sqrt{-\Delta_Z}$ in the Stabler and Decay Hamilton operators be related to the strong force.

Wave $\tilde{\Psi}(Z,t) = \exp(\mathbf{i}mc^2 t/h)\Psi(Z,t)$ satisfies:

$$(\Delta_Z + \mathbf{i}\frac{2m}{h}\frac{\partial}{\partial t} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\tilde{\Psi}(Z,t) = 0 \tag{16}$$

(cf. [2.11] in [P], Vol. 5, pages 4), which, due to $\tilde{\Psi}\tilde{\Psi}^* = \Psi\Psi^*$, determines the same probabilistic density than Ψ. That is, Ψ and $\tilde{\Psi}$ describe the very same quantum state.

The transformation of Yukawa's original equation (15) to (16) allows to express the static Monistic Wave Operator (which originally contains only second order partial differentiations: ∂_{tt} regarding the time variable) as the sum of operator standing in (16) and the parabolic Schrödinger operator containing only first order differentiations regarding the time variable. The decomposition is simply implemented by introducing the trivial term $\frac{2m\mathbf{i}}{\hbar}\frac{\partial}{\partial t} - \frac{2m\mathbf{i}}{\hbar}\frac{\partial}{\partial t}$ into the static Monistic Wave Operator such that the first term is incorporated into Yukawa's and the second one into Schrödinger's operator. It is used below for establishing a new spectral mass-assigning procedure without using symmetry breaking. As an other important point, the mass is also related to the spectrum of the Hamilton operator appearing in the second part of the decomposed Monistic Wave Laplacian.

3.1.3. Spectral mass-assigning procedure In order to carry out this scheme, the static waves are introduced by:

$$\Psi_{\text{st}}(X,Z,t) = \int_{\mathbb{R}^l} e^{\mathbf{i}(\langle Z,K\rangle - \frac{\omega}{c}t)}\phi_{\mathbf{s}}(|X|,|K|)\Pi_{K_u}^{\mathbf{s}}(\psi(X,K_u))dK, \tag{17}$$

where the ψ can be linear combination of generation functions. They are eigenfunctions of Yukawa's original operator which appears in the form $\Delta_Z - \partial_{tt}^2$ on the (Z,t)-space, over each X-vector. (As compared with (15), this one does not contains factor $1/c^2$ before ∂_{tt}^2 but which difference is compensated by considering ω/c instead of ω in the above formula.) The mass appears in the eigenvalue $m^2 c^2/h^2$ of this operator, which, however, has no relation to the spectrum of Hamilton operators also inbuilt into the Monistic wave Laplacian. To build up these connections, consider waves

$$\tilde{\Psi}_{st}(X,Z,t) = e^{\frac{\mathbf{i}mc^2}{\hbar}t}\Psi_{st}(X,Z,t), \tag{18}$$

which are harmonic regarding the so called exhausted Yukawa operator:

$$\Delta_Z + \frac{2m\mathbf{i}}{\hbar}\partial_t - \partial_{tt}^2. \tag{19}$$

The name (coined in this paper) is ment to indicate that the mass is exhausted regarding the Yukawa operator and the action of the whole Monistic Wave Laplacian on $\tilde{\Psi}_{st}(X, Z, t)$ is completely determined by the action of the Schrödinger operator. That is to say, exhausting waves (18) exhaust the mass just regarding the Yukawa operator which is furnished then into the Schrödinger operator in order to relate the mass to the spectrum of the Monistic Hamilton Operator. Due to the fact that the mass and time t appear both outside and inside of integration, the Schrödinger operator decomposes into the sum:

$$
\Delta_X + \frac{1}{4}|X|^2\Delta_Z + \sum \partial_\alpha D_\alpha \bullet - \frac{2m\mathbf{i}}{h}\partial_t =
$$
$$
= \frac{2m^2c^2}{ch^2}\sqrt{1 - \frac{h^2}{m^2c^2}\Delta_Z} + \tag{20}
$$
$$
\Delta_X + \frac{1}{4}|X|^2\Delta_Z + \sum \partial_\alpha D_\alpha \bullet - \frac{2m\mathbf{i}}{h}\partial_t - \frac{2m^2c^2}{ch^2}\sqrt{1 - \frac{h^2}{m^2c^2}\Delta_Z}) \tag{21}
$$

where the parts are called universal gravitation and actual Schrödinger operators, respectively. By this reason, the second in the decomposed Monistic Wave Laplacian (12) is called "mammoth" Schrödinger operator. Identity

$$
\frac{1}{c}\frac{2m^2c^2}{h^2}\sqrt{1 - \frac{h^2}{m^2c^2}\Delta_Z}(\Psi_{\text{st}})(X, Z, t) = \tag{22}
$$
$$
= -\frac{2m}{h}\int_{\mathbb{R}^l}\frac{\omega}{c}e^{\mathbf{i}(\langle Z, K\rangle - \frac{\omega}{c}t)}\phi_{\mathbf{s}}\Pi^{\mathbf{s}}_{K_u}(\psi)(X, K)dK = \tag{23}
$$
$$
= \frac{2m}{h}\Omega(\Psi_{\text{st}})(X, Z, t) \tag{24}
$$

regarding the universal gravitation operator is established in [Sz2, Sz6]. It primarily appears as an integral operator, but only its differential operator version indicates its relation to the strong, spool, and winch operators. It tells us as to how is the common mass m, defined for all particles participating in the system, establishes additional gravitational relation to those associated with the spool and winch operators, which can justifiably called "Mass-relating Gravitational Wave Operator". Its universality means that this kind of gravitation also exists among particles being in different components of the Cartesian product of H-type groups whose extension, into the time direction, defines the Zeeman space-time. Such universality, however, is not possessed by the spool and winch gravitations which do not involve mass-terms into their original definitions.

The harmonic solutions of (21) relate the mass to the eigenvalues of the actual Schrödinger operator according to the mass-energy equivalence. This spectral data depends on boundary conditions and refers to all waves produced by repeated actions of the Stabler and Decay operators. Only those mass values are physically appropriate which emerge from these spectral relations and include the masses of all constituent particles. Thus the possible masses are discrete and strongly relating to the boundary conditions. These computations actually determine the mass in relation to the sizes of particles. This mass-size relation allows the spectral establishments of the coupling constants defined insofar by Lagrangian means. There can also be explained as to why is the proton much heavier than the electron. This phenomenon is due to the parity violating spectrum and the extendedness of protons, as opposed to the electrons which are point particles.

The eigenfunctions of the actual Schrödinger operator are not eigenfunctions of the universal gravitation operator, whose action rather defines unbalanced waves which can be seen as being produced by the interaction existing between the commonly defined mass and the balanced system of particles associated with the eigenfunction-waves. During this interaction the boundary conditions are destroyed, after which effect the particles are compelled to seek their ways back to the eigenstate. There are two opposite tendencies inbuilt into the mammoth Schrödinger operator. One of them is mobilized by the actual Schrödinger operator which determines the possible eigenstates and compels the particles to move into those states. The other is orchestrated by the universal gravitation operator which tips out the system from the eigenstates. The cooperative actions of these two operators gives rise to motion – the most fundamental feature manifesting everywhere in Nature. This is why, the actual Schrödinger and the universal gravitation operators are called Dynamic Duo in the mammoth Schrödinger operator. Their interaction is resulted by that that they are two non-commutative operators. The duality is manifested also by the particular form how the time appears in the exhausting Yukawa waves. The exponential term outside of integration relates the wave to the actual Schrödinger operator, whereas the one standing inside relates it to the universal gravitation operator. This is called dynamical timing of waves, allowing multilateral exploration of the Monistic Wave Laplacian. This is contrary to Schrödinger's original waves, which only one-laterally explore his original equation.

3.1.4. Relations to symmetry breaking   Although the spectral mass assignment does not use the idea of symmetry breaking, it can be related to it, particularly to the chiral symmetry breaking that was used by Nambu [N] to explain as to how do the protons and neutrons gain masses in the Yang-Mills theories. Chiral symmetry means no difference between right and lefthanded systems. In Nambu's theory the orientation of an object is relative to the observer and it depends on as to whether the observer is moving behind or ahead of the object. Thus the laws describing non-bypassable objects – because they are moving with speed of light – obey chiral symmetry. The laws controlling the movements of protons and neutrons in Yang-Mills theory are chiral symmetric, originally, thus they are massless in that stage. They gain mass by breaking the chiral symmetry, meaning slowing down to speed less than c.

The spectral mass assignment procedure evolves according to a very similar scenario. In the first step, the mass is generated as an eigenvalue of the time-symmetric Yukawa operator. For photons, the eigenvalue and therefore the mass is zero and they are moving with speed of light. They are the unbypassable objects whose tangents of their world-lines point into the direction of the light-cone of the (Z, t) space-time and the laws describing their motion obey chiral symmetry. If the eigenvalue is greater than zero, the tangents point into the inside of the light-cone, indicating speed less than c and the corresponding laws disobey chiral symmetry which is exhibited in relation to photons. This gives rise to a Lagrangian explanation as to why do these objects have positive masses. But they still are the products of the time-symmetric Yukawa operator and the mass does not have any relation to the spectrum of the rest part of the Monistic Wave Operator. This incompleteness is straiten out in the second step when the implementation of $0 = \frac{2m\mathbf{i}}{\hbar}\partial_t - \frac{2m\mathbf{i}}{\hbar}\partial_t$ into the undecomposed Operator breaks the time-symmetry of the Yukawa operator and provides the mass to the actual Schrödinger operator in order to bring it in connection with its spectrum. The implemented term also determines the time direction with respect to which the

masses appear to be positive. By this reason, it is called Chiral Mass-Time Switch. The chiral and other symmetries are further explored in [Sz7].

3.2. Accelerating Zeeman space-times.

The prototype accelerating Zeeman space-times are carried out on the solvable extensions of H-type groups defined on the half-space $N \times \mathbb{R}_+$. The Lie algebra, $\mathcal{S} = \mathcal{N} \oplus \mathcal{T}$, is completely determined by:

$$[\partial_t, X] = \frac{1}{2}X; \ [\partial_t, Z] = Z; \ [T(N), T(N)]_{/SN} = [T(N), T(N)]_{/N}, (25)$$

where $X \in \mathcal{X}$ and $Z \in \mathcal{Z}$. By the traditional interpretation, this Lie algebra is identified with the tangent space at the unity $(0,0,1)$. The indefinite metric tensor is introduced by the left-invariant extension of the indefinite inner product, $\langle , \rangle$, defined on the solvable Lie algebra S by $\langle \partial_t, \partial_t \rangle = -1$, $\langle \partial_t, T(N) \rangle = 0$, and $\langle T(N), T(N) \rangle = \langle T(N), T(N) \rangle_N$.

The left-invariant extensions $\mathbf{Y}_i; \mathbf{V}_\alpha; \mathbf{T}$ of unit tangent vectors $E_i = \partial_i; e_\alpha = \partial_\alpha; \epsilon = \partial_t$ picked up at $(0,0,1)$, are the vector fields:

$$\mathbf{Y}_i = t^{\frac{1}{2}}\mathbf{X}_i \quad ; \quad \mathbf{V}_\alpha = t\mathbf{Z}_\alpha \quad ; \quad \mathbf{T} = t\partial_t, \tag{26}$$

where $\mathbf{X}_i$ and $\mathbf{Z}_\alpha$ are invariant vector fields on the nilpotent subgroup N.

It follows that not t but T defined by $\partial_T = \mathbf{T}$ is the correct physical time-parameterization on the t parameter lines. Endowed with this parameterization, they become geodesics on SN. Transformation law $\partial_T = (dt/dT)\partial_t$ yields: $(dt/dT) = t$; $\ln t = T$; and $t = e^T$, thus the t-level sets are the same as the $T = \ln t$-level sets and subgroup N corresponds both to $t = 1$ and $T = 0$. The reversed time is $\tau = -T$.

Let $\mathbf{X}$ and $\mathbf{Z}$ be invariant vector fields on the subgroup N, furthermore, $c_x(s)$ and $c_z(s)$ be their integral curves of finite length denoted by $||c_x||$ and $||c_z||$, respectively. The flow generated by ∂_τ moves the curves to $c_x^\tau(s)$ resp. $c_z^\tau(s)$. By the above formulas, the tangent vectors $\dot{c}_x(s)$ and $\dot{c}_z(s)$ are of unit length just on N, defined as level set corresponding to $T = 0$, and they are changing according to the formulas $||c_x^\tau|| = ||c_x||e^{q\tau/2}$ resp. $||c_z^\tau|| = ||c_z||e^{q\tau}$. That is, by considering them as functions of the time-variable τ, the length is increasing such that the rate of change (derivative with respect to τ) is proportional to the length of the curves. In other words, this space-time represents an expanding universe where the distance between objects is growing according to a law similar to Hubble's.

But the law yielded on Zeeman space-time is very different from that of Hubble. Namely, on the Friedmann model, where the original Hubble law is mathematically established, the expansion is not accelerating. By contrast, the Zeeman space-time is expanding at an exponentially accelerating rate, meaning that the acceleration, together with the higher order ones, are also accelerating, and each of these higher order accelerations are proportional to the distance. That is, the farther is the galaxy, the bigger are the accelerations - of any order - by which it moves away from us. (To see the statement, consider the higher order derivatives of the arc-length formulas.)

3.3. The expanding Monistic Wave Laplacian.

The Monistic Wave Laplacian on accelerating Zeeman space-times is:

$$\Delta_E = \{(e^{-2\tau}\Delta_Z + \frac{2m\mathbf{i}}{\hbar}e^{-\tau}\partial_\tau - \partial_{\tau\tau}^2) - (\frac{k}{2} + l)\partial_\tau\} + \tag{27}$$

$$+e^T\{\Delta_X + \frac{1}{4}|X|^2\Delta_Z + \sum\partial_\alpha D_\alpha \bullet + \frac{2m\mathbf{i}}{\hbar}\partial_T\}. \tag{28}$$

Without $\frac{2m\mathbf{i}}{\hbar}e^{-\tau}\partial_\tau - \frac{2m\mathbf{i}}{\hbar}e^{-\tau}\partial_\tau = 0$, one has the undecomposed Δ_E in which

$$EY = e^{-2\tau}\Delta_Z - \partial^2_{\tau\tau} - (\frac{k}{2}+l)\partial_\tau \tag{29}$$

is the expanding version of the original Yukawa operator. The first term in the decomposed operator (27) could be the candidate for the expanding exhausted Yukawa operator, however, it turns out that it is a much more complicated operator. The operator on the last line is the static mammoth Schrödinger operator multiplied by e^T on the left side. Thus, its action and waves can completely be described by the static operator. Since it is non-expanding even in the accelerating universe, this operator must be identified with the Monistic Wave Operator of ordinary matter.

3.4. Mass-assignment on accelerating Zeeman space-times

As compared with the static case, the mass-acquiring process evolves in a much more complicated way on expanding Zeeman space-times. The details of the following brief outline are established in [Sz2, Sz4].

The expanding wave functions are introduced by:

$$\Psi_{\text{ex}}(X,Z,\tau) = \int_{\mathbb{R}^l} e^{\mathbf{i}(\langle Z,K\rangle - \omega e^{-\tau})}e^{Q\tau}\phi_{\mathbf{s}}\Pi^{\mathbf{s}}_{K_u}\psi^{(g)}(X,K_u)dK, \tag{30}$$

where the Q is determined such that Ψ_{ex} is an eigenfunction of (29). This is satisfied if and only if $Q = \frac{1}{2}(1 - \frac{k}{2} - l)$, when the eigenvalue is:

$$\frac{m^2c^2}{h^2} - Q^2 - (\frac{k}{2}+l)Q = \frac{m^2c^2}{h^2} - (\frac{1}{4} + \frac{1}{4}(\frac{k}{2}+l)^2) + \frac{1}{2}(\frac{k}{2}+l)^2. \tag{31}$$

Then, the expanding exhausting waves are defined by:

$$\hat{\Psi}_{\text{ex}}(X,Z,\tau) = e^{\frac{\mathbf{i}mc^2 e^{-\tau}}{\hbar}}\int_{\mathbb{R}^l} e^{\mathbf{i}(\langle Z,K\rangle - \frac{\omega e^{-\tau}}{c})}e^{Q\tau}\phi_{\mathbf{s}}\Pi^{\mathbf{s}}_{K_u}\psi^{(g)}dK, \tag{32}$$

on which, the action of the first operator:

$$(e^{-2\tau}\Delta_Z - \frac{2m\mathbf{i}}{\hbar}e^{-\tau}\partial_\tau - \partial^2_{\tau\tau}) - (\frac{k}{2}+l)\partial_\tau. \tag{33}$$

appearing in the decomposed Monistic Wave Laplacian (27) results:

$$-(\frac{1}{4} + \frac{1}{4}(\frac{k}{2}+l)^2 - \frac{1}{2}(\frac{k}{2}+l)^2 + \frac{\mathbf{i}mc^2}{h}e^{-\tau})\hat{\Psi}_{\text{ex}}. \tag{34}$$

That is, unlike in the static case, the candidate for the expanding exhausted Yukawa operator is not completely exhausted by $\hat{\Psi}_{\text{ex}}$. They are not even eigenfunctions regarding this operator. The reason is that only the ordinary matter has been exhausted but the dark matter and dark energy are still there. This gives rise to the opportunity for finding their Wave Operators in (33) such that eigenvalue $\frac{1}{4} + \frac{1}{4}(\frac{k}{2}+l)^2$ is associated with the dark matter and the rest with the dark energy. The corresponding decomposition is:

$$\{(e^{2T}\Delta_Z + (\frac{2m\mathbf{i}}{h}e^T + \frac{\frac{k}{2}+l}{1-\frac{k}{2}-l})\partial_T - \partial^2_{TT} + \frac{\mathbf{i}(\frac{k}{2}+l)^2 e^T}{1-\frac{k}{2}-l}\Omega\} + \tag{35}$$

$$-\{(\frac{k}{2}+l - \frac{\frac{k}{2}+l}{1-\frac{k}{2}-l})\partial_\tau + \mathbf{i}(\frac{m}{h}e^{-\tau} + \frac{(\frac{k}{2}+l)^2}{1-\frac{k}{2}-l}e^{-\tau}\Omega)\}, \tag{36}$$

where (35) is the dark matter and (36) the dark energy operator. The action of the universal gravitation operator Ω on expanding waves is defined by:

$$-\frac{2m}{h}\int_{\mathbf{R}^l}\frac{\omega}{c}e^{\mathbf{i}(\langle Z,K\rangle-\frac{\omega}{c}e^{-\tau})}e^{Q\tau}(\phi_{\mathbf{s}}\Pi^{\mathbf{s}}_{K_u}(\psi))(X,K))dK = \qquad (37)$$

$$=\frac{1}{c}\frac{2m^2c^2}{h^2}\sqrt{1-\frac{h^2}{m^2c^2}\Delta_Z}(\Psi_{\mathrm{ex}})(X,Z,\tau)=\frac{2m}{h}\Omega(\Psi_{\mathrm{ex}})(X,Z,\tau). \quad (38)$$

Terms including Ω and $p(k,l) = ((k/2)+l)/(1-(k/2)-l)$ terminate the relations of the dark matter to those represented by $-\frac{1}{2}(\frac{k}{2}+l)^2$ and $\frac{\mathbf{i}mc^2}{h}e^{-\tau}$ in (34). They entirely show up in the dark energy operator. The dark matter, dark energy, and the mammoth Schrödinger operators (the latter appears as the second long term on line (28)) fully establish the wave operators of the three matter-energy formations in terms of the expanding Monistic Wave Laplacian. In order to associate positive energy terms with them, the dark and ordinary matter operators should be considered regarding the shrinking time direction T, whereas, the dark energy regarding $\tau = -T$. The considered expanding waves are eigenfunctions of the dark matter operator with eigenvalue $-(\frac{1}{4}+\frac{1}{4}(\frac{k}{2}+l)^2)$. It represents positive energy level regarding the shrinking direction. The rest in (34) is counted for the dark energy operator. They form a Dynamic Duo in EY, displayed by the dynamical timing of expanding exhausted waves (32). The mammoth Schrödinger operator should be treated according to the static case. It acts on the static exhausting waves decomposes like a Dynamic Duo with components corresponding to the actual Schrödinger operator and the static universal gravitation operator. The orientations clearly indicate that the dark energy operator is associated with pushing away and the dark matter operator with pulling together force acting on the space-section, that is, on Zeeman manifolds. Namely, the accelerations are positive regarding τ and negative regarding T. Since the ordinary matter is described by static waves, this category also behaves according to the expectations.

3.5. Historical retrospect and outlook to the developments

The primary goal in this paper is to establish the relativistic wave operators of the dark energy, dark matter, and ordinary matter in the normal (non-anomalous) case by a new spectral mass-assignment procedure not using the idea of symmetry breaking, the main tool to establish the Higgs mechanism. Nonetheless, this new mechanism (purely based on Wave Operators) can be related to those evolved in SM and it can also be bridged to classical wave operators but which appear in new forms having contrasting features with the originals. Both relations need further clarifications.

3.5.1. Relations to pristine quantum operators
It is known that Klein-Gordon's relativistic wave equation of the electron was also discovered by Schrödinger, who, however, was not able to handle the second order term ∂_{tt}^2 present there which led to spectral computations not matching the experiments. He could restore the agreement with the reality by exchanging it for the first order differential operator $\mathbf{i}\hbar\partial_t$, which costed the price that the equation became non-relativistic valid only for small speeds $v << c$. The relativity was restored by Dirac, by considering appropriate first order partial derivatives regarding the space-variables, as well.

Operator called above as "static exhausted Yukawa operator" has also been well known in the quantum theory. The equation yielding the harmonic functions regarding

this operator is called "non-relativistic wave equation". It is noteworthy that Pauli determined solutions of this equation by means of de Broglie's waves such that he considered only lower order terms in the power series of ω (cf. [P], Vol. 5., pages 3-4). Such approximating solutions are really non-relativistic which describe physical reality just for small speeds $v \ll c$. This equation has always been associated with Yukawa's pion operator and never with the Schrödinger equation.

All these non-relativistic treatments are not to be confused with those applied on the static Zeeman space-time, where the two operators appear together after the relativistic Monistic Wave Laplacian is decomposed by introducing the trivial term $\frac{2m\mathbf{i}}{\hbar}\partial_t - \frac{2m\mathbf{i}}{\hbar}\partial_t$ there and establishing the exhausted operator by Yukawa's and with $\frac{2m\mathbf{i}}{\hbar}\partial_t$. The mammoth Schrödinger operator is equal to the rest of the Monistic Wave Laplacian. No operator exchange or any kind of negligence has been implemented, thus all these operators remain relativistic. They arise as two facets of the relativistic static Monistic Wave Laplacian, representing the unification of strong force interactions with the electromagnetic, weak, and gravitational. It should also be emphasized that they are written up in terms of a fixed coordinate system, and, in order to comply with relativity, in other coordinate system, they should be computed in accordance with Lorentz transformations. In this theory, term ∂_{tt}^2 has not gotten rid of but remains there in the Yukawa operator as one of the contributors for establishing the new spectral mass assigning process. This program can be carried through just by the two operator together, which situation is present just on Zeeman space-time where the exterior and the common interior space appear as partners of each other. This tool was not available for Schrödinger who therefore was compelled to use non-relativistic tools in order to keep his equation close to physical reality.

There arise differences also between the ways how the harmonic solutions of the actual Schrödinger operator, defined on the static Zeeman space-time, and Schrödinger's original operator are obtained, respectively. To see this notice that the exhausting wave $\tilde{\Psi}$ - which is harmonic regarding the exhausted Yukawa operator, thus the result, due to the action of the complete Monistic Wave Laplacian on this wave, depends on the action of the mammoth Schrödinger operator which really contributes an idiosyncratic "mammoth" structure to the solution. Namely, the time is involved into two exponential functions standing both in the insides and outsides of integration. This feature actually results the decomposition into the actual Schrödinger operator and the universal mass-relating gravitation operator.

Also notice that the actual Schrödinger operator is related not to the linear mass m but to the squared mass, m^2. This is one of the reasons as to why should the Dirac operators apply square root in order to have linear masses for the fermions. An other reason why Dirac searched for quantum operators different from Schrödinger's was that the latter did not explain the observed double spectral lines which are due to the mutual spin of particles. In [Sz2, Sz3, Sz4, Sz5, Sz6], the proper spin operators are established by responding to all these requirements. To see the difference between the two approaches, it is enough to mention that, for seeking out the harmonic solutions of his equation, Schrödinger considered waves of the form $\Psi(x,t) = \psi(x)e^{-\mathbf{i}Et/\hbar}$, which neither respond to the universal gravitation nor have any relation to the exhausted Yukawa operator. Thus they are non-relativistic also on the Zeeman space-time. The anomalous case will also be different from that of Dirac whose operator has not been related to any kind of gravitational wave operator.

3.5.2. Densities of dark energy, dark matter, and ordinary matter The wave operators of dark energy, dark matter, and ordinary matter in the accelerating universe are completely unexplored in the literature, insofar. There are new arguments developed in [Sz2, Sz3, Sz4, Sz5, Sz6, Sz7] which demonstrate in consent with the experiments as to why are (28), (35), and (36) the appropriate wave operators for the three matter-energy formations. One of them is the explicit computations of the participation ratios which is carried out by relating these operators to the energy densities involved to the time-time component of the stress energy tensor $E(A, A^*) = Ri(A, A^*) - \frac{1}{2}\mathcal{R}\langle A, A^*\rangle$, where $A = X + Z + \mathbf{T}$ and $A^* = X^* + Z^* + \mathbf{T}$. These objects are considered both in the static and expanding cases, in [Sz2, Sz5, Sz6]. The point is that the above decomposition uniquely determines a corresponding decomposition of the energy densities appearing in the time-time component of the stress-energy tensor and their ratios determine the participation ratios of the three matter-energy formations on the Zeeman space-time model. Thus the model passes a very serious reality test if these participation ratios agree with those measured in Nature.

To carry out these computations, the density corresponding to the ordinary matter is the first to be determined. Explicit computations show that the static stress energy tensor $E_{St}(A, A^*)$ also appears in the expanding stress energy tensor with which the only non-expanding matter formation - the ordinary matter - must be associated. But there is a little confusion at this point, because a negative number, $-kl/8$, appears for the ordinary matter, whereas, the time-time component of the rest of the tensor, with which both the dark matter and dark energy is associated, is positive. But this enigmatic appearance only indicates that all matter-energy can not appear together with the ordinary matter such that everything is positively detected. Since the time directions for the operators are chosen such that they represent positive energies, the non-expanding ordinary matter must evidently be associated with density. $kl/8$. By the above arguments, the dark matter density is $\frac{1}{4} + \frac{1}{4}(\frac{k}{2} + l)^2$, directed to the shrinking direction. What is left in the total:

$$DE(k, l) = \frac{1}{4} + \frac{1}{2}(\frac{1}{2}(\frac{k}{2} + l)^2 + \frac{3k}{4} + 3l) \tag{39}$$

is the dark energy density which is counted regarding the expending direction.

These computations actually give rise to a new correspondence principle, associating energy densities, involved into the time-time component of the stress energy tensor, to the wave operators of the three energy-matter formations. This is the tool by which the participation ratios $(DER : DMR : OMR) = (70\% : 25\% : 5\%)$ on the dynamical Eternal Whizz model can be established. The computations are demonstrated on the most simple one particle model, defined by $k = 2$ and $l = 1$, where the corresponding densities are:

$$DE(2, 1) = \frac{14}{4}, \quad DM(2, 1) = \frac{5}{4}, \quad OM(2, 1) = \frac{1}{4}. \tag{40}$$

Thus the total density is:

$$TOT(2, 1) = \frac{14}{4} + \frac{5}{4} + \frac{1}{4} = \frac{20}{4}, \tag{41}$$

and the corresponding participation ratios are:

$$DER(2, 1) = \frac{DE(2, 1)}{TOT(2, 1)}100\% = \frac{14}{20}100\% = 70\%, \tag{42}$$

$$DMR(2,1) = \frac{DM(2,1)}{TOT(2,1)}100\% = \frac{5}{20}100\% = 25\%, \qquad (43)$$

$$OMR(2,1) = \frac{OM(2,1)}{TOT(2,1)}100\% = \frac{1}{20}100\% = 5\%. \qquad (44)$$

That is, the participation ratios measured in Nature are surprisingly accurately represented by one particle systems and by all those models which are solvable extensions of Zeeman manifolds defined by Cartesian products of one particle Zeeman manifold models. They constitute very complex highly realistic physical models, that are made up by heavy protons, neutrons, electrons, and bosons associated with the mass assignment and the action of the Stabler. Although the spool gravitation is individualistic, holding together just the splinters derived from the individual particles, the Ω-gravity wraps them up into a physically connected system where two elements are related to each other at least by this gravitation. But one thing is still missing from this world, namely, there is no decay associated with higher dimensional Z-spaces and the actions of the **W**-operators. The above correspondence principle is a working tool for computing the participation ratios when the system of independent 1-particles is mixed up with multiparticles having common higher dimensional Z-spaces. They represent a universe of such high complexity which can correspond to that observed today. All quantum physical laws are engraved into the Monistic Wave Laplacian. These ideas give rise to the Eternal Whizz model of the universe which started out infinitely long time ago and last forever. It is a dynamical model which represented independent 1-particles for an infinitely long time till the system got cool enough to admit multiparticles systems having common higher dimensional Z-spaces. The time when this took place is the Zeeman Big Bang, when the universe became of such high complexity as is today.

3.5.3. CERN's scalar boson The theory also explores as to how does CERN's scalar boson appear in this quantum universe and how are the particles of proper spin created there. The scalar boson emerges, in strong relation with the the participation ratios, as a compound Monistic Boson, having inner structure, which has components in each of the three matter-energy formations. Its mass overwhelmingly origins from dark matter and dark energy, which formations are completely made up by these two components of the scalar boson. The Z and W also appear as compound particles, originated from dark energy and ordinary matter only. Besides them, there are splinter W's of the same origin. The ordinary matter components of the scalar boson relating to these particles are called "Zolly" and "Willey", respectively, which together make up the complete amount of ordinary scalar matter from which all Bosonic and Fermionic ordinary matters are originated by the corresponding spin operators acting on spinors.

3.5.4. Zeeman space-time vs. Higgs field The Zeeman space-time theory does not contradict SM or the mass-assigning procedure established by the Higgs field. In case of Higgs mechanism, the particles - considered as tiny billiard balls - acquire mass through their interactions with an external field, the Higgs field. Whereas, on the Zeeman space-time, the particles are represented by waves, endowed with masses by the above described spectral procedure, which originates its name for relating the mass to the spectrum of the Hamilton operators involved into the Monistic Wave Laplacian. But, all these differences notwithstanding, the Higgs field and Zeeman space-time are not contradictory concepts, which point of view can be explained as follows.

The contradiction between general relativity and quantum theory origins from the different representations of matter. In general relativity, Einstein's field equation completely embeds it into the fabric of the curved space-time. While in quantum theory, the matter is released from there and is described, instead, by waves or wave-packets, living in a flat Minkowski space. Notice, for instance, that all $SU(n)$ models are defined over a Minkowski space, supporting these arguments. Additionally, there has also been created the Higgs field, a particular matter - a kind of plasma - that fills up the flat Minkowski space and provides masses to the released matter particles. Exactly this plasma matter brings up a question which helps us to build up connections between the SM and the Zeeman space-time theories. Namely, the question arises, as to whether there exists a curved space-time which realizes this plasma-matter according to the principles of general relativity.

The Zeeman space-time seems to be the relativistic space-time realization of this plasma-matter. But it should be pointed out that the theory breaks off both from general relativity and classical quantum theory and stands in the middle between them. The space-time does not absorbs into its fabric all the matter existing in the universe but only that much which presents the Monistic Wave Operator of the Zeeman space-time. The rest of the matter is represented by waves defined by Z-Fourier transforms. The Monistic Wave Operator governs all physical processes taking place there. In this theory, the only field which exists there is the pseudo Riemannian metric of Lorentz signature defined on the Zeeman space-time. This is it what corresponds to the Higgs plasma, which have been explored in SM by means available for Lagrangian theories.

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