



No hair for spherically symmetric neutral reflecting stars: Nonminimally coupled massive scalar fields



Shahar Hod ^{a,b,*}

^a The Ruppin Academic Center, Emek Hefer 40250, Israel

^b The Hadassah Academic College, Jerusalem 91010, Israel

ARTICLE INFO

Article history:

Received 30 July 2017

Received in revised form 12 August 2017

Accepted 15 August 2017

Available online 24 August 2017

Editor: M. Cvetič

ABSTRACT

Recent no-hair theorems have revealed the intriguing fact that horizonless stars with compact reflecting surfaces *cannot* support non-linear matter configurations made of scalar, vector, and tensor fields. In the present paper we extend the regime of validity of these no-hair theorems by explicitly proving that spherically symmetric compact reflecting stars cannot support static configurations made of *massive* scalar fields with *non*-minimal coupling to gravity. Interestingly, our no-hair theorem is valid for *generic* values of the dimensionless field-curvature coupling parameter ξ .

© 2017 The Author. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

The influential no-scalar-hair theorems of Bekenstein [1] and Teitelboim [2] (see also [3–8] and references therein) have revealed the interesting fact that asymptotically flat classical black-hole spacetimes, which are characterized by absorbing one-way membranes (event horizons), cannot support static non-linear matter configurations (external ‘hair’) made of spatially regular massive scalar fields with minimal coupling to gravity [9–12].

The rigorous derivation of a no-hair theorem for scalar matter configurations *nonminimally* coupled to gravity turns out to be a mathematically challenging task. In particular, the interesting no-hair theorems presented by Bekenstein and Mayo [7,8] can be used to rule out the existence of spherically symmetric asymptotically flat black-hole spacetimes supporting neutral nonminimally coupled massive scalar fields in the restricted physical regimes $\xi < 0$ and $\xi \geq 1/2$ [13–18].

Recently, we have raised [19] the following physically intriguing question: Can the canonical no-scalar-hair theorems presented in [1–8] for classical black-hole spacetimes with absorbing event horizons be extended to the physical regime of asymptotically flat *horizonless* curved spacetimes?

It is worth mentioning that this physically important question is also motivated by the recent scientific interest in the physical and mathematical properties of regular spacetimes describing highly

compact horizonless objects (see [20–23] and references therein) which have been proposed as possible exotic quantum-gravity alternatives to the canonical classical black-hole spacetimes.

Motivated by the above stated physically interesting question, in Ref. [19] we have explored the physical implications of replacing the standard ingoing (*absorbing*) boundary condition which characterizes the behavior of matter fields at the classical horizon of a black-hole spacetime [1,8] by a reflecting (*repulsive*) boundary condition at the surface of a spherically symmetric horizonless compact star. Intriguingly, the analysis presented in [19] has revealed the fact that horizonless compact stars with reflecting surfaces [24] share the no-scalar-hair property with the more familiar classical black-hole spacetimes with absorbing one-way membranes (event horizons). In particular, it has been explicitly proved [19] that spherically symmetric compact objects with reflecting boundary conditions cannot support spatially regular matter configurations made of non-linear scalar (spin-0) fields with *minimal* coupling to gravity. Moreover, in a very interesting paper, Bhattacharjee and Sarkar [25] have extended the results of [19] and proved that horizonless stars with reflecting boundary conditions cannot support vector (spin-1) and tensor (spin-2) fields. Recently, we have explicitly proved that neutral reflecting stars [24] cannot support static spherically symmetric configurations of *nonminimally* coupled *massless* scalar fields [26].

It is physically important to explore the regime of validity of the intriguing no-hair behavior recently observed for *horizonless* regular spacetimes describing compact reflecting stars [19,25,26]. The main goal of the present paper is to analyze the physical and mathematical properties of the non-linearly coupled Einstein-

* Correspondence to: The Ruppin Academic Center, Emek Hefer 40250, Israel.
E-mail address: shaharhod@gmail.com.

scalar field equations for *massive* scalar fields with *nonminimal* ($\xi \neq 0$) coupling to gravity. In particular, below we shall present a novel no-hair theorem which explicitly proves that static spherically symmetric neutral stars with reflecting (that is, *repulsive* rather than *absorbing*) boundary conditions cannot support spatially regular matter configurations made of non-linear *massive* scalar fields with *generic* values of the dimensionless nonminimal coupling parameter ξ .

2. Description of the system

We shall analyze the static sector of a physical system which is composed of a spherically symmetric neutral reflecting object (a compact reflecting star [24]) of radius R_s which is non-linearly coupled to a massive scalar field. The scalar field is assumed to be non-minimally coupled to gravity. The curved line element of the static and spherically symmetric spacetime can be expressed in the form [7,27,28]

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $\nu = \nu(r)$ and $\lambda = \lambda(r)$. Asymptotic flatness of the spacetime is characterized by the asymptotic large- r behaviors [7]

$$\nu \sim M/r \text{ and } \lambda \sim M/r \text{ for } r \rightarrow \infty \quad (2)$$

of the metric functions, where M is the total ADM mass (as measured by asymptotic observers) of the composed static star-field configurations.

The action of the non-minimally coupled massive scalar field ψ in the curved spacetime of the spherically symmetric reflecting star is given by [7,29]

$$S = S_{EH} - \frac{1}{2} \int (\partial_\alpha \psi \partial^\alpha \psi + \mu^2 \psi^2 + \xi R \psi^2) \sqrt{-g} d^4x, \quad (3)$$

where μ is the proper mass of the field [30] and $R(r)$ is the scalar curvature of the spacetime. An asymptotically flat spacetime is characterized by the large- r asymptotic behavior [7]

$$R(r \rightarrow \infty) \rightarrow 0. \quad (4)$$

The dimensionless physical parameter ξ in the action (3) quantifies the strength of the nonminimal coupling of the massive scalar field to the spacetime curvature.

The action (3) yields the characteristic differential equation [7]

$$\partial_\alpha \partial^\alpha \psi - (\mu^2 + \xi R) \psi = 0 \quad (5)$$

for the nonminimally coupled massive scalar field. Substituting into (5) the metric components of the spherically symmetric curved spacetime (1), one finds that the spatial behavior of the static nonminimally coupled massive scalar field is governed by the non-linear radial differential equation [31]

$$\psi'' + \frac{1}{2} \left(\frac{4}{r} + \nu' - \lambda' \right) \psi' - (\mu^2 + \xi R) e^\lambda \psi = 0. \quad (6)$$

The nonminimally coupled massive scalar field ψ is assumed to vanish on the surface $r = R_s$ of the central spherically symmetric compact reflecting star [32,33]:

$$\psi(r = R_s) = 0. \quad (7)$$

In addition, as discussed in [7], a curved spacetime describing a physically acceptable system must be characterized by an asymptotic finite and positive value of the effective gravitational constant $G_{\text{eff}} = G(1 - 8\pi G \xi \psi^2)$ [7]. This physical requirement enforces the asymptotic spatial bounds

$$-\infty < 8\pi \xi \psi^2 < 1 \text{ for } r \rightarrow \infty \quad (8)$$

on the characteristic radial eigenfunctions of the nonminimally coupled massive scalar fields.

From the action (3) one also finds the following expressions [7]

$$T_t^t = e^{-\lambda} \frac{\xi(4/r - \lambda') \psi \psi' + (2\xi - 1/2)(\psi')^2 + 2\xi \psi \psi''}{1 - 8\pi \xi \psi^2} - \frac{\mu^2 \psi^2}{2(1 - 8\pi \xi \psi^2)}, \quad (9)$$

$$T_t^t - T_r^r = e^{-\lambda} \frac{(2\xi - 1)(\psi')^2 + 2\xi \psi \psi'' - \xi(\nu + \lambda)' \psi \psi'}{1 - 8\pi \xi \psi^2}, \quad (10)$$

and

$$T_t^t - T_\phi^\phi = e^{-\lambda} \frac{\xi(2/r - \nu') \psi \psi'}{1 - 8\pi \xi \psi^2} \quad (11)$$

for the components of the energy-momentum tensor which characterizes the nonminimally coupled massive scalar field in the static curved spacetime (1) of the star.

In addition, as explicitly proved by Bekenstein and Mayo [7], causality requirements imply that the components of the energy-momentum tensors of physically acceptable systems must respect the following inequalities:

$$|T_\theta^\theta| = |T_\phi^\phi| \leq |T_t^t| \geq |T_r^r|. \quad (12)$$

For later purposes, we note that the simple relations [7]

$$\text{sgn}(T_t^t) = \text{sgn}(T_t^t - T_r^r) = \text{sgn}(T_t^t - T_\phi^\phi) \quad (13)$$

provide necessary conditions for the validity of the energy conditions (12) which, as proved in [7], characterize the energy-momentum components of physically acceptable systems.

3. The no-hair theorem for the composed reflecting-stars-nonminimally-coupled-massive-scalar fields configurations

In the present section we shall explicitly prove that spatially regular static matter configurations made of nonminimally coupled massive scalar fields with the action (3) *cannot* be supported by spherically symmetric *horizonless* reflecting stars.

We shall first prove that, for physically acceptable systems, the characteristic radial eigenfunction ψ of the massive scalar fields must approach zero at spatial infinity. Taking cognizance of the asymptotic functional relations (2) and (4), which characterize asymptotically flat spacetimes with finite ADM masses, one finds that, in the asymptotic far region $M/r \ll 1$, the radial equation (6) of the nonminimally coupled massive scalar fields can be approximated by

$$\psi'' + \frac{2}{r} \psi' - \mu^2 \psi = 0. \quad (14)$$

The general mathematical solution of the asymptotic radial equation (14) is given by

$$\psi(r) = A \cdot (\mu r)^{-1} e^{-\mu r} + B \cdot (\mu r)^{-1} e^{\mu r} \text{ for } r \gg M, \quad (15)$$

where $\{A, B\}$ are dimensionless constants.

One immediately realizes that the asymptotic radial solution (15) with $B \neq 0$ violates the upper bound (8) which, as discussed in [7], characterizes physically acceptable systems. We therefore find that spatially regular massive scalar field configurations supported in physically acceptable asymptotically flat spacetimes [that is, spacetimes which are characterized by finite and positive asymptotic values of the effective gravitational constant

$G_{\text{eff}} = G(1 - 8\pi G\xi\psi^2)$ [7]] are characterized by the asymptotic radial eigenfunction [see Eq. (15) with $B = 0$]

$$\psi(r) = A \cdot (\mu r)^{-1} e^{-\mu r} \quad \text{for } r \gg M. \quad (16)$$

For later purposes, we note that the characteristic asymptotic behavior (16) of the massive scalar fields together with the inner boundary condition (7) at the surface of the horizonless spherically symmetric compact reflecting star imply that the radial scalar eigenfunction ψ must have (at least) one extremum point, $r = r_{\text{peak}}$, which is located in the interval $r_{\text{peak}} \in (R_s, \infty)$. At this extremum point the radial eigenfunction of the static nonminimally coupled massive scalar fields is characterized by the simple relations

$$\{\psi \neq 0 ; \quad \psi' = 0 ; \quad \psi \cdot \psi'' < 0\} \quad \text{for } r = r_{\text{peak}}. \quad (17)$$

3.1. The no-massive-scalar-hair theorem for generic inner boundary conditions in the physical regimes $\xi < 0$ and $\xi > 1/4$

Interestingly, as we shall now prove, one can use the characteristic asymptotic behavior (16) of the radial scalar eigenfunction in order to exclude the existence of asymptotically flat static matter configurations made of nonminimally coupled massive scalar fields in the regimes $\xi < 0$ and $\xi > 1/4$.

Substituting the spatially regular asymptotic eigenfunction (16) of the massive scalar fields into (9) and (11), and using the asymptotic spatial behavior (2) of the metric components of the asymptotically flat static spacetime (1), one finds the simple functional expressions

$$T_t^t = (4\xi - 1)\mu^2\psi^2 \cdot [1 + O(M/r)] \quad (18)$$

and

$$T_t^t - T_\phi^\phi = -\xi \frac{2\mu}{r} \psi^2 \cdot [1 + O(M/r)] \quad (19)$$

for the components of the energy-momentum tensor. From Eqs. (18) and (19) one immediately deduces the far-region relation

$$\text{sgn}(T_t^t) = -\text{sgn}(T_t^t - T_\phi^\phi) \quad \text{for } \xi < 0 \text{ or } \xi > 1/4. \quad (20)$$

We point out that the scalar field relation (20) violates the characteristic relation (13) which, as discussed in [7], is imposed by causality on the energy-momentum components of physically acceptable spacetimes. One therefore concludes that spherically symmetric reflecting stars *cannot* support static massive scalar fields nonminimally coupled to gravity in the regimes $\xi \leq 0$ [34] and $\xi > 1/4$.

Before proceeding, we would like to stress the fact that in the present subsection we have made *no* reference to a particular physical boundary condition at the surface of the central compact object. Thus, the compact no-massive-scalar-hair theorem presented in this subsection rules out the existence of spatially regular static matter configurations made of nonminimally coupled massive fields in the regimes $\xi < 0$ and $\xi > 1/4$ for both classical black-hole spacetimes with absorbing event horizons and for horizonless regular spacetimes describing compact stars with reflecting surfaces.

3.2. The no-massive-scalar-hair theorem for spherically symmetric neutral reflecting stars with positive values of the physical coupling parameter ξ

In the present subsection we shall prove that spherically symmetric horizonless compact stars with reflecting boundary conditions cannot support static matter configurations made of nonminimally coupled massive scalar fields with generic positive values of the physical coupling parameter ξ .

Using the Einstein relation $R = -8\pi T$ [35], one obtains from Eqs. (9), (10), and (11) the functional expression

$$R = -\frac{8\pi}{1 - 8\pi\xi\psi^2} \left\{ e^{-\lambda} \left[\xi \left(\frac{12}{r} + 3\nu' - 3\lambda' \right) \psi\psi' + 6\xi\psi\psi'' + (6\xi - 1)(\psi')^2 \right] - 2\mu^2\psi^2 \right\} \quad (21)$$

for the Ricci scalar curvature which characterizes the static curved spacetime (1). Substituting the radial functional expression (21) into the characteristic differential equation (6) of the nonminimally coupled massive scalar fields, one finds the radial scalar equation

$$\begin{aligned} \psi'' \cdot [1 + 8\pi\xi(6\xi - 1)\psi^2] \\ + \psi' \cdot \left[\frac{1}{2} \left(\frac{4}{r} + \nu' - \lambda' \right) [1 + 8\pi\xi(6\xi - 1)\psi^2] \right. \\ \left. + 8\pi\xi(6\xi - 1)\psi\psi' \right] - \mu^2 e^\lambda (1 + 8\pi\xi\psi^2) \psi = 0. \end{aligned} \quad (22)$$

The (rather cumbersome) non-linear differential equation (22) determines the spatial behavior of the static nonminimally coupled massive scalar fields in the spherically symmetric curved spacetime.

We shall now prove that the radial function

$$\mathcal{F}(r; \xi) \equiv 1 + 8\pi\xi(6\xi - 1)\psi^2 \quad (23)$$

that appears on the l.h.s of (22) is positive definite. This is obviously true in the physical regimes $\xi \geq 1/6$ and $\xi \leq 0$. As we shall now show explicitly, the physical regime $0 < \xi < 1/6$ is also characterized by the inequality $\mathcal{F} > 0$. We first note that the radial function (23) is characterized by the simple relation [see Eq. (7)]

$$\mathcal{F}(r = R_s) = 1 \quad (24)$$

at the reflecting surface of the horizonless compact star. Now, let us assume that $\mathcal{F}(r)$ vanishes at some point $r = r_0$. Then, from Eq. (22) one finds the simple functional relation

$$8\pi\xi(6\xi - 1)(\psi')^2 = \mu^2 e^\lambda (1 + 8\pi\xi\psi^2) \quad \text{at } r = r_0 \quad (25)$$

at the assumed root of $\mathcal{F}(r)$. A simple inspection of Eq. (25) reveals that, in the physical regime $0 < \xi < 1/6$, the functional expression on the l.h.s of (25) is non-positive whereas the functional expression on the r.h.s of (25) is positive definite. One therefore concludes that the radial function $\mathcal{F}(r)$ *cannot* switch signs. In particular, taking cognizance of the boundary relation (24), we find the characteristic inequality

$$\mathcal{F}(r) > 0. \quad (26)$$

Interestingly, from Eqs. (17), (22), and (23) one finds the compact functional relation

$$\mathcal{F} \cdot \psi\psi'' = \mu^2 e^\lambda (1 + 8\pi\xi\psi^2) \psi^2 \quad \text{at } r = r_{\text{peak}} \quad (27)$$

at the extremum point $r = r_{\text{peak}}$ of the scalar eigenfunction (where $\psi' = 0$). Taking cognizance of the analytically derived inequality (26) and the characteristic relation $\psi\psi'' < 0$ at the extremum point of the static nonminimally coupled massive scalar field configuration [see Eq. (17)], one finds that the functional expression on the l.h.s of (27) is negative definite whereas, for $\xi \geq 0$, the functional expression on the r.h.s of (27) is positive definite. Thus, the characteristic differential relation (22) for the nonminimally coupled massive scalar fields *cannot* be respected at the extremum point $r = r_{\text{peak}}$ of the scalar eigenfunction. We therefore conclude that spherically symmetric reflecting stars *cannot* support spatially regular static configurations made of massive scalar fields nonminimally coupled to gravity in the physical regime $\xi \geq 0$ [36].

4. Summary

The physically important and mathematically elegant no-hair theorems of Bekenstein and Mayo [7,8] have revealed the interesting fact that spherically symmetric black holes with absorbing horizons cannot support asymptotically flat non-linear matter configurations made of massive scalar fields [18] nonminimally coupled to gravity in the physical regimes $\xi < 0$ and $\xi \geq \frac{1}{2}$.

Intriguingly, it has recently been revealed that asymptotically flat horizonless spacetimes may share the no-hair property with the more familiar absorbing black-hole spacetimes [19,25,26]. In particular, the no-hair theorems presented in [19,25,26] have explicitly proved that regular curved spacetimes describing compact stars with repulsive (*reflecting*) boundary conditions [as opposed to the attractive (*ingoing*) boundary conditions which characterize the horizons of the classical black-hole spacetimes considered in the original no-hair theorems [7,8]] cannot support non-linear matter configurations made of minimally coupled scalar, vector, and tensor fields [19,25], as well as nonminimally coupled massless scalar fields [26].

It is certainly of physical interest to explore the regime of validity of the no-hair behavior recently observed for regular horizonless compact objects [19,25,26]. In the present paper, by studying analytically the non-linear Einstein-scalar field equations, we have explicitly proved that horizonless neutral stars with reflecting (that is, *repulsive* rather than *absorbing*) boundary conditions cannot support spatially regular non-linear matter configurations made of massive scalar fields *nonminimally* coupled to gravity [37].

Finally, we would like to emphasize the interesting fact that, while existing no-hair theorems for the composed black-hole-massive-scalar-field system are valid in the restricted physical regimes $\xi < 0$ and $\xi \geq 1/2$ [7,8,14,18], the no-hair theorem presented in this paper for the composed compact-reflecting-star-massive-scalar-field system is valid for *generic* values of the dimensionless nonminimal coupling parameter ξ .

Acknowledgements

This research is supported by the Carmel Science Foundation. I would like to thank Yael Oren, Arbel M. Ongo, Ayelet B. Lata, and Alona B. Tea for helpful discussions.

References

- [1] J.D. Bekenstein, Phys. Rev. Lett. 28 (1972) 452.
- [2] C. Teitelboim, Lett. Nuovo Cimento 3 (1972) 326.
- [3] J.E. Chase, Commun. Math. Phys. 19 (1970) 276.
- [4] J.D. Bekenstein, Phys. Today 33 (1980) 24.
- [5] M. Heusler, J. Math. Phys. 33 (1992) 3497; M. Heusler, Class. Quantum Gravity 12 (1995) 779.
- [6] J.D. Bekenstein, Phys. Rev. D 51 (1995) R6608.
- [7] A.E. Mayo, J.D. Bekenstein, Phys. Rev. D 54 (1996) 5059.
- [8] J.D. Bekenstein, arXiv:gr-qc/9605059.
- [9] It is worth noting that the no-scalar-hair property deduced in [1–8] can be extended to the physically interesting regime of non-linear external matter configurations with monotonically increasing self-interaction scalar potentials [7,8].
- [10] It is worth mentioning that, while the elegant and physically important no-hair theorems of [1] and [2] have ruled out the existence of hairy black-hole spacetimes supporting external static configurations made of massive scalar fields, it has been explicitly proved in [11,12] that spinning black-hole spacetimes can support spatially regular *stationary* (that is, non-decaying in time) massive scalar (bosonic) fields.
- [11] S. Hod, Phys. Rev. D 86 (2012) 104026, arXiv:1211.3202; S. Hod, Eur. Phys. J. C 73 (2013) 2378, arXiv:1311.5298; S. Hod, Phys. Rev. D 90 (2014) 024051, arXiv:1406.1179; S. Hod, Phys. Lett. B 739 (2014) 196, arXiv:1411.2609; S. Hod, Class. Quantum Gravity 32 (2015) 134002, arXiv:1607.00003; S. Hod, Phys. Lett. B 751 (2015) 177; S. Hod, Class. Quantum Gravity 33 (2016) 114001; S. Hod, Phys. Lett. B 758 (2016) 181, arXiv:1606.02306; S. Hod, O. Hod, Phys. Rev. D 81 (2010) 061502, Rapid communication, arXiv:0910.0734; S. Hod, Phys. Lett. B 708 (2012) 320, arXiv:1205.1872; S. Hod, J. High Energy Phys. 01 (2017) 030, arXiv:1612.00014.
- [12] C.A.R. Herdeiro, E. Radu, Phys. Rev. Lett. 112 (2014) 221101; C.L. Benone, L.C.B. Crispino, C. Herdeiro, E. Radu, Phys. Rev. D 90 (2014) 104024; C.A.R. Herdeiro, E. Radu, Phys. Rev. D 89 (2014) 124018; C.A.R. Herdeiro, E. Radu, Int. J. Mod. Phys. D 23 (2014) 1442014; Y. Brihaye, C. Herdeiro, E. Radu, Phys. Lett. B 739 (2014) 1; J.C. Degollado, C.A.R. Herdeiro, Phys. Rev. D 90 (2014) 065019; C. Herdeiro, E. Radu, H. Rúnarsson, Phys. Lett. B 739 (2014) 302; C. Herdeiro, E. Radu, Class. Quantum Gravity 32 (2015) 144001; C.A.R. Herdeiro, E. Radu, Int. J. Mod. Phys. D 24 (2015) 1542014; C.A.R. Herdeiro, E. Radu, Int. J. Mod. Phys. D 24 (2015) 1544022; P.V.P. Cunha, C.A.R. Herdeiro, E. Radu, H.F. Rúnarsson, Phys. Rev. Lett. 115 (2015) 211102; B. Kleihaus, J. Kunz, S. Yazadjiev, Phys. Lett. B 744 (2015) 406; C.A.R. Herdeiro, E. Radu, H.F. Rúnarsson, Phys. Rev. D 92 (2015) 084059; C. Herdeiro, J. Kunz, E. Radu, B. Subagyo, Phys. Lett. B 748 (2015) 30; C.A.R. Herdeiro, E. Radu, H.F. Rúnarsson, Class. Quantum Gravity 33 (2016) 154001; C.A.R. Herdeiro, E. Radu, H.F. Rúnarsson, Int. J. Mod. Phys. D 25 (2016) 1641014; Y. Brihaye, C. Herdeiro, E. Radu, Phys. Lett. B 760 (2016) 279; Y. Ni, M. Zhou, A.C. Avendano, C. Bambi, C.A.R. Herdeiro, E. Radu, J. Cosmol. Astropart. Phys. 1607 (2016) 049; M. Wang, arXiv:1606.00811.
- [13] Here ξ is a dimensionless physical parameter which quantifies the strength of the nonminimal coupling of the field to the scalar curvature of the spacetime [see Eq. (3)].
- [14] It is worth stressing the fact that, to the best of our knowledge, at present there is no mathematically rigorous no-hair theorem which rules out the possible existence of hairy black-hole spacetimes supporting static matter configurations made of neutral nonminimally coupled massive scalar fields in the physical regime $0 < \xi < 1/2$ [7].
- [15] In this context, it is important to mention the existence of the extremal Bocharova–Bronnikov–Melnikov–Bekenstein (BBMB) [16,17] hairy black-hole solution of the non-linearly coupled Einstein-scalar field equations. This spherically symmetric hairy black-hole spacetime is characterized by a conformally coupled massless scalar field with $\xi = 1/6$. Obviously, this value of the dimensionless nonminimal coupling parameter ξ is outside the regimes $\xi < 0$ and $\xi \geq 1/2$ covered by the influential no-hair theorems presented in [7,8] for nonminimally coupled scalar fields.
- [16] N. Bocharova, K. Bronnikov, V. Melnikov, Vestn. Mosk. Univ. Fiz. Astron. 6 (1970) 706.
- [17] J.D. Bekenstein, Ann. Phys. (NY) 82 (1974) 535; J.D. Bekenstein, Ann. Phys. (NY) 91 (1975) 72.
- [18] It is worth noting that the no-hair theorems presented in [7,8] are valid for scalar fields with positive semidefinite self-interaction potentials.
- [19] S. Hod, Phys. Rev. D 94 (2016) 104073, arXiv:1612.04823.
- [20] E. Maggio, P. Pani, V. Ferrari, arXiv:1703.03696.
- [21] R. Brito, V. Cardoso, P. Pani, Lect. Notes Phys. 906 (2015) 1.
- [22] S. Hod, J. High Energy Phys. 06 (2017) 132, arXiv:1704.05856.
- [23] S. Hod, Phys. Lett. B 770 (2017) 186.
- [24] The term ‘reflecting star’ is used here to describe a physical compact object for which the external matter fields vanish on its outer reflecting surface [see Eq. (7)].
- [25] S. Bhattacharjee, S. Sarkar, Phys. Rev. D 95 (2017) 084027.
- [26] S. Hod, Phys. Rev. D 96 (2017) 024019.
- [27] Here we use the Schwarzschild spacetime coordinates (t, r, θ, ϕ) .
- [28] We shall use natural units in which $G = c = 1$.
- [29] Here S_{EH} is the Einstein–Hilbert action.
- [30] Note that the mass parameter μ , which characterizes the nonminimally coupled massive scalar field, stands for $\mu/\hbar$. Thus, this physical parameter has the dimensions of $(\text{length})^{-1}$.
- [31] Here a prime ‘ $\prime$ ’ denotes a spatial derivative with respect to r .
- [32] It is worth mentioning that, following the influential work of Press and Teukolsky [33] on the ‘black-hole bomb’ phenomenon, many researches have explored the physical properties of the composed black-hole-scalar-field-reflecting-mirror system. In this composed physical system, one places a reflecting surface around the black hole whose role is to prevent the scalar field from radiating its energy to infinity. On the other hand, in the present paper (see also [19,25,26]) we consider a spherically symmetric reflecting surface whose role is to prevent the nonminimally coupled massive scalar field from radiating its energy into the central horizonless compact star.
- [33] W.H. Press, S.A. Teukolsky, Nature 238 (1972) 211; W.H. Press, S.A. Teukolsky, Astrophys. J. 185 (1973) 649.

- [34] The no-hair theorem of [19] can be used to rule out the existence of external static matter configurations made of minimally coupled ($\xi = 0$) massive scalar fields.
- [35] Here T is the trace of the energy-momentum tensor.
- [36] Thus far we have considered horizonless reflecting stars with Dirichlet [that is, $\psi(r = R_s) = 0$] boundary conditions. It is interesting to note that one can extend the regime of validity of the no-hair behavior to the case of compact stars with Neumann [that is, $\psi'(r = R_s) = 0$] boundary conditions. In particular, as emphasized above, the no-hair theorem presented in section IIIA is valid in the regimes $\xi < 0$ and $\xi > 1/4$ for *generic* inner boundary conditions. In addition, one finds from Eq. (22) that, for the case of Neumann boundary conditions, the functional relation (27) is valid at the surface $r = R_s$ of the reflecting star. Furthermore, using the fact that $\mathcal{F}(r)$ cannot switch signs together with the asymptotic behavior $\mathcal{F}(r/M \rightarrow \infty) \rightarrow 1$ [see Eqs. (16) and (23)], one concludes that $\mathcal{F}(r) > 0$ for the Neumann case as well [that is, the characteristic inequality (26) is valid for both Dirichlet and Neumann boundary conditions]. Now, if $\psi\psi'' > 0$ at the surface of the star, then from the characteristic asymptotic behavior (16) of the nonminimally coupled massive scalar field one learns that the radial eigenfunction $\psi(r)$ must have (at least) one extremum point $r_{\text{peak}} \in (R_s, \infty)$ with the functional properties (17). At this ex-

tremum point, the l.h.s of (27) is negative definite whereas, for $\xi \geq 0$, the r.h.s of (27) is positive definite. Thus, one concludes that the characteristic relation (22) *cannot* be respected at the extremum point $r = r_{\text{peak}}$ of the scalar eigenfunction. Likewise, if $\psi\psi'' < 0$ at the boundary $r = R_s$ of the star (where $\psi' = 0$ for the Neumann boundary conditions), then one finds that, on the reflecting surface, the l.h.s of (27) is negative definite whereas, for $\xi \geq 0$, the r.h.s of (27) is positive definite. Again, one realizes that the characteristic relation (22) *cannot* be respected at the compact reflecting surface $r = R_s$ of the star. We therefore conclude that spherically symmetric horizonless reflecting stars with Neumann boundary conditions *cannot* support static configurations made of massive scalar fields nonminimally coupled to gravity.

- [37] It is worth stressing the fact that the no-hair theorem presented in section IIIA for massive scalar fields nonminimally coupled to gravity in the physical regimes $\xi < 0$ and $\xi > \frac{1}{4}$ is valid for *generic* inner boundary conditions. In particular, this theorem rules out the existence of massive scalar hair with non-minimal coupling parameter in the dimensionless regimes $\xi < 0$ and $\xi > 1/4$ for both horizonless curved spacetimes describing compact regular stars and for black-hole spacetimes with absorbing event horizons.