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MEASUREMENT OF THE RATIO OF THE PROTON FORM FACTORS, G_E/G_M, AT HIGH MOMENTUM TRANSFERS AND THE QUESTION OF SCALING*

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SUMMARY

Electron-proton elastic scattering cross sections have been measured at the Stanford Linear Accelerator Center at four-momentum transfers squared (q^2) of 1.0, 1.5, 2.0, 2.5 and 3.75 $(GeV/c)^2$. The angular distributions at $q^2 = 2.5$ and 3.75 $(GeV/c)^2$ are sufficient to provide values of the ratio G_E/G_M independent of the results from other laboratories. Our results are compatible with scaling, $G_E(q^2) = G_M(q^2)/\mu$, within the experimental errors.

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The scaling relationship between the electric (G_E) and magnetic (G_M) form factors of the proton is:

$$G_E(q^2) = G_M(q^2)/\mu \qquad (1)$$

where μ is the magnetic moment of the proton and q is the four-momentum transfer. This relation is true by definition in the limit of $q^2 = 0$, and has been shown[1] to hold within the experimental errors at q^2 values up to 1 $(\mathrm{GeV/c})^2$. Recently[2,3] a significant deviation from this relation has been reported for q^2 values between 1 and 2 $(\mathrm{GeV/c})^2$.

In the one-photon exchange approximation, the Rosenbluth cross section for elastic electron-proton scattering is

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{NS} \left\{\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right\} \qquad (2)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{NS} = \left(\frac{e^2}{2 E_0 \sin^2\frac{\theta}{2}}\right)^2 \frac{E}{E_0} \cos^2\frac{\theta}{2}$$

$$\tau = \frac{q^2}{4M^2}, \qquad q^2 = 2E_0E(1-\cos\theta)$$

in which E_0, E are the incident and scattered energy of the electron, θ is the electron scattering angle and M is the proton rest mass. The separation of the contributions of G_E and G_M is performed by measuring the angular distribution of electron-proton elastic scattering at a fixed value of four-momentum transfer. This separation becomes increasingly difficult as the momentum transfer increases since the contribution of G_E to elastic electron-proton scattering becomes smaller. If we assume relation (1) to be true, the maximum percentage contribution of G_E^2 to the cross section is 31% at $q^2 = 1(\mathrm{GeV/c})^2$, 15% at $2.5(\mathrm{GeV/c})^2$ and 10% at $4(\mathrm{GeV/c})^2$.

Electron-proton elastic scattering cross sections have been made at the Stanford Linear Accelerator Center at q^2 values from 1.0 to 3.75 $(GeV/c)^2$. The angular distribution measurements at $q^2 = 2.5$ and 3.75 $(GeV/c)^2$ provide values of the ratio G_E/G_M independent of the results from other laboratories. Small angle measurements at q^2 values of 1.0, 1.5 and 2.0 $(GeV/c)^2$ serve as a useful check of previous external beam measurements from DESY.[4] The experimental details and method of analysis were similar to that of previous elastic electron-proton measurements by this group.[5]

The electron beam was momentum analyzed and passed through a 23 cm long liquid hydrogen target. The scattered particles were momentum analyzed with the SLAC 8-GeV/c magnetic spectrometer[6] and detected in two banks of scintillation counter hodoscopes located at the focal planes of the spectrometer. Scattered electrons were identified from the pulse height information from a lead-lucite total absorption shower counter.

The energy of the incident beam was known to better than $\pm 0.2\%$, and a total momentum spread $\Delta p/p$ of 0.4% was used for the data runs. The incident beam direction was defined to within ± 0.05 mrad by alignment of the beam spot on two fluorescent screens. Incident beam currents were integrated using a toroid induction monitor[7] and secondary emission monitors. Frequent intercalibration of the beam-current monitors were made using a Faraday cup.[8]

Heating tests of our condensation-type liquid hydrogen target[9] showed that, for average beam currents of about $15\mu A$ and beam spot sizes of a few square millimeters, hydrogen density changes of about 20% could be induced. To be certain of keeping density changes below 1%, the average beam currents were restricted to below $1\mu A$ for the data runs. The beam spot sizes were typically 3 mm high and 6 mm wide.

It is usual to separate G_E and G_M by writing Eq. (2) as:

$$R_1 = \left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{NS} = I + S \tan^2 \theta/2 \qquad (3)$$

A plot of R_1 versus $\tan^2 \theta/2$ will have a slope $S = 2\tau G_M^2$ and an intercept $I = \left(G_E^2 + \tau G_M^2\right) / (1+\tau)$. However, at high q^2 values, the contribution of G_E to the intercept is small, and this type of plot does not give a clear indication of how well G_E is being determined. We prefer to rearrange the variables of Eq. (2) to produce a type of plot which gives an immediate visual impression of the accuracy of both G_E and G_M.

For convenience we use the "dipole" expression for the form factors:

$$\left[G_E(q^2)\right]_{DIPOLE} = \left[G_M(q^2)/\mu\right]_{DIPOLE} = (1 + q^2/0.71)^{-2} \qquad (4a)$$

and redefine the form factors at a fixed q^2 value as:

$$g_E = G_E(1 + q^2/0.71)^2 \quad \text{and} \quad g_M = (G_M/\mu)(1 + q^2/0.71)^2 \qquad (4b)$$

We define a kinematic factor $A = \epsilon/\mu^2\tau$, where ϵ is given by $\epsilon = 1/\left[1+2(1+\tau)\tan^2 \theta/2\right]$. As the quantity ϵ is limited to values between 0 and 1, A may vary from 0 (at $\theta = 180^o$) to $A_{MAX} = 1/\mu^2 \tau$ (at $\theta = 0^o$). Thus we can rewrite the Rosenbluth formula as:

$$R_2 = (1 + A) \left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{DIPOLE} = g_M^2 + A\, g_E^2 \qquad (5)$$

where the dipole cross section (which assumes scaling), $(d\sigma/d\Omega)_{DIPOLE}$, is obtained by substituting Eq. (4a) into Eq. (2).

Plotting R_2 versus A, therefore, gives a slope of value g_E^2 and an extrapolated intercept (at A = 0) of g_M^2. Thus, if scaling is true, the slope and intercept will be equal; if the dipole relation is also true, they will both equal unity. In this representation the statistical correlation between G_E and G_M is more easily seen, and small deviations from the dipole prediction are readily apparent. We emphasize that the values of G_E and G_M obtained by using this procedure do not depend on the mathematical relationships in Eq. (4) being true in nature.

At high q^2 values, the separation of G_E and G_M is very sensitive to the absolute normalization of the measurements. For this reason, it is usual to extract values of the ratio G_E/G_M from the data, as this quantity is insensitive to an absolute normalization, provided all cross sections are changed by the same factor. Thus, for a single experiment, one need only include the random errors in determining G_E/G_M. If data from several laboratories are combined, then the normalization differences must also be included.

The raw data have been corrected for radiative losses due to straggling of the electron beam in the target and beam windows[10] and for losses due to radiation in the scattering process.[11] For the latter, the formulae[11] of Tsai and of Meister and Yennie have been applied, assuming exponentiation. Different analytic methods involving differential or integral radiative corrections have been used. These different methods cause variations of ±1% in the results at a given q^2 value. The variation of the corrections with q^2 led us to assign an absolute uncertainty of ±1.5% to all cross sections.

The measured spectra after radiative correction had about 1% of the counts outside the expected elastic scattering peaks. This amount varied from run to run and may reflect uncertainties in the empty target subtractions (the subtractions were typically 2-4%), or may have been from pole-tip scattering in the spectrometer. This effect introduced an uncertainty of about one-fifth of the size of the errors in our form factor ratios at $q^2 = 2.5$ and 3.75 $(GeV/c)^2$.

A detailed check of the spectrometer optics was made by ray-tracing with electron beams of different momenta. Our cross section values were corrected to allow for the 2% variation of the solid angle of the spectrometer over the momentum range of 1.7 - 8.0 GeV/c used for the data runs. The absolute value of the solid angle is known to within ±3%. A random uncertainty of ±0.5% was included in the experimental errors to allow for the accuracy in centering the elastic scattering peak on the detectors.

The final cross section values are given in Table 1. Several runs have been combined at each angle and q^2 value. Two series of runs performed at $q^2 = 2.5$ $(GeV/c)^2$ were separated by several months and provided a useful check on the reproducibility of the data.

Table 1 contains only the random errors which directly affect the ratio G_E/G_M. These errors are due to counting statistics, empty target subtractions, fluctuations in beam monitoring (±0.5%), density uncertainties of the hydrogen target (± 1%), uncertainties in the radiative corrections (± 1%), solid angle (± 0.5%), and calibrations of the incident energy and scattering angle (± 0.4%).

There is an overall normalization uncertainty (not included in the values given in Table 1) which is estimated to be about ±4%. This includes the uncertainty in the absolute value of the solid angle (± 3%), beam monitor normalization (± 0.5%), calibration of the incident energy and scattering angle (± 0.75%), the accuracy of the radiative corrections (± 1.5%), the uncertainty in data event selection (± 1%) and the density of the liquid hydrogen (± 1.5%). This error does not in good approximation affect the values of G_E/G_M obtained using SLAC data alone, as any normalization effect cancels in the ratio.

In Fig. 1, we compare the elastic electron-proton cross sections from DESY[4] and this experiment which have been measured at values of the electron scattering angle less than 25°. The measured cross sections divided by the dipole cross sections are shown. Normalization errors have not been included in the data. At all q^2 values, the data agree within a spread of about ±1.5%. The quoted normalization errors in the cross sections are 3% from DESY[4] and the 4% from this experiment.

Our data at $q^2 = 1.0$, 1.5, and 2.0 $(GeV/c)^2$ are plotted in Figs. 2a-2c together with the external beam data from Bonn[3] and DESY.[4] Normalization errors have not been included. The data are plotted according to the method outlined above (Eq. (5)). For each graph we adjusted the data points to a common, central q^2 value by multiplying the cross sections by the relative change in the dipole cross section. In the figures we include straightline fits to the data obtained (a) by adjusting G_E/G_M for a best fit and (b) by assuming scaling to be true, that is, by setting $\mu G_E/G_M$ equal to unity. The inclusion of normalization errors would reduce the significance of the deviation from scaling that is suggested by the data at $q^2 = 1.5$ $(GeV/c)^2$. For example, a 1.5% normalization shift of the SLAC and DESY points relative to the Bonn data would change the ratio of $\mu G_E/G_M$ by about 5% at this q^2 value.

At higher q^2 values it becomes more difficult to make an accurate determination of G_E/G_M due to the expected small size of the contribution of G_E. As shown in Figs. 2c, 2d, and 2e, the data at $q^2 = 2.0$, 2.5 and 3.75 $(GeV/c)^2$ are consistent with scaling within the experimental errors, as was a previous DESY measurement[12] at $q^2 = 3\,(GeV/c)^2$ which gave $\mu G_E/G_M = 0.90 \pm 0.24$.

We have calculated values of the ratio $\mu G_E/G_M$ in the range of q^2 from 0.4 to 4 $(GeV/c)^2$ using the data from Refs. 3, 4, 12 and this experiment. These values are shown in Fig. 3, where the error bars represent only the effect of random errors in the data. We have used all the published data within ± 0.03 $(GeV/c)^2$ of each central value of q^2 and have adjusted the cross sections to that q^2. This method avoids the inherent uncertainties involved in interpolating the data over a sizeable q^2 interval and simplifies the propagation of statistical errors, because each measured cross section is used only once.

We have made a straightline fit to the data of Fig. 3 according to the relation

$$\mu G_E/G_M = 1 + \alpha q^2 \tag{6}$$

where q^2 is in $(\mathrm{GeV/c})^2$. We find $\alpha = -.051$ with an uncertainty, arising from the random errors in the published data, of $\delta\alpha_{\mathrm{random}} = \pm .018$. For this fit $\chi^2 = 12.9$ for 11 degrees of freedom. To estimate the effect of normalization differences between the laboratories where data from several experiments are combined, we have refitted with Eq. (6) after changing the normalization of the small angle data (DESY or SLAC) by ±1.5% with respect to the large angle data (Bonn). This relative normalization shift is about half of the quoted normalization uncertainties of the experiments. These shifts cause α to change by about $\delta\alpha_{\mathrm{normalization}} = \pm .030$.

We conclude that the existence of a significant deviation from the scaling rule is still in doubt.

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TABLE 1

Final values of the electron-proton elastic scattering cross sections. Only random errors are shown. There is an overall normalization uncertainty not included here, which we estimate to be ± 4%.

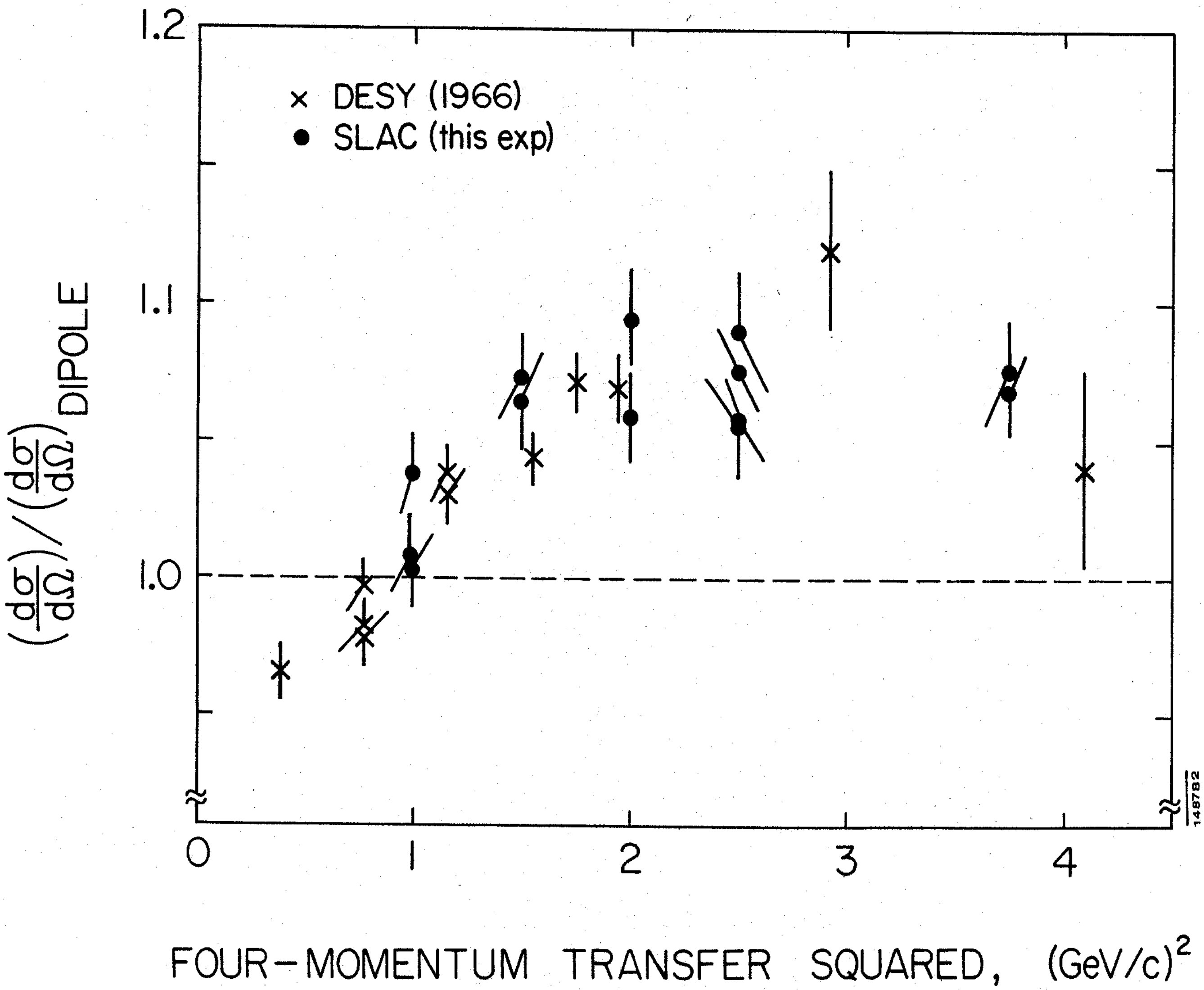
Nominal q^2 $(GeV/c)^2$	Incident energy E_0 (GeV)	Electron scattering angle θ (deg)	Final cross section $\frac{d\sigma}{d\Omega}$ (cm^2/sr)
1.00	3.996	15.44	$(.6593 \pm .0092) \times 10^{-31}$
	3.296	19.06	$(.4065 \pm .0061) \times 10^{-31}$
	2.998	21.18	$(.3308 \pm .0051) \times 10^{-31}$
1.50	6.197	12.15	$(.3307 \pm .0049) \times 10^{-31}$
	3.296	24.64	$(.635 \pm .011) \times 10^{-32}$
	2.998	27.58	$(.4818 \pm .0075) \times 10^{-32}$
2.00	6.197	14.40	$(.830 \pm .014) \times 10^{-32}$
	3.996	23.84	$(.2414 \pm .0039) \times 10^{-32}$
	3.296	30.22	$(.1334 \pm .0032) \times 10^{-32}$
	2.998	34.14	$(.999 \pm .017) \times 10^{-33}$
2.50 (Run 1)	7.909	12.59	$(.4708 \pm .0067) \times 10^{-32}$
	5.253	20.09	$(.1538 \pm .0026) \times 10^{-32}$
	3.802	29.96	$(.565 \pm .010) \times 10^{-33}$
	3.294	36.20	$(.3532 \pm .0065) \times 10^{-33}$
2.50 (Run 2)	7.909	12.59	$(.4777 \pm .0066) \times 10^{-32}$
	6.197	16.55	$(.2566 \pm .0052) \times 10^{-32}$
	3.996	28.04	$(.688 \pm .012) \times 10^{-33}$
	3.296	36.17	$(.3475 \pm .0065) \times 10^{-33}$
	2.998	41.40	$(.2469 \pm .0045) \times 10^{-33}$
3.75	9.998	12.42	$(.973 \pm .014) \times 10^{-33}$
	7.911	16.26	$(.5112 \pm .0089) \times 10^{-33}$
	3.996	40.03	$(.4710 \pm .0092) \times 10^{-34}$

FIGURE CAPTIONS

1. A comparison of electron-proton elastic cross sections from DESY[4] and this experiment for electron scattering angles less than 25°. We plot the cross sections divided by the cross sections calculated using the dipole relation (Eq. (4a)) and the Rosenbluth relation (Eq. (2)). Normalization errors are not included in the data.

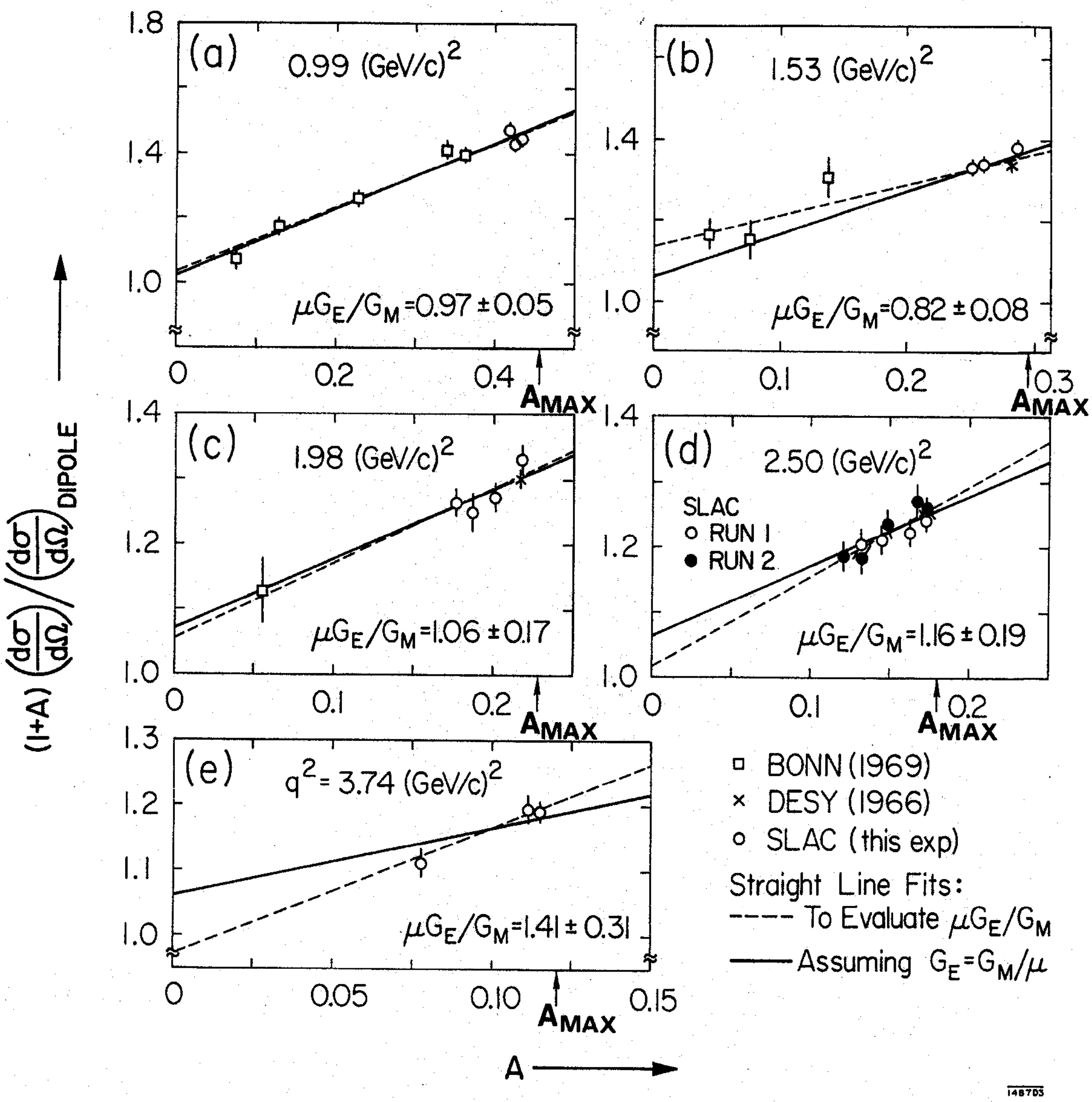
2a-e. Electron-proton elastic scattering data from Bonn,[3] DESY[4] and this experiment are shown on the type of plot described in the text. The abscissa is $A = \epsilon/\mu^2\tau$, and $A_{MAX} = 1/\mu^2\tau$. Data are plotted only for the q^2 values of this experiment. Straightline fits producing values of the ratio $\mu G_E/G_M$ are shown together with fits assuming that the scaling relation $G_E = G_M/\mu$ is true.

3. Values of $\mu G_E/G_M$ calculated from data in Refs. 3, 4, 12 and this experiment are shown for the range of four-momentum transfer squared, q^2, from 0.4 to 4 $(GeV/c)^2$. For each value we have used all data points which are close to the q^2 value. The errors are calculated from random errors in the published data points. No allowance has been made for normalization differences between measurements at different laboratories. A fit $\mu G_E/G_M = 1 - 0.051\ q^2$ is indicated by the solid line.



× DESY (1966)
● SLAC (this exp)
1.2
1.1
1.0
0
1
2
3
4
$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_{DIPOLE}$
FOUR-MOMENTUM TRANSFER SQUARED, $(GeV/c)^2$
1487B2

Fig. 1



(a) 0.99 (GeV/c)²
μG_E/G_M=0.97 ± 0.05
(b) 1.53 (GeV/c)²
μG_E/G_M=0.82 ± 0.08
(c) 1.98 (GeV/c)²
μG_E/G_M=1.06 ± 0.17
(d) 2.50 (GeV/c)²
SLAC
RUN 1
RUN 2
μG_E/G_M=1.16 ± 0.19
(e) q²= 3.74 (GeV/c)²
μG_E/G_M=1.41 ± 0.31
A_MAX
(1+A) (dσ/dΩ)/(dσ/dΩ)_DIPOLE
A
BONN (1969)
DESY (1966)
SLAC (this exp)
Straight Line Fits:
To Evaluate μG_E/G_M
Assuming G_E=G_M/μ

Fig. 2

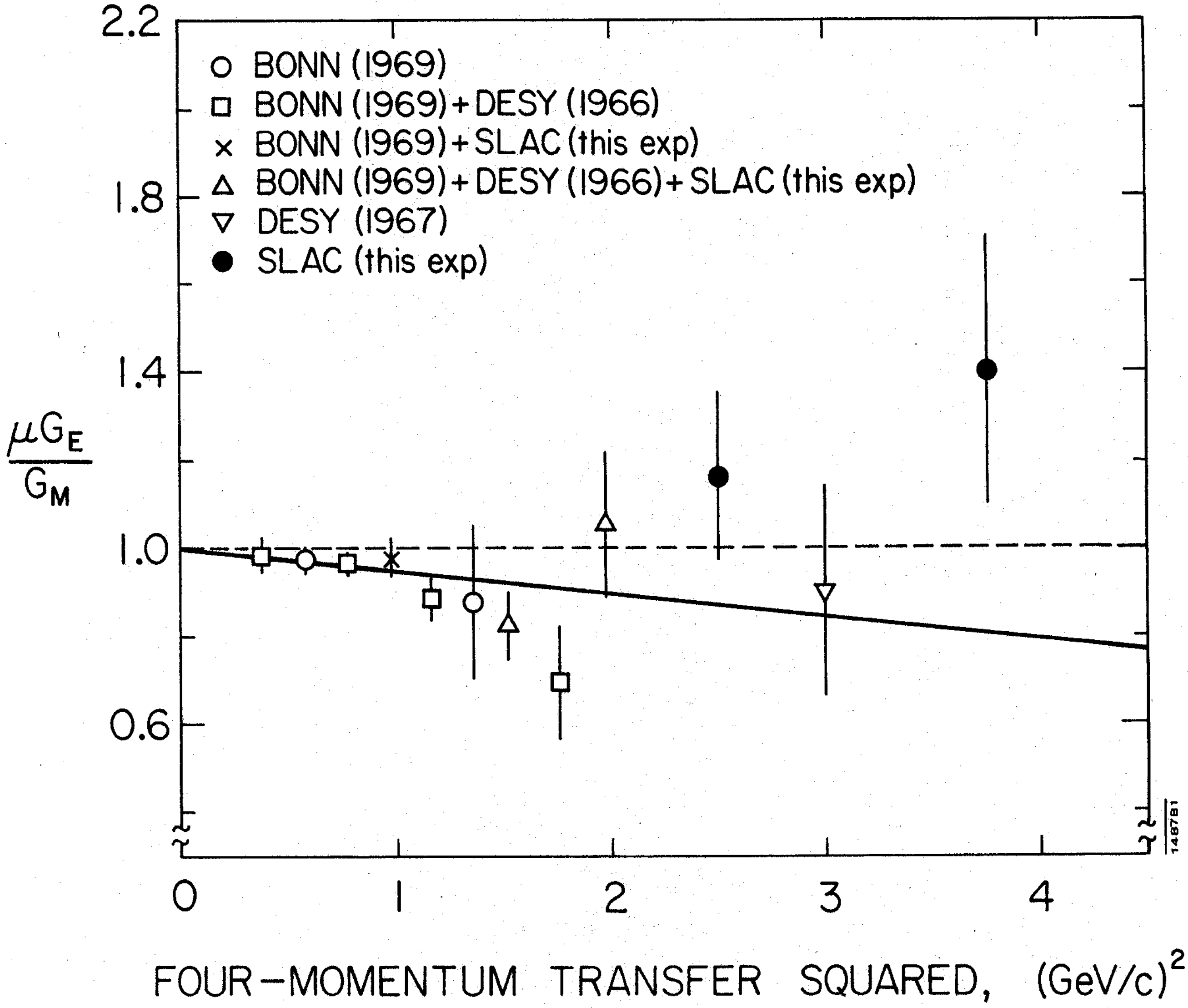

2.2
1.8
1.4
1.0
0.6
$\frac{\mu G_E}{G_M}$
BONN (1969)
BONN (1969) + DESY (1966)
BONN (1969) + SLAC (this exp)
BONN (1969) + DESY (1966) + SLAC (this exp)
DESY (1967)
SLAC (this exp)
0
1
2
3
4
FOUR-MOMENTUM TRANSFER SQUARED, $(GeV/c)^2$

Fig. 3