

# Use of GEANE for tracking in Virtual Monte Carlo

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## Abstract.

The concept of Virtual Monte Carlo (VMC) allows to use different Monte Carlo programs to simulate particle physics detectors without changing the geometry definition and the detector response simulation. In this context, to study the reconstruction capabilities of a detector, the availability of a tool to extrapolate the track parameters and their associated errors due to magnetic field, straggling in energy loss and Coulomb multiple scattering plays a central role: GEANE is an old program written in Fortran 15 years ago that performs this task through dense materials and that is still successfully used by many modern experiments in its native form. Among its features there is the capability to read directly the geometry and the magnetic field map from the simulation and to use different track representations. In this work we have "rediscovered" GEANE in the context of the Virtual Monte Carlo: we will show how GEANE has been integrated in the FairROOT framework, firmly based on the VMC, by keeping the old features in the new ROOT geometry modeler. Moreover new features have been added to GEANE that allow one to use it also for low density materials, i.e. for gaseous detectors, and preliminary results will be shown and discussed. The tool is now used by the PANDA and CBM collaborations at GSI as the first step for the global reconstruction algorithms, based on a Kalman filter which is currently under development.

## 1. Track following in Virtual Monte Carlo

The concept of Virtual Monte Carlo (VMC) provides a unique tool to collect in a common ROOT-based framework the most popular Monte Carlo (MC) codes available today for the simulation of particle physics detectors, i.e. Geant3, Geant4 and Fluka. The advantage of this approach is the possibility to use different simulation packages without writing different versions of the geometry in different formats. This is extremely useful in the context of simulation, in particular for the validation of new MC routines by comparison with old and tested ones, but it can also be exploited in the context of reconstruction where the main task is track fitting, the first step of which is the so called 'track following'. With the words 'track following', one usually intends two main tasks:

- (i) the transport of the track parameters (particle momentum, position and direction) from one point to another in the apparatus, forward and backward;

- (ii) the propagation of the errors on the track parameters together with the mean values. This is usually obtained by calculating, step by step, the  $5 \times 5$  covariance matrix of the track. This mathematical part is analytically rather complicated: some general ideas are given in the current literature [1] but the detailed formulae have been published only very recently [2].

The Monte Carlo and track fitting tasks have been treated jointly by the CERN community in the nineties. The GEANT3 program was used for the point (i), that is for the determination of the track mean values. For the point (ii), the routines for the calculation and the transport of the error matrix, written by the CERN European Muon Collaboration (EMC) [2], were interfaced with the GEANT3 structure, giving rise to a Fortran package called GEANE [3, 4]. The great advantage of this approach is that the track following is automatically obtained *with exactly the same geometry of the Monte Carlo*, i.e. the GEANT3 geometry, without the necessity to write *ad hoc* codes.

The main purpose of this work is to preserve the old functionality of the GEANT3-GEANE programs in the framework of the Virtual Monte Carlo: we have reintroduced the possibility to call the GEANE functions into the main VMC class, TGeant3, so that now it is possible to access them from within ROOT. Moreover we have added new features to GEANE to allow its use also for low density materials, i.e. for gaseous detectors, and to build a Kalman filter for non-planar detectors like the PANDA Time Projection Chamber or Straw Tubes Tracker (STT) [5]. Finally we have written a C++ interface to Geane for the FairROOT framework, which is the simulation environment developed at GSI and used by the PANDA and CBM Collaborations (see the talk 156 by D. Bertini and talk 403 by S. Spataro at this conference). The main advantage of this work is the immediate possibility of using GEANE in reconstruction with the ROOT geometry modeler, i.e. with the same geometry used for the VMC simulation.

## 2. Track representations

The physical path of a particle of assigned mass  $m$  and momentum  $p$  is a six-fold entity of parameters  $x, y, z, p_x, p_y, p_z$ . The track is defined as a set of points in the detectors, corresponding to the physical path of a particle: the track points are obtained as the intersections of the particle path with detector planes, that can be real detector planes or ideal planes, in this case usually chosen perpendicularly to the particle direction. If we translate the Master Reference System (MARS) and make the  $xy$  plane to coincide with the detector plane, the  $z$ -coordinate is blocked: so, the track is an entity defined by five parameters.

Usually there are different choices of the five track parameters. The most common ones, that are also codified in the GEANE package, are (see Fig. 1) [3, 4]:

- the transverse SC system:

$$1/p, \lambda, \phi, y_{\perp}, z_{\perp}, \quad (1)$$

where  $\lambda$  and  $\phi$  are the dip and azimuthal angle in MARS and  $y_{\perp}$  and  $z_{\perp}$  are the coordinate of the trajectory in a frame with  $x_{\perp}$  along the particle direction and  $y_{\perp}$  parallel to the  $xy$  plane.

- the detector SD system:

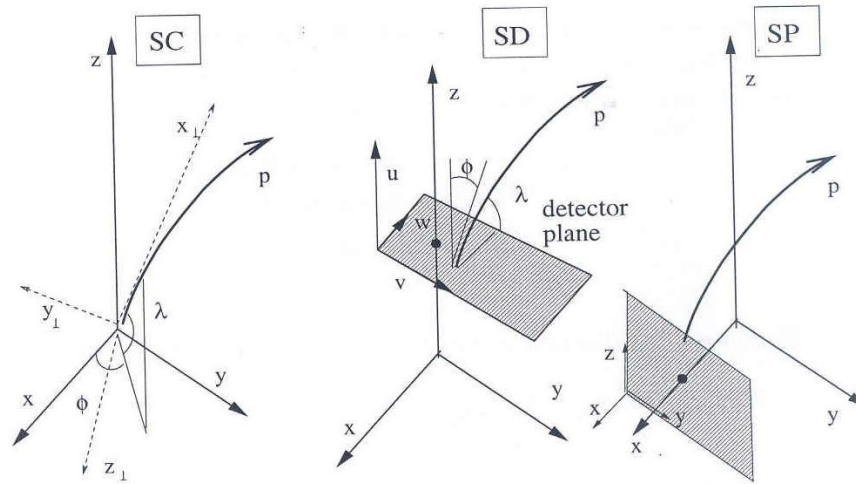
$$1/p, v', w', v, w, \quad (2)$$

where  $(u, v, w)$  is an orthonormal reference system with the  $vw$  plane coincident with the detector one. The derivatives indicate the momentum direction variation in the new system.

- the 'spline' SP system:

$$1/p, y', z', y, z. \quad (3)$$

This representation is used when the detector arrays are placed along the  $x$ -axis.



**Figure 1.** Systems of reference for the track following.

The most used representations are the SD and the SC ones: the SD representation follows the trajectory on any detector plane, real or ideal, while the SC representation gets the track parameters at any point along the track, at a given length or entering/exiting a given volume. The GEANE package can pass from one representation to another one through a set of routines [3]: this involves the calculation of the transport Jacobians between planes of arbitrary orientations and is not a trivial task, as explained in [2]. These routines are written in single precision and we have rewritten them in C++ using double precision variables since the problem of the calculation precision is crucial in track following [2].

### 3. New features of Geane

GEANE predicts the trajectory of a charged particle in terms of mean values and errors of the track parameters both in the forward and in the backward direction. Three physical effects are taken into account: Coulomb multiple scattering (which affects errors only), energy loss (which affects mean values and errors) and the magnetic field (which affects mean values only). In this work we have updated the original code with respect to the following items:

- new parametrization for the variance of the Coulomb multiple scattering angle;
- extension of the parametrization for the fluctuations of energy loss to the case of low density media, by including the Vavilov, Landau and sub-Landau regimes to the tracking routines;
- new option for track parameters extrapolation to the point of closest approach to a space point or to a wire.

In this section we will briefly motivate these changes and we will illustrate the new features.

#### 3.1. Coulomb multiple scattering parametrization

The two quantities of interest in the multiple scattering process are the displacement  $\Delta$  and the scattering angle  $\theta$ : we have considered here the latter for which the theory of Molière finds a statistical distribution in agreement with the experimental data. This distribution can be approximated as the sum of two Gaussians, a core one that takes into account the bulk process and a flat one that describes the tails [6].

The  $\theta_p$  total projected *variance* for a particle of momentum  $p$  (in GeV/c) that travels through an absorber of thickness  $d$  (in cm) is expressed as [6]:

$$\langle \theta_p^2 \rangle = \frac{225 \cdot 10^{-6}}{p^2} \frac{d}{\beta^2 X_s}, \quad X_s = X_0 \frac{Z+1}{Z} \frac{\ln(287 Z^{-1/2})}{\ln(159 Z^{-1/3})} \quad (4)$$

where  $X_0$  is the radiation length of the absorber.

The standard practice is to parametrize the core variance as [7]:

$$\langle \theta_p^2 \rangle = \frac{(0.0136)^2 d}{p^2 \beta^2 X_0} \left[ 1 + 0.038 \ln \left( \frac{d}{X_0} \right) \right]^2 \quad (5)$$

However, this variance should not be used in track following for two reasons:

- it is not the whole variance, so that the pull quantities show an underestimation of the multiple scattering errors;
- the thickness contained in the logarithmic term makes the calculation dependent on the tracking steps. Indeed, to make the calculation independent of the layers in which an absorber is divided, the variance should be directly proportional to the thickness  $d$ , as in (4).

Hence, we emphasize that the variance (4) should be used, which is the result of the accurate treatment of [6]. In this case, slight deviations from the gaussian form of the pull distribution have to be expected, since the angle distribution has non gaussian tails.

In GEANE the following formula is used:

$$\langle \theta_p^2 \rangle = \frac{184.96 \cdot 10^{-6}}{p^2} \frac{d}{\beta^2 X_0}. \quad (6)$$

We substituted this formula with eq. (4) in the `ermcsc.f` function of GEANE. This results in a slight improvement of the pull distribution for the dip angle since for most light scatterers, the ratio  $X_s/X_0 \simeq 1.15$  is near to the ratio  $225/185 = 1.21$ .

### 3.2. Energy loss straggling

The fluctuations in ionization for one particle of charge  $z$ , mass  $m$ , velocity  $\beta$ , are characterized by the parameter  $\kappa$ ,

$$\kappa = \frac{\xi}{E_{\max}} \quad (7)$$

which is proportional to the ratio of mean energy loss  $\xi$  to the maximum allowed energy transfer  $E_{\max}$  in a single collision with an atomic electron:

$$E_{\max} = \frac{2m_e \beta^2 \gamma^2}{1 + 2\gamma m_e/m + (m_e/m)^2} \quad (8)$$

where  $\gamma = 1/\sqrt{1-\beta^2} = E/m$  and  $m_e$  is the electron mass. The parameter  $\xi$  comes from the Rutherford scattering cross section and is defined as [7]:

$$\xi = 153.4 \frac{z^2 Z}{\beta^2 A} \rho d \quad (\text{keV}) \quad (9)$$

where  $\rho$ ,  $d$ ,  $Z$  and  $A$  are the density (g/cm<sup>3</sup>), thickness, atomic and mass number of the medium.

The parameter  $\kappa$  takes into account both the projectile energy and the geometrical thickness of the absorber; it defines univocally the absorber characteristics, that is the straggling conditions [8]:

$\lambda_{\max}$	$\alpha$	Mean	$\sigma_{\alpha}$
11.1	0.90	1.61	2.83
22.4	0.95	2.40	4.23
110.0	0.99	4.19	10.16
200.0	0.995	4.82	13.88
256.0	0.996	5.08	15.76
339.0	0.997	5.37	18.19
507.0	0.998	5.78	22.33
1007.0	0.999	6.48	31.59

**Table 1.** Result of the integration  $\alpha = \int_{\lambda_{\min}}^{\lambda_{\max}} f(\lambda)d\lambda$  of the Landau distribution from  $\lambda_{\min} \simeq -3.5$  to  $\lambda_{\max}$  of the table. The mean and the standard deviation of the truncated distribution are also shown. For this distribution the full mean and the variance are infinite, only the cumulative can be calculated.

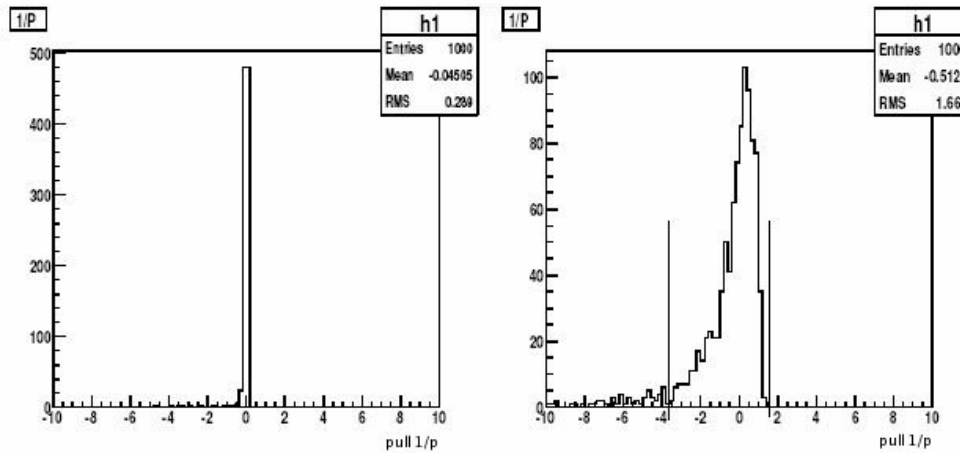
- 1 for heavy absorbers,  $\kappa > 10$  and the distribution is gaussian;
- 2 for moderate absorbers,  $0.01 < \kappa < 10$  and the distribution follows the function of Vavilov [8], that tends smoothly to the gaussian by increasing the thickness;
- 3 when  $\kappa < 0.01$ , we are in the presence of thin absorbers. When the number of collisions  $N_c > 50$ , the distribution follows the Landau function;
- 4 for very thin absorbers,  $N_c < 50$  (the condition  $\kappa \ll 0.01$  is implicitly fulfilled) there are no universal straggling functions, but only approximated models [9]. We will call this as the *sub-Landau* regime: it is the dominant in a gaseous detector.

For the cases 1. and 2. the straggling problem has a definite solution, both in simulation and in track following, because the general theory of the moments of the energy straggling distribution, based on the transport equation [10], allows one to calculate a finite variance. However, for the Landau distribution, that assumes  $E_{\max} = \infty$ , *both the average and the variance are infinite* [1, 11]. Since GEANE only treats the gaussian case, the results are completely unreliable for thin layers, where the Landau or sub-Landau conditions 3. and 4. often are verified. To remedy to this situation, we have updated the original code of GEANE with the procedure explained below, that has been inserted in the routine `erland.f`.

The main point is that for thin and very thin absorbers a rigorous solution exists for the simulation but not for track following. Indeed, whereas in the simulation the sampling and the tracking of the  $\delta$ -electrons reproduce correctly some rare effects in the detectors or the noise characteristics, in track following the long tail of the energy lost by the particle, due to the  $\delta$ -electron emission, makes the energy straggling variance infinite (for the Landau distribution [1, 11]) or so big (in sub-Landau models [9]) that the uncertainty in the track momentum is meaningless, because these fluctuations refer to 'enormous' energy losses occurring with very low probability. Since a universally accepted solution of this problem at present does not exist, we use some approximations based on truncated distributions.

In the Landau regime we cut the Landau distribution to an area  $\alpha$ , corresponding to a value  $\sigma_{\alpha}$  given by table 1: then for the Landau case we assume  $\sigma(E) = \xi \sigma_{\alpha}$ . For example, for  $\alpha = 0.95$  we have, from table 1,  $\sigma(E) = 4.23 \xi$ .

In the sub-Landau case we have the further difficulty due to the non existence of a straggling density in a closed analytical form [9]. In this case we decided to use a variance value obtained from the Urban model [12], which is one of the models used to sample the energy lost by the particle in very thin absorbers both in GEANT3 and GEANT4 [8]. The variance is calculated again by truncating the Urban distribution and considering a standard deviation representing a



**Figure 2.** Pull distribution  $\Delta(1/p)/\sigma$  for 1 GeV muons after passing through the PANDA straw tube detector. Left: Standard GEANE result ( $RMS \simeq 0.3$  in the displayed window); right: result after the modification with  $\alpha = 0.995$  (see the text). The region between the vertical lines has  $RMS = 1.03$ .

percentage  $\alpha$  of the area of the  $\delta$ -electron energy distribution: for values  $\alpha > 0.99$ , this variance goes smoothly towards the Landau value while increasing the absorber thickness.

Now we have to choose for  $\alpha$  the values that are useful for our track following: we note that the  $1/p$  pull spectra obtained with GEANE, in the case of thin absorbers, are composed of a peak, a shoulder near the peak and a very long and flat zone with very few events extending to the left (or the right) of the distribution. The area of the peak plus the shoulder is greater than the 99% of the total area. We choose  $\alpha$  to have a unitary RMS ( $\sigma \simeq 1$ ) for the peak plus the shoulder (see Fig. 2). Using this criterion, after many tracking tests through the PANDA apparatus, we found that a meaningful error propagation, taking into account the core of the distribution and excluding the long  $\delta$ -electron tail, is obtained with values  $0.995 \leq \alpha \leq 0.998$ .

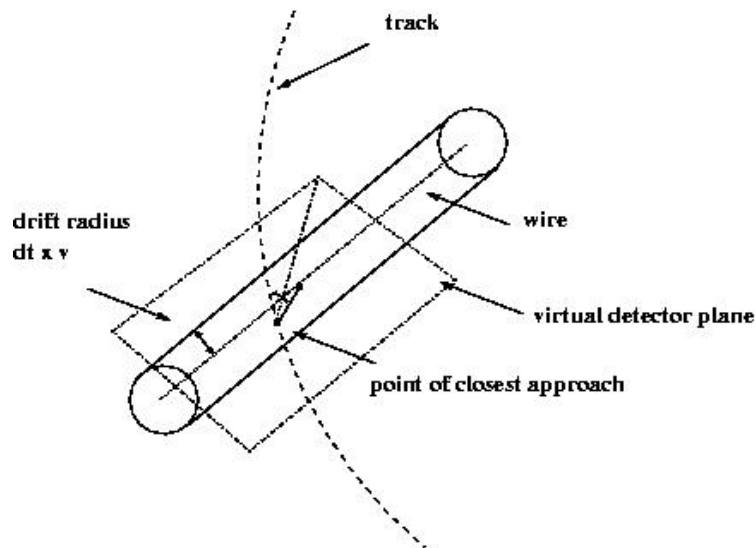
In the new GEANE version, the parameter  $\alpha$  is under the control of the user: if one sets  $\alpha = 1$  the run uses the old standard GEANE for backward compatibility. It is important to note that the treatment of the uncertainties due to the energy loss reported here is somewhat arbitrary and based on approximations: therefore, it has to be tuned, carefully checked and progressively improved in the different experimental conditions.

### 3.3. Extrapolation to the point of closest approach

The third feature added to GEANE is a new extrapolation option which extends the extrapolation to a given length along the track. The idea is to give the user the possibility to extrapolate to the Point of Closest Approach (PCA), along the track, to a given space point or to a given segment: this may represent a hit in a TPC or a wire in a gaseous detector, like the PANDA Straw Tubes Tracker (STT).

The purpose is to use GEANE as a track follower in a global reconstruction algorithm, like for instance in a Kalman filter. In a planar tracking detector, the track is first extrapolated into the detector plane, then it is updated by taking the weighted mean between the position of the hit before the update and the measurement point of the detector. The weights for this mean are given by the inverse spatial resolution matrix of the detector and by the inverse covariance matrix of the track. In a non-planar detectors there are not reference planes and the way of treating this problem is to calculate the PCA between the extrapolated track and the space

point measured by a TPC or the position of the wire of a straw tube which fired. The plane perpendicular to the track at this point is called the 'virtual detector plane'. It is in this virtual detector plane that one can make use of the track and the position measurement to calculate refined track parameters with the weighted mean technique.



**Figure 3.** Point of closest approach of track and straw tube wire. The measurement of the straw tube defines that the particle traversed somewhere on the drift cylinder.

The PCA finding algorithms that we developed make use, at each step of the tracking, of the last 3 extrapolations of the track parameters. This works as a two phases process, due to the stepping of GEANE: first, in the `eustep.f` function, an approximation of the PCA is found which is bound to the size of the GEANE steps, then, in the new interface, the exact PCA is calculated by using these last 3 points with the use of different algorithms, since 2 of the 3 extrapolations may coincide. Great care must be taken in choosing the tracking step size depending on the medium.

#### 4. New C++ interface to Geane for FairROOT

Taking advantage of the structure of the VMC and of its integration in ROOT, GEANE can be easily put to work and used also in modern C++ simulations with the ROOT geometry navigation tools. To allow this we have made some changes to the code at three different levels:

- inclusion of the new features described in the previous section: this has been done at the level of the FORTRAN code and consisted in updates to the functions `ermcsc.f`, `erland.f` and `eustep.f` and in the addition of a new common block, called `GCMORE`, for storing the new parameters like the  $\alpha$  cut value and the results of the PCA extrapolation.
- modification of the main VMC class, the `TGeant3` class: here we have reintroduced all the GEANE common blocks as C structures and C++ wrapper functions to the standard GEANE routines, to which we have added methods to access data in the new common `GCMORE`.
- development of a new C++ user interface to GEANE for use with the FairROOT framework: we have designed new classes that have been collected in two packages of the framework, `trackbase` and `geane`. The package `trackbase` contains classes for track representations in

the SC and SD systems and a utility class that is used to transform between representations. The package `geane` is used to initialize the use of GEANE and to handle the routines to find the PCA for all the possible cases.

It is not mandatory to work within the FairROOT framework to use GEANE in the VMC: the first 2 points of the update list allow to use GEANE with the standard user interface and with access to the new features. The FairROOT interface is an optional way to use GEANE in a more easier and rationalized way, but it also contains the new methods to handle the extrapolation to the PCA. The code for the first 2 points is available in the VMC CVS repository at CERN and allows the use of GEANE via the old interface with C++ wrappers to the FORTRAN functions. The C++ interface instead is available in the FairROOT SVN repository at GSI and is currently used by the PANDA and CBM collaborations for the global reconstruction.

## 5. Results for the PANDA Straw Tubes Tracker

This new code is currently used in the design of a tracking Kalman filter for the PANDA detector: we present here some results obtained in this work.

The first result is a check of the GEANE performances with a set of simulated data with the PANDA STT geometry. The multiple scattering has been simulated with the Molière distribution. The pull or standard variables are  $T = \frac{(x_{\text{GEANE}} - x_{\text{MC}})}{\sigma_{\text{GEANE}}}$  where  $x = [1/p, \lambda, \phi, y, z]$  in the SC system or  $x = [1/p, v', w', v, w]$  in the SD system for the 5 track parameters are shown in the case of 1 GeV muons: in the first case for each track the extrapolation has been performed from the origin to the volume of a detector plane external to the apparatus, while for the second from the inner layer of the STT to the PCA to the tube that fired in the outer layer.

The shape of the distributions, apart from that of  $1/p$ , is near to the standard gaussian with  $\sigma[T] \simeq 1$ , as expected: it must be stressed that this is obtained in the sub-Landau regime using the Urban model.

The second result comes from the use of GEANE as first step in a Kalman filter algorithm developed for the local track reconstruction in the PANDA STT: the results are compared with the reconstruction obtained with a traditional fit based on the conformal mapping method [13] and show an improvement of the momentum resolution of  $\approx 30\%$ . Conforted by this results, we are now working with the PANDA computing group on a global Kalman filter for the whole PANDA apparatus called `genfit` (generic fitter).

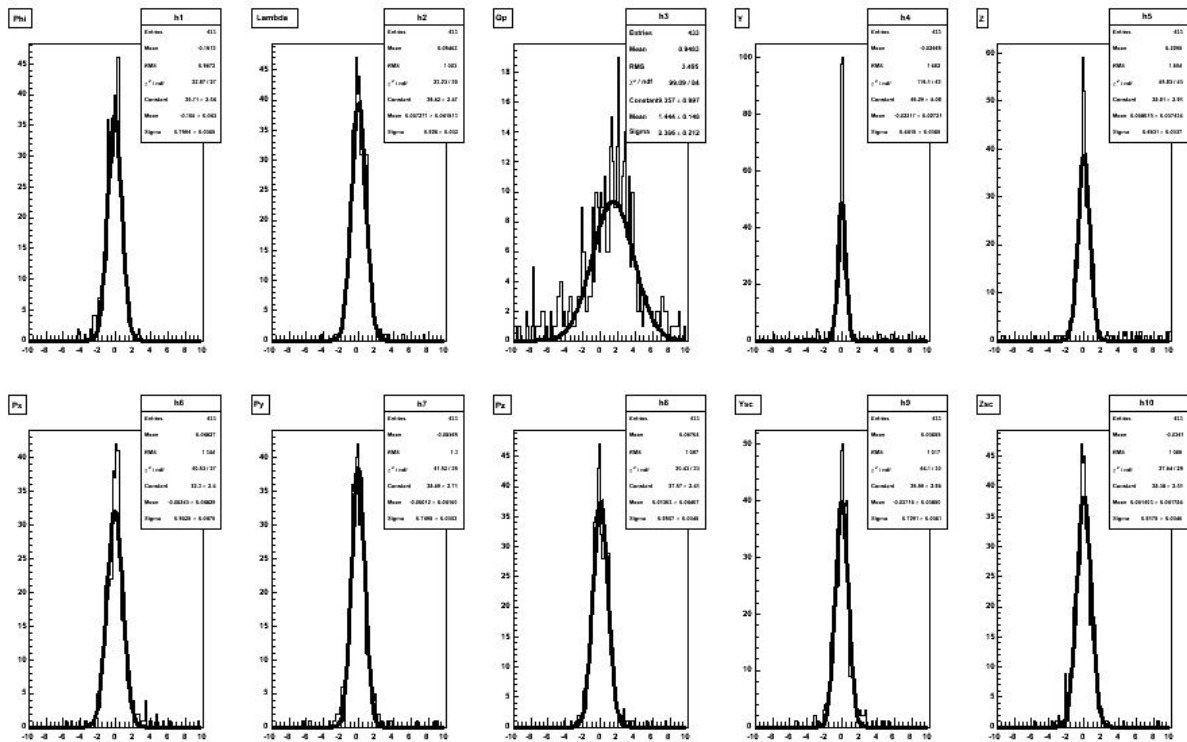
Similar application of this tool are also being developed for the global tracking by the CBM and HADES collaborations.

## 6. Conclusions

We have restored and improved the old program GEANE and we have integrated it with the VMC classes to allow its use with the ROOT geometry modeler. We have added new features and simplified its use via a new C++ interface for the FairROOT framework. This was done to preserve the knowledge about track following from the experiments of the nineties and to allow the new experiments to make use of it.

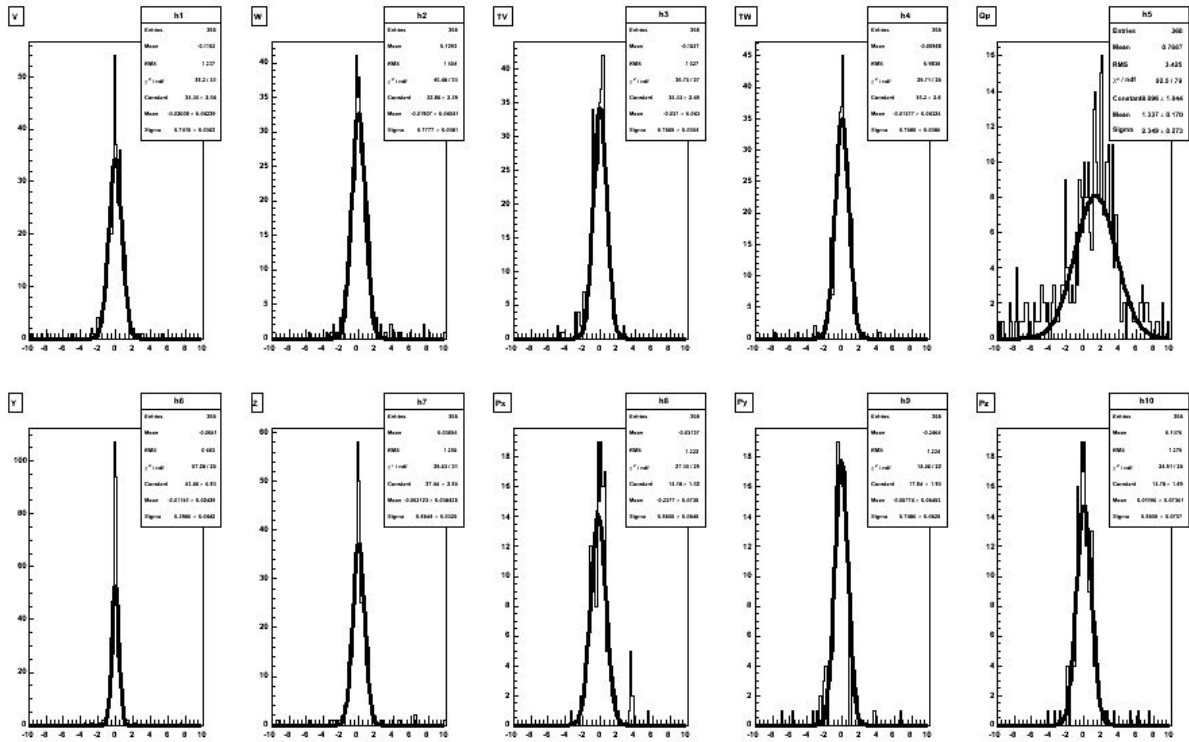
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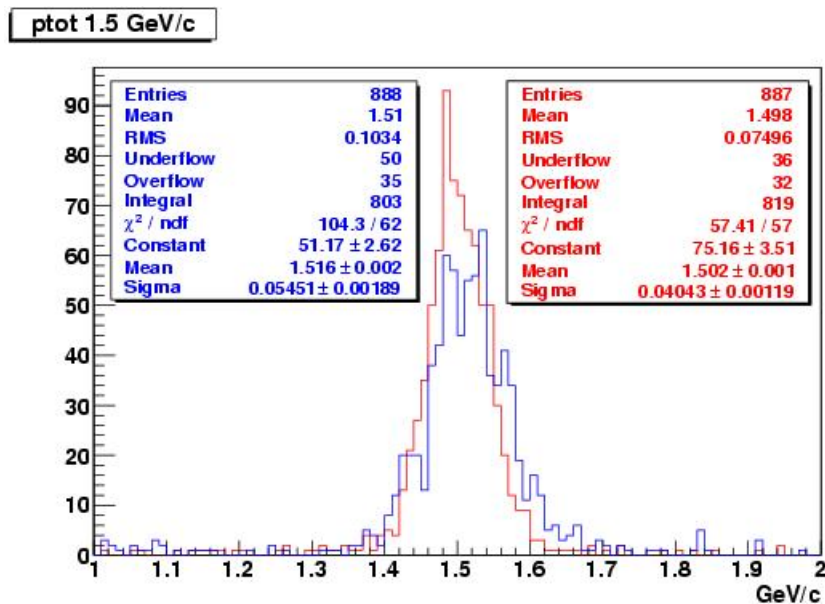


**Figure 4.** Pulls of the 5 track parameters in the SC representation, of the 3 momentum components in MARS and of the coordinates  $y$  and  $z$  in the SC system for a 1 GeV muon crossing the PANDA STT detector from the origin to a detector plane external to the apparatus.

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**Figure 5.** Pulls of the 5 track parameters in the SD representation, of the 3 momentum components in MARS and of the coordinates  $u$  and  $v$  in the SD system for a 1 GeV muon crossing the PANDA STT detector from the inner to the outer layer of the STT.



**Figure 6.** Comparison between the total momentum reconstructed with the PANDA STT using a track fit based on the conformal mapping method (blue line) and the total momentum reconstructed using a GEANE based Kalman filter (red line) for 1.5 GeV/c muons.