Supersymmetry at the Dawn of the LHC Era

by

Zachary Talbott Thomas

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Abstract

In this thesis, I explore various experimental, theoretical, and observational consequences of supersymmetry (SUSY). I show how copious production of Higgs bosons in SUSY events at the LHC can be a striking signal of multiple SUSY-breaking. In the context of anomaly mediation in supergravity, I demonstrate how goldstino couplings can be used as a probe of the underlying symmetry structure of unbroken SUSY in anti-de Sitter space. When mutiple SUSY-breaking occurs and goldstini comprise most of the dark matter in the universe, I find a new two-body decay mode of a goldstini to a gravitino and a single photon that could be a striking indirect detection of dark matter if it were seen at gamma-ray telescopes such as FERMI.

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Chapter 1

Introduction

My studies here at MIT have coincided with one of the most exciting periods in modern particle physics—the opening and first major operational period of the CERN Large Hadron Collider (LHC). The first collisions took place in September 2008, barely a month after my matriculation, and the first running period began in 2010 and continued until 2013. The largest achievement of the first running period came in July 2012 with the discovery of the Higgs boson, the last heretofore-undiscovered particle in the Standard Model. Operating at center-of-mass energies of 7 and 8 TeV (with an upgrade to at least 13 TeV due for completion in 2015), the LHC has been pushing the high energy frontier, and continues to be well-poised to discover any Beyond the Standard Model (BSM) physics that may exist at the TeV scale.

A well-motivated and extensively-studied model of such BSM physics is supersymmetry (SUSY). In this thesis, I will discuss a number of phenomenological and theoretical surprises of supersymmetry as it may be realized in nature. The only allowed non-trivial extension of the Poincaré group, supersymmetry is a fermionic symmetry that transforms bosons into fermions and vice versa. For every fermion (boson) in the Standard Model, it predicts a partner boson (fermion) with the same gauge quantum numbers and, for unbroken supersymmetry, the same mass. As such superpartners have yet to be discovered, if supersymmetry is realized in nature it must be spontaneously broken, with (almost) all of the superpartners acquiring masses of at least the weak scale. In Ch. 2 (based in part on the TASI lectures of Ref. [21]), I will review the basics of supersymmetry necessary for the remainder of the thesis.

Supersymmetry provides a particularly appealing solution to the hierarchy problem, the puzzle of why the Higgs boson mass (and its vacuum expectation value) are so small compared to other expected scales in high-energy physics when it generically receives radiative corrections to its mass-squared proportional to the square of such scales. Supersymmetry protects scalar masses from such quadratic corrections, as supersymmetry relates the renormalization of bosons to that of fermions, whose masses are in turn protected by chiral symmetries. As a result, if supersymmetry is to resolve the hierarchy problem, one expects to find superpartners near or not too much above the weak scale, which is precisely the range that the LHC is currently probing. Already after its first full run, the ATLAS and CMS collaborations at the LHC have managed to place impressive bounds on various superpartner masses, ranging upwards of a TeV for the colored squarks and gluinos.

Supersymmetric theories also often provide a good particle candidate for the dark matter which comprises nearly a quarter of the energy density of the universe. Viable SUSY theories usually require the imposition of a discrete symmetry called R-parity, in order to avoid dangerous lepton- and baryon-violating couplings that would mediate unacceptably short proton decay lifetimes in contradiction to experiment. Such a symmetry mandates that superpartners can only be created or destroyed in pairs, which implies that the lightest supersymmetric particle (the LSP) will be absolutely stable, and as such is a good dark matter candidate if it is electrically neutral and a color singlet. This is especially true if the LSP is the superpartner of a Standard Model particle (such as the Z or the Higgs), in which case it would be expected to have a weak-scale mass and interaction cross section, and would thus be produced with the correct abundance in the early universe (the so-called 'WIMP miracle'). At the LHC, this would imply that collisions can only produces SUSY particles in pairs, each of which would decay (perhaps in a cascade) to the invisible LSP, resulting in a signal with many Standard Model particles and considerable missing transverse energy.

The breaking of SUSY necessarily features particles that couple only very weakly to the Standard Model—through heavy messengers and/or non-renormalizable interactions. If this were not the case, supertrace sum rules would imply that at least one scalar with electric or color charge would be considerably lighter than the corresponding fermions, in contradiction with experiment. As a result, SUSY-breaking is, in the usual paradigm, said to take place in a hidden sector. As particles in the hidden sector couple so weakly to the supersymmetric Standard Model (SSM), their interactions are usually irrelevant to physics at LHC scales—unless one of them is lighter than all the SSM superpartners, in which case the lightest observable-sector SUSY particle (LOSP) would be expected to decay to such a hidden sector particle.

The best-motivated candidate for such a light hidden sector SUSY state is the goldstino—a particle whose presence figuratively permeates all the work in this thesis.[1] As with any other spontaneously broken symmetry, spontaneously broken SUSY will have a massless Goldstone mode in its spectrum that obeys a shift symmetry. As SUSY is a fermionic symmetry, the Goldstone mode is the fermionic *goldstino*. The goldstino necessarily couples derivatively to the Noether current of supersymmetry—that is, to a fermion and its superpartner boson, with a strength that can be readily shown to be proportional to the partners' mass(-squared) difference (itself a measure of SUSY-breaking). If the goldstino is lighter than the LOSP (as is guaranteed for global SUSY, where it is always massless), one expects LHC SUSY events to always feature two cascade decays ending with the decay of a LOSP to its SM partner and an invisible goldstino. For example, a bino LOSP (the superpartner of the $U(1)$ hypercharge gauge boson) would predominantly decay into a single photon or Z boson and a goldstino, yielding signals at the LHC that would include photons and missing energy.

If SUSY is independently broken in multiple hidden sectors, then each sector will have its own goldstino. One linear combination of these goldstini will be the true goldstino, which couples derivatively to the supercurrent, but the other goldstini may have strikingly different couplings in general. As a result, LOSP decays to these other goldstini may result in unconventional LHC phenomenology. In Ch. 3, based on work in Ref. [146], I will discuss a scenario in which a bino LOSP would primarily decay to a Higgs boson and one of the goldstini, contrary to the usual expectation for a single hidden sector. This would result in copious production of potentially boosted Higgses in LHC SUSY events, which would give us considerable insight into the structure of both SUSY breaking and the Higgs sector.

If SUSY is a local symmetry, the resulting gauge degree of freedom is a spin-3/2 fermion—the gravitino, the superpartner of the graviton. When this supergravity (SUGRA) is broken, the gravitino acquires a mass $m_{3/2}$, and it eats the goldstino degrees of freedom. This is called the super-Higgs mechanism, by analogy with the Higgs mechanism for ordinary bosonic internal symmetries in which gauge bosons acquire mass by eating goldstone

[1] Literally, too, should it comprise the dark matter of the universe.

bosons. At energies well above $m_{3/2}$, the interactions of the longitudinal polarizations of the gravitino are well described by those of the goldstino—a goldstino equivalence theorem much like the goldstone equivalence theorem for internal symmetries.

In local SUSY, a ubiquitous cause of boson-fermion mass differences is anomaly mediation, which occurs when SUSY-breaking is communicated to the visible sector at loop level by the regulators of the theory. In order to elucidate this theory, which has been the subject of much confusion in the literature, in Ch. 4, drawing from work in Ref. [53], I approach anomaly mediated gaugino masses from the point of view of (eaten) goldstino couplings. I find that there are two fundamentally different 'faces' of anomaly mediation Kähler mediation, in which gaugino masses and the corresponding goldstino couplings are identical (as expected), and gravitino mediation, in which gaugino masses occur without corresponding goldstino couplings. This quite surprising result suggests that anomaly mediation is not a SUSY-breaking effect.

In Ch. 5, based on Ref. [54], I show explicitly that anomaly mediation does not break SUSY, as it exists for unbroken SUGRA, whose background metric is that of anti-de Sitter (AdS) space. In AdS space, spacetime translations and supersymmetry transformations no longer commute, and there can be boson-fermion mass differences proportional to $r_{\mathrm{AdS}}^{-1} = m_{3/2}$ even in the absence of SUSY breaking. This is clearest for scalar masses and B-terms at tree level—there are both B-terms without corresponding goldstino couplings, and goldstino couplings without corresponding scalar masses for SUGRA in flat space. I find the usual anomaly-mediated effects arise starting at one- or two-loop level by carefully considering a 1PI effective action—the running of couplings in AdS SUSY are necessarily associated with corresponding boson-fermion mass differences, which are preserved when SUSY-breaking uplifts the background metric to flat space. As a result, anomaly-mediated effects, as they do not break SUSY, do not have associated goldstino couplings, while there are additional loop-level goldstino couplings without associated mass differences arising from the uplifting to flat space caused by SUSY breaking.

In models with multiple SUSY-breaking, one of the corresponding goldstini can easily comprise most of the dark matter in the universe. Such dark matter will not be absolutely stable, however, as the uneaten goldstini can decay to the gravitino on cosmological timescales. In Ch. 6, drawing off Ref. [126], I discuss a striking two-body decay mode that can occur in the presence of electroweak symmetry breaking—that of an uneaten goldstino to a single photon and a gravitino. The resulting signal would be a monochromatic gamma (or X-) ray line visible at telescopes such as FERMI, an impressive indirect detection of dark matter. I find that such a mode can generically be the first sign of such dark matter for goldstini masses at or below the TeV scale.

Chapter 2

Supersymmetry

Supersymmetry (SUSY) is a well-motivated extension of the Standard Model (SM), with rich implications for collider physics and cosmology. In this chapter, I will introduce SUSY in 3+1 dimensions using the language of superspace (in both Minkowski and anti-de Sitter spacetimes), and briefly discuss some of the more prominent phenomenological ramifications of both global and local SUSY. Certain portions of this chapter draw heavily from parts of Ref. [21].

First, a brief note on notation—in this thesis I will use exclusively two-component spinor notation, also known as Weyl spinors, largely following the conventions of Ref. [154].[1] While it is possible to do superspace manipulations using four-component notation (as in Refs. [152, 73]), Weyl spinors are far more convenient, since they are true irreducible representations of the Lorentz group.

2.1 Superspace

SUSY relates the properties of bosons and fermions, but in ordinary relativistic quantum field theory, bosons and fermions are represented by very different objects. For example, a spin-0 boson is described by a complex-valued scalar field $\phi(x)$, while a spin-1/2 fermion is described by a Grassmann-valued Weyl field $\psi_\alpha(x)$ (with a Lorentz spinor index, no less). In order to make SUSY manifest, we want to somehow package bosons and fermions into a single object.

[1] We differ in that we use daggers instead of bars to denote the hermitian conjugates of spinors, and our gauginos are normalized such that $\boldsymbol{W}_\alpha| = \lambda_\alpha$.

To do so, we introduce the technique of superspace, which augments the ordinary space-time coordinates with an additional Grassmann spinor θ^α (and its complex conjugate $\theta^{\dagger\dot{\alpha}}$):

$$x^\mu \to \{x^\mu, \theta^\alpha, \theta^{\dagger\dot{\alpha}}\}. \tag{2.1}$$

A field that depends on $\{x^\mu, \theta^\alpha, \theta^{\dagger\dot{\alpha}}\}$ is called a superfield, which automatically packages boson and fermion fields into multiplets. While it is possible to describe SUSY theories using ordinary space-time alone, superspace makes it simpler to identify SUSY-invariants and write SUSY Lagrangians.

A generic scalar supermultiplet is

$$\boldsymbol{S}(x^\mu, \theta^\alpha, \theta^{\dagger\dot{\alpha}}), \tag{2.2}$$

which depends on the Grassmann spinor placeholders/coordinates θ^α. Throughout this thesis we will use boldface letters to indicate superfields. This object is an overall Lorentz scalar, but it contains spin-0, spin-1/2, and spin-1 components. Because of the Grassmann nature of our placeholders, the Taylor expansion is exact:

$$\begin{aligned} \boldsymbol{S} = \quad & a && + \ \theta\xi && + \ \theta^2 b \\ + \ & \theta^\dagger\chi^\dagger && + \ \theta^\dagger\overline{\sigma}^\mu\theta v_\mu && + \ \theta^2\theta^\dagger\zeta^\dagger \\ + \ & \theta^{\dagger 2} c && + \ \theta^{\dagger 2}\theta\eta && + \ \theta^4 d. \end{aligned} \tag{2.3}$$

Invariance of the action under shifts in x^μ corresponds to translation invariance, an important subset of the full Poincaré space-time symmetry. It is natural to explore what corresponds to translation invariance in superspace under the shift

$$\theta^\alpha \to \theta^\alpha + \epsilon^\alpha, \tag{2.4}$$

where ϵ^α is an infinitesimal Grassmann parameter. This (passive) transformation of the coordinate can be interpreted instead as an (active) transformation of the components. For example, starting with

$$\boldsymbol{\Phi}(\theta^\alpha) = \phi + \theta^\alpha\psi_\alpha + \ldots, \tag{2.5}$$

translations yield

$$\mathbf{\Phi}(\theta^\alpha + \epsilon^\alpha) = \phi + (\theta^\alpha + \epsilon^\alpha)\psi_\alpha + \ldots \tag{2.6}$$

$$= (\phi + \epsilon^\alpha \psi_\alpha) + \theta^\alpha \psi_\alpha + \ldots, \tag{2.7}$$

so the components transform as $\phi \to \phi + \epsilon^\alpha \psi_\alpha$ (with ψ_α left fixed). As desired, we have related bosons to fermions! However, we know that boson and fermion kinetic terms have differing numbers of derivatives, so in order to successfully build a SUSY Lagrangian, we must somehow combine θ^α translations with space-time derivatives.

The key to SUSY is that the shift of the fermionic coordinate θ^α is accompanied by a translation of the ordinary bosonic coordinate x_μ as well

$$\begin{aligned} \theta^\alpha &\to \theta^\alpha + \epsilon^\alpha, \\ \theta^{\dagger\dot\alpha} &\to \theta^{\dagger\dot\alpha} + \epsilon^{\dagger\dot\alpha}, \\ x^\mu &\to x^\mu + \Delta^\mu, \end{aligned} \tag{2.8}$$

where

$$\Delta^\mu \equiv -i\epsilon\sigma^\mu\theta^\dagger - i\epsilon^\dagger\overline{\sigma}^\mu\theta. \tag{2.9}$$

We could have guessed the form of Δ^μ, since this is the unique real four-vector object one can build that is linear in ϵ and has the right dimension.[2]

Let us now act this SUSY coordinate shift on a generic supermultiplet $\boldsymbol{S}$:

$$\begin{aligned} \boldsymbol{S}(x^\mu, \theta^\alpha, \theta^{\dagger\dot\alpha}) \to\ & \boldsymbol{S}(x^\mu + \Delta x^\mu, \theta^\alpha + \epsilon^\alpha, \theta^{\dagger\dot\alpha} + \epsilon^{\dagger\dot\alpha}) \\ &= \boldsymbol{S}(x^\mu, \theta^\alpha, \theta^{\dagger\dot\alpha}) \\ &\quad + \left(\Delta^\mu\partial_\mu + \epsilon^\alpha\partial_\alpha + \epsilon^\dagger_{\dot\alpha}\partial^{\dagger\dot\alpha}\right) \boldsymbol{S}(x^\mu, \theta^\alpha, \theta^{\dagger\dot\alpha}), \end{aligned} \tag{2.10}$$

where we have used the Taylor expansion up to the first order, both for ordinary and Grassmann coordinates. Here, we are using the notation $\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha}$ and $\partial^{\dagger\dot\alpha} \equiv \frac{\partial}{\partial\theta^\dagger_{\dot\alpha}}$.

We see immediately that translations in superspace act in two different ways. First, the shift $\theta^\alpha \to \theta^\alpha + \epsilon^\alpha$ relates higher components of the superfield to lower components as in Eq. (2.6). Second, because Δ^μ contains factors of θ^α, the translation $x^\mu \to x^\mu + \Delta^\mu$

[2] Note that from Eq. (2.5), θ (and thus ϵ) must have mass dimension $[\theta] = -1/2$.

relates *derivatives* of lower components to higher components. This is crucial for relating the kinetic terms for bosons and fermions.

2.2 The SUSY Algebra

Thus far, we have talked about SUSY transformations without ever mentioning the underlying SUSY algebra. Indeed, one advantage of superspace is that Eq. (2.8) contains the full structure of SUSY. However, it is instructive to turn the superspace translation picture into an operator picture to show that the SUSY algebra closes.

Recall that ordinary space-time translations are generated by the energy-momentum operator

$$e^{ia_\mu P^\mu} f(x^\mu) = f(x^\mu + a^\mu), \tag{2.11}$$

where

$$P_\mu \equiv -i\partial_\mu. \tag{2.12}$$

Translations are part of the full Poincaré group that includes Lorentz transformations generated by $M_{\mu\nu}$, with algebra

$$[M_{\mu\nu}, M_{\rho\tau}] = -i\left(\eta_{\nu\rho}M_{\mu\tau} + \eta_{\mu\tau}M_{\nu\rho} - \eta_{\mu\rho}M_{\nu\tau} - \eta_{\nu\tau}M_{\mu\rho}\right), \tag{2.13}$$

$$[P_\mu, M_{\nu\rho}] = -i\left(\eta_{\mu\nu}P_\rho - \eta_{\mu\rho}P_\nu\right), \tag{2.14}$$

$$[P_\mu, P_\nu] = 0. \tag{2.15}$$

Note that the explicit expression of $M_{\mu\nu}$ depends on the spin of the field it acts on. For a scalar field, for example,

$$M_{\mu\nu} = -i\left(x_\mu\partial_\nu - x_\nu\partial_\mu\right). \tag{2.16}$$

We want to introduce new SUSY generators that implement Eq. (2.10), namely operators Q and $Q^\dagger$ such that

$$e^{-i\epsilon Q - i\epsilon^\dagger Q^\dagger}\boldsymbol{S}(x^\mu, \theta^\alpha, \theta^{\dagger\dot\alpha}) = \boldsymbol{S}(x^\mu + \Delta^\mu, \theta^\alpha + \epsilon^\alpha, \theta^{\dagger\dot\alpha} + \epsilon^{\dagger\dot\alpha}). \tag{2.17}$$

In analogy with Eq. (2.11), we see immediately that

$$Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} + (\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \tag{2.18}$$

$$Q^{\dagger\dot{\alpha}} = i\frac{\partial}{\partial\theta^\dagger_{\dot{\alpha}}} + (\overline{\sigma}^\mu\theta)^{\dot{\alpha}}\partial_\mu. \tag{2.19}$$

It is straightforward to show that these generators satisfy the SUSY algebra

$$\begin{aligned}
\{Q_\alpha, Q^\dagger_{\dot{\beta}}\} &= 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu, \\
\{Q_\alpha, Q_\beta\} &= \{Q^\dagger_{\dot{\alpha}}, Q^\dagger_{\dot{\beta}}\} = 0, \\
[Q_\alpha, P_\mu] &= \left[Q^\dagger_{\dot{\alpha}}, P_\mu\right] = 0, \\
[M_{\mu\nu}, Q_\alpha] &= -i\sigma_{\mu\nu\alpha}{}^\beta Q_\beta, \\
\left[M_{\mu\nu}, Q^{\dagger\dot{\alpha}}\right] &= -i\overline{\sigma}_{\mu\nu}{}^{\dot{\alpha}}{}_{\dot{\beta}}Q^{\dagger\dot{\beta}},
\end{aligned} \tag{2.20}$$

thus extending the Poincaré algebra. In this way, two SUSY translations are equivalent to one ordinary space-time translation, and we sometimes refer to SUSY as being the "square root" of translations. The non-trivial commutator between SUSY and Lorentz generators just indicates that the SUSY generator is a Lorentz spinor. The SUSY algebra indeed closes, and accounting for the possibility of higher $\mathcal{N}$, one can show that SUSY is the unique extension of the Poincaré algebra.[89]

The SUSY algebra allows us to write the Hamiltonian $H = P^0$ as a sum of squares of SUSY generators:

$$H = \frac{1}{4}(Q_1^\dagger Q_1 + Q_1 Q_1^\dagger + Q_2^\dagger Q_2 + Q_2 Q_2^\dagger). \tag{2.21}$$

If SUSY is unbroken (i.e. it is a symmetry of the vacuum, $Q^\alpha|0\rangle = 0$), then

$$\langle H\rangle = \langle 0| H |0\rangle = 0 \quad \text{(SUSY vacuum)}, \tag{2.22}$$

so the vacuum energy is zero. The converse is also true, such that a zero vacuum energy implies $Q^\alpha|0\rangle = 0$ and SUSY is unbroken. In contrast, if the vacuum energy is non-zero, then SUSY is spontaneously broken in the vacuum. In fact, because each term in Eq. (2.21) is an operator squared,

$$\langle H\rangle > 0 \quad \text{(SUSY-breaking vacuum)}, \tag{2.23}$$

so spontaneous SUSY breaking corresponds to a strictly positive vacuum energy. It should be stressed that this statement only holds for global SUSY in flat space, in which gravity is not dynamical; SUSY-breaking is indeed compatible with the (nearly) vanishing cosmological constant in our universe.

As Q and $Q^\dagger$ commute with P^2, we can immediately make a very powerful statement about unbroken SUSY—every component of a superfield satisfies the same Klein-Gordon equation and therefore has the same mass. The fact that boson and fermion masses are related by SUSY has extremely important implications for the "hierarchy problem" of the Standard Model—the question as to why the Higgs boson mass-squared is so small compared to any putative higher energy scales in physics (the Planck scale, if nothing else) as it receives quantum corrections quadratic in those scales. SUSY protects scalar mass-squareds by relating them to fermion masses, which are themselves protected by chiral symmetries. Of course, there is no 511 keV bosonic partner of the electron, so if SUSY is realized in nature, it must be spontaneously broken. However, even in spontaneously broken SUSY, scalar mass-squareds receive at most corrections logarithmic in new energy scales. The coefficients of such logarithms are generically scalar mass-squareds themselves—so for a natural theory, one expects the mass scale of superpartners to be not that far above the weak scale, exactly the scale that the LHC is probing.

One can augment the SUSY algebra with an additional $U(1)_R$ symmetry that does not commute with Q:

$$[Q, R] = -Q \qquad\qquad [Q^\dagger, R] = Q. \tag{2.24}$$

Effectively, this gives a charge of $+1$ to θ and a charge of -1 to $\theta^\dagger$, so different components of a superfield will have different R-charges.

2.3 SUSY-Covariant Derivatives and Chiral Multiplets

Since superspace includes both ordinary spacetime coordinates and the new Grassman coordinates, it is natural to consider derivatives with respect to these new coordinates. The most obvious choice would be $\frac{\partial}{\partial\theta_\alpha}\boldsymbol{S}$, but note that it does not commute with SUSY transformations, rendering its use very impractical. We can, however, construct SUSY-covariant derivatives

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta_\alpha} + i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu, \tag{2.25}$$

$$\mathcal{D}^{\dagger\dot\alpha} = \frac{\partial}{\partial\theta^{\dagger\dot\alpha}} + i(\overline{\sigma}^\mu\theta)^{\dot\alpha}\partial_\mu. \tag{2.26}$$

that do commute with Q_α and $Q^\dagger_{\dot\beta}$ as desired. As a result, SUSY-covariant derivatives of superfields transform as sensible superfields. Note also that

$$\{\mathcal{D}_\alpha, \mathcal{D}^\dagger_{\dot\beta}\} = -2i\sigma^\mu_{\alpha\dot\beta}\partial_\mu. \tag{2.27}$$

Like any sensible derivative, these obey a Leibniz (product) rule

$$\mathcal{D}_\alpha(\boldsymbol{XY}) = (\mathcal{D}_\alpha\boldsymbol{X})\boldsymbol{Y} + \boldsymbol{X}(\mathcal{D}_\alpha\boldsymbol{Y}). \tag{2.28}$$

There is one subtlety, however, because the $\mathcal{D}$s pick up a minus sign if you move them across an odd number of spinor indices:

$$\mathcal{D}_\alpha(\boldsymbol{X}_\beta\boldsymbol{Y}) = (\mathcal{D}_\alpha\boldsymbol{X}_\beta)\boldsymbol{Y} - \boldsymbol{X}_\beta(\mathcal{D}_\alpha\boldsymbol{Y}). \tag{2.29}$$

Note that $\mathcal{D}^3\boldsymbol{S} = \mathcal{D}^{\dagger 3}\boldsymbol{S} = 0$, because $\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0$.[3] Like the Qs, the $\mathcal{D}$s also commute with ∂_μ.

These SUSY-covariant derivatives are useful in a large variety of circumstances. We can use them to constrain superfields. For example, we define a chiral superfield $\boldsymbol{\Phi}$ by the constraint

$$\mathcal{D}^{\dagger\dot\alpha}\boldsymbol{\Phi} = 0. \tag{2.30}$$

Similarly, an anti-chiral superfield $\boldsymbol{\Phi}^\dagger$ satisfies $\mathcal{D}_\alpha\boldsymbol{\Phi} = 0$; they are hermitian conjugates of each other. We can also use SUSY-covariant derivatives to construct chiral superfields; $\mathcal{D}^{\dagger 2}\boldsymbol{V}$ is automatically chiral for any $\boldsymbol{V}$ as $\mathcal{D}^{\dagger 3} = 0$.

SUSY-covariant derivatives can also be used to extract components of a superfield. For

[3] This fact is not true in AdS_4 space or SUGRA, which is part of the reason why SUGRA is so complicated.

example, a chiral superfield has the components

$$\boldsymbol{\Phi}| = \phi, \tag{2.31}$$

$$\frac{1}{\sqrt{2}}\mathcal{D}_\alpha \boldsymbol{\Phi}| = \chi_\alpha, \tag{2.32}$$

$$-\frac{1}{4}\mathcal{D}^2 \boldsymbol{\Phi}| = F. \tag{2.33}$$

where $|$ means to take the lowest component of a superfield. Note that any other components of $\boldsymbol{\Phi}$ (i.e. by applying $\mathcal{D}^\dagger$ to these) will be proportional to derivatives of these components due to Eq. (2.27) and the chirality of $\boldsymbol{\Phi}$. For chiral superfields, we will often write this as $\phi + \sqrt{2}\chi\theta + F\theta^2$, suppressing the $\theta^\dagger$ components (or, more technically, incorporating the $\theta^\dagger$ dependence into a modified spacetime coordinate).

In the presence of an R-symmetry, $\mathcal{D}$ ($\mathcal{D}^\dagger$) has charge -1 ($+1$), so for a chiral multiplet of charge r, ϕ has charge r, χ has charge $r-1$, and Φ has charge $r-2$.

SUSY-covariant derivatives provide a different and generally more useful means for finding SUSY transformations of a superfield:

$$\delta \boldsymbol{V} = 2i(\theta\sigma^\mu\epsilon^\dagger - \epsilon\sigma^\mu\theta^\dagger)\mathcal{D}_\mu \boldsymbol{V} + \epsilon^\alpha \mathcal{D}_\alpha \Phi + \epsilon^\dagger_{\dot\alpha}\mathcal{D}^{\dagger\dot\alpha}\boldsymbol{V}, \tag{2.34}$$

Brief calculations then give the SUSY transformations of fields in a chiral multiplet:

$$\delta\phi = \epsilon^\alpha \mathcal{D}_\alpha \Phi| \qquad = \sqrt{2}\epsilon\chi, \tag{2.35}$$

$$\delta\chi_\alpha = \frac{1}{\sqrt{2}}\epsilon^\beta \mathcal{D}_\beta \mathcal{D}_\alpha \Phi| + \frac{1}{\sqrt{2}}\epsilon^\dagger_{\dot\alpha}\mathcal{D}^{\dagger\dot\alpha}\mathcal{D}_\alpha\Phi| \qquad = -\sqrt{2}\epsilon_\alpha F - i\sqrt{2}(\sigma^\mu\epsilon^\dagger)_\alpha \partial_\mu\phi, \tag{2.36}$$

$$\delta F = -\frac{1}{4}\epsilon^\dagger_{\dot\alpha}\mathcal{D}^{\dagger\dot\alpha}\mathcal{D}^2\Phi| - \frac{1}{4}\epsilon^\alpha\mathcal{D}_\alpha\mathcal{D}^2\Phi| \qquad = i\sqrt{2}\epsilon^\dagger\bar\sigma^\mu\partial_\mu\chi. \tag{2.37}$$

The component ϕ is a complex scalar, χ its partner Weyl fermion, and F is another complex scalar that can be shown to be an auxiliary field—its kinetic term has no derivatives, so it does not propagate and may be completely integrated out of the Lagrangian. If $\langle F\rangle \neq 0$, note that χ transforms as a shift—χ is then a goldstone fermion and SUSY is spontaneously broken. Therefore, F is an order parameter for SUSY-breaking.

Note that since $\mathcal{D}^3 = 0$ and $\boldsymbol{\Phi}$ is chiral, $F = -\frac{1}{4}\mathcal{D}^2\boldsymbol{\Phi}|$ transforms as a total derivative. This makes it a good candidate for a SUSY-invariant action. The same is also true of $\frac{1}{16}\mathcal{D}^2\mathcal{D}^{\dagger 2}\boldsymbol{V}|$ since $\mathcal{D}^3 = \mathcal{D}^{\dagger 3} = 0$. In fact, any SUSY-invariant action can be written in the form

$$\mathcal{L} = \int d^4\theta\, \boldsymbol{V}_{\text{comp}} + \left(\int d^2\theta\, \boldsymbol{\Phi}_{\text{comp}} + \text{h.c.} \right) \tag{2.38}$$

for some composite superfields $\boldsymbol{V}$ and $\boldsymbol{\Phi}_{\text{comp}}$ satisfying $\boldsymbol{V}^\dagger = \boldsymbol{V}$, $\mathcal{D}^\dagger \boldsymbol{\Phi} = 0$, with $\int d^4\theta$ and $\int d^2\theta$ shorthands for $\frac{1}{16}\mathcal{D}^2\mathcal{D}^{\dagger 2}|$ and $-\frac{1}{4}\mathcal{D}^2|$, respectively.[4] It is often useful to write this purely as an integral over half of superspace

$$\mathcal{L} = \int d^2\theta\, \boldsymbol{\Phi}_{\text{comp}} - \frac{1}{8}\mathcal{D}^{\dagger 2}\boldsymbol{V}_{\text{comp}} + \text{h.c.} \tag{2.39}$$

If the theory obeys an R-symmetry, it is clear that $\boldsymbol{V}$ must have R-charge 0 and $\boldsymbol{\Phi}_{\text{comp}}$ must have R-charge 2.

If our theory only has chiral superfields $\boldsymbol{\Phi}^i$, a candidate renormalizable Lagrangian is the Wess-Zumino Lagrangian

$$\mathcal{L} = \int d^4\theta\, \boldsymbol{\Phi}^{\dagger i}\boldsymbol{\Phi}^i + \left(\int d^2\theta\, \boldsymbol{W}(\boldsymbol{\Phi}^i) + \text{h.c.} \right), \tag{2.40}$$

where $\boldsymbol{W}$ is some (holomorphic) function of the superfields $\boldsymbol{\Phi}^i$, which we call the superpotential. For a renormalizable theory, the superpotential is at most cubic in fields. A brief calculation gives the corresponding Lagrangian in components:

$$\begin{aligned} \mathcal{L} = &-\partial_\mu \phi^i \partial^\mu \phi^{*\bar{\imath}} - i\chi^{\dagger\bar{\imath}}\bar{\sigma}^\mu \partial_\mu \chi^i + F^i F^{*\imath} \\ &+ F^i W_i - \frac{1}{2} W_{ij}\chi^i\chi^j + \text{h.c.} \end{aligned} \tag{2.41}$$

As promised, F^i is an auxiliary field as it has no kinetic terms. As a result, we can immediately perform the path integral over its possible field configurations, solving its (full quantum) equation of motion:

$$F^i = -W_i^*, \tag{2.42}$$

$$\begin{aligned} \mathcal{L} = &-\partial_\mu \phi^i \partial^\mu \phi^{*\bar{\imath}} - i\chi^{\dagger\imath}\bar{\sigma}^\mu \partial_\mu \chi^i + F^i F^{*\imath} \\ &- W_i W_i^* - \frac{1}{2} W_{ij}\chi^i\chi^j + \text{h.c.} \end{aligned} \tag{2.43}$$

[4]Due to their Grassmann nature, integration and differentiation with respect to $\theta^{(\dagger)}$ are equivalent.

A generic (non-renormalizable) SUSY effective field theory will in general feature terms higher order in fields in the superpotential, terms higher order in fields in the integrand of $\int d^4\theta$ (then called the Kähler potential $\boldsymbol{K}$, which was just $\boldsymbol{\Phi}^{\dagger i}\boldsymbol{\Phi}^i$ in the Wess-Zumino model), as well as terms with arbitrary numbers of spacetime or SUSY-covariant derivatives.

Note that the SUSY Lagrangian is invariant under the Kähler transformation $\boldsymbol{K} \to \boldsymbol{K} + \boldsymbol{P} + \boldsymbol{P}^\dagger$ for $\boldsymbol{P}$ chiral, as the resulting terms either vanish or are a total derivative.

2.4 Gauge Interactions

Chiral multiplets do not contain a vector degree of freedom, so SUSY gauge interactions will require another sort of multiplet. Let us begin by considering the transformation of a charged chiral multiplet under a $U(1)$ gauge transformation:

$$\boldsymbol{\Phi} \to e^{iq\alpha(x)}\boldsymbol{\Phi}. \tag{2.44}$$

Due to the possible spacetime dependence of α, such a gauge transformation does not commute with SUSY unless α is promoted to a full superfield:

$$\boldsymbol{\Phi} \to e^{q\boldsymbol{\Omega}}\boldsymbol{\Phi}, \tag{2.45}$$

with $\boldsymbol{\Omega}$ chiral. As (non-constant) chiral and anti-chiral superfields cannot cancel against each other, this means that the kinetic term $\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}$ is not gauge-invariant. We rectify this by introducing a real vector superfield $\boldsymbol{V} = \boldsymbol{V}^\dagger$ that transforms under a gauge transformation as

$$\boldsymbol{V} \to \boldsymbol{V} - \frac{\boldsymbol{\Omega} + \boldsymbol{\Omega}^\dagger}{2}, \tag{2.46}$$

so that the Lagrangian

$$\mathcal{L} = \int d^4\theta\, \boldsymbol{\Phi}^\dagger e^{2q\boldsymbol{V}}\boldsymbol{\Phi} \tag{2.47}$$

is now gauge-invariant. The superfield $\boldsymbol{V}$ has a large number of components, but many of them are pure gauge, as they can be removed by the gauge transformation of Eq. (2.46). The physical components are those which are not present in a chiral or anti-chiral superfield—a gauge boson A_μ (the $\theta\theta^\dagger$ component), a fermionic gaugino λ_α (the $\theta\theta^{\dagger 2}$ component) and its hermitian conjugate, and real scalar D (the θ^4 component) which will prove to be auxiliary. The latter two components do not transform under a gauge transformation, while the vector transforms as expected.

The superfield $\boldsymbol{V}$ of course needs its own gauge-invariant kinetic term. In order to build one, we need a gauge-invariant superfield containing $\boldsymbol{V}$:

$$\boldsymbol{W}_\alpha \equiv -\frac{1}{4}\mathcal{D}^{\dagger 2}\mathcal{D}_\alpha \boldsymbol{V}, \tag{2.48}$$

$$= \lambda_\alpha + D\theta_\alpha - i(\sigma^{\mu\nu}\theta)_\alpha F_{\mu\nu} - i\theta^2(\sigma^\mu\partial_\mu\lambda^\dagger)_\alpha. \tag{2.49}$$

In addition to being gauge invariant, $\boldsymbol{W}_\alpha$ is also chiral as $\mathcal{D}^{\dagger 3} = 0$. Note that D is also an order parameter for SUSY-breaking, as λ_α will transform as a shift (and will thus be a Goldstone fermion) if D obtains a vev. It is generally true, however, that a D-term can only be non-zero if a F-term is also non-zero (with one exception for Abelian groups which is difficult to generalize to local SUSY).

A suitable kinetic Lagrangian is then[5]

$$\mathcal{L} = \int d^2\theta\, \frac{1}{4}\boldsymbol{W}^\alpha \boldsymbol{W}_\alpha + \text{h.c.} \tag{2.50}$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2. \tag{2.51}$$

This can be generalized to non-abelian gauge groups, in which the components of $\boldsymbol{V}$ are in the adjoint representation of the gauge group:

$$\boldsymbol{W}_\alpha \equiv -\frac{1}{8}\mathcal{D}^{\dagger 2}\left(e^{-2\boldsymbol{V}}\mathcal{D}_\alpha e^{2\boldsymbol{V}}\right), \tag{2.52}$$

$$\boldsymbol{W}_\alpha \to e^{\boldsymbol{\Omega}}\boldsymbol{W}_\alpha e^{-\boldsymbol{\Omega}}, \tag{2.53}$$

$$e^{2\boldsymbol{V}} \to e^{-\boldsymbol{\Omega}^\dagger}e^{2\boldsymbol{V}}e^{-\boldsymbol{\Omega}}, \tag{2.54}$$

$$\boldsymbol{\Phi} \to e^{\boldsymbol{\Omega}}\boldsymbol{\Phi}, \tag{2.55}$$

$$\mathcal{L} = \int d^2\theta\, \frac{1}{2}\text{Tr}\left[\boldsymbol{W}^\alpha \boldsymbol{W}_\alpha\right]. \tag{2.56}$$

If our theory obeys an R-symmetry, $\boldsymbol{V}$ must have vanishing R-charge (as it is real),

[5]Here, we switch to a canonical normalization for the gauge multiplet, in which g is in the covariant derivative rather than the kinetic term.

which implies that the gaugino λ must have R-charge 1.

The most general renormalizable SUSY theory with gauge interactions is then[6]

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\, \boldsymbol{\Phi}^{\dagger\bar{i}} e^{2g_a \boldsymbol{V}^a T^a} \boldsymbol{\Phi}^i + \left(\int d^2\theta\, \boldsymbol{W}(\boldsymbol{\Phi}^i) + \frac{1}{4} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^a_\alpha + \text{h.c.} \right) &(2.57)\\ &= -\mathcal{D}_\mu \phi^{*\bar{i}} \mathcal{D}^\mu \phi^i - i\chi^{\dagger\bar{i}} \bar{\sigma}^\mu \mathcal{D}_\mu \phi^i + F^{*\bar{i}} F^i \\ &\quad - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - i\lambda^{\dagger a} \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \frac{1}{2} D^a D^a \\ &\quad + W_i F^i - \frac{1}{2} W_{ij} \chi^i \chi^j + \text{h.c.} \\ &\quad + g_a (\phi^{*\bar{i}} (T^a)_{\bar{i}j} \phi^j) D^a - \left(\sqrt{2} g_a (\phi^{*\bar{i}} (T^a)_{\bar{i}j} \chi^j) \lambda^a + \text{h.c.} \right), &(2.58)\end{aligned}$$

where the derivatives are the usual gauge-covariant ones, and where we work in Wess-Zumino gauge, where all the pure gauge degrees of freedom in $\boldsymbol{V}^a$ (apart from the standard one the vector component has) are transformed to zero. Note that D^a, like F^i, is an auxiliary field that may simply be integrated out.

This Lagrangian also reinforces the fact that $\langle D^a \rangle$ and $\langle F^i \rangle$ are order parameters for SUSY-breaking, as $\langle H \rangle \neq 0$ only when at least one of them is non-vanishing.

2.5 SUSY Breaking and Goldstinos

As we noted in Sec. 2.2, SUSY must be spontaneously broken as there are no 511 keV selectrons in nature. We can make this statement even stronger by consider a supertrace sum rule valid for renormalizable theories at tree level that spontaneously break SUSY [68]:

$$\text{STr}\left[m^2\right] \equiv \sum_s (-1)^{2s} (2s+1) \text{Tr}\left[m_s^2\right] = -2g_a \text{Tr}\left[T^a\right] \langle D^a \rangle = 0, \tag{2.59}$$

where s represents the spin of the particle.[7] Consider the MSSM with flavor conservation (i.e. no mixing between scalars of different generations) and with no additional broken U(1) symmetries involved in SUSY breaking. For the first generation of squarks, for example,

[6] We have omitted here the possibility of Fayet-Iliopoulos terms, $\int d^4\theta\, \boldsymbol{V}$, which are gauge-invariant for Abelian groups but difficult to incorporate into local SUSY theories in flat space. We have also omitted Θ-terms for the gauge fields, which correspond to an imaginary coefficient of the gauge kinetic term; they only yield the usual $F\tilde{F}$ term in components.

[7] This last equality is obvious for a non-Abelian gauge theory with $\text{Tr}(T^a) = 0$. For a U(1) gauge group, the sum of the hypercharges must vanish to avoid the gravitational anomaly.

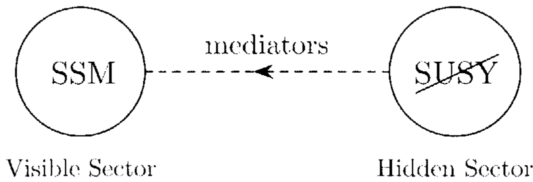

Figure 2-1: Standard paradigm of a SUSY-breaking hidden sector coupled to the SUSY SM via mediators.

since

$$\mathrm{Tr}(\sigma^3) = 0 \quad \text{and} \quad Y_{\tilde{u}_L} + Y_{\tilde{u}_R^*} + Y_{\tilde{d}_L} + Y_{\tilde{d}_R} = 0, \tag{2.60}$$

Eq. (2.59) decouples, leading to the relation

$$m^2_{\tilde{u}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{d}_R} + m^2_{\tilde{d}_L} = 2(m_u^2 + m_d^2). \tag{2.61}$$

If $\mathrm{SU}(3)_C$ is to remain unbroken, this would imply light (MeV) scale superpartners, in conflict with observation. Similar arguments exist in the presence of large flavor mixings[57], even apart from the dangerous flavor-changing neutral currents they would introduce.

For these reasons, the standard SUSY-breaking paradigm is for SUSY to be broken in a "hidden sector", and the effects of SUSY breaking communicated to the SUSY SM (the "visible sector") via loop processes or higher-dimension operators. We draw this schematically as in Fig. 2-1. The effect of SUSY breaking on the visible sector is obviously important, but the phenomenological implications of the SUSY-breaking sector itself are typically meager (with one important exception); as a result, I will try to abstract the most important features of the hidden sector.

There is one hidden sector state with broad phenomenological relevance. As with any other spontaneously broken global symmetry, SUSY will have a gapless goldstone mode. However, since SUSY is a fermionic symmetry, this mode will be a fermion—the goldstino $\widetilde{G}_L$. To abstract hidden sector dynamics as much as possible, we will consider the goldstino residing in a non-linear superfield $\boldsymbol{X}$ satisfying $\boldsymbol{X}^2 = 0$:

$$\boldsymbol{X} = \left(\theta + \frac{\widetilde{G}_L}{\sqrt{2}F}\right)^2 F. \tag{2.62}$$

Note that we have effectively integrated out the sgoldstino by using this multiplet. For the purposes of this thesis (in which we never care about Lagrangian terms with more than two goldstinos), we can safely consider F to be a (real) non-dynamical background field—that is, we never solve its equation of motion, and we just consider it to be a constant.

The Lagrangian for the goldstino multiplet itself takes the simple form

$$\mathcal{L} = \int d^4\theta \boldsymbol{X}^\dagger \boldsymbol{X} - \int d^2\theta F \boldsymbol{X} + \text{h.c.} \tag{2.63}$$

The constraint $\boldsymbol{X}^2 = 0$ forbids any additional terms without SUSY-covariant or spacetime derivatives. The coefficient of the superpotential is constrained to be $-F$ by consistency (if we were to treat F as dynamical, the solution to its equation of motion should yield F). Note that this Lagrangian by itself obeys an R-symmetry under which $\boldsymbol{X}$ has R-charge 2 (and thus the goldstino $\widetilde{G}_L$ has R-charge 1).

Coupling the goldstino superfield $\boldsymbol{X}$ to visible sector fields will communicate SUSY-breaking to the visible sector in the form of soft-SUSY breaking parameters. By SUSY, these come with goldstino couplings that are exactly proportional to those soft terms. The full set of (relevant) soft terms for the visible sector is given by[8]

$$\begin{aligned}\mathcal{L}_{\text{soft}} = &-\int d^4\theta \,\frac{m^2_{i\bar{j}}}{F^2}\boldsymbol{X}^\dagger \boldsymbol{X}\boldsymbol{\Phi}^{\dagger\bar{j}}\boldsymbol{\Phi}^i - \int d^2\theta \,\frac{M_a}{2F}\boldsymbol{X}\boldsymbol{W}^\alpha \boldsymbol{W}_\alpha \\ &- \int d^2\theta \,\frac{B_{ij}}{2F}\boldsymbol{X}\boldsymbol{\Phi}^i\boldsymbol{\Phi}^j + \frac{A_{ijk}}{6F}\boldsymbol{X}\boldsymbol{\Phi}^i\boldsymbol{\Phi}^j\boldsymbol{\Phi}^k - \frac{M_a}{2F}\boldsymbol{X}\boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha,\end{aligned} \tag{2.64}$$

$$\begin{aligned}\mathcal{L} = &-m^2_{i\bar{j}}\phi^{*\bar{j}}\phi^i + \frac{m^2_{ij}}{F}\widetilde{G}_L\chi^i\phi^{*j} + \text{h.c.} \\ &- \frac{1}{2}M_a\lambda^a\lambda^a + \frac{iM_a}{\sqrt{2}F}\widetilde{G}_L\sigma^{\mu\nu}\lambda^a F^a_{\mu\nu} + \frac{M_a}{\sqrt{2}F}\widetilde{G}_L\lambda^a D^a + \text{h.c.} \\ &- \frac{1}{2}B_{ij}\phi^i\phi^j + \frac{B_{ij}}{F}\widetilde{G}_L\chi^i\phi^j + \text{h.c.} \\ &- \frac{1}{6}A_{ijk}\phi^i\phi^j\phi^k + \frac{A_{ijk}}{2F}\widetilde{G}_L\chi^i\phi^j\phi^k + \text{h.c.}\end{aligned} \tag{2.65}$$

Note that if the theory obeys an R-symmetry, Majorana gaugino masses of the type shown here are forbidden, as are A- and B-terms corresponding to allowed superpotential terms.

[8]This is not an exhaustive list, for two reasons. First, many different terms in superspace give the same soft terms (e.g. $\boldsymbol{X}\boldsymbol{\Phi}\boldsymbol{\Phi}^\dagger$ or $\boldsymbol{X}^\dagger\boldsymbol{X}\boldsymbol{\Phi}^2$ can both give B-terms). Secondly, we have omitted soft terms corresponding to tadpoles for scalars or Dirac masses for gauginos and adjoint fermions, neither of which occur in the Minimal SUSY SM (MSSM).

2.5.1 The Supercurrent

Using the non-linear goldstino multiplet, we found that goldstino couplings were directly related to soft terms. There is a more formal way of seeing this same effect using conservation of the supercurrent.

The supercurrent is the Noether current associated with SUSY transformations[155, 50]

$$\begin{aligned} j^\mu_\alpha = & \sqrt{2}(\sigma^\nu \bar{\sigma}^\mu \psi^i)_\alpha \mathcal{D}_\nu \phi^{*i} - i\sqrt{2}(\sigma^\mu \psi^{\dagger i})_\alpha W_i^* \\ & - \frac{1}{2}(\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^{\dagger a}) F^a_{\nu\rho} - i g_a \phi^{*i} (T^a \phi)_i (\sigma^\mu \lambda^{\dagger a})_\alpha . \end{aligned} \tag{2.66}$$

Note that the supercurrent has an extra α-index to match the SUSY generator Q_α. Conservation of the supercurrent implies $\partial_\mu j^\mu_\alpha = 0$.

We can isolate the goldstino contribution to the supercurrent via[64, 65]

$$j^\mu_\alpha = j^{\mu,\text{matter}}_\alpha + i\sqrt{2} F_{\text{tot}} \left(\sigma^\mu \bar{\eta}\right)_\alpha , \tag{2.67}$$

where $F_{\text{tot}} = \sqrt{|F_{\text{vis}}|^2 + |F_{\text{hid}}|^2}$ also includes any SUSY breaking in the visible sector. Conservation of the full supercurrent implies

$$\partial_\mu j^\mu_\alpha = 0 = \partial_\mu j^{\mu,\text{matter}}_\alpha + i\sqrt{2} F_{\text{tot}} \left(\sigma^\mu \partial_\mu \bar{\eta}\right)_\alpha . \tag{2.68}$$

As expected, because SUSY in the visible sector is broken, $\partial_\mu j^{\mu,\text{matter}}_\alpha \neq 0$. If we interpret Eq. (2.68) as an equation of motion for the goldstino, this implies that in addition to a kinetic term, the goldstino Lagrangian must contain

$$\mathcal{L}_\eta \supset -\frac{1}{\sqrt{2} F_{\text{tot}}} \eta \partial_\mu j^{\mu,\text{matter}} + \text{h.c.} \tag{2.69}$$

This is called a Goldberger-Treiman relation from the analogous relation for couplings of Goldstone bosons of spontaneously broken global symmetries to matter currents.[85]

For a massless on-shell goldstino

$$(\partial_\mu \eta) \sigma^\mu = 0, \tag{2.70}$$

so after integration by parts, the second and fourth terms in Eq. (2.66) are irrelevant for

Table 2.1: Quantum Numbers of the MSSM.

	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$
$\boldsymbol{Q}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-\frac{1}{6}$
$\boldsymbol{U}$	$\mathbf{3}$	$\mathbf{1}$	$+\frac{2}{3}$
$\boldsymbol{D}$	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$
$\boldsymbol{L}$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$
$\boldsymbol{E}$	$\mathbf{1}$	$\mathbf{1}$	-1
$\boldsymbol{H}_u$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
$\boldsymbol{H}_d$	$\mathbf{1}$	$\mathbf{2}$	$+\frac{1}{2}$

Eq. (2.69), which reduces to

$$\begin{aligned}\mathcal{L}_\eta \supset \frac{1}{F_{\mathrm{tot}}}\eta\Big(&- (\Box\psi^i)\phi_i^\dagger + \psi^i\Box\phi_i^\dagger + \frac{1}{\sqrt{2}}\sigma^{\nu\rho}(\sigma^\mu\partial_\mu\lambda^{\dagger a})F^a_{\nu\rho} \\ &- \frac{1}{\sqrt{2}}\sigma^\rho\lambda^{\dagger a}\partial^\nu F^a_{\nu\rho}\Big) + \text{h.c.}\end{aligned} \tag{2.71}$$

Using equations of motion for the visible sector fields, we find that the three-point couplings of the goldstino are proportional to *physical* mass differences:

$$\mathcal{L} \supset \frac{m^2_{\phi^i} - m^2_{\psi^i}}{F_{\mathrm{tot}}}\eta\psi^i\phi_i^\dagger + \frac{B_{ij}}{F_{\mathrm{tot}}}\eta\chi^i\psi^j + \frac{im_\lambda}{\sqrt{2}F_{\mathrm{tot}}}\eta\sigma^{\mu\nu}\lambda^a F^a_{\mu\nu}, \tag{2.72}$$

where, for simplicity, we have assumed unbroken gauge groups and dropped terms with more than three particles. We see that this result exactly reproduces Eq. (2.65) in a non-trivial way.[9]

2.6 The MSSM

With all of these ingredients, we can finally write down the field content and Lagrangian of the Minimal Supersymmetric Standard Model (MSSM). The chiral superfields in the MSSM are given in Table 2.1. The quantum numbers may differ by a sign from those in other references; this is because in our conventions, chiral superfields are *right*-handed fields. As a result, $\boldsymbol{Q}$ and $\boldsymbol{L}$ should be interpreted as the conjugates of the left-handed fields we normally deal with.

The leading relevant and marginal interactions are given by the gauge interactions of

[9]The terms with D^a would show up here among the terms with more than three particles, on the D^a equation of motion.

these fields, and by terms in the superpotential of up to dimension 3, which could include

$$\boldsymbol{W} \supset \mu \boldsymbol{H}_u \boldsymbol{H}_d + \lambda^u_{ij} \boldsymbol{Q}^i \boldsymbol{U}^j \boldsymbol{H}_u + \lambda^d_{ij} \boldsymbol{Q}^i \boldsymbol{D}^j \boldsymbol{H}_d + \lambda^e_{ij} \boldsymbol{E}^i \boldsymbol{L}^j \boldsymbol{H}_d. \tag{2.73}$$

The fermions and scalars in the $\boldsymbol{H}$ superfields (the Higgs bosons and Higgsinos) receive a supersymmetric mass from the μ mass term. If $\boldsymbol{H}_u$ and $\boldsymbol{H}_d$ get vevs in their lowest component, the other three superpotential terms then yield masses for the up-type quarks, down-type quarks, and leptons, respectively. As the superpotential must be holomorphic, we need at least two Higgs doublets in order to give masses to all fermions. Two Higgs doublets are also required to avoid the Higgsinos introducing gauge anomalies.

As written, this superpotential obeys two additional global U(1) symmetries: a baryon number symmetry under which $\boldsymbol{Q}$ and $\boldsymbol{U}/\boldsymbol{D}$ have opposite charges, and a lepton number symmetry under which $\boldsymbol{L}$ and $\boldsymbol{E}$ have opposite charges. However, we can easily write down terms that do not respect these symmetries:

$$\boldsymbol{W} \supset \rho_i \boldsymbol{L}^i \boldsymbol{H}_u + \lambda^{(1)}_{ijk} \boldsymbol{Q}^i \boldsymbol{D}^j \boldsymbol{L}^k + \lambda^{(2)}_{ijk} \boldsymbol{U}^i \boldsymbol{D}^j \boldsymbol{D}^k + \lambda^{(3)}_{ijk} \boldsymbol{L}^i \boldsymbol{L}^j \boldsymbol{E}^k. \tag{2.74}$$

Note that the last two terms must involve particles from multiple generations, since the SU(2) and SU(3) indices are contracted antisymmetrically. If one wanted to forbid all of these terms, one could impose baryon and lepton number symmetries explicitly.

Alternatively, one could use a $\mathrm{U}(1)_R$ symmetry to forbid the terms in Eq. (2.74). If one gives the Higgs doublets an R-charge of 1, and all other superfields an R-charge of $+1/2$, the problematic terms are forbidden since the resulting superpotential would not have an R-charge of 2. We do not even need a full R-symmetry to achieve the same effect, which is desirable as we generally expect continuous R-symmetries to be broken by SUSY-breaking effects (or by $m_{3/2}$ if nothing else; see Sec. 2.7). The R-symmetry contains a discrete $\mathbb{Z}_2$ subgroup called R-parity (R_p), under which the Higgs doublets have R-parity $+1$ and the other multiplets have R-parity -1. This is sufficient to forbid the terms in Eq. (2.74).

SUSY-breaking will generically give scalar mass-squareds for all scalars, Majorana gaugino masses for all gauginos, and A- and B-terms corresponding to each term in the superpotential. R-parity (under which $\boldsymbol{X}$ is even) will also forbid soft SUSY breaking terms corresponding to the dangerous terms in Eq. (2.74).

Note that the imposition of R-parity means that R-parity odd particles (all the super-

Table 2.2: Particle Content of the MSSM.

R_p-even	R_p-odd	
q	$\widetilde{q}_L$, $\widetilde{q}_R$	(squarks)
l	$\widetilde{l}_L$, $\widetilde{l}_R$	(sleptons)
ν	$\widetilde{\nu}$	(sneutrinos)
γ, Z, h, A^0, H^0	χ_i^0	(neutralinos)
$W^{\pm}$, $H^{\pm}$	$\chi_i^{\pm}$	(charginos)
g	$\widetilde{g}$	(gluino)
G (graviton)	ψ_μ	(gravitino)

partners of SM fields) may only interact in pairs. This has wide-ranging phenomenological implications. The decay of any SUSY particle must feature another SUSY particle as one of the decay products; this implies that the lightest SUSY particle (the LSP) will be absolutely stable. If the LSP is electrically neutral and a color singlet, this could be a good dark matter candidate. This is especially true if the LSP is in the visible sector, as then one would expect it to have weak-strength interactions, allowing it to be produced in the early universe with the right relic abundance via the so-called 'WIMP miracle'.

R-parity also implies that all SUSY particles must be created in pairs. Therefore, SUSY events at colliders such as the LHC will generally feature the production of two SUSY particles, each of which will undergo a series of decays down to the LSP, which, if it is the dark matter, is stable and exits the collider invisibly. Such signals of multiple SM particles (most typically jets at the LHC) and missing energy is a striking signal for SUSY that can be seen at colliders that has relatively little background from SM processes.

A full discussion of the MSSM spectrum and phenomenology is beyond the scope of this introduction, but a brief enumeration of the particle content of the MSSM should prove useful for the reader, and is given in Table 2.2. Every SM fermion has a partner scalar; for quarks and leptons, there are superpartners for both the left- and right-handed fermions. After electroweak symmetry breaking (EWSB), the superpartners of left- and right-handed fermions can mix, but this mixing is generally only important for the third generation. Sfermions of different generations can of course mix, but there are often stringent limits on such mixing from the absence of flavor-changing neutral currents (FCNC).

Since the MSSM has two Higgs doublets, there are four additional Higgs bosons—the charged $H^{\pm}$, a scalar H^0, and a pseudoscalar A^0. After EWSB, the superpartners of the Higgses and the electroweak gauge bosons (the Higgsinos, binos, and winos) can mix, so generically there are four neutral R_p-odd fermions (the neutralinos χ_0^i) and four charged ones (the charginos $\chi_i^\mp$). The gluon has a partner color adjoint fermion, the gluino, which does not have the correct quantum numbers to mix with any other states.

The only visible sector R-parity odd particles that are neutral and colorless (and thus potential LSP dark matter candidates) are the sneutrinos and neutralinos. In the MSSM, sneutrino dark matter has long been ruled out by a combination of direct detection experiments and (for low masses) the invisible width of the Z [31, 32, 136, 128]. Neutralino dark matter has thus been the standard paradigm for SUSY dark matter, but it is by no means the only possibility. A non-minimal visible sector can feature additional candidates, such as singlinos (gauge singlet fermions) or the scalar superpartner of a right-handed neutrino [88, 13].

The LSP could of course be in a hidden sector, such as one responsible for SUSY-breaking. The most obvious candidate would be the goldstino, but the goldstino is exactly massless in global SUSY. This drawback can be rectified in local SUSY or supergravity by considering the gravitino ψ_μ, the spin-3/2 partner of the graviton—a possibility we'll consider in detail in Sec. 2.8.

2.7 SUSY in AdS

The previous sections considered rigid SUSY only in Minkowski space. Rigid SUSY can be considered in other spacetime backgrounds, as well—the case of anti-de Sitter (AdS) space is especially relevant, as it forms the global limit of unbroken SUGRA. AdS space is a space of negative uniform curvature:

$$R_{\mu\nu\rho\lambda} = m_{3/2}^2(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}), \tag{2.75}$$

$$R_{\mu\nu} = 3m_{3/2}^2 g_{\mu\nu}, \tag{2.76}$$

$$R = 12m_{3/2}^2, \tag{2.77}$$

with $m_{3/2} = r_{\rm AdS}^{-1}$ the inverse radius of AdS curvature.[10] Naively, one might think that SUSY in AdS might just consist of promoting flat-space derivatives to spacetime-covariant derivatives. However, recall that in such a curved space, spacetime derivatives no longer

[10]The notation $m_{3/2}$ comes from SUGRA, where the same parameter also plays the role of mass parameter for the spin-3/2 gravitino.

commute when not acting on Lorentz scalars. As a result, the Poincaré algebra is modified in AdS:

$$[P_\mu, P_\nu] = -im_{3/2}^2 M_{\mu\nu}. \tag{2.78}$$

As SUSY is an extension of the Poincaré algebra, the SUSY algebra is also modified in AdS. We can see this immediately by considering such Jacobi identities as

$$0 = [Q_\alpha, [P_\mu, P_\nu]] + [P_\mu, [P_\nu, Q_\alpha]] + [P_\nu, [Q_\alpha, P_\mu]]. \tag{2.79}$$

As the first term is non-vanishing in AdS (recall that Q, as a spinor, does not commute with M), the consistency of the SUSY algebra in AdS requires $[P_\mu, Q_\alpha] \neq 0$ as well. Using such Jacobi identities, one can easily show that the SUSY algebra in AdS must include [4, 51, 104, 156, 95]

$$[Q, P_\mu] = -\frac{1}{2}m_{3/2}\sigma_\mu Q^\dagger, \qquad \{Q_\alpha, Q^\beta\} = 2im_{3/2}\sigma^{\mu\nu}{}_\alpha{}^\beta M_{\mu\nu}. \tag{2.80}$$

Note that this modification of the SUSY algebra for AdS precludes the existence of R-symmetries.

The fact that SUSY and spacetime translations no longer commute is one of the most important features of SUSY in AdS, and has many unexpected implications which we will see throughout this thesis. One of them follows from the requirement that $\delta_{\text{SUSY}}\partial_\mu\phi = \partial_\mu\delta_{\text{SUSY}}\phi$ (so that the algebra is consistent); this implies a non-trivial requirement on the infinitesimal SUSY transformation parameter:

$$0 = \mathcal{D}_\mu\epsilon^\dagger + \frac{1}{2}im_{3/2}\bar{\sigma}_\mu\epsilon. \tag{2.81}$$

2.7.1 SUSY Covariant Derivatives

Just as in flat space, one can define superspace-covariant derivatives. These obey the following commutation relations, which follow straightforwardly from the SUSY algebra in

AdS:

$$\begin{aligned}
\{\mathcal{D}_\alpha, \mathcal{D}^\dagger_{\dot\beta}\} &= -2i\sigma^\mu_{\alpha\dot\beta}\mathcal{D}_\mu, & \{\mathcal{D}_\alpha, \mathcal{D}_\beta\}V^\gamma &= -2m_{3/2}(V_\alpha\delta_\beta{}^\gamma + V_\beta\delta_\alpha{}^\gamma),\\
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\}V^{\dagger\dot\gamma} &= 0, & \{\mathcal{D}_\alpha, \mathcal{D}^\beta\}V^\mu &= m_{3/2}(\sigma^{\mu\nu})_\alpha{}^\beta V_\nu,\\
[\mathcal{D}_\mu, \mathcal{D}_\beta] &= -\frac{1}{2}im_{3/2}(\sigma_\mu\mathcal{D}^\dagger)_\beta, & [\mathcal{D}_\mu, \mathcal{D}_\nu]\,V^\rho &= m_{3/2}^2(V_\mu\delta_\nu{}^\rho - V_\nu\delta_\mu{}^\rho),\\
[\mathcal{D}_\mu, \mathcal{D}_\nu]\,V^\gamma &= -m_{3/2}^2(V\sigma_{\mu\nu})^\gamma. & & \qquad (2.82)
\end{aligned}$$

One can yet again use these SUSY covariant derivatives to construct constrained superfields. Just as before, a chiral superfield is defined by the requirement

$$\mathcal{D}^{\dagger\dot\alpha}\mathbf{\Phi} = 0. \qquad (2.83)$$

Rather than thinking about an explicit expansion in θ, y, we'll just define components with the SUSY covariant derivatives themselves

$$\Phi| = \phi, \qquad \frac{1}{\sqrt{2}}\mathcal{D}_\alpha\Phi| = \chi_\alpha, \qquad -\frac{1}{4}\mathcal{D}^2\Phi| = F. \qquad (2.84)$$

We can think of this schematically as $\Phi = \phi + \sqrt{2}\chi\Theta + F\Theta^2$. These Θ are really and truly placeholders; one can treat them as normal for addition and multiplication of chiral multiplets, but not much else.

In flat-space SUSY, $\mathcal{D}^{\dagger 2}V$ is chiral since $\mathcal{D}^{\dagger 3} = 0$. This is no longer true for AdS SUSY, as $\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} \neq 0$. However, this can be easily remedied, and (anti-)chiral projectors still exist:

$$\mathcal{D}_\alpha(\mathcal{D}^2 - 4m_{3/2})\boldsymbol{V} = 0, \qquad \mathcal{D}^{\dagger\dot\alpha}(\mathcal{D}^{\dagger 2} - 4m_{3/2})\boldsymbol{V} = 0. \qquad (2.85)$$

Note again that $[\mathcal{D}_\mu, \mathcal{D}_\beta] \neq 0$. One immediate consequence of this is that bosons and fermions have differing masses, even for unbroken SUSY:

$$\begin{aligned}
\mathcal{D}_\alpha \mathcal{D}^\mu \mathcal{D}_\mu \mathbf{\Phi} &= [\mathcal{D}_\alpha, \mathcal{D}_\mu]\mathcal{D}^\mu \mathbf{\Phi} + \mathcal{D}^\mu [\mathcal{D}_\alpha, \mathcal{D}_\mu]\mathbf{\Phi} + \mathcal{D}^\mu \mathcal{D}_\mu \mathcal{D}_\alpha \mathbf{\Phi} \\
\mathcal{D}_\alpha (m_\phi^2 \mathbf{\Phi} + B\mathbf{\Phi}^\dagger) &= \frac{1}{2} i m_{3/2} \sigma_{\mu\alpha\dot{\alpha}} [\mathcal{D}^{\dagger\dot{\alpha}}, \mathcal{D}^\mu]\mathbf{\Phi} - (\sigma \cdot \mathcal{D}\bar{\sigma} \cdot \mathcal{D} + 3m_{3/2}^2)\mathcal{D}_\alpha \mathbf{\Phi} \\
m_\phi^2 \mathcal{D}_\alpha \mathbf{\Phi} &= m_{3/2}^2 \mathcal{D}_\alpha \mathbf{\Phi} + (m_\chi^2 - 3m_{3/2}^2)\mathcal{D}_\alpha \mathbf{\Phi} \\
m_\phi^2 &= m_\chi^2 - 2m_{3/2}^2. \qquad (2.86)
\end{aligned}$$

We've performed some sleight of hand here on the right side of the equation, by applying the Klein-Gordon equation to the superfield rather than its lowest component. One can easily show that this is legitimate, though there are some subtleties at loop level that we will see in Sec. 5. We can easily do the same trick for B-terms:

$$\begin{aligned}
\mathcal{D}^{\dagger\dot{\alpha}} \mathcal{D}^\mu \mathcal{D}_\mu \mathbf{\Phi} &= [\mathcal{D}^{\dagger\dot{\alpha}}, \mathcal{D}^\mu]\mathcal{D}_\mu \mathbf{\Phi} + \mathcal{D}_\mu [\mathcal{D}^{\dagger\dot{\alpha}}, \mathcal{D}^\mu]\mathbf{\Phi} \\
\mathcal{D}^{\dagger\dot{\alpha}} (m_\phi^2 \mathbf{\Phi} + B\mathbf{\Phi}^\dagger) &= \frac{1}{2} i m_{3/2} \bar{\sigma}^{\mu\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \mathcal{D}_\mu]\mathbf{\Phi} + i m_{3/2} \bar{\sigma}^{\mu\dot{\alpha}\alpha} \mathcal{D}_\mu \mathcal{D}_\alpha \mathbf{\Phi} \\
B &= -m_{3/2} m_\chi, \qquad (2.87) \\
m_{s,p}^2 &= m_\chi^2 - 2m_{3/2}^2 \mp m_{3/2} m_\chi, \qquad (2.88)
\end{aligned}$$

where the final line shows the masses of the scalar and pseudoscalar in the chiral multiplet, respectively. We see that even for unbroken SUSY at tree level, there are 'soft' scalar masses and B-terms in AdS. The sign of the scalar mass squared may worry the reader; for certain values of m_χ, one or both of the scalars may have a negative mass-squared, which naively would correspond to a tachyonic instability. However, defining mass in AdS is a subtle proposition, as P^2 is no longer a Casimir of the algebra. Breitenlohner and Freedman [25] showed that negative scalar masses are stable in AdS, so long as they are not too negative ($m^2 \geq -\frac{9}{4} m_{3/2}^2$). This bound is saturated for $m_\chi = \pm m_{3/2}/2$, but is never exceeded; unbroken SUSY in AdS is always stable.

We can also use the SUSY covariant derivatives to find the SUSY transformations of the components of chiral superfields. The following holds in both flat-space and AdS SUSY:

$$\delta\Phi = -2i(\theta\sigma^\mu \epsilon^\dagger - \epsilon\sigma^\mu \theta^\dagger)\mathcal{D}_\mu \Phi - \epsilon^\alpha \mathcal{D}_\alpha \Phi - \epsilon^\dagger_{\dot{\alpha}} \mathcal{D}^{\dagger\dot{\alpha}} \Phi. \qquad (2.89)$$

We can evaluate this for the individual components. Since we're always taking the lowest component, the first term never matters. The lowest and fermionic components follow through exactly as in the flat space case:

$$\delta\phi = -\epsilon^{\alpha}\mathcal{D}_{\alpha}\Phi| \tag{2.90}$$

$$= -\sqrt{2}\epsilon\chi, \tag{2.91}$$

$$\delta\chi_{\alpha} = -\frac{1}{\sqrt{2}}\epsilon^{\beta}\mathcal{D}_{\beta}\mathcal{D}_{\alpha}\Phi| - \frac{1}{\sqrt{2}}\epsilon^{\dagger}_{\dot{\alpha}}\mathcal{D}^{\dagger\dot{\alpha}}\mathcal{D}_{\alpha}\Phi| \tag{2.92}$$

$$= -\sqrt{2}\epsilon_{\alpha}F - i\sqrt{2}(\sigma^{\mu}\epsilon^{\dagger})_{\alpha}\partial_{\mu}\phi. \tag{2.93}$$

The F-component, however, transforms differently:

$$\delta F = \frac{1}{4}\epsilon^{\dagger}_{\dot{\alpha}}\mathcal{D}^{\dagger\dot{\alpha}}\mathcal{D}^{2}\Phi| + \frac{1}{4}\epsilon^{\alpha}\mathcal{D}_{\alpha}\mathcal{D}^{2}\Phi| \tag{2.94}$$

The first term is exactly what you would expect in flat space (except of course now with an appropriately-covariant derivative). The $\{\mathcal{D}^{\dagger\dot{\alpha}}, \mathcal{D}^{\alpha}\}$ anti-commutators are exactly the same as in flat space; there's a possible $[\mathcal{D}^{\alpha}, \mathcal{D}^{\mu}]$ commutator to perform, but this yields a $\mathcal{D}^{\dagger}$, which vanishes on the chiral superfield. The other term vanishes in flat space, but in AdS gives a new contribution arising from Eq. (2.89):

$$\mathcal{D}_{\alpha}\mathcal{D}^{2}\Phi = 4m_{3/2}\mathcal{D}_{\alpha}\Phi \tag{2.95}$$

$$\delta F = \sqrt{2}m_{3/2}\epsilon\chi - i\sqrt{2}\epsilon^{\dagger}\overline{\sigma}^{\mu}\mathcal{D}_{\mu}\chi. \tag{2.96}$$

2.7.2 The AdS SUSY Lagrangian

Note that the F component of a chiral superfield does *not* transform as a total derivative,[11] both due to the first term and due to the fact that $\mathcal{D}_{\mu}\epsilon \neq 0$. However, the following combination can easily be shown to transform as a total derivative:

$$F + 3m_{3/2}\phi \tag{2.97}$$

We can define a 'chiral density' $2\mathcal{E} \equiv e(1 + 3m_{3/2}\Theta^{2})$—this is the equivalent of e, the determinant of the vielbein, for chiral superspace. The following is then is a good choice

[11]We do not consider here terms on the boundary of AdS, though they can play an important role in general.

for a SUSY-invariant Lagrangian:

$$\mathcal{L} = \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \boldsymbol{\Phi} + \text{h.c.} \tag{2.98}$$

with $\boldsymbol{\Phi}$ any chiral superfield (elementary or composite), and $\int d^2\Theta = -\frac{1}{4}\mathcal{D}^2|$, as usual. This corresponds to superpotential-style terms. We can write down kinetic terms in a similar fashion, by making use of the fact that $(\mathcal{D}^{\dagger 2} - 4m_{3/2})$ is automatically chiral.

$$\mathcal{L} = \int d^2\Theta\, 2\boldsymbol{\mathcal{E}} \left[-\frac{1}{8}(\mathcal{D}^{\dagger 2} - 4m_{3/2})\boldsymbol{K} + \boldsymbol{W} \right] + \text{h.c.}, \tag{2.99}$$

where $\boldsymbol{K}$ is an arbitrary function of chiral and anti-chiral superfields, and $\boldsymbol{W}$ is a holomorphic function of chiral superfields alone. There may also be terms with additional covariant derivatives which we omit here.

Note that holomorphic terms in the Kähler potential no longer give a vanishing contribution to the Lagrangian, so the Kähler transformation $\boldsymbol{K} \to \boldsymbol{K} + \boldsymbol{P} + \boldsymbol{P}^\dagger$ (for $\boldsymbol{P}$ some chiral superfield) is no longer an invariance of the theory. However, the Kähler invariance can be restored by including in the Kähler transformations $\boldsymbol{W} \to \boldsymbol{W} - m_{3/2}\boldsymbol{P}$. We can use this freedom to kill $\boldsymbol{W}$ entirely, leaving our entire Lagrangian in terms of $\boldsymbol{G} \equiv \boldsymbol{K} + \boldsymbol{W}/m_{3/2} + \boldsymbol{W}^\dagger/m_{3/2}$:

$$\mathcal{L} = \int d^2\Theta\, 2\boldsymbol{\mathcal{E}} \left[-\frac{1}{8}(\mathcal{D}^{\dagger 2} - 4m_{3/2})\boldsymbol{G} \right] + \text{h.c.} \tag{2.100}$$

It may occasionally be more convenient to rewrite the Lagrangian as the following integral over all of superspace:

$$\mathcal{L} = \int d^4\Theta\, \boldsymbol{E}\, \boldsymbol{G}, \tag{2.101}$$

$$\boldsymbol{E} \equiv e(1 + m_{3/2}\Theta^2 + m_{3/2}\Theta^{\dagger 2} + 3m_{3/2}^2\Theta^4). \tag{2.102}$$

We have only discussed chiral superfields here. One can easily include vector superfields, as well, but the resulting component Lagrangian can be shown to be the same as in flat space. I note briefly here that Fayet-Iliopoulos terms are no longer gauge-invariant for SUSY in AdS, as $\int d^4\theta\, \boldsymbol{E}\, (\boldsymbol{\Omega} + \boldsymbol{\Omega}^\dagger) \neq 0$. This obstruction persists in SUGRA, where Fayet-Iliopoulos terms correspond to a gauged $U(1)_R$ symmetry [18, 67], which is difficult to reconcile with the explicit violation of R-symmetry here.

2.7.3 The Component Lagrangian

A short calculation yields the Lagrangian of Eq. (2.101) in components:

$$\mathcal{L} = G_{i\bar{j}}F^i F^{*\bar{j}} - iG_{i\bar{j}}\chi^{\dagger\bar{j}}\bar{\sigma}^\mu D_\mu\chi^i - G_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\phi^{*\bar{j}} + 3m_{3/2}^2 G \tag{2.103}$$

$$-\frac{1}{2}G_{i\bar{j}k}F^{*\bar{j}}\chi^i\chi^k - \frac{1}{2}G_{\bar{i}j\bar{k}}F^j\chi^{\dagger\bar{i}}\chi^{\dagger\bar{k}} + \frac{1}{4}G_{i\bar{j}k\bar{l}}\chi^i\chi^k\chi^{\dagger\bar{j}}\chi^{\dagger l} \tag{2.104}$$

$$+ m_{3/2}G_iF^i - \frac{1}{2}m_{3/2}G_{ij}\chi^i\chi^j + \text{h.c.} \tag{2.105}$$

After integrating out the auxiliary fields:

$$\mathcal{L} = -iG_{i\bar{j}}\chi^{\dagger\bar{j}}\bar{\sigma}^\mu D_\mu\chi^i - G_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\phi^{*\bar{j}} \tag{2.106}$$

$$+ \frac{1}{4}R_{i\bar{j}k\bar{l}}\chi^i\chi^k\chi^{\dagger\bar{j}}\chi^{\dagger l} \tag{2.107}$$

$$- \frac{1}{2}m_{3/2}\nabla_i G_j\chi^i\chi^j + \text{h.c.} \tag{2.108}$$

$$- m_{3/2}^2(G_iG^i - 3G) \tag{2.109}$$

This is written in a 'Kahler-covariant' manner. We can think of $G_{i\bar{j}}$ as a metric of sorts, with a well-defined inverse $G^{\bar{j}i}$. We raise and lower indices with the metric and its inverse. The metric has associated Christoffel symbols $\Gamma^i_{jk} = G^{i\bar{l}}G_{jk\bar{l}}$, $\Gamma^{\bar{i}}_{\bar{j}\bar{k}} = G^{\bar{i}l}G_{\bar{j}\bar{k}l}$, sensible covariant derivatives $\nabla_i G_j = G_{ij} - \Gamma^k_{ij}G_k$, and a curvature tensor $R_{i\bar{j}k\bar{l}} = G_{i\bar{j}k\bar{l}} - g_{m\bar{n}}\Gamma^m_{ik}\Gamma^{\bar{n}}_{\bar{j}\bar{l}}$. All of this of course simplifies if the Kahler metric is just $\delta_{i\bar{j}}$ (as it must be if we don't want any irrelevant operators), in which case $\nabla_i G_j = G_{ij}$ and the four-fermion term vanishes.

From this Lagrangian, we can easily find the mass spectrum

$$M_{ij} = m_{3/2}\nabla_i G_j \tag{2.110}$$

$$V_i = m_{3/2}^2(G^k\nabla_i G_k - 2G_i) \tag{2.111}$$

$$V_{i\bar{j}} = m_{3/2}^2(\nabla_i G_k\nabla_{\bar{j}}G^k - R_{i\bar{j}k\bar{l}}G^kG^{\bar{l}} - 2G_{i\bar{j}}) \tag{2.112}$$

$$\nabla_i V_j = m_{3/2}^2(G^k\nabla_i\nabla_j G_k - \nabla_i G_j) \tag{2.113}$$

$$m_{i\bar{j}}^2 = M_{ik}M^k{}_{\bar{j}} - 2m_{3/2}^2 G_{i\bar{j}} - m_{3/2}^2 R_{i\bar{j}k\bar{l}}G^kG^l \tag{2.114}$$

$$m_{ij}^2 = -m_{3/2}M_{ij} + m_{3/2}^2 G^k\nabla_i\nabla_j G_k \tag{2.115}$$

The last term on each of the last two lines contributes if there is SUSY-breaking ($\langle G_i \rangle \neq 0$).

2.7.4 The Goldstino

If SUSY is spontaneously broken in AdS, one still has a goldstino that transforms as a shift under SUSY transformations. However, since $\mathcal{D}_\mu \epsilon \neq 0$ in AdS, the kinetic goldstino Lagrangian by itself is no longer invariant under a shift symmetry. This can be rectified by the addition of a goldstino mass term

$$\mathcal{L} = -i\eta^\dagger \bar{\sigma}^\mu D_\mu \eta - m_{3/2}\eta\eta - m_{3/2}\eta^\dagger \eta^\dagger. \tag{2.116}$$

In AdS, the goldstino has a Lagrangian mass term, $m_\eta = 2m_{3/2}$.

We can also write down a superspace Lagrangian for the same $\boldsymbol{X}_{\rm NL} = \frac{1}{2F}(\eta + \sqrt{2}F\Theta)^2$ superfield:

$$\mathcal{L} = \int d^2\Theta\, 2\boldsymbol{\mathcal{E}} \left[-\frac{1}{8}(D^{\dagger 2} - 4m_{3/2})(\boldsymbol{X}^\dagger \boldsymbol{X} - F m_{3/2}^{-1} \boldsymbol{X} - F m_{3/2}^{-1} \boldsymbol{X}^\dagger) \right] + \text{h.c.} \tag{2.117}$$

If I couple the goldstino to matter, its couplings are proportional to deviations from the unbroken results—*not*, as in flat space, the boson-fermion mass difference (which is non-zero for unbroken SUSY). In particular, suppose I have a massless fermion and enough SUSY-breaking to make the scalars massless as well; then there is a goldstino coupling $\sim \frac{2m_{3/2}^2}{F}$ despite no boson-fermion mass difference. The same results can also be derived from supercurrent conservation, as in Sec. 2.5.1; we will consider this more thoroughly in Sec. 5.2.2.

2.8 Supergravity

So far, we have been considering global (or rigid) SUSY, in which the allowed spacetime dependence of the SUSY transformations is extremely constrained. Since SUSY is an extension of the Poincaré algebra, making SUSY a local symmetry requires making the rest of the Poincaré symmetry local as well—i.e. one needs to consider general relativity (GR). As a result, the terms 'local SUSY' and 'supergravity' (SUGRA) are used interchangeably in the literature. Most of the features of supergravity are well beyond the scope of this chapter, but we will discuss a few phenomenologically relevant features here.

The gauge fermion of local SUSY is spin-3/2 gravitino ψ_μ, so called because it is the superpartner of the spin-2 graviton of GR. Along with these, there are two auxiliary fields in the gravity multiplet,[12] a real vector b^μ and a complex scalar M. In SUGRA, the superpotential serves as a source for M^* and a mass term for the gravitino:

$$\mathcal{L} \supset -\frac{1}{3}M^*M - WM^* - \frac{W^*}{M_{\rm Pl}^2}\psi_\mu\sigma^{\mu\nu}\psi_\nu + \text{h.c.} \tag{2.118}$$

Defining $m_{3/2} \equiv \langle W \rangle / M_{\rm Pl}^2$, the gravitino gets a mass term of $m_{3/2}$, M^* obtains a vev of $-3m_{3/2}$, and there is a cosmological constant term $\langle V \rangle = -3m_{3/2}^2 M_{\rm Pl}^2$. The Einstein equation then yields an AdS spacetime, and we can finally see that $r_{\rm AdS}^{-1} = m_{3/2}$, as promised. This is just the local version of the SUSY in AdS considered in Sec. 2.7, so we see that a vev for M does not break SUSY (though it does break any $U(1)_R$ symmetry). F^i and D^a are still the SUSY-breaking order parameters, and since we are considering a theory with dynamical gravity, large enough amounts of SUSY breaking can lift the spacetime from AdS to flat space:

$$\langle V \rangle = F^i F_i^* + \frac{1}{2}D_a D^a - \frac{3|W|^2}{M_{\rm Pl}^2} \tag{2.119}$$

This is shown schematically in Fig. 2-2. As our universe is flat, this lets us express $m_{3/2}$ in terms of $M_{\rm Pl}$ and the amount of SUSY-breaking:

$$m_{3/2} = \frac{\sqrt{3}F_{\rm tot}}{M_{\rm Pl}}. \tag{2.120}$$

It should be stressed that this relation is a fine-tuning (the same fine-tuning as for the cosmological constant), and does not hold outside of flat space.

The gravitino transforms under SUSY as

$$\psi_\alpha^\mu \to \psi_\alpha^\mu + 2\left(\mathcal{D}^\mu\epsilon + \frac{i}{2}m_{3/2}\sigma^\mu\epsilon\right)_\alpha + \cdots \tag{2.121}$$

This is highly reminiscent of Eq. (2.81), the rigidity constraint on ϵ in AdS. This is no surprise, as rigid SUSY transformations had better not introduce a gravitino where there was not one to begin with [70, 114]. As with any other gauge field, the leading coupling of

[12]The question of what auxiliary fields are present in the gravity multiplet depends on what formalism one uses; I predominantly use that of Ref. [154] in this thesis.

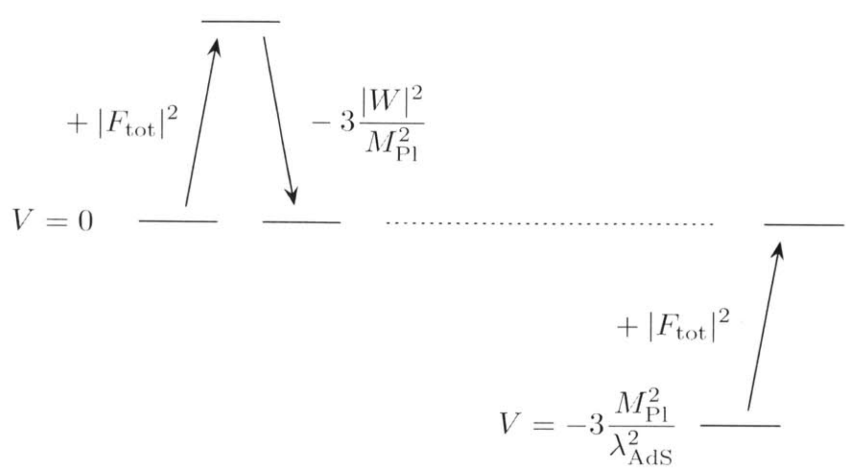

Figure 2-2: Two ways to think about achieving SUSY breaking with $V \simeq 0$. The second picture makes clear the underlying AdS_4 algebra.

the gravitino is to the corresponding Noether current, in this case the supercurrent:

$$\mathcal{L} \supset -\frac{1}{2M_{\rm Pl}}\psi_\mu j^\mu + \text{h.c.} \tag{2.122}$$

Naively, one might think that couplings of the gravitino are Planck-suppressed. However, this is not the case in practice, due to the gravitino's longitudinal polarizations, which come with inverse powers of $m_{3/2}$. When SUSY is broken, Eq. (2.122) contains a goldstino-gravitino mixing:

$$\mathcal{L} \supset \frac{i}{\sqrt{2}}\frac{F_{\rm tot}}{M_{\rm Pl}}\widetilde{G}_L^\dagger \overline{\sigma}^\mu \psi_\mu + \text{h.c.} \tag{2.123}$$

We can use SUSY transformations to eliminate the goldstino from the spectrum—in this unitarity gauge, the gravitino has eaten the goldstino to acquire its longitudinal degrees of freedom. This is the super-Higgs mechanism [55, 149, 71, 72], and it comes with an associated goldstino equivalence theorem [64, 65, 33, 34]—at energies well above $m_{3/2}$, the couplings of longitudinal gravitinos are well described by the couplings of goldstinos. As a result, the technical difficulties involved in performing calculations involving a spin-3/2 fermion can avoided in many cases, and gravitino couplings are really only suppressed by F^{-1}, not $M_{\rm Pl}^{-1}$.

Couplings suppressed by F^{-1} can still be quite weak, however, which has important implications for both collider and dark matter phenomenology. Gravitinos are unlikely to be phenomenologically relevant unless they are the LSP, in which case the lightest observable-sector SUSY particle (the LOSP) would be expected to decay to the gravitino [56, 8, 58, 7]. Therefore, SUSY cascade decays would be expected to always terminate in a LOSP to gravitino decay, making the identity of the LOSP quite important. If the coupling is weak enough, the LOSP lifetime may be long enough to see displaced vertices—or for it to be stable on collider time scales, which would be very striking for a charged or colored LOSP.

As a dark matter candidate, the weak couplings of the gravitino mean it does not annihilate efficiently in the early universe, and therefore would tend to be overproduced; the gravitino is most emphatically not a WIMP. This can be avoided if the reheating temperature of the universe is sufficiently low [129, 48]. The correct relic abundance can arise naturally by the super-WIMP mechanism [66] LOSPs are produced in the early universe with roughly the correct relic abundance (by the usual WIMP miracle), which then later decay to gravitinos after Big Bang Nucleosynthesis (BBN).

Chapter 3

Goldstini Give the Higgs a Boost

3.1 Introduction

As discussed in Sec. 2.5, most SUSY theories consist of an "observable sector" coupled to one or more "hidden sectors." The observable sector contains the fields of the supersymmetric standard model (SSM), in particular the lightest observable-sector supersymmetric particle (LOSP). The hidden sectors are responsible for breaking SUSY and generating soft masses for SM superpartners, and may contain light states accessible to colliders.

A typical SUSY collider event involves production of two heavy SM superpartners which then undergo cascade decays to a pair of LOSPs. If there are hidden sector particles lighter than the LOSP, then the subsequent LOSP decays—if they occur inside the detector—can dramatically impact SUSY collider phenomenology. In Sec. 2.8, we discussed the most well-known example of a decaying LOSP, which occurs when the light hidden sector particle is a gravitino [56, 8, 58, 7]. In that case, the LOSP decays to its superpartner and a longitudinal gravitino via interactions constrained by the conserved supercurrent and the goldstino equivalence theorem [64, 65, 33, 34]. For example, a mostly bino LOSP will decay to a photon, Z, or—through its small Higgsino fraction—a Higgs boson.

In this chapter, we will show how changes in the couplings between the observable and hidden sectors can have a dramatic impact on the decay modes of the LOSP, shown generically in Fig. 3-1. Our case study will be a nearly pure bino LOSP λ with an order one branching fraction to Higgs bosons, a very counterintuitive decay pattern from the point of view of the standard decay of a bino LOSP to a γ/Z plus a longitudinal gravitino. In fact, in this example, the LOSP branching ratio to Higgs bosons is *enhanced* with increasing

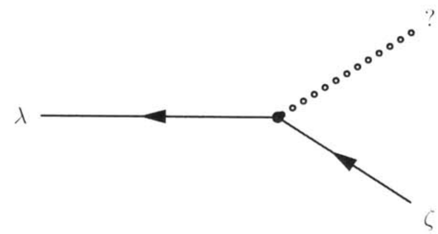

Figure 3-1: A generic LOSP decay. We will focus on the case where λ is a bino-like LOSP, and ζ is a (pseudo-)goldstino from spontaneous SUSY breaking. Contrary to the naive expectation, λ can decay dominantly to Higgs bosons, even if λ has negligible Higgsino fraction.

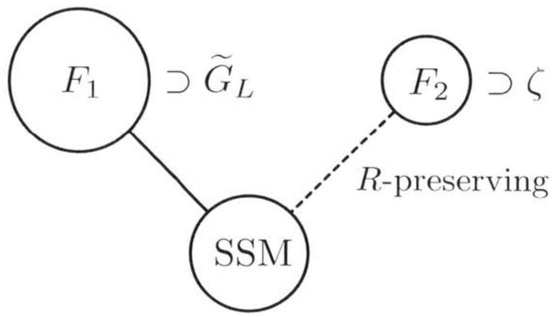

Figure 3-2: The R-symmetric setup that will be the focus of this chapter. Here, sector 1 has a higher SUSY breaking scale than sector 2, i.e. $F_1 \gg F_2$, so the LOSP preferentially decays to the pseudo-goldstino ζ coming mostly from sector 2. Since sector 2 preserves an R symmetry, the decay $\lambda \to \gamma/Z + \zeta$ is highly suppressed, and the mode $\lambda \to h^0 + \zeta$ can dominate.

Higgsino mass μ, approaching 100% in the small $(m_\lambda \tan\beta)/\mu$ limit. This is unlike the case of a Higgsino LOSP, which generically has equal branching fractions to Higgs and Z bosons.

These novel bino LOSP decays are possible in the presence of multiple sectors which break supersymmetry, yielding a corresponding multiplicity of "goldstini" [42]. While the couplings of the true goldstino (eaten by the gravitino) are constrained by the supercurrent, the orthogonal uneaten goldstini can have different couplings from the naive expectation. The spectrum of goldstini exhibits a number of fascinating properties [42, 45, 10], and they may play a role in cosmology or dark matter [40, 37]. Here, we will focus on properties of goldstini relevant for their collider phenomenology.

For our case study, we consider two sectors which break SUSY, both of which communicate to the SSM, but one of which preserves an $U(1)_R$ symmetry, as in Fig. 3-2.[1] For the

[1]There have been recent studies where the entire SUSY breaking and SSM sectors preserve a $U(1)_R$ symmetry [112, 9].

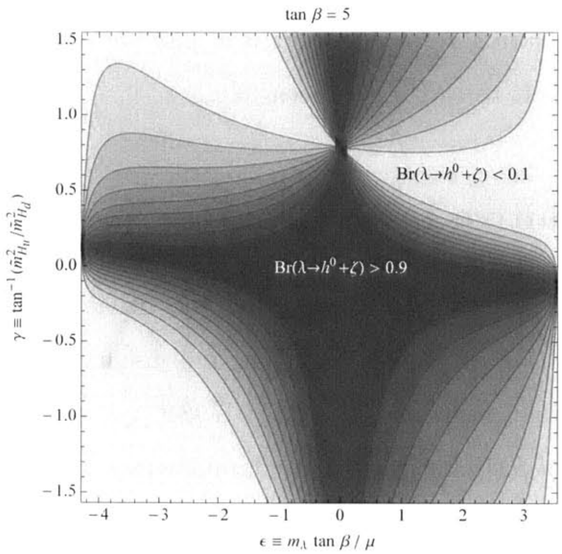

Figure 3-3: Branching ratio $\lambda \to h^0 + \zeta$ for a bino LOSP in the R-symmetric setup from Fig. 3-2. Throughout this parameter space, the remaining branching ratio is dominated by $\lambda \to Z + \zeta$. The expected mode $\lambda \to \gamma + \zeta$ is almost entirely absent. Shown is $\mathrm{Br}(\lambda \to h^0\zeta)$ as a function of $\epsilon \equiv m_\lambda \tan\beta/\mu$ and $\gamma \equiv \tan^{-1}(\widetilde{m}^2_{H_u}/\widetilde{m}^2_{H_d})$, fixing $\tan\beta = 5$ and $M_1 = 165$ GeV. The plot terminates on the left and right side at the kinematic bound $m_\lambda < m_{h^0}$.

appropriate hierarchy of SUSY breaking scales, the LOSP will couple more strongly to the uneaten goldstino ζ than to the longitudinal gravitino $\widetilde{G}_L$. Since the uneaten goldstino ζ is charged under the $U(1)_R$ symmetry, the R-violating decay $\lambda \to \gamma/Z+\zeta$ is suppressed, letting the counterintuitive decay $\lambda \to h^0 + \zeta$ dominate.[2] This fascinating result is demonstrated in Fig. 3-3.

In this way, goldstini can give the Higgs a boost: a boost in production cross section since most LOSP decays yield a Higgs boson; and a boost in kinematics since the Higgses are produced with relatively large gamma factors in SUSY cascade decays. This example gives further motivation to identify boosted Higgses using jet substructure techniques [30, 111, 2]. This example also motivates searches for other counter-intuitive LOSP decay patterns, where there is a mismatch between the identity of the LOSP and its decay products.

In the next section, we summarize and explain the main results of this chapter. We then describe the framework of goldstini in Sec. 3.3, and derive the low energy effective goldstini interactions and resulting LOSP decay widths in Sec. 3.4. We explain in more detail why the goldstini case differs from the more familiar gravitino case in Sec. 3.5. Plots

[2]In Ref. [42], it was erroneously claimed that in the presence of an R symmetry, the dominant decay is $\lambda \to \psi\bar{\psi} + \zeta$, where ψ is a SM fermion. This chapter corrects that error.

of the LOSP branching ratios appear in Sec. 3.6, and we conclude in Sec. 3.7. Various calculational details are left to the appendices.

3.2 Counterintuitive LOSP Decays

Throughout this chapter, we will be considering the situation where a LOSP decays to a lighter neutral fermion as in Fig. 3-1, and we will assume the minimal SSM (MSSM) field content. The possible decay patterns of a LOSP are constrained by symmetries, at minimum conservation of SM charges. In the familiar case where the LOSP decays to its superpartner and a gravitino, there are further constraints imposed by conservation of the supercurrent. We will see that these constraints can be significantly relaxed in the presence of multiple SUSY breaking sectors.

3.2.1 A Conventional Goldstino

In the conventional setup with a single SUSY breaking sector and a light gravitino, the couplings of the helicity-1/2 components of the gravitino are linked via the goldstino equivalence theorem to the couplings of the goldstino $\widetilde{G}_L$. Recall from Sec. 2.5.1 that supercurrent conservation implies that, at leading order in the inverse SUSY breaking scale $1/F$, the goldstino couples only derivatively to observable sector fields via the supercurrent:

$$\mathcal{L}_{\text{eff}} = -i\widetilde{G}_L^\dagger \bar{\sigma}^\mu \partial_\mu \widetilde{G}_L + \frac{1}{\sqrt{2}F} \partial_\mu \widetilde{G}_L j^\mu, \tag{3.1}$$

$$j^\mu = \sqrt{2}\sigma^\nu \bar{\sigma}^\mu \psi_i D_\nu \phi^{*i} - \frac{1}{2}\sigma^\nu \bar{\sigma}^\rho \sigma^\mu \lambda^{\dagger a} F^a_{\nu\rho}, \tag{3.2}$$

where we have elided terms that vanish on the goldstino equation of motion. Here, ϕ_i is a scalar and ψ_i is its fermionic superpartner, and $F^a_{\mu\nu}$ is a gauge field strength with λ^a its corresponding gaugino. In particular, the only possible LOSP decays are to its superpartner and a gravitino. This implies, for example, that a pure right-handed stau LOSP $\tilde{\tau}_R$ can only decay to a gravitino and a right-helicity tau τ_R, despite the fact that after electroweak symmetry breaking, there is no symmetry forbidding the decay to a left-helicity tau τ_L.

For concreteness we will focus on a bino-like LOSP throughout this chapter, though many of the following arguments hold with only minor modifications for a wino, as well. In that case, the supercurrent in Eq. (3.2) permits the decay $\lambda \to \gamma/Z + \widetilde{G}_L$ via the second term

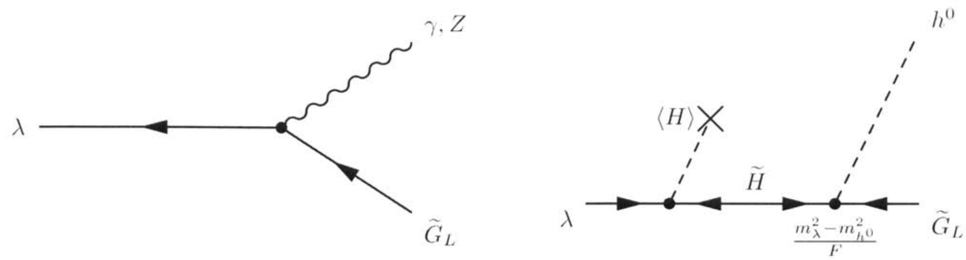

Figure 3-4: The standard decays of a bino-like LOSP to the longitudinal gravitino. They are primarily to a photon or Z (left), though the bino may also decay to a Higgs via its Higgsino component (right). The derivatives in Eq. (3.2) yield the Yukawa coupling labeled here, proportional to the mass-squared difference of the on-shell bino and Higgs. A cancellation between the two possible intermediate Higgsinos means the propagator contributes a factor of μ^{-2} to the amplitude at leading order, leading to a very large suppression of this channel in the Higgsino decoupling limit. Feynman diagrams throughout follow the framework of Ref. [61].

in the supercurrent. There is also a possible decay $\lambda \to h^0 + \widetilde{G}_L$ where h^0 is the physical Higgs boson, but since this occurs entirely through the Higgsino fraction of the LOSP, it will be comparatively suppressed.[3] Explicitly, to leading order in m_λ/μ, the dominant LOSP partial widths are

$$\Gamma_\gamma = \frac{m_\lambda^5 \cos^2\theta_W}{16\pi F^2}, \tag{3.3}$$

$$\Gamma_Z = \frac{m_\lambda^5 \sin^2\theta_W}{16\pi F^2}\left(1 - \frac{M_Z^2}{m_\lambda^2}\right)^4, \tag{3.4}$$

where $m_\lambda \simeq M_1$ is the bino-like LOSP mass, and θ_W is the weak mixing angle. The subdominant width to Higgs bosons is

$$\Gamma_{h^0} = \frac{m_\lambda^2 M_Z^2}{\mu^4}\frac{m_\lambda^5 \sin^2\theta_W \cos^2 2\beta}{32\pi F^2}\left(1 - \frac{m_{h^0}^2}{m_\lambda^2}\right)^2, \tag{3.5}$$

where $\tan\beta \equiv v_u/v_d$. Feynman diagrams for these standard decays are shown in Fig. 3-4.

3.2.2 Additional Operators?

In the case of the true goldstino $\widetilde{G}_L$, its couplings are saturated by Eq. (3.2). But if the LOSP were to decay not to a true goldstino but to a generic neutral fermion ζ, then there are many more operators that might mediate LOSP decay instead. For example, the dimension

[3] See Ref. [127] for a recent discussion of more general neutralino decays.

5 operator

$$\mathcal{O}^5_R = C^5_R \frac{\mu}{F} \lambda \zeta (H_u \cdot H_d)^* \qquad (\lambda \to h^0 + \zeta) \tag{3.6}$$

mediates the decay $\lambda \to h^0 + \zeta$ after electroweak symmetry breaking. Here, the coefficient μ/F has been chosen with malice aforethought, as this will turn out to be the approximate scaling behavior for the *eaten* goldstino. The subscript R indicates that this operator will preserve a $U(1)_R$ symmetry once we identify ζ with an uneaten goldstino of R-charge 1.

There are also additional operators at dimension 5 which violate this $U(1)_R$ symmetry,

$$\mathcal{O}^5_{\not{R},u\cdot d} = C^5_{\not{R},u\cdot d} \frac{\mu}{F} \lambda \zeta (H_u \cdot H_d), \tag{3.7}$$

$$\mathcal{O}^5_{\not{R},u} = C^5_{\not{R},u} \frac{\mu}{F} \lambda \zeta H_u^\dagger H_u, \tag{3.8}$$

$$\mathcal{O}^5_{\not{R},d} = C^5_{\not{R},d} \frac{\mu}{F} \lambda \zeta H_d^\dagger H_d. \tag{3.9}$$

Considering these $\mathcal{O}^5$ operators together, the partial width for the decay $\lambda \to h^0 + \zeta$ is

$$\Gamma_{h^0} = \left(C^5_{\text{net}}\right)^2 \frac{\mu^2 M_Z^2}{m_\lambda^4} \frac{m_\lambda^5 \sin^2\theta_W}{32\pi F^2} \left(1 - \frac{m_{h^0}^2}{m_\lambda^2}\right)^2. \tag{3.10}$$

Here, we have defined

$$C^5_{\text{net}} = \frac{\sqrt{2}}{g'} \left(\left(C^5_R + C^5_{\not{R},u\cdot d} \right) \cos(\alpha + \beta) - 2C^5_{\not{R},u} \sin\beta \cos\alpha + 2C^5_{\not{R},d} \cos\beta \sin\alpha \right), \tag{3.11}$$

with α being the physical Higgs mixing angle. Thus, if somehow the $\mathcal{O}^5$ operators were dominant over operators like those in Eq. (3.2), then the decay of a pure bino LOSP to a Higgs would dominate over the decay to a γ/Z. Note that the $\mathcal{O}^5$ operators only mediate a decay to one or more Higgs bosons, and not to a longitudinal Z, due to the gauge invariance of the scalar portion of the operators.

Now, in the conventional goldstino case, there is a sense in which the $\mathcal{O}^5$ operators are indeed generated after integrating out the Higgsino as in Fig. 3-5. This occurs not in the derivatively-coupled basis, but rather in the non-linear goldstino basis described in Sec. 3.3. The pertinent combination of Wilson coefficients attains the value

$$C^5_{\text{net}} = \frac{(m_{H_u}^2 - m_{H_d}^2)\sin 2\beta + 2B_\mu \cos 2\beta}{\mu^2} + \mathcal{O}\left(\frac{m_\lambda}{\mu}\right), \tag{3.12}$$

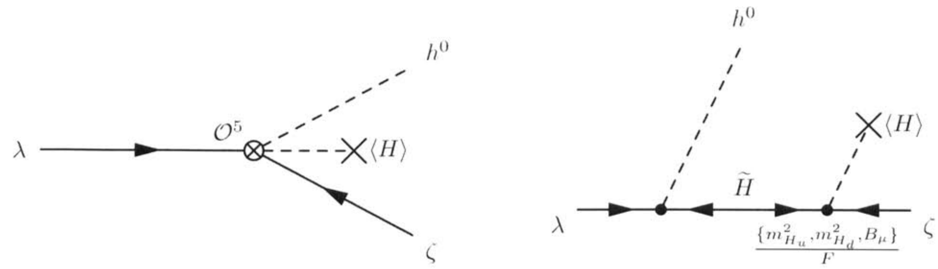

Figure 3-5: Additional diagrams which could contribute to LOSP decay. The dimension 5 operator (left) can be generated by integrating out an intermediate Higgsino (right). There is also a diagram with h^0 and $\langle H\rangle$ reversed. However, if ζ is a longitudinal gravitino $\widetilde{G}_L$, then the width $\Gamma(\lambda \to h^0 + \zeta)$ vanishes in the Higgsino decoupling limit.

which MSSM aficionados will recognize as being *zero* for the tree-level Higgs potential in the decoupling limit $|\mu| \gg M_Z$—the same limit in which it was legitimate to integrate out the Higgsinos in the first place (see App. A.1 for an explanation of this cancellation). This is as it must be; the physical predictions in this field basis must agree with those of the basis corresponding to the supercurrent picture of Eq. (3.2), in which the decay rate to Higgs bosons is highly suppressed.

However, because $C^5_{\text{net}} = 0$ arises only because of a delicate cancellation in the true goldstino case, any deviation will give rise to additional LOSP decays beyond the supercurrent prediction. In particular, if there are multiple sectors that break SUSY [42], each of which contributes only partially to the SSM soft masses, then the couplings of the uneaten goldstini cannot be determined by supercurrent considerations.[4] In general, the goldstini will have very different couplings from the gravitino; concretely, the goldstini need not be derivatively coupled to observable-sector particles. For a generic uneaten goldstino

$$C^5_{\text{net}} = \frac{(\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d})\sin 2\beta + 2\widetilde{B}_\mu \cos 2\beta}{\mu^2} + \mathcal{O}\left(\frac{m_\lambda}{\mu}\right), \tag{3.13}$$

where the tildes indicate the linear combination, appropriate to the given goldstino, of contributions from the SUSY-breaking sectors to the corresponding soft mass. These parameters need not cancel and thus a pure bino LOSP can exhibit the counterintuitive decay to a Higgs boson and an uneaten goldstino.

[4]This fact was recently exploited in Ref. [37] to arrange for goldstini dark matter with leptophilic decays.

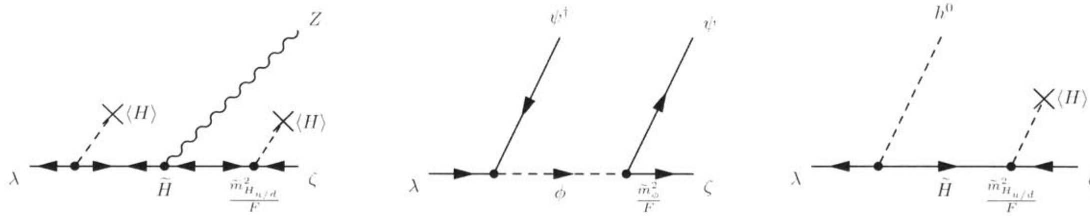

Figure 3-6: Representative diagrams contributing to the dimension 6 operators. After integrating out the intermediate Higgsinos or sfermions, these diagrams mediate LOSP decays to Z bosons and SM difermions, as well as generating additional LOSP decays to h^0.

3.2.3 Goldstini and R Symmetries

The differences between LOSP decays to an eaten goldstino versus an uneaten goldstino become especially striking in the presence of a $U(1)_R$ symmetry, and they will be the main example in this chapter. Consider the case of two SUSY breaking sectors as in Fig. 3-2 where the uneaten goldstino is associated with a sector 2 that preserves an R-symmetry. As we will argue in Sec. 3.3, if the scale of SUSY breaking in sector 1 is much higher than in sector 2, i.e. $F_1 \gg F_2$, then we can ignore the standard LOSP decay to a gravitino, since it will be overwhelmed by the LOSP decay to the uneaten goldstino from sector 2.

The gaugino soft mass terms violate the R-symmetry, so a bino LOSP cannot undergo the associated decay to a γ/Z and the uneaten goldstino ζ. Instead, it *must* (at tree level) decay to the uneaten goldstino via a virtual Higgsino or sfermion as in Figs. 3-5 and 3-6, producing a Higgs h^0, an arbitarily-polarized Z, or two SM fermions $\psi\bar{\psi}$ in the process.

To understand this effect more clearly, note there are only a limited number of R-symmetric operators that can mediate the decay of a bino LOSP to an uneaten goldstino and standard model particles once the Higgsinos and sfermions are integrated out. At dimension 5, only $\mathcal{O}^5_R$ respects the R-symmetry; the other $\mathcal{O}^5$ operators are associated with the R-symmetry-violating B_μ term. At dimension 6, we will show that the only operators consistent with gauge symmetries, R-parity, and our imposed R-symmetry are

$$\mathcal{O}^6_{\Phi,1} = \frac{C^6_{\Phi,1}}{F} i\zeta^\dagger\bar{\sigma}^\mu\lambda\Phi^\dagger D_\mu\Phi \qquad (\lambda \to h^0/Z + \zeta), \tag{3.14}$$

$$\mathcal{O}^6_{\Phi,2} = \frac{C^6_{\Phi,2}}{F} i\zeta^\dagger\bar{\sigma}^\mu\lambda(D_\mu\Phi^\dagger)\Phi \qquad (\lambda \to h^0/Z + \zeta), \tag{3.15}$$

$$\mathcal{O}^6_{\psi} = \frac{C^6_{\psi}}{F} (\zeta^\dagger\psi^\dagger)(\psi\lambda) \qquad (\lambda \to \psi\bar{\psi} + \zeta), \tag{3.16}$$

where Φ stands for either H_u or H_d, ψ is an SM fermion, and we have indicated in parentheses the corresponding LOSP decay mode. The values of the Wilson coefficients C^6 are omitted here for clarity; they are given explicitly in Eq. (3.33). Despite the fact that we have integrated out a Higgsino/sfermion, these operators are not suppressed by the Higgsino/sfermion mass as there is a cancellation between the propagator of the virtual heavy particle and its coupling to the goldstino. We will explain this fact in more detail in Sec. 3.4; it is sufficient to note for now that the $\mathcal{O}^6_\Phi$ are suppressed by a power of μ relative to $\mathcal{O}^5_R$.

The relative importance of $\mathcal{O}^5_R$, $\mathcal{O}^6_{\Phi,i}$, and $\mathcal{O}^6_\psi$ for LOSP decays depend sensitively on the SSM parameters. In general, the three-body decay $\lambda \to \psi\bar{\psi} + \zeta$ is subdominant to the two-body decays $\lambda \to h^0/Z + \zeta$. As mentioned already, $\mathcal{O}^5_R$ only mediates a decay to Higgs bosons, not to longitudinal Z bosons, whereas $\mathcal{O}^6_{\Phi,i}$ can yield either, or even a transverse Z. One might naively expect $\mathcal{O}^5_R$ to dominate over $\mathcal{O}^6_{\Phi,i}$, since the dimension 6 operator has a decay amplitude suppressed by m_λ/μ. However, $\mathcal{O}^5_R$ contains $H_u \cdot H_d$ which involves an additional $1/\tan\beta$ suppression in the large $\tan\beta$ limit, while the operators $\mathcal{O}^6_{H_u,i}$ have no such suppression. Thus, the dimension 6 decays are only suppressed by

$$\epsilon \equiv \frac{m_\lambda \tan\beta}{\mu} \tag{3.17}$$

compared to the dimension 5 decays, which may not even be a suppression at large $\tan\beta$.

In Fig. 3-3, we showed the LOSP branching ratios as a function of both ϵ and the most important other free parameter in the theory

$$\tan\gamma \equiv \frac{\widetilde{m}^2_{H_u}}{\widetilde{m}^2_{H_d}}, \tag{3.18}$$

which is the ratio of the contributions to $m^2_{H_u}$ and $m^2_{H_d}$ from the sector containing the uneaten goldstino. For special values of γ, the decay mode $\lambda \to Z+\zeta$ can either be completely suppressed or enhanced relative to $\lambda \to h^0+\zeta$ due to cancellations. Our main interest will be in the Higgsino decoupling limit with small ϵ, where the Higgs mode generically dominates.

Thus, in the presence of a R-symmetry, the LOSP decay to an uneaten goldstino gives a boost to Higgs boson production, even if (and especially if) the LOSP has a negligible Higgsino fraction. Moreover, the decays to the uneaten goldstino, whether featuring a Higgs boson or not, will completely dominate over any decays to the gravitino if there is an appropriate hierarchy between the two SUSY-breaking scales, as we will describe in the next section.

3.3 Goldstino and Gravitino Couplings

Having understood the possibility of enhanced $\lambda \to h^0 + \zeta$ decays from an operator perspective, the remainder of this chapter will show how precisely this works in the explicit example of multiple SUSY breaking sectors.

3.3.1 The General Framework

As in Ref. [42], we consider two sequestered sectors, each of which spontaneously breaks SUSY. Each sector has an associated goldstino (η_1 and η_2, respectively), and we characterize the size of SUSY breaking via the goldstino decay constants (F_1 and F_2, respectively). Each SUSY breaking sector can be parametrized in terms of a non-linear goldstino multiplet [109, 42]

$$X_i = \frac{\eta_i^2}{2F_i} + \sqrt{2}\theta\eta_i + \theta^2 F_i, \tag{3.19}$$

for $i = 1, 2$. We define the quantities

$$F \equiv \sqrt{F_1^2 + F_2^2}, \qquad \tan\theta \equiv \frac{F_2}{F_1}, \tag{3.20}$$

and we take $\tan\theta \leq 1$ ($F_1 \geq F_2$) without loss of generality.

The combination $\widetilde{G}_L = \sin\theta\,\eta_1 + \cos\theta\,\eta_2$ is eaten by the gravitino to become its longitudinal components via the super-Higgs mechanism, but the orthogonal goldstino $\zeta = \cos\theta\,\eta_1 - \sin\theta\,\eta_2$ remains uneaten and will be the focus of our study. For simplicity, we will work in the $M_{\text{Pl}} \to \infty$ limit where the uneaten goldstino remains massless, though in general ζ will get a mass proportional to $m_{3/2}$ via SUGRA effects, in particular $m_\zeta = 2m_{3/2}$ in the minimal goldstini scenario [42]. In addition, variations in the SUSY-breaking dynamics [45] or induced couplings between the two sectors [42, 10] can modify the mass term for ζ.[5]

[5] At minimum, one expects loops of SM fields to generate $m_\zeta \simeq m_{\text{soft}}/(16\pi^2)^n$ [42], where n depends on the number of loops necessary to effectively connect sectors 1 and 2 and transmit the needed $U(1)_R$ breaking. The uneaten goldstino will also obtain a tree-level mass due to mixing with the neutralinos, but this is of order $1/F^2$ and is comparatively negligible.

Supersymmetry breaking is communicated from the two hidden sectors to the visible sector by means of a non-trivial Kähler potential and gauge kinetic function (presumably coming from integrating out heavy messenger fields). Some representative terms contributing to the SSM soft masses are[6]

$$K = \Phi^\dagger \Phi \sum_i \frac{m_{\phi,i}^2}{F_i} X_i^\dagger X_i, \tag{3.21}$$

$$f_{ab} = \frac{1}{g_a^2}\delta_{ab}\left(1 + \sum_i \frac{2M_{a,i}}{F_i} X_i\right), \tag{3.22}$$

where $i = 1, 2$, and Φ stands for a general SSM multiplet. These yield the following terms in the Lagrangian up to order $1/F$ [23]:

$$\begin{aligned}\mathcal{L} = & -\sum_i m_{\phi,i}^2 \phi^* \phi + \sum_i \frac{m_{\phi,i}^2}{F_i} \eta_i \psi \phi^* \\ & -\frac{1}{2}\sum_i M_{a,i}\lambda^a\lambda^a + \sum_i \frac{iM_{a,i}}{\sqrt{2}F_i}\eta_i \sigma^{\mu\nu}\lambda^a F^a_{\mu\nu} + \sum_i \frac{M_{a,i}}{\sqrt{2}F_i}\eta_i \lambda^a D^a. \end{aligned} \tag{3.23}$$

Thus, the parameter $m_{\phi,i}^2$ ($M_{a,i}$) is the contribution to the SUSY-breaking scalar (gaugino) mass from each respective sector. Note that they are intrinsically related to the coupling of the SSM fields to the goldstini.

Rotating to the $\widetilde{G}_L$–ζ basis yields similar interaction terms for the eaten goldstino $\widetilde{G}_L$ and the uneaten goldstino ζ,

$$\mathcal{L}_{\widetilde{G}_L} = \frac{m_\phi^2}{F}\widetilde{G}_L\psi\phi^* + \frac{iM_a}{\sqrt{2}F}\widetilde{G}_L\sigma^{\mu\nu}\lambda^a F^a_{\mu\nu} + \frac{M_a}{\sqrt{2}F}\widetilde{G}_L\lambda^a D^a, \tag{3.24}$$

$$\mathcal{L}_\zeta = \frac{\widetilde{m}_\phi^2}{F}\zeta\psi\phi^* + \frac{i\widetilde{M}_a}{\sqrt{2}F}\zeta\sigma^{\mu\nu}\lambda^a F^a_{\mu\nu} + \frac{\widetilde{M}_a}{\sqrt{2}F}\zeta\lambda^a D^a, \tag{3.25}$$

where the untilded and tilded mass parameters associated with gauginos denote

$$M_a = M_{a,1} + M_{a,2}, \tag{3.26}$$

$$\widetilde{M}_a = M_{a,2}\cot\theta - M_{a,1}\tan\theta, \tag{3.27}$$

with the analogous notation for the scalar mass-squared parameters. Throughout, we will

[6] We only give the Kähler potential for a single species of scalar; more general A and B-terms involving multiple species can also be formed.

work in the limit $\cot\theta \gg 1$, for which we can take

$$\frac{\widetilde{M}_a}{F} = \frac{M_{a,2}}{F_2}, \qquad \frac{\widetilde{m}_\phi^2}{F} = \frac{m_{\phi,2}^2}{F_2}. \tag{3.28}$$

In this limit, as long as any of the $M_{a,2}$ or $m_{\phi,2}^2$ are at least on the order of the weak scale, LOSP decays to gravitinos are very suppressed and can be ignored for collider purposes. We see that as predicted via the supercurrent, the true goldstino $\widetilde{G}_L$ couples to SSM fields in proportion to the physical soft masses. In contrast, ζ couples via the *tilded* mass parameters which in the $\cot\theta \gg 1$ limit are proportional just to the contribution of sector 2 to the SSM soft masses.

3.3.2 The Decoupling and R-symmetric Limit

In this chapter, we will focus on the Higgsino decoupling and R-symmetric limits. That is, we will be considering the limit where μ is large compared to m_λ, and the limit where sector 2 preserves a $U(1)_R$ symmetry. There are a number of important features of this limit.

When the Higgsinos are decoupled, the soft terms $m_{H_u}^2$, $m_{H_d}^2$, and B_μ must scale as $\mathcal{O}(\mu^2)$ in order to get successful electroweak symmetry breaking.[7] We can see from the above Lagrangian that the coupling of $\widetilde{G}_L$ to a Higgsino and a Higgs is proportional to these $\mathcal{O}(\mu^2)$ soft SUSY-breaking masses. The same is true for the couplings of ζ if we make the additional simplifying assumption that the *tilded* mass parameters scale in the same fashion, so long as this is not forbidden by a symmetry. With one noted exception in Sec. 3.6.2, however, our results do not depend on this assumption. From the diagrams in Fig. 3-5, one would naively expect the amplitudes for the decay of a bino LOSP to the physical Higgs and either goldstino via a virtual Higgsino to be of order μ and thus dominant over other decays to the same goldstino in the decoupling limit. As we will argue in Sec. 3.5.2, there is a cancellation in the $\widetilde{G}_L$ case which renders the decay $\lambda \to h^0 + \widetilde{G}_L$ small, whereas for ζ, the decay $\lambda \to h^0 + \zeta$ can indeed dominate.

In the limit where sector 2 is R-symmetric, the contribution from sector 2 to SSM A-terms, B-terms, and gaugino masses is zero. Most relevant for our purposes, this implies

[7] Strictly speaking, this is only true for the combinations $m_{H_u}^2$ and $m_{H_d}^2 + B_\mu \tan\beta$ (working in the large $\tan\beta$ limit). However, if one simultaneously decouples the heavy Higgs scalars in the same way, so that $m_{A^0}^2$ is of order μ^2, then all three soft mass parameters scale as μ^2 barring accidental cancellations. Our later results for the uneaten goldstino are robust against this assumption, since $m_{H_u}^2$ has the desired scaling properties regardless.

that $\widetilde{B}_\mu$ and $\widetilde{M}_1$ are nearly zero. The absence of a $\widetilde{B}_\mu$ term implies that the cancellation in Eq. (3.12) seen for $\widetilde{G}_L$ cannot persist for the uneaten goldstino ζ. The absence of a $\widetilde{M}_1$ term means that the LOSP decay to a γ/Z and ζ is highly suppressed.[8] Both of these facts imply a large $\lambda \to h^0 + \zeta$ branching fraction. Depending on the relative importance of the dimension 5 or dimension 6 operators, the mode $\lambda \to Z + \zeta$ can be large as well.

3.4 Higgsino Decoupling Limit Effective Field Theory

Starting from the above goldstini framework, we can now systematically describe which operators contribute to bino LOSP decay in the Higgsino decoupling and R-symmetric limits. We will then give the resulting decay rates for the three main decay modes: $\lambda \to h^0 + \zeta$, $\lambda \to Z + \zeta$, and $\lambda \to \psi\bar{\psi} + \zeta$.

3.4.1 Leading R-symmetric Operators

In the Higgsino decoupling limit, it is convenient to organize the LOSP decay operators in terms of the small parameter m_λ/μ. This may be accomplished practically by integrating out the heavy Higgsino degrees of freedom, yielding an effective field theory with successively higher-dimension operators suppressed by additional powers of μ. Away from the decoupling limit, App. A.3 describes how to calculate the LOSP branching fractions for arbitrary μ. For simplicity, we will take $F_1 \gg F_2$, in which case the couplings of the uneaten goldstino are completely determined by sector 2.

Recall that in the MSSM, gauginos have R-charge 1, Higgs multiplets have R-charge 1, and matter multiplets have R-charge 1/2. For an R-symmetric SUSY breaking sector, the corresponding goldstino has R-charge 1. Putting this together, at dimension 5, there is only a single operator contributing to bino LOSP decay consistent with the symmetries of the theory (including the imposed R-symmetry):

$$\mathcal{O}_R^5 = C_R^5 \frac{\mu}{F} \lambda\zeta (H_u \cdot H_d)^*. \tag{3.29}$$

This operator may mediate the decay of a bino LOSP to the uneaten goldstino and one or two physical Higgs bosons h^0.[9]

[8] In the alternative limit where sector 1 preserves an R-symmetry, one expects $\lambda \to \gamma/Z + \zeta$ to still be relevant, but that will not be the focus of this chapter.

[9] Gauge invariance of the scalar portion of the operator forbids production of goldstone bosons (i.e. lon-

At dimension 6, there are three sorts of additional operators:[10]

$$\begin{aligned}
\mathcal{O}^6_{\Phi,1} &= \frac{C^6_{\Phi,1}}{F} i\zeta^\dagger\bar{\sigma}^\mu\lambda\Phi^\dagger D_\mu\Phi, &(3.30)\\
\mathcal{O}^6_{\Phi,2} &= \frac{C^6_{\Phi,2}}{F} i\zeta^\dagger\bar{\sigma}^\mu\lambda(D_\mu\Phi^\dagger)\Phi, &(3.31)\\
\mathcal{O}^6_{\psi} &= \frac{C^6_{\psi}}{F}(\zeta^\dagger\psi^\dagger)(\psi\lambda), &(3.32)
\end{aligned}$$

where Φ stands for either H_u or H_d, and ψ is a standard model fermion. The dimension 6 operators $\mathcal{O}^6_{\Phi,i}$ may produce a Z boson (longitudinal or otherwise) instead of or in addition to any Higgs boson production. The dimension 6 operator $\mathcal{O}^6_{\psi}$ will produce a difermion pair instead.[11] The effects of $\mathcal{O}^6_{\psi}$, but not the others, were considered in Ref. [42].

We have omitted two possible R-symmetric operators, $\partial^\mu\zeta\sigma^\nu\lambda^\dagger F_{\mu\nu}$ and $\partial^\mu\zeta\sigma^\nu\lambda^\dagger\widetilde{F}_{\mu\nu}$, which could mediate the decay of the bino to a photon or Z and the goldstino. It is clear by examining the original Lagrangian of Eq. (3.25) that in the R-symmetric limit with $\widetilde{M}_1 = 0$, a decay to a photon cannot occur at tree-level, so that any effects of such operators will be suppressed compared to the others of the same mass dimension.

The values of the Wilson coefficients for the above operators can be found by matching onto the original Lagrangian of Eq. (3.25):

$$\begin{aligned}
C^5_R &= \frac{g'\left(\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d}\right)}{\mu^2}, & C^6_{H_u,1} &= -\frac{g'\widetilde{m}^2_{H_u}}{\sqrt{2}\mu^2}, & C^6_{H_d,1} &= \frac{g'\widetilde{m}^2_{H_d}}{\sqrt{2}\mu^2},\\
C^6_\psi &= -\sqrt{2}g'Y_\psi\frac{\widetilde{m}^2_\phi}{m^2_\phi}, & C^6_{H_u,2} &= 0, & C^6_{H_d,2} &= 0. \qquad (3.33)
\end{aligned}$$

Here, g' is the hypercharge gauge coupling, Y_ψ is the hypercharge of the relevant SM fermion, and the tilded mass parameters are defined in Eq. (3.27). Inverse powers of the Higgsino mass-squared μ^2 and scalar mass-squared m^2_ϕ appear as expected, since these are the masses of the fields we are integrating out.

gitudinal W/Z bosons), and the heavier Higgs bosons A^0, H^0, and $H^\pm$ are of course kinematically excluded in the decoupling limit.

[10]We have used integration by parts to move all derivatives off of λ, and used field redefinitions to eliminate terms proportional to the equations of motion of the goldstino and gauge bosons. We elect not to use field redefinitions to eliminate terms proportional to $\bar{\sigma}^\mu\partial_\mu\lambda$, as the resulting operators (arising from the gaugino mass term) would violate the R-symmetry.

[11]This operator arises from integrating out intermediate sfermions as opposed to Higgsinos, so our power counting may be spoiled if there are any relatively light sfermions. We will later explicitly calculate the decay rate for $\lambda \to \bar{\psi}\psi + \zeta$ at tree level to all orders in m^2_λ/m^2_ϕ to account for this possibility.

The key observation is that the above Wilson coefficients are still order $\mathcal{O}(\mu^0)$ in the Higgsino decoupling limit,[12] since the soft masses scale as $\mathcal{O}(\mu^2)$. Thus, even if the LOSP has negligible Higgsino fraction, there are relevant bino-goldstino-Higgs couplings. As advertised, the leading decays in the Higgsino-decoupling and R-symmetric limits are

$$\lambda \to h^0 + \zeta, \qquad \lambda \to Z + \zeta, \qquad \lambda \to \psi\bar{\psi} + \zeta. \tag{3.34}$$

Now, using the effective operators of Sec. 3.4.1, we can calculate the various bino LOSP decay widths in the Higgsino decoupling and R-symmetric limits. Possible R-violating decays are described in App. A.2.

3.4.2 Decay to Higgs Bosons

The contributions to the $\lambda \to h^0 + \zeta$ decay from the dimension 5 and dimension 6 operators may be expressed in terms of an effective Yukawa interaction for on-shell states:[13]

$$\mathcal{L}_{\text{eff}} = -\frac{M_Z \mu \sin\theta_W}{\sqrt{2}F}\left(C^5_{\text{net}} + \frac{m_\lambda}{\mu}C^6_{\text{net}}\right)\lambda\zeta h^0. \tag{3.35}$$

The coefficients C^5_{net} and C^6_{net} are appropriate linear combinations of the Wilson coefficients of the dimension 5 and 6 operators, respectively, and are given explicitly in App. A.2. In the decoupling and R-symmetric limits, they take on the values

$$C^5_{\text{net}} = \frac{(\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d})\sin 2\beta}{\mu^2}, \tag{3.36}$$

$$C^6_{\text{net}} = \frac{\widetilde{m}^2_{H_u}\sin^2\beta - \widetilde{m}^2_{H_d}\cos^2\beta}{\mu^2}. \tag{3.37}$$

The decay rate via this channel is

$$\Gamma = \frac{m_\lambda \mu^2 M_Z^2 \sin^2\theta_W}{32\pi F^2}\left(C^5_{\text{net}} + \frac{m_\lambda}{\mu}C^6_{\text{net}}\right)^2\left(1 - \frac{m^2_{h^0}}{m^2_\lambda}\right)^2. \tag{3.38}$$

[12]Note that they are not of order 1, but rather of order $\cot\theta$. We have chosen to leave such dependence in the Wilson coefficients, rather than replacing F with F_2 everywhere, so that the only modification needed to describe the couplings of the *eaten* goldstino is to remove tildes from all soft mass parameters.

[13]This is not strictly speaking the whole story; the bino may also decay via two local dimension 5 operators ($\mathcal{O}^5_R$ and $\lambda\lambda(H_u \cdot H_d)^*$ or their wino equivalents) connected by a virtual wino or bino. However, their contributions to the decay amplitude are suppressed by $m_\lambda/(\mu\tan\beta)$ compared to that of O^5_R alone, or $1/\tan^2\beta$ to those of the dimension 6 operators, and can be safely ignored in most limits.

In the extreme decoupling limit, we would expect the $C^5_{\rm net}$ term, arising from the dimension 5 operator, to dominate over the effects of any dimension 6 operators, which are naturally suppressed by a factor of m_λ/μ. However, our power counting may be spoiled for large $\tan\beta$, due to the factor of $\sin 2\beta \approx 2/\tan\beta$ in $C^5_{\rm net}$. In the event that $\tan\beta$ is of the same order as μ/m_λ, we cannot neglect the dimension 6 operators. There are no such complications for the dimension 7 or higher operators, which may be safely ignored in the decoupling limit.

As a side note, there are only a few changes to the above calculation if we consider a wino LOSP. There are now *two* allowed operators at dimension 5—namely, $\lambda^a\zeta(H_u T^a\cdot H_d)^*$ and $\lambda^a\zeta(H_u\cdot T^a H_d)^*$—but the results throughout are almost identical, requiring only the replacement $g'\to -g$ or $\sin\theta_W\to -\cos\theta_W$ (as the neutral Higgsinos have T^3 and Y differing only by a sign). In particular, one can verify that there is no net coupling to the Z boson from the dimension 5 operators,[14] so the neutral wino LOSP decays dominantly to Higgs bosons in the small $(m_\lambda\tan\beta)/\mu$ limit.

3.4.3 Decay to Z Bosons

The dimension 5 operator does not contribute to Z decay. The dimension 6 operators mediate the decay $\lambda\to Z+\zeta$ due to the presence of covariant derivatives. Expanding the Lagrangian in unitarity gauge, we find a relatively simple coupling to the Z boson:

$$\mathcal{L} = \frac{M_Z^2\sin\theta_W}{\sqrt{2}F}C^6_{{\rm net},Z}\zeta^\dagger\bar\sigma^\mu\lambda Z_\mu, \tag{3.39}$$

with $C^6_{{\rm net},Z}$ being a different linear combination of the Wilson coefficients of the dimension 6 operators. The definition of $C^6_{{\rm net},Z}$ is given explicitly in App. A.2, and attains the value

$$C^6_{{\rm net},Z} = -\frac{\widetilde m^2_{H_u}\sin^2\beta+\widetilde m^2_{H_d}\cos^2\beta}{\mu^2} \tag{3.40}$$

in the decoupling and R-symmetric limit. The resulting decay rate is

$$\Gamma_Z = \frac{M_Z^2 m_\lambda^3\sin^2\theta_W}{32\pi F^2}\left(C^6_{{\rm net},Z}\right)^2\left(1-\frac{M_Z^2}{m_\lambda^2}\right)^2\left(1+2\frac{M_Z^2}{m_\lambda^2}\right). \tag{3.41}$$

[14] Dimension 5 operators can, however, induce a $\lambda^\pm\to W^\pm\zeta$ decay. Such decays may well be phenomenologically interesting, as the competing observable-sector decays ($\lambda^\pm\to l^\pm\nu\lambda_3$, $\lambda^\pm\to\pi^\pm\lambda_3$, et al. [36]) can be highly suppressed due to the near-degeneracy of the chargino and wino. However, such decays are certainly not specific to this R-symmetric limit, or even to the multiple goldstino model.

3.4.4 Decay to Difermions

Finally, the operator O_ψ^6 mediates the decay of a bino LOSP to a goldstino and a fermion pair. The decay rate from just this operator is

$$\Gamma_{\psi\bar{\psi}} = \frac{m_\lambda^5 \sec^2\theta_W}{32\pi F^2} \frac{\alpha_{\mathrm{EM}} Y_\psi^2}{12\pi} \frac{\widetilde{m}_\phi^4}{m_\phi^4} \tag{3.42}$$

in the limit of vanishing fermion masses.[15]

As argued in Ref. [42], the decay rate is non-zero even in the limit of very large scalar masses. However, due to the factor of $\alpha_{\mathrm{EM}} Y_\psi^2/(12\pi)$, the decay rate to fermions is typically subdominant to the Higgs and Z modes, even after summing over all possible fermion final states. One might wonder whether there could be an enhancement at moderate values of the scalar masses. Calculating the explicit tree-level decay rate for this mode to all orders in the scalar mass (while still working in the Higgsino decoupling limit), the result in Eq. (3.42) is multiplied by a function $f[m_\phi^2/m_\lambda^2]$:

$$f[x] = 6x^2\left(-5 + 6x + 2(x-1)(3x-1)\log\left[1 - \frac{1}{x}\right]\right) \simeq 1 + \frac{4}{5x} + \mathcal{O}\left(\frac{1}{x^2}\right). \tag{3.43}$$

This function never grows larger than 6 (at $m_\phi = m_\lambda$), and drops off quite sharply from that value as m_ϕ increases. For example, m_ϕ must be less than $1.25 m_\lambda$ for f to be greater than 2. Thus, the difermion mode is indeed subdominant. The sole exception occurs when $\widetilde{m}_{H_u}^2$ and $\widetilde{m}_{H_d}^2$ are both close to zero, where the Higgs and Z decay modes are suppressed.

3.5 Comparisons to the Gravitino Case

Before showing results for bino LOSP branching ratios in the next section, it is instructive to compare the R-symmetric goldstino results in Sec. 3.4 to the more familiar case of a gravitino. Indeed, the existence of a bino-goldstino-Higgs coupling in the Higgsino decoupling limit is quite surprising from the point of view of the more familiar longitudinal gravitino couplings, where it is known that the decay $\lambda \to h^0 + \widetilde{G}_L$ is highly suppressed. In this section, we will go to the Higgsino decoupling limit and calculate the effective interactions

[15] We also neglect here possible contributions from interference between diagrams featuring this operator and diagrams in which the fermions originate from an off-shell Higgs or Z produced by one of the other dimension 6 operators.

for a longitudinal gravitino. In the decoupling limit effective theory, we will find seemingly miraculous cancellations enforced by supercurrent conservation.

3.5.1 Additional Operations for the Gravitino

In the Higgsino decoupling limit for a longitudinal gravitino, the operators from Sec. 3.4.1 persist after the replacement $\zeta \to \widetilde{G}_L$, and they have the same Wilson coefficients as Eq. (3.33) after removing the tildes from the soft mass parameters. In addition, there are eight R-symmetry-violating operators at dimension 5 and 6 which contribute to bino LOSP decay. Their associated Wilson coefficients can again be found by matching[16]

$$\mathcal{O}^5_{\not{R},B} = C^5_{\not{R},B}\frac{M_1}{F}i\lambda\sigma^{\mu\nu}\widetilde{G}_L F_{\mu\nu}, \qquad C^5_{\not{R},B} = \frac{1}{\sqrt{2}}, \tag{3.44}$$

$$\mathcal{O}^5_{\not{R},H_u\cdot H_d} = C^5_{\not{R},H_u\cdot H_d}\frac{\mu}{F}\lambda\widetilde{G}_L(H_u\cdot H_d), \qquad C^5_{\not{R},H_u\cdot H_d} = 0, \tag{3.45}$$

$$\mathcal{O}^5_{\not{R},H_u} = C^5_{\not{R},H_u}\frac{\mu}{F}\lambda\widetilde{G}_L H_u^\dagger H_u, \qquad C^5_{\not{R},H_u} = \frac{g'}{\sqrt{2}}\left(\frac{B_\mu}{\mu^2} - \frac{M_1}{2\mu}\right), \tag{3.46}$$

$$\mathcal{O}^5_{\not{R},H_d} = C^5_{\not{R},d}\frac{\mu}{F}\lambda\widetilde{G}_L H_d^\dagger H_d, \qquad C^5_{\not{R},H_d} = -\frac{g'}{\sqrt{2}}\left(\frac{B_\mu}{\mu^2} - \frac{M_1}{2\mu}\right), \tag{3.47}$$

$$\mathcal{O}^6_{\not{R},1} = \frac{C^6_{\not{R},1}}{F}i\widetilde{G}_L^\dagger\bar{\sigma}^\mu\lambda(H_u\cdot D_\mu H_d)^*, \qquad C^6_{\not{R},1} = \frac{g'B_\mu}{\sqrt{2}\mu^2}, \tag{3.48}$$

$$\mathcal{O}^6_{\not{R},2} = \frac{C^6_{\not{R},2}}{F}i\widetilde{G}_L^\dagger\bar{\sigma}^\mu\lambda(D_\mu H_u\cdot H_d)^*, \qquad C^6_{\not{R},2} = -\frac{g'B_\mu}{\sqrt{2}\mu^2}, \tag{3.49}$$

$$\mathcal{O}^6_{\not{R},3} = \frac{C^6_{\not{R},3}}{F}i\widetilde{G}_L^\dagger\bar{\sigma}^\mu\lambda(H_u\cdot D_\mu H_d), \qquad C^6_{\not{R},3} = 0, \tag{3.50}$$

$$\mathcal{O}^6_{\not{R},4} = \frac{C^6_{\not{R},4}}{F}i\widetilde{G}_L^\dagger\bar{\sigma}^\mu\lambda(D_\mu H_u\cdot H_d), \qquad C^6_{\not{R},4} = 0. \tag{3.51}$$

The first operator $\mathcal{O}^5_B$ is exactly the second term in Eq. (3.24). The terms proportional to M_1 in $C^5_{\not{R},H_u}$ and $C^5_{\not{R},H_d}$ derive from the third term in Eq. (3.24), which contains the auxiliary field D. The remaining contributions arise from the R-symmetry-violating B_μ term.

Looking at these Wilson coefficients, one might (erroneously) conclude that in the Higgsino decoupling limit, a bino LOSP should dominantly decay to a gravitino via a physical Higgs instead of via a γ/Z. After all, the leading order bino-goldstino-Higgs couplings come

[16] There are also analogous results in the case of an uneaten goldstino in the absence of an R-symmetry, as long as tildes are added to the soft mass parameters and $\widetilde{G}_L$ is replaced with ζ. See App. A.2.

from four dimension-5 operators—$\mathcal{O}^5_R$, $\mathcal{O}^5_{\not{R},H_u\cdot H_d}$, $\mathcal{O}^5_{\not{R},H_u}$, and $\mathcal{O}^5_{\not{R},H_d}$—which are enhanced by a factor of μ/m_λ compared to the bino-goldstino-γ/Z coupling from $\mathcal{O}^5_B$.

However, we know this not to be the case for the gravitino. From conservation of the supercurrent, the decay rate for $\lambda \to h^0 + \widetilde{G}_L$ given in Eq. (3.5) is suppressed in the decoupling limit by a factor of $\mathcal{O}(m_\lambda^2 M_Z^2/\mu^4)$ from the decay rates for $\lambda \to \gamma/Z + \widetilde{G}_L$ given in Eqs. (3.3) and (3.4). Apparently, when calculating the decay rate of a bino LOSP to a Higgs boson and a gravitino using the decoupling limit effective field theory, the contributions to the amplitude from the dimension 5, dimension 6, and dimension 7 operators yield cancellations up to three orders in the m_λ/μ expansion.

3.5.2 Miraculous Cancellations

The easiest way to see that there must be a cancellation is to go back to the gravitino coupling from Eq. (3.24) before integrating out the Higgsino. We can make a standard SUSY transformation on all of our visible sector fields with infinitesimal parameter $\widetilde{G}_L/F$,

$$\phi \quad \to \quad \phi + \frac{1}{F}\psi \widetilde{G}_L, \tag{3.52}$$

with similar expressions for other fields. This is an allowed field redefinition since it leaves the one-particle states unchanged. Since the coefficients of the SUSY-breaking mass terms and the couplings of $\widetilde{G}_L$ are identical up to a sign, the coupling terms (at order $1/F$) *cancel* under this transformation. This cancellation is special to the eaten goldstino and does not in general occur for an uneaten goldstino. The SUSY-respecting part of the Lagrangian will clearly remain unchanged under this field redefinition except for terms proportional to $\partial_\mu \widetilde{G}_L$. Thus, $\widetilde{G}_L$ only couples derivatively to MSSM particles, and does so in exactly the manner described by the supercurrent formalism of Eq. (3.2).

It is also instructive to see how this cancellation works in the decoupling limit effective field theory. The $\lambda \to h^0 + \widetilde{G}_L$ decay may still be completely parametrized as a Yukawa interaction as in Eq. (3.35) for the leading two orders in m_λ/μ:[17]

$$\mathcal{L} \quad = \quad -\frac{M_Z \mu \sin\theta_W}{\sqrt{2}F}\left(C^5_{\text{net}} + \frac{m_\lambda}{\mu} C^6_{\text{net}}\right)\lambda \widetilde{G}_L h^0. \tag{3.53}$$

[17]The diagrams featuring two dimension 5 operators connected by a virtual bino or wino cancel separately.

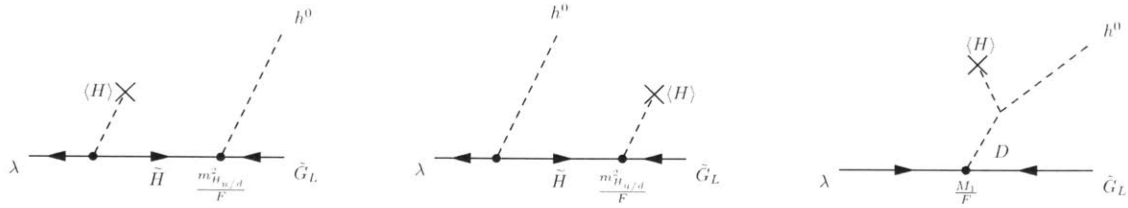

Figure 3-7: These diagrams, which we would expect to yield $\mathcal{O}(\mu^0)$ contributions to the $\lambda \to h^0 + \widetilde{G}_L$ amplitude, cancel among themselves.

The coefficients C^5_{net} and C^6_{net} have new contributions proportional to B_μ and M_1:

$$C^5_{\text{net}} = \frac{(m^2_{H_u} - m^2_{H_d})\cos(\alpha+\beta) - 2B_\mu \sin(\alpha+\beta)}{\mu^2} + \frac{M_1}{\mu}\sin(\alpha+\beta), \tag{3.54}$$

$$C^6_{\text{net}} = \frac{m^2_{H_d}\cos\beta\sin\alpha + m^2_{H_u}\sin\beta\cos\alpha - B_\mu\cos(\beta-\alpha)}{\mu^2}. \tag{3.55}$$

If one uses the tree-level relations for the parameters in the Higgs potential (see App. A.1), these simplify considerably:

$$C^5_{\text{net}} = -\frac{M_1\cos 2\beta}{\mu} + \mathcal{O}\left(\frac{M_Z^2}{\mu^2}\right), \qquad C^6_{\text{net}} = \cos 2\beta + \mathcal{O}\left(\frac{M_Z^2}{\mu^2}\right). \tag{3.56}$$

We see that the $\mathcal{O}(1)$ term in C^5_{net} have cancelled entirely, and the $\mathcal{O}(M_1/\mu)$ term, which arose from the $\lambda\widetilde{G}_L D$ term in Eq. (3.25), cancels against C^6_{net} since $M_1 = m_\lambda$ at this order. Diagrammatically, the first cancellation is among the diagrams in Fig. 3-5, and the second cancellation is among those in Fig. 3-7. There is yet another cancellation at the next order in μ involving dimension 7 operators, but it is not instructive to show it explicitly here; it may be verified using the methods of App. A.3 after diagonalizing the neutralino mass matrix order by order in μ.

3.5.3 Why Goldstini are Different

These miraculous cancellations for the gravitino case, removing the leading three orders of contributions to the bino LOSP decay to Higgs, are very specific to the gravitino and the values of its associated Wilson coefficients. There is much more freedom in choosing the couplings of the uneaten goldstino. Concretely, the Wilson coefficients feature the *tilded* versions of soft SUSY-breaking mass parameters, recalling $\widetilde{M}_i = M_{i,2}\cot\theta - M_{i,1}\tan\theta$ from Eq. (3.27). These tilded parameters need not satisfy any a priori relation among themselves, and thus the cancellations above will not occur in general for a goldstino.

Said another way, the mechanisms which ensured the cancellations for the gravitino are not applicable in the goldstino case. The field redefinition of Eq. (3.52) made it manifest that the gravitino couples derivatively to observable sector fields, but the same cannot be done in general for the uneaten goldstino. We could attempt to remove one such coupling with the same sort of SUSY transformation, with

$$\phi \to \phi + \frac{1}{F}\frac{\widetilde{m}_\phi^2}{m_\phi^2}\psi\zeta, \tag{3.57}$$

but unless $\widetilde{m}_\phi^2/m_\phi^2 = \widetilde{M}_a/M_a$ for all scalar and gaugino mass terms, there is no transformation that will remove all such couplings and make ζ purely derivatively coupled.

Thus, one expects a variety of counterintuitive LOSP decay patterns in the presence of goldstini, such as wrong-helicity decays like $\tilde{\tau}_R \to \tau_L + \zeta$, flavor-violating decays, or reshuffled neutralino/chargino branching fractions. Of course, the phenomenological differences between a longitudinal gravtino and an uneaten goldstino are highlighted when the "standard" decay is forbidden. This is precisely the case for our bino LOSP in the Higgsino decoupling and R-symmetric limit, where the standard γ/Z decay is suppressed and the novel h^0 mode can dominate.

3.6 Branching Ratio Results

We now discuss the bino LOSP branching ratios in the presence of multiple SUSY breaking sectors, using the R-symmetric setup from Fig. 3-2. In the bulk of parameter space, the decay mode $\lambda \to \psi\bar{\psi} + \zeta$ is suppressed, so we will first focus on the branching ratios to Higgs and Z bosons, neglecting any three-body decays. A brief discussion of what happens away from the R-symmetric limit appears in Sec. 3.6.3.

3.6.1 Higgs and Z Boson Branching Ratios

When three-body decays can be neglected, the dominant phenomenology is determined by the two parameters

$$\epsilon \equiv \frac{m_\lambda \tan\beta}{\mu}, \qquad \tan\gamma \equiv \frac{\widetilde{m}_{H_u}^2}{\widetilde{m}_{H_d}^2}, \tag{3.58}$$

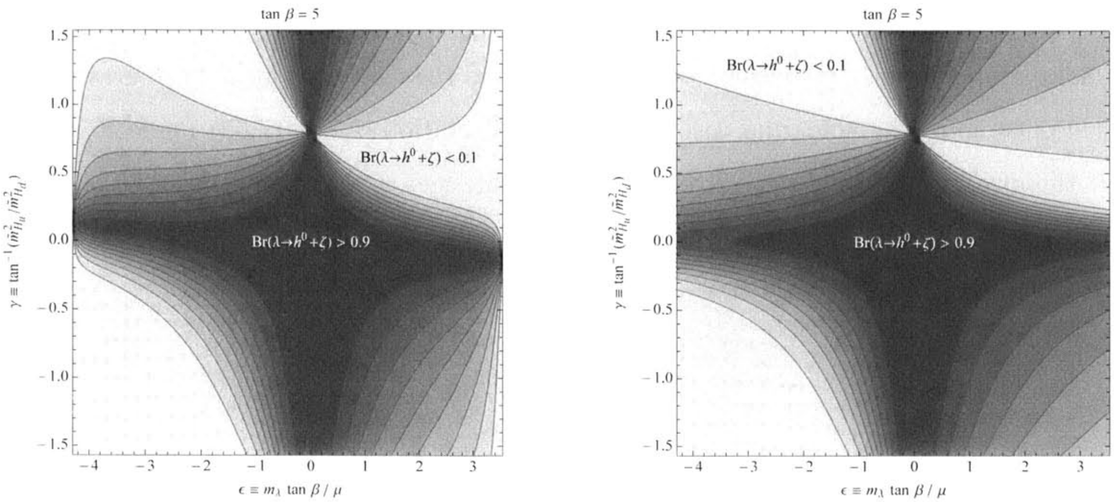

Figure 3-8: Branching ratios for $\lambda \to h^0 + \zeta$ in the ϵ–γ plane for $\tan\beta = 5$ (left, same as Fig. 3-3) and $\tan\beta = 20$ (right), respectively. The remaining branching ratio is dominated by $\lambda \to Z + \zeta$. The main differences between the two plots arise because at larger $\tan\beta$, the kinematically excluded region $m_\lambda < m_h^0$ (which bounds the left plot) is not encountered until larger ϵ. In this and the remaining plots, we have fixed $M_1 = 165$ GeV, which is mainly relevant for setting the phase space factors in the partial widths.

previously mentioned in Eqs. (3.17) and (3.18). Using the partial widths calculated in Eq. (3.38) and Eq. (3.41), the branching ratio for the bino LOSP decay to h^0 or Z, assuming both are kinematically allowed, may be expressed in the relatively compact form:

$$\mathrm{Br}(\lambda \to h^0\zeta) = \frac{\left(\frac{\epsilon^{-1}-\epsilon_0^{-1}}{\omega}\right)^2}{1+\left(\frac{\epsilon^{-1}-\epsilon_0^{-1}}{\omega}\right)^2}, \qquad \mathrm{Br}(\lambda \to Z\zeta) = \frac{1}{1+\left(\frac{\epsilon^{-1}-\epsilon_0^{-1}}{\omega}\right)^2}. \tag{3.59}$$

In particular, the branching ratio to Z bosons is a Lorentzian in ϵ^{-1} and is thus negligible for small ϵ, as expected. The Lorentzian is centered at ϵ_0^{-1} with a width ω,

$$\epsilon_0^{-1} = \frac{1-\tan\gamma\tan^2\beta}{2\tan^2\beta(\tan\gamma-1)}, \tag{3.60}$$

$$\omega = \frac{1+\tan\gamma\tan^2\beta}{2\tan^2\beta(\tan\gamma-1)}\left(\frac{m_\lambda^2-M_Z^2}{m_\lambda^2-m_{h^0}^2}\sqrt{1+2\frac{M_Z^2}{m_\lambda^2}}\right), \tag{3.61}$$

where the precise values of ϵ_0^{-1} and ω depend on the Higgs soft mass ratio $\tan\gamma$, $\tan\beta$, and various kinematic factors. Of course, additional three-body decays, whether to fermions or to multiple Higgs or Z bosons, will spoil the simplicity of these expressions.

Plots of the branching ratio to Higgs in the ϵ–γ plane are shown in Fig. 3-8, and slices

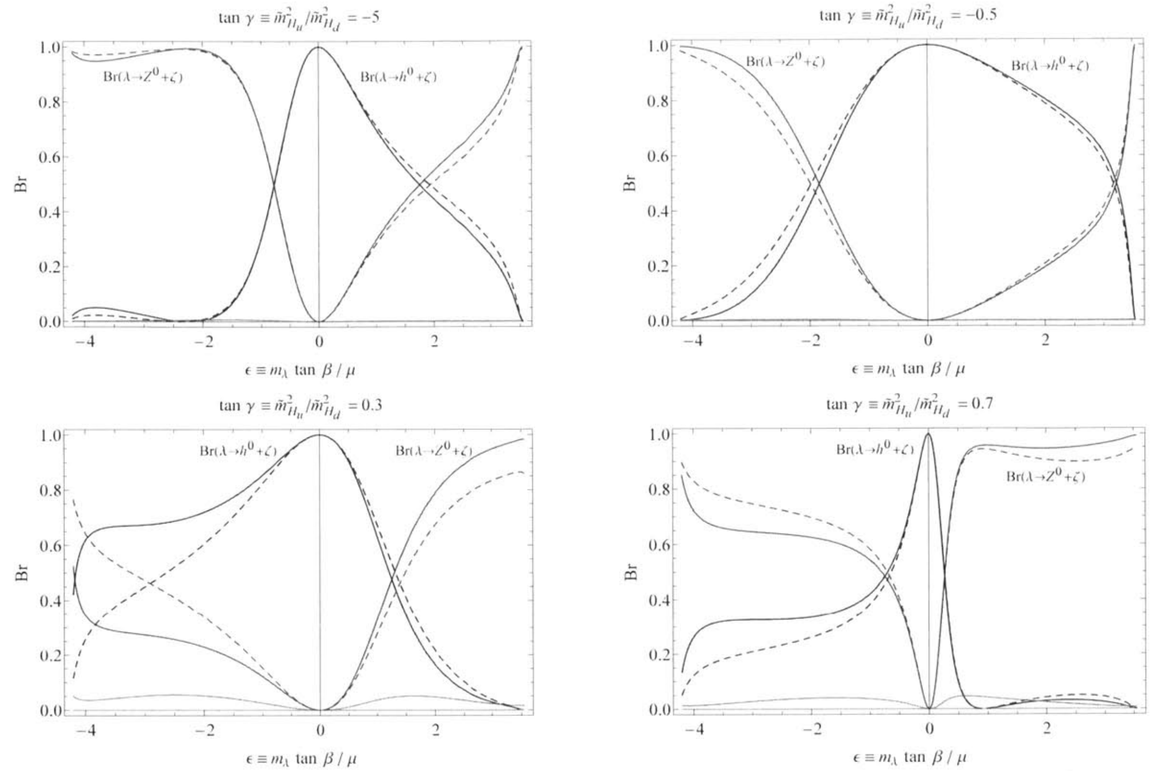

Figure 3-9: Branching ratios for the bino LOSP as a function of ϵ for fixed values of $\tan\gamma$. These are all slices of the left plot in Fig. 3-8 with $\tan\beta = 5$ and $M_1 = 165$ GeV. The solid curves are the all-orders result from App. A.3, while the dashed curves are from the Higgsino decoupling effective theory in Sec. 3.4. The curves are $\mathrm{Br}(\lambda \to h^0\zeta)$ (blue), $\mathrm{Br}(\lambda \to Z\zeta)$ (red), $\mathrm{Br}(\lambda \to \psi\bar{\psi}\zeta)$ (green), and $\mathrm{Br}(\lambda \to \gamma\zeta)$ (yellow). The decay to Higgses dominates in the small ϵ limit, with the next most relevant mode being the Z. The branching ratio to difermions is calculated using the results of Sec. 3.6.2, taking the parameter ρ defined in Eq. (3.64) to be 1.0. As advertised, this branching ratio to difermions is very suppressed, and the branching ratio to photons is essentially zero.

through that plane are shown in Figs. 3-9 and 3-10. In the latter plots, the solid lines are the all-orders tree-level calculations from App. A.3, while the dashed lines are the analytic results obtained using the Higgsino decoupling effective theory from Sec. 3.4 (while still using the all-orders result for the physical LOSP mass m_λ).

The small ϵ limit corresponds to the extreme Higgsino decoupling regime, where not only $|\mu| \gg m_\lambda$, but the $\tan\beta$ suppressed dimension 5 operator $\mathcal{O}^5_R$ dominates over the dimension 6 operators. Thus, generically, for small ϵ, the decay is overwhelmingly to Higgs bosons, as expected. However, there is an exception for the region around $\tan\gamma = 1$. When $\tan\gamma = 1$, $\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d}$ and $C^5_{\rm net}$ are both zero and the branching ratios to Higgs and Z bosons should be roughly equal up to phase space factors. For $\tan\gamma$ slightly removed from

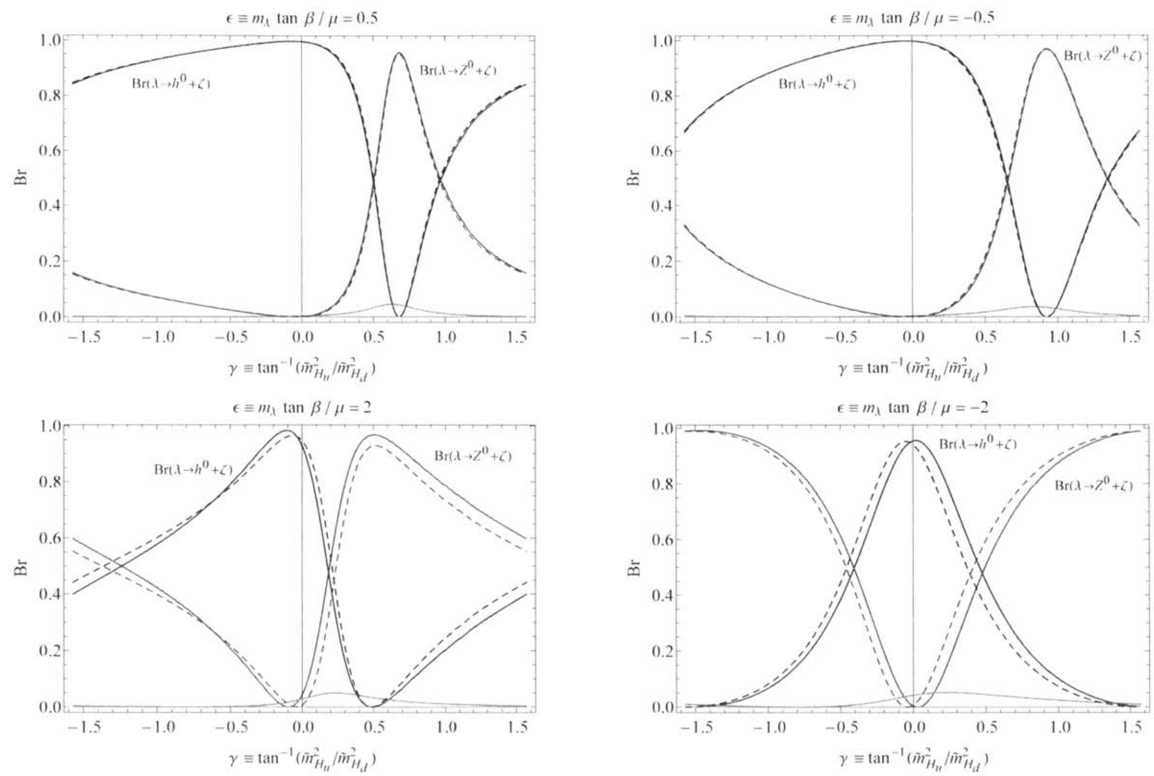

Figure 3-10: Same as Fig. 3-9, but with branching ratios given as a function of γ for fixed values of ϵ.

unity (downwards for $\epsilon > 0$, upwards for $\epsilon < 0$), C^5_{net} will destructively interfere with C^6_{net} and the Z mode will dominate.

Moving away from small ϵ, we expect the Z branching ratio to increase, as the contributions from dimension 6 operators to bino decay are roughly equal for the Higgs and Z modes. This is shown in Fig. 3-9. The effects of interference between the dimension 5 and dimension 6 operators on the Higgs amplitude also grow more pronounced for larger ϵ. For $\epsilon > 0$, the interference is destructive for $\tan\gamma \in (1/\tan^2\beta, 1)$, and vice versa for $\epsilon < 0$.[18]

For extremely large ϵ, the approximations based on being in the Higgsino decoupling limit break down as m_λ/μ approaches $\mathcal{O}(1)$. Ultimately, the Higgs mode is kinematically excluded once the mass of the lightest neutralino (by now predominantly Higgsino) drops below the Higgs mass.

3.6.2 Difermion Branching Ratio

In most of parameter space, the decay mode $\lambda \to \psi\bar{\psi} + \zeta$ is suppressed. We can see this most clearly by comparing the decay rate to all fermion species to the decay rate to a Z:

$$\frac{\sum_\psi \Gamma_{\psi\bar{\psi}}}{\Gamma_Z} = \frac{\alpha_{\text{EM}}}{3\pi \sin^2 2\theta_W} \frac{\sum_i Y_i^2 \tau_i^2 f_i}{\left(C^6_{\text{net},Z}\right)^2} \frac{m_\lambda^2}{M_Z^2} \left(1 - \frac{M_Z^2}{m_\lambda^2}\right)^{-2} \left(1 + 2\frac{M_Z^2}{m_\lambda^2}\right)^{-1}, \tag{3.62}$$

where $\tau_i \equiv \widetilde{m}^2_{\phi_i}/m^2_{\phi_i}$ and $f_i \equiv f[m^2_{\phi_i}/m^2_\lambda]$, with the function f defined in Eq. (3.43).

For concreteness, consider the limit where $\tan\beta \gg 1$, $|\mu|, m_{\phi_i} \gg m_\lambda$, and the τ_i are all equal to a common value τ_0. The sum over SM fermion hypercharges (excluding the presumably kinematically inaccessible top) is 103/12. Assuming that the tree-level relations between the soft masses approximately hold, $C^6_{\text{net},Z} = \tau_0$. All the τ_i values then cancel, and the net result is

$$\frac{\sum_\psi \Gamma_{\psi\bar{\psi}}}{\Gamma_Z} \approx \frac{1}{107} \frac{m_\lambda^2}{M_Z^2} \left(1 - \frac{M_Z^2}{m_\lambda^2}\right)^{-2} \left(1 + 2\frac{M_Z^2}{m_\lambda^2}\right)^{-1}. \tag{3.63}$$

This ratio obtains a minimum of around 1/28 at $m_\lambda \approx 140$ GeV, and it is smaller than 1/10 for m_λ in the approximate range 100–300 GeV.

Of course, there is one somewhat contrived region of parameter space for which the

[18] The operative relative sign is that between μ and m_λ. The $\mathcal{O}^5$ operator features an odd power of μ, while the m_λ factor comes from the C^6 operators, whose only non-vanishing contributions feature the Dirac equation applied to the external bino spinor.

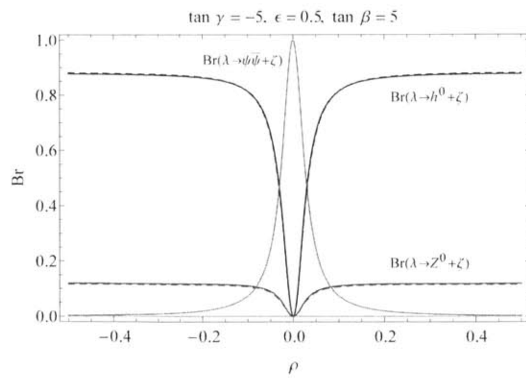

Figure 3-11: Branching ratios for the bino LOSP as a function of the parameter ρ defined in Eq. (3.64) below, measuring in effect the relative contributions to the Higgs and sfermion mass terms by sector 2. If this parameter is tuned close to zero, then the Higgs and Z modes shut off, leaving only the difermion channel. For larger values of ρ, the difermion channel is suppressed; this occurs generically when the tilded Higgs soft mass parameters scale with μ^2, as mentioned in Sec. 3.3.2. For concreteness, all prior figures have used $\rho = 1.0$.

decay to fermions can dominate; if the sector containing the uneaten goldstino gives no contribution to any of the Higgs soft masses, then $\widetilde{m}^2_{H_u}$ and $\widetilde{m}^2_{H_d}$ vanish and the decay via an off-shell sfermion are the only ones allowed. Fig. 3-11 shows that the decay to fermions can indeed dominate if the parameter

$$\rho \equiv \frac{\widetilde{m}^2_{H_u} + \widetilde{m}^2_{H_d}}{2\mu^2 \cot\theta} \frac{\sum_i Y_i^2}{\sum_i Y_i^2 \tau_i f_i}, \tag{3.64}$$

with sums taken over all appropriate sfermion species, is tuned close enough to zero.

3.6.3 The R-violating Regime

Though not the focus of this chapter, we wish to briefly comment on possible R-violating decays, for which calculations are given in App. A.2. As we move away from the R-symmetric limit, the LOSP decay to photons is now allowed at tree level, and will generally garner a branching ratio that is at least of the same order as of those to Higgs or Z. In Fig. 3-12, we show branching ratios as a function of a parameter δ which measures the amount of deviation from the R-symmetric limit,

$$\delta \equiv \frac{2}{3} \frac{\tau_1 + \tau_2 + \tau_{B_\mu}}{\tau_{H_u} + \tau_{H_d}}, \tag{3.65}$$

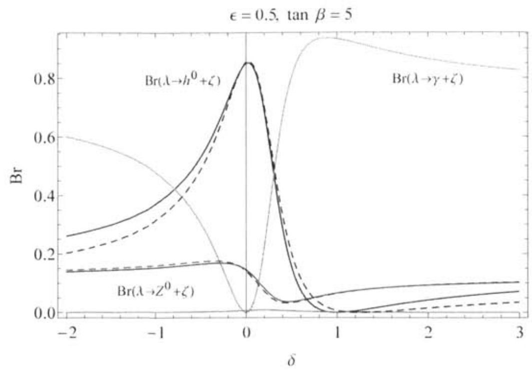

Figure 3-12: Branching ratios for the bino LOSP as a function of the parameter δ, defined in Eq. (3.65), that measures the deviation from the R-symmetric limit. When $\delta = 1$, we are in the R-symmetric limit of the previous figures. When $\delta = 1$, the branching ratios for $\lambda \to X + \zeta$ are exactly what one would predict for $\lambda \to X + \widetilde{G}_L$ in the more conventional model with only one hidden sector; the photon mode dominates and the Higgs mode is highly suppressed.

with $\tau_i \equiv \widetilde{M}_i/M_i$ for any soft mass(-squared) parameter M_i. In Fig. 3-12, we hold $\tau_1 = \tau_2 = \tau_{B_\mu}$ and $\tau_{H_d} = \tau_{H_u} = \tau_{\phi_i}$ for simplicity. When $\delta = 0$ we have the exact R-symmetric limit; when $\delta = 1$ we have the "aligned" limit in which the uneaten goldstino couples simply as a rescaled version of the gravitino (i.e. there is a basis, obtained by making the field redefinition Eq. (3.57), in which it couples only derivatively). Note in the latter limit the Higgs branching ratio effectively shuts off, as expected.

The diversity of possible LOSP decay branching ratios shown in Fig. 3-12 is reminiscent of mixed neutralino LOSP scenarios, where the LOSP has comparable bino, wino, and Higgsino fractions. Here, however, we are still working in the Higgsino decoupling limit, so the interesting pattern of LOSP widths come not from varying the identity of the LOSP but rather from varying how the hidden sectors couple to the SSM.

3.7 Conclusion

SUSY breaking scenarios with a light gravitino offer fascinating phenomenological possibilities. With the LOSP no longer stable, gravitinos could comprise part or all of the dark matter of the universe, and collider experiments could discover extended SUSY cascade decays. However, the gravitino need not be the only SUSY state lighter than the LOSP. In the context of multiple SUSY breaking, there is a corresponding multiplicity of goldstini whose masses are all typically proportional to $m_{3/2}$ (or loop suppressed compared SSM soft masses). Thus, the LOSP may dominantly decay to an uneaten goldstino instead of the gravitino. Since the couplings of the uneaten goldstino are unconstrained by supercurrent conservation, the LOSP can exhibit counterintuitive decay patterns.

In this chapter, we have focused on the case of a bino-like LOSP which decays dominantly to Higgs bosons despite having negligible Higgsino fraction. This effect is particularly pronounced in the presence of a $U(1)_R$ symmetry, which suppresses the expected $\lambda \to \gamma + \zeta$ decay. By studying which low energy effective operators are generated in the Higgsino decoupling limit, we have understood why the mode $\lambda \to h^0 + \zeta$ dominates in the limit of small $(m_\lambda \tan\beta)/\mu$, and also why there is a non-standard $\lambda \to Z + \zeta$ decay mode away from that limit. We have seen explicitly that there are delicate cancellations in the decay width of the LOSP to a gravitino, and the counterintuitive decays of a LOSP to an uneaten goldstino arise from incomplete cancellations.

Similar counterintuitive decay patterns would be present for a wino-like LOSP, and in general, one should contemplate the possibility of any LOSP decay pattern consistent with SM charges. Those LOSP decays might involve an uneaten goldstino as in this chapter, but could also be present with a light axino [105, 134] or a new light hidden sector [15, 49, 41]. To our mind, the most intriguing possibilities involve copious Higgs boson production in the final stages of a SUSY cascade decay, which may offer new Higgs production modes and give further motivation for boosted Higgs searches. Studying these phenomena is particularly relevant given the expected LHC sensitivity to SUSY scenarios in the 13 TeV run to begin next year.

Chapter 4

The Two Faces of Anomaly Mediation

4.1 Introduction

As we discussed in Ch. 2, if SUSY is realized in nature, then it must be spontaneously broken and the effects of SUSY breaking must be mediated to the supersymmetric standard model (SSM). In the context of supergravity (SUGRA), the most ubiquitous form of mediation is "anomaly mediation" [135, 84, 133, 81], which persists even when (and especially when) a SUSY-breaking hidden sector is sequestered from the visible sector. Of course, anomaly mediation need not be the dominant source of SSM soft masses, and there are theories where anomaly mediation is suppressed or absent [115, 120, 119]. But given its ubiquity, it is worth better understanding the physics of anomaly mediation and the circumstances which give rise to sequestering.

Indeed, anomaly mediation has been the source of much theoretical confusion, and various papers have aimed to clarify the underlying mechanism [35, 16, 17, 60, 46, 99, 44, 140]. The original description of anomaly mediation involved the super-Weyl anomaly [135, 84], and the most straightforward derivation of anomaly-mediated soft masses uses the conformal compensator formalism of SUGRA [80]. As discussed in Ref. [16], anomaly mediation really involves three different anomalies: a super-Weyl anomaly, a Kähler anomaly, and a sigma-model anomaly. More recently, Ref. [60] emphasized that SUGRA is not even a necessary ingredient, as a version of anomaly mediation (corresponding to the sigma-model anomaly) appears even in the $M_{\rm Pl} \to \infty$ limit.[1]

In this chapter, we will show that the phenomenon known as "anomaly mediation" really consists of two physically distinct effects. This realization clarifies a number of confusions surrounding anomaly mediation, and leads to a physical definition of sequestering in terms of goldstino couplings. Throughout this chapter, we will use "goldstino" to refer to the longitudinal gravitino mode in the high energy limit ($E \gg m_{3/2}$) [64, 34, 33].[2] The two effects are as follows.

- **Gravitino Mediation**. As we showed in Sec. 2.7, bosonic and fermionic modes in the same multiplet have SUSY mass splittings in the bulk of four-dimensional anti-de Sitter (AdS) space [25, 132, 87].[3] These mass splittings are proportional to the AdS curvature, and thus to the gravitino mass $m_{3/2}$. If SUSY AdS space is minimally uplifted to Minkowski space via SUSY breaking, these mass splittings are preserved, leading to SSM soft masses from "gravitino mediation". These soft masses do not have associated couplings to the goldstino, naively violating the (flat space) goldstino equivalence theorem [64, 34, 33]. Nevertheless, the absence of goldstino couplings is necessary for conservation of the AdS_4 supercurrent. Gravitino mediation closely resembles traditional anomaly mediation [135, 84], and is related to the super-Weyl anomaly. Gravitino mediation can never be turned off, since it arises from the infrared symmetry structure of SUSY AdS space.

- **Kähler Mediation**. If visible sector fields have linear couplings to SUSY-breaking fields in the Kähler potential, then this gives rise to "Kähler mediation", where SSM fields get mass splittings proportional to beta function coefficients. Linear couplings are ubiquitous in the presence of modulus fields, so Kähler mediation typically accompanies (and sometimes cancels) gravitino mediation in explicit SUGRA constructions [115, 120, 119, 11]. As expected from the (flat space) goldstino equivalence

[1] Ref. [60] also emphasized that the language of "anomalies" is not necessary, as the effect can be alternatively described in terms of gaugino counterterms. These gaugino counterterms are necessary to maintain SUSY in the 1PI effective action, including all anomaly contributions.

[2] For $M_{\rm Pl} \to \infty$ and $m_{3/2} \to 0$, this mode is the true goldstino from spontaneous SUSY breaking [148, 137, 139]. Here, we will keep $m_{3/2}$ fixed by considering the goldstino mode in rigid AdS space [104, 156, 94, 95]. In particular, the familiar relation $m_{3/2} = F_{\rm eff}/\sqrt{3}M_{\rm Pl}$ is only true after adjusting the cosmological constant to zero, so we can still take $M_{\rm Pl} \to \infty$ while preserving effects proportional to $m_{3/2}/F_{\rm eff}$.

[3] These splittings are required by the global AdS SUSY algebra. The case of massless gauge multiplets is subtle, since physical gauginos are massless in AdS_4. Crucially, a bulk gaugino mass term is required to cancel an infrared contribution to the gaugino mass in the 1-loop 1PI effective action, arising from boundary conditions in AdS_4 [87].

	Anomaly?	$m_\lambda \propto$?	SUGRA?	Goldstino?
Gravitino Mediation	Super-Weyl	$(3T_G - T_R)m_{3/2}$	✓	
Kähler Mediation	Super-Weyl	$\frac{1}{3}(3T_G - T_R)K_i F^i$	✓	✓
	Kähler	$-\frac{2}{3}T_R K_i F^i$	✓	✓
	Sigma-Model	$2\frac{T_R}{d_R}(\log\det K\|''_R)_i F^i$		✓

Table 4.1: The two faces of anomaly mediation. Shown are the corresponding anomalies and their contributions to gaugino masses, with a notation to be explained in the body of the text. (All the masses have an overall factor of $g^2/16\pi^2$.) We indicate whether the effect requires SUGRA and whether there is an associated gauge boson-gaugino-goldstino coupling. Gravitino mediation can be distinguished from Kähler mediation by the goldstino coupling. If SUSY breaking couples directly to gauginos, then there is an additional anomaly contribution discussed in App. B.1, which yields both one-loop gaugino masses and goldstino couplings.

principle, these soft masses have a corresponding coupling to the goldstino. In the $M_{\text{Pl}} \to \infty$ limit, Kähler mediation appears via the sigma-model anomaly (as emphasized in Ref. [60]). It also receives $1/M_{\text{Pl}}$ corrections due to the super-Weyl and Kähler anomalies. Unlike gravitino mediation, Kähler mediation is sensitive to the ultraviolet couplings of the theory.

These two contributions to anomaly mediation are summarized in Table 4.1, focusing on the case of gaugino soft masses. Full anomaly mediation is simply the sum of gravitino mediation and Kähler mediation.[4]

One might naively expect that no physical measurement could distinguish between gravitino mediation and Kähler mediation, since they only appear in combination in SSM soft masses. However, there is a crucial physical distinction in terms of goldstino couplings.[5] In usual SUSY breaking scenarios, gaugino soft masses are accompanied by a corresponding coupling between the gaugino λ^a, the gauge boson A^a_μ, and the goldstino $\widetilde{G}_L$,

$$\mathcal{L} \supset -\frac{1}{2}m_\lambda \lambda_a \lambda^a + \frac{i c_\lambda}{\sqrt{2}F_{\text{eff}}}\widetilde{G}_L \sigma^{\mu\nu}\lambda_a F^a_{\mu\nu}, \tag{4.1}$$

[4]As pointed out in Ref. [44] in the context of string theory, there is an additional anomaly-mediated gaugino mass which arises from an anomalous rescaling of the gauge multiplets. We discuss this effect in App. B.1 and show that it yields a corresponding goldstino coupling consistent with (flat space) supercurrent conservation.

[5]Our results can be interpreted as describing goldstino couplings in the analog of Landau gauge where the gravitino field is purely transverse. At the end of Sec. 4.4.2, we explain the same effect in unitary gauge.

where F_{eff} is the scale of SUSY breaking.[6] For global SUSY, the relation $c_\lambda = m_\lambda$ is required by the (flat space) goldstino equivalence theorem. In contrast, gravitino mediation is dictated by SUSY in AdS space, and generates a contribution to m_λ *without* a corresponding contribution to c_λ. Indeed, the difference $m_\lambda - c_\lambda$ is necessarily proportional to $m_{3/2}$ by conservation of the AdS supercurrent, and this gives a physical way to measure gravitino mediation as distinct from all other sources of SSM soft masses. We will show this explicitly in Eq. (4.30).

This result allows us to give an unambiguous definition of sequestering [135], which is the condition necessary for traditional anomaly mediation (i.e. gravitino mediation) to be the sole source of SSM soft masses.

- Visible sector fields are sequestered from SUSY breaking if they do not have linear couplings to the goldstino.[7]

In other words, c_λ is measure of how well the visible sector is sequestered from the goldstino. Previously, sequestering was known to occur when the Kähler potential K and superpotential W took a special "factorized" form [135]

$$-3e^{-K/3} = \Omega_{\text{vis}} + \Omega_{\text{hid}}, \qquad W = W_{\text{vis}} + W_{\text{hid}}. \tag{4.2}$$

However, Eq. (4.2) is ambiguous, since the separation into "visible" and "hidden" sectors is not robust to Kähler transformations by a chiral multiplet X with $K \to K + X + X^\dagger$ and $W \to e^{-X} W$. Also, sequestering usually (but not always) requires moduli to be stabilized [86, 121, 14, 122, 125, 63]. Sequestering does have an unambiguous extra-dimensional interpretation in terms of geometric separation [135]. Here we can use the absence of goldstino couplings as a purely four-dimensional definition of sequestering. Since physical couplings are invariant to Lagrangian manipulations such as Kähler transformations, this definition does not suffer from the ambiguities of Eq. (4.2), and gives a robust criteria for determining when traditional anomaly mediation is dominant.

We can highlight the distinction between gravitino mediation alone and anomaly me-

[6] There is also an additional coupling between the gaugino, goldstino, and the auxiliary field D^a, $\frac{ic_\lambda}{\sqrt{2}F_{\text{eff}}} \lambda_a \widetilde{G}_L D^a$. The c_λ of this coupling is guaranteed to be identical to the c_λ in Eq. (4.1), so we omit this term for brevity throughout.

[7] Strictly speaking this is only true for gauginos. As we will explain below, scalar soft masses are more subtle because of irreducible couplings to the goldstino, but sequestering for scalars can still be defined as the absence of any further couplings to the goldstino.

diation more generally by comparing models with strict sequestering [135] to models with warped [121, 14, 122, 119, 125, 63, 144] or conformal sequestering [130, 108, 131, 123, 107, 118, 106, 59, 92, 93, 141]. In the case of strict sequestering, SUSY breaking is confined to a brane which is geometrically separated from the visible sector brane. This geometric separation *forbids* couplings between the goldstino and the visible sector. The only source of visible sector soft masses comes from gravitino mediation, which can be captured by the conformal compensator[8]

$$\langle \mathbf{\Phi} \rangle = 1 + \theta^2 m_{3/2}. \tag{4.3}$$

In the case of warped sequestering, visible sector fields on the IR brane feel an "effective" conformal compensator

$$\boldsymbol{\omega} = \mathbf{\Phi} e^{-k\boldsymbol{T}}, \tag{4.4}$$

where $\boldsymbol{T}$ is the radion superfield. Visible sector fields obtain anomaly-mediatied soft masses proportional to

$$\left\langle \frac{F_\omega}{\omega} \right\rangle = m_{3/2} - kF_T, \tag{4.5}$$

but because the radion has overlap with the goldstino direction, there are visible sector couplings to the goldstino proportional to kF_T. In the language of this chapter, warped sequestering exhibits a cancellation between gravitino mediation and Kähler mediation.[9]

Throughout this chapter, we focus on gaugino masses, leaving a full description of anomaly-mediated scalar soft masses to Ch. 5. As a preview, there is a mass splitting between scalars and matter fermions in the bulk of AdS$_4$, analogous to the gaugino case, which includes the familiar two-loop anomaly-mediated scalar masses. However, already at tree-level in AdS$_4$, scalars have tachyonic scalar soft masses equal to $-2m_{3/2}^2$ [25, 132]. While tachyonic scalar masses do not destabilize the theory in AdS space, they do in flat space. Thus, the SUSY breaking that uplifts the theory from AdS to flat space must remove these tree-level tachyonic soft masses, resulting in irreducible goldstino couplings which complicate the definition of sequestering.[10]

[8]The relation $\langle F_\Phi \rangle = m_{3/2}$ is special to strict sequestering. See Eq. (4.19) for a more general expression.

[9]This cancellation is not a fine tuning, since it arises from the geometry of the warped (AdS$_5$) space. The curvature of AdS$_5$ should not be confused with the curvature of AdS$_4$, which is responsible for gravitino mediation.

[10]There is a related subtlety involving tree-level holomorphic B-terms, since B terms arising from the superpotential have different associated goldstino couplings than B terms arising from the Giudice-Masiero mechanism [82]. Previously, both phenomena were considered to occur in the sequestered limit, but Giudice-Masiero secretly violates the conditions for sequestering, as we will see in Ch. 5.

In the remainder of this chapter, we derive the gaugino soft masses and goldstino couplings arising from anomaly mediation, emphasizing the distinction between gravitino mediation and Kähler mediation. The soft masses are well-known in the literature, but to the best of our knowledge, the goldstino couplings have never been calculated explicitly. In Sec. 4.2, we give a straightforward derivation of how Kähler mediation arises in global SUSY. We then turn to full SUGRA in Sec. 4.3, applying the improved SUGRA gauge fixing of Ref. [38]. This is the simplest way to isolate gravitino mediation, since this gauge automatically decouples the (transverse) gravitino, leaving the goldstino coupling manifest. In Sec. 4.4, we describe the same physics using a more conventional SUGRA notation of Ref. [16]. We also explain the connection to the AdS_4 supercurrent conservation and the goldstino equivalence theorem. We conclude in Sec. 4.5.

4.2 Kähler Mediation in Global SUSY

Before deriving full anomaly mediation in Sec. 4.3, it is useful to focus on the case of pure Kähler mediation, which arises in the limit of global SUSY. This example was emphasized in Ref. [60], but in order to connect to the (perhaps) more familiar language of Ref. [16], we will derive the result using chiral anomalies (instead of gaugino counterterms).

Consider a field redefinition acting on a chiral superfield $\boldsymbol{Q}$ of the form

$$\boldsymbol{Q} \to e^{\boldsymbol{\alpha}} \boldsymbol{Q}, \tag{4.6}$$

where $\boldsymbol{\alpha}$ is another chiral superfield.[11] This field redefinition changes the Lagrangian in a classical way, but it also introduces a term related to the Konishi anomaly [43, 110]. If $\boldsymbol{Q}$ is in the representation R of non-Abelian gauge field, then the Lagrangian shifts as

$$\mathcal{L}(\boldsymbol{X}) \to \mathcal{L}(e^{\boldsymbol{\alpha}} \boldsymbol{X}) + \frac{g^2 T_R}{16\pi^2} \int d^2\theta \, \boldsymbol{\alpha} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^a_\alpha, \tag{4.7}$$

where T_R is the Dynkin index of the representation R. In the language of Ref. [83, 12], Eq. (4.7) is simply the chiral anomaly analytically continued into superspace.

[11] Throughout this chapter, we will use the notation of Ref. [38], where boldface ($\boldsymbol{X}$) indicates a superfield and regular typeface (X) indicates the lowest component of the corresponding superfield. Superscripts are field labels and subscripts indicate derivatives with respect to chiral fields. As needed, we raise and lower indices using the Kähler metric. We will use $\boldsymbol{Q}$ to indicate visible sector fields and $\boldsymbol{X}$ to indicate hidden sector SUSY-breaking fields.

In global SUSY, Kähler mediation arises whenever charged matter has linear couplings to SUSY breaking in the Kähler potential. This is easiest to understand using a non-linear representation $\boldsymbol{X}_{\text{NL}}$ of a SUSY-breaking field which obeys the constraint $\boldsymbol{X}_{\text{NL}}^2 = 0$ [137, 116, 109, 42, 39]. Consider a Lagrangian which contains a matter field $\boldsymbol{Q}$ coupled to SUSY breaking as

$$\mathcal{L} \supset \int d^4\theta \, \boldsymbol{Q}^\dagger \boldsymbol{Q} \left(1 + \frac{\boldsymbol{X}_{\text{NL}} + \boldsymbol{X}_{\text{NL}}^\dagger}{\Lambda}\right). \tag{4.8}$$

We can remove the linear couplings of $\boldsymbol{X}_{\text{NL}}$ by performing an (anomalous) field redefinition

$$\boldsymbol{Q} \to \boldsymbol{Q}\left(1 - \frac{\boldsymbol{X}_{\text{NL}}}{\Lambda}\right) = \boldsymbol{Q} e^{-\boldsymbol{X}_{\text{NL}}/\Lambda}, \tag{4.9}$$

where the last equality relies on $\boldsymbol{X}_{\text{NL}}^2 = 0$. From the Konishi anomaly, this yields

$$\mathcal{L} \supset \int d^4\theta \, \boldsymbol{Q}^\dagger \boldsymbol{Q} \left(1 - \frac{\boldsymbol{X}_{\text{NL}} \boldsymbol{X}_{\text{NL}}^\dagger}{\Lambda^2}\right) - \frac{g^2 T_R}{16\pi^2} \int d^2\theta \, \frac{\boldsymbol{X}_{\text{NL}}}{\Lambda} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^a_\alpha, \tag{4.10}$$

After the field redefinition, $\boldsymbol{X}_{\text{NL}}$ only has quadratic couplings to $\boldsymbol{Q}$, at the expense of introducing new couplings between $\boldsymbol{X}_{\text{NL}}$ and the gauge multiplet. This is the essence of Kähler mediation.

Expanding out $\boldsymbol{X}_{\text{NL}}$ in terms of F_X and the goldstino $\widetilde{G}_L$ [137, 116, 109, 42, 39]

$$\boldsymbol{X}_{\text{NL}} = \left(\theta + \frac{1}{\sqrt{2}} \frac{\widetilde{G}_L}{F_X}\right)^2 F_X, \tag{4.11}$$

Eq. (4.10) contains a soft mass for the gauginos and a corresponding coupling to the goldstino, as anticipated in Eq. (4.1)

$$\mathcal{L} \supset -\frac{1}{2} m_\lambda \lambda_a \lambda^a + \frac{i c_\lambda}{\sqrt{2} F_{\text{eff}}} \widetilde{G}_L \sigma^{\mu\nu} \lambda_a F^a_{\mu\nu}, \tag{4.1}$$

where $F_{\text{eff}} \equiv F_X$ in this example, and

$$m_\lambda = c_\lambda = \frac{g^2 T_R}{8\pi^2} \frac{F_X}{\Lambda}. \tag{4.12}$$

As expected from the goldstino equivalence theorem (see Sec. 4.4.2), the goldstino couplings are proportional to the gaugino mass.

In the above derivation, the matter superfield $\boldsymbol{Q}$ was assumed to be massless, which was crucial for seeing a physical effect from the sigma-model anomaly. Indeed, without massless "messengers" to communicate SUSY breaking, one does not expect Kähler potential terms to affect holomorphic quantities like gaugino masses. To see what happens for massive matter, consider vector-like chiral superfields with a supersymmetric mass term $\mu\boldsymbol{Q}\boldsymbol{Q}^c$. In this case, the chiral rescaling in Eq. (4.9) yields a new superpotential term $-\frac{\mu}{\Lambda}\boldsymbol{Q}\boldsymbol{Q}^c\boldsymbol{X}_{\mathrm{NL}}$. For large μ, $\boldsymbol{Q}$ and $\boldsymbol{Q}^c$ are just heavy messenger fields, generating a gauge-mediated contribution to the gaugino masses which exactly cancels Eq. (4.12), as explicitly shown in Refs. [142, 62]. This insensitivity to heavy supersymmetric thresholds is a well-known feature of anomaly mediation, and persists in SUGRA as well; we may in general evaluate anomaly or beta-function coefficients at the scale of interest. For simplicity, we will take all matter superfields to be massless in the remainder of this chapter.

The chiral rescaling procedure in Eq. (4.9) can be generalized to an arbitrary Kähler potential $\boldsymbol{K}$.

$$\mathcal{L} \supset \int d^4\theta \boldsymbol{K}. \tag{4.13}$$

Consider a set of chiral multiplets $\boldsymbol{Q}$ in the representation R with the Kähler metric $\boldsymbol{K}|''_R$.[12] In general, $\boldsymbol{K}|''_R$ will be a function of SUSY-breaking fields $\boldsymbol{X}^i$, but as shown in App. B.2, there is a field redefinition that removes all linear couplings of $\boldsymbol{X}^i$ in $\boldsymbol{K}|''_R$ but generates the anomalous term

$$\delta\mathcal{L} = -\int d^2\theta \frac{g^2}{16\pi^2}\boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha \frac{D^{\dagger 2}D^2}{16\,\Box}\left[\frac{T_R}{d_R}\log\det \boldsymbol{K}|''_R\right], \tag{4.14}$$

where d_R is dimension of the representation R. This form makes explicit use of the chiral projection operator $(D^{\dagger 2}D^2/16\,\Box)$, which is overkill for our purposes. Since we are only interested in soft masses and goldstino couplings, we will assume that all SUSY-breaking fields have been shifted such that $\langle X^i \rangle = 0$, and focus on a subset of terms from expanding Eq. (4.14) to first order in $\boldsymbol{X}^i$:

$$\delta\mathcal{L} \supset -\int d^2\theta \frac{g^2}{16\pi^2}\frac{T_R}{d_R}(\log\det K|''_R)_i \boldsymbol{X}^i \boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha. \tag{4.15}$$

In each SUSY-breaking multiplet $\boldsymbol{X}^i$, the fermionic component χ^i has overlap with the

[12]The Kähler metric $\boldsymbol{K}|''_R$ is just $\boldsymbol{K}_{i\bar{\jmath}}$ where $\boldsymbol{Q}^i$ and $\boldsymbol{Q}^{\bar{\jmath}}$ transform in R.

goldstino direction as

$$\chi^i \to \frac{F^i}{F_{\text{eff}}} \widetilde{G}_L, \tag{4.16}$$

where F_{eff} is the total amount of SUSY breaking (in the absence of D terms, $F_{\text{eff}} = \sqrt{F_i F^i}$). We see that Eq. (4.15) contains a gaugino mass and corresponding goldstino coupling

$$m_\lambda = c_\lambda = \frac{g^2}{16\pi^2} \frac{2T_R}{d_R} (\log \det K|''_R)_i F^i. \tag{4.17}$$

Once we sum over representations R, this is the general expression for Kähler mediation in global SUSY. As we will see, this chiral field scaling procedure will persist when we go to SUGRA, but the equality between m_λ and c_λ will be broken.

4.3 Gravitino and Kähler Mediation in SUGRA

Having derived Kähler mediation in global SUSY, we can now understand the analogous effect in full SUGRA. Now, the goldstino is eaten by the super-Higgs mechanism to become the longitudinal component of the gravitino, but it is still convenient to isolate the goldstino mode by using goldstino equivalence in the high energy limit. For simplicity, we will use "anomaly mediation" to refer to the combined effect of gravitino and Kähler mediation. As we will see, these two effects are physically distinct from the perspective of goldstino couplings.

The improved SUGRA gauge fixing of Ref. [38] is particularly convenient for understanding anomaly mediation, both in terms of soft masses and goldstino couplings. In this gauge, matter multiplets (including the goldstino multiplet) are decoupled from the gravity multiplet up to $1/M_{\text{Pl}}^2$ suppressed effects. This allows calculations involving the matter fields alone to be performed in *global* superspace. After giving a brief description of the SUGRA Lagrangian and the gauge fixing of Ref. [38], we will calculate gaugino masses and goldstino couplings to see the two faces of anomaly mediation.

4.3.1 The SUGRA Lagrangian

The conformal compensator formalism arises from gauge fixing conformal SUGRA using the conformal compensator field $\mathbf{\Phi}$. As reviewed in Ref. [38], the tree-level SUGRA Lagrangian can be written as

$$\mathcal{L} = -3 \int d^4\theta\, \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi} e^{-\boldsymbol{K}/3} + \int d^2\theta\, \boldsymbol{\Phi}^3\, \boldsymbol{W} + \text{h.c.} + \frac{1}{4} \int d^2\theta\, \boldsymbol{f}_{ab} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^b_\alpha + \text{h.c.} + \ldots, \quad (4.18)$$

where the ellipsis $(\ldots)$ corresponds to terms involving the graviton and gravitino. In general, the ellipsis contains quadratic mixing terms between matter multiplets and the graviton multiplet, but Ref. [38] showed that there is an improved gauge fixing for $\boldsymbol{\Phi}$ where this mixing is absent:

$$\boldsymbol{\Phi} = e^{\boldsymbol{Z}/3}(1 + \theta^2 F_\phi), \quad (4.19)$$

$$\boldsymbol{Z} = \langle K/2 - i \operatorname{Arg} W \rangle + \langle K_i \rangle\, \boldsymbol{X}^i. \quad (4.20)$$

In this gauge, one can simply drop the ellipsis terms in Eq. (4.18) for any calculation not involving gravitons or gravitinos, allowing one to study matter multiplets in SUGRA using global superspace manipulations.

There are a few important caveats to this gauge fixing. First, Eq. (4.19) only removes mixing terms at tree level, so strictly speaking, one can only study tree-level and one-loop effects using this formalism. This is sufficient for understanding anomaly-mediated gaugino masses at one loop, but we will have to postpone a study of two-loop scalar soft masses for future work. Second, this gauge fixing assumes that the cosmological constant has been adjusted to zero to yield a Minkowski vacuum, a necessary assumption for phenomenology. Third, Eq. (4.19) explicitly contains vacuum expectation values (vevs), which is perhaps unfamiliar but conceptually sound.

A nice feature of this gauge is that after adjusting the cosmological constant to zero

$$\langle F_\phi \rangle = m_{3/2}, \quad (4.21)$$

making it easy to identify terms proportional to the gravitino mass [38]. In particular, note that the $(1 + \theta^2 m_{3/2})$ part of $\boldsymbol{\Phi}$ has a SUSY-breaking F-component *without* any coupling to fermions. This will be the origin of gravitino mediation, which yields soft masses proportional to $m_{3/2}$ without a corresponding goldstino coupling.

In addition to the tree-level terms in Eq. (4.18), there is a contribution to the Lagrangian coming from anomaly matching. Before introducing (and gauge fixing) $\boldsymbol{\Phi}$, conformal SUGRA contained a non-anomalous $U(1)_R$ gauge symmetry with gauge field b_μ, so the corresponding global $U(1)_R$ must also be non-anomalous. Under this $U(1)_R$, $\mathbf{\Phi}$ (which we have yet to gauge fix) has R-charge 2/3 and matter fields have R-charge 0. Since chiral fermions have R-charge -1 and gauginos have R-charge $+1$, the gauge kinetic function for each gauge field must contain

$$\boldsymbol{f}_{ab} \supset \delta_{ab} \frac{g_a^2}{4\pi^2} \left(\frac{3T_R - 3T_G}{2} \right) \log \mathbf{\Phi}, \tag{4.22}$$

such that these anomalies can be cancelled by a $U(1)_R$ shift of $\log \mathbf{\Phi}$ [103] (see also Ref. [17]). Note that this is *not* the familiar expression for $\mathbf{\Phi}$ coupling involving the beta function (see e.g. Ref. [135]). This will arise after appropriate field redefinitions of the matter fields.

4.3.2 Field Redefinitions in SUGRA

The Lagrangian shift in Eq. (4.7) appears for any field rescaling of chiral multiplets, including rescalings involving the conformal compensator. With the improved gauge fixing, there is no mixing between matter multiplets and the gravity multiplet, and this lack of mixing persists (at least at one loop) after field rescalings.[13] In addition to the appearance of Eq. (4.22), the main difference between Kähler mediation in global SUSY and full anomaly mediation in SUGRA is that $\boldsymbol{K}$ in Eq. (4.13) is replaced by $\mathbf{\Phi}^\dagger \mathbf{\Phi}\, \mathbf{\Omega}$, with

$$\mathbf{\Omega} \equiv -3e^{-\boldsymbol{K}/3}. \tag{4.23}$$

We can now use the same fields manipulation as in Sec. 4.2, treating $\mathbf{\Phi}$ as one of the SUSY-breaking fields. First, to remove linear couplings to the conformal compensator, we can perform the field redefinition

$$\boldsymbol{Q}^i \to \frac{\boldsymbol{Q}^i}{\mathbf{\Phi}}. \tag{4.24}$$

Combined with Eq. (4.22), this leads to the familiar anomaly-mediated term

$$\delta \mathcal{L} = -\frac{g^2}{16\pi^2} \int d^2\theta \left(\frac{3T_G - T_R}{2} \right) \log \mathbf{\Phi} \boldsymbol{W}^{a\alpha} \boldsymbol{W}^a_\alpha, \tag{4.25}$$

which is proportional to the beta function $b_0 \equiv 3T_G - T_R$ as expected. To remove linear

[13]This rescaling does induce a gravitational anomaly term, but this is irrelevant for our present purposes.

couplings to SUSY-breaking fields in $\boldsymbol{\Omega}$, we use Eq. (4.14), replacing $\boldsymbol{K}$ with $\boldsymbol{\Omega}$

$$\frac{1}{d_R}(\log\det\boldsymbol{\Omega}|''_R) \Rightarrow -\frac{1}{3}\boldsymbol{K} + \frac{1}{d_R}(\log\det\boldsymbol{K}|''_R). \tag{4.26}$$

Here, we have used the fact that for unbroken gauge symmetries, the vev of K_i (and of any derivatives of K_i with respect to the SUSY-breaking fields) is zero for charged fields $\boldsymbol{Q}^i$. Combined with Eq. (4.25), we arrive at the final anomaly-mediated expression[14]

$$\delta\mathcal{L} = -\int d^2\theta\,\frac{g^2}{16\pi^2}\left(\left(\frac{3T_G - T_R}{2}\right)\log\boldsymbol{\Phi} + \frac{D^{\dagger 2}D^2}{16\Box}\left[-\frac{T_R}{3}\boldsymbol{K} + \frac{T_R}{d_R}(\log\det\boldsymbol{K}|''_R)\right]\right)\boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha. \tag{4.27}$$

Using the improved gauge fixing, anomaly mediation in SUGRA has essentially the same origin as Kähler mediation in global SUSY, arising from performing anomalous chiral rescalings to remove linear couplings to SUSY breaking in the Kähler potential.

As emphasized in Ref. [16], anomaly mediation is associated with three different anomalies—a super-Weyl anomaly, a Kähler anomaly, and a sigma-model anomaly—corresponding to the three terms in Eq. (4.27). In our rescaling procedure, the Kähler and sigma-model anomalies in SUGRA have a common origin, and arise from taking the global sigma-model anomaly involving the Kähler potential $\boldsymbol{K}$ and replacing it with an "effective" Kähler potential $\boldsymbol{\Omega}$. In this way, the Kähler anomaly should be regarded as a $1/M_{\text{Pl}}$ correction to the sigma-model anomaly. The super-Weyl anomaly is truly a SUGRA effect, and depends crucially on the fact that prior to gauge fixing, there was an anomaly-free global $U(1)_R$ symmetry.[15]

4.3.3 Soft Masses and Gaugino Couplings

Before expanding Eq. (4.27) in components, there is no apparent difference between gravitino mediation and Kähler mediation. This difference only becomes visible after identifying the gaugino soft masses and corresponding gaugino couplings in Eq. (4.1), repeated for convenience:

$$\mathcal{L} \supset -\frac{1}{2}m_\lambda\lambda_a\lambda^a + \frac{ic_\lambda}{\sqrt{2}F_{\text{eff}}}\widetilde{G}_L\sigma^{\mu\nu}\lambda_a F^a_{\mu\nu}. \tag{4.1}$$

[14] As discussed in App. B.1, there is an additional anomaly-mediated contribution arising from rescaling gauge multiplets from a holomorphic basis to a canonical basis. This effect is not captured by Ref. [16] since it requires direct couplings between SUSY breaking and gauginos, but it does appear in Ref. [44].

[15] As a side note, the derivation of anomaly mediation in Ref. [60] focused only on an Abelian gauge theory, so it does not capture the T_G dependence in non-Abelian theories which arises from Eq. (4.22).

The gaugino mass from expanding Eq. (4.27) is

$$m_\lambda = \frac{g^2}{16\pi^2}\left((3T_G - T_R)\left(m_{3/2} + \frac{K_i F^i}{3}\right) - 2\frac{T_R}{3}K_i F^i + 2\frac{T_R}{d_R}(\log\det K|''_R)_i F^i\right). \quad (4.28)$$

Note that both the super-Weyl and Kähler anomaly pieces have contributions proportional to $K_i F^i$, and we have used the fact that $\langle F_\phi \rangle = m_{3/2}$ in the improved gauge fixing from Eq. (4.19). We can extract the goldstino coupling c_λ from Eq. (4.27), using Eq. (4.16) to identify the goldstino direction:

$$c_\lambda = \frac{g^2}{16\pi^2}\left((3T_G - T_R)\frac{K_i F^i}{3} - 2\frac{T_R}{3}K_i F^i + 2\frac{T_R}{d_R}(\log\det K|''_R)_i F^i\right). \quad (4.29)$$

Crucially, c_λ differs from m_λ by terms proportional to $m_{3/2}$, owing to the fact that the $(1 + \theta^2 m_{3/2})$ piece of $\mathbf{\Phi}$ has a SUSY-breaking F-component without a corresponding goldstino components. These terms are summarized in Table 4.1.

We can rewrite the gaugino mass and goldstino coupling in the following suggestive way:

$$m_\lambda = m_{\mathrm{AdS}} + c_\lambda, \quad (4.30)$$

where

$$m_{\mathrm{AdS}} = \frac{g^2}{16\pi^2}\left(m_{3/2}(3T_G - T_R)\right), \quad (4.31)$$

$$c_\lambda = \frac{g^2}{16\pi^2}\left(\frac{K_i F^i}{3}(3T_G - 3T_R) + 2\frac{T_R}{d_R}(\log\det K|''_R)_{,i} F^i\right). \quad (4.32)$$

This is the primary result of this chapter. Here, m_{AdS} is the gaugino mass splitting from the bulk of SUSY AdS space (derived in Ref. [87] and discussed further in Sec. 4.4.2), and gives rise to a gravitino-mediated soft mass with no associated goldstino coupling. The remaining part of anomaly mediation c_λ is Kähler mediation, which generalizes the global SUSY results from Sec. 4.2. As advertised, c_λ is an effective measure of sequestering—in particular, sequestering of visible sector gauginos from the goldstino—and the limit $c_\lambda = 0$ corresponds to pure gravitino mediation.

4.4 Alternative Descriptions

Having seen the two faces of anomaly mediated in the conformal compensator formalism, it is worth repeating the calculation in the (perhaps) more familiar language of Ref. [16]. We first rederive Eq. (4.30) in components, and then explain the connection to the AdS supercurrent and the goldstino equivalence theorem.

4.4.1 Anomaly Mediation in Components

As shown in Refs. [27, 117, 16], after lifting the super-Weyl, Kähler, and sigma-model anomalies to superspace, the 1PI effective action contains

$$\mathcal{L}_{\text{SF}} \supset -\frac{g^2}{256\pi^2}\int d^2\Theta\, 2\mathcal{E}\, \boldsymbol{W}^\alpha \boldsymbol{W}_\alpha \boldsymbol{C}, \tag{4.33}$$

where for convenience, we have defined a chiral superfield $\boldsymbol{C}$ as

$$\boldsymbol{C} = \frac{1}{\Box}\left(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R}\right)\left[4(T_R - 3T_G)\boldsymbol{R}^\dagger - \frac{1}{3}T_R\mathcal{D}^2\boldsymbol{K} + \frac{T_R}{d_R}\mathcal{D}^2 \log\det \boldsymbol{K}|''_R\right], \tag{4.34}$$

where $\boldsymbol{R}$ is the curvature superfield. This expression is valid in "supergravity frame" where the Einstein-Hilbert term has the non-canonical normalization $e^{-K/3}R_{\text{EH}}$.[16] By taking the lowest component of $\boldsymbol{C}$ in App. B.3, we recover (non-local) terms in the Lagrangian that express the three anomalies.

In order to derive physical couplings and masses from the other components of $\boldsymbol{C}$, we need to transform to "Einstein frame" where the graviton (and gravitino) have canonical

[16] It may be confusing that Eq. (4.33) is only a function of the Kähler potential K and not the Kähler invariant $G \equiv K + \log W + \log W^*$. Because of the Kähler anomaly, there is a physical distinction between the superpotential W and the holomorphic terms in K. See Refs. [16, 17].

kinetic terms. This can be accomplished by performing the field redefinitions [16, 154]

$$e_c^\mu \to e^{-K/6} e_c^\mu, \tag{4.35}$$

$$\psi_\mu \to e^{+K/12}(\psi_\mu + i\sqrt{2}\sigma_\mu \chi^{\dagger\bar{i}} K_{\bar{i}}/6), \tag{4.36}$$

$$M^* \to e^{-K/6}(M^* - F^i K_i), \tag{4.37}$$

$$\lambda^a \to e^{-K/4}\lambda^a, \tag{4.38}$$

$$\chi^i \to e^{-K/12}\chi^i, \tag{4.39}$$

$$F^i \to e^{-K/6} F^i, \tag{4.40}$$

$$b_\mu \to b_\mu + \frac{i}{2\sqrt{2}}(\psi_\mu^\dagger \chi^{\dagger\bar{i}} K_i - \psi_\mu \chi^i K_i) \tag{4.41}$$

Note that the gravitino ψ_μ, scalar auxiliary field M^*, and vector auxiliary field b_μ transform inhomogeneously under this redefinition.[17] With this field redefinition and adjusting the cosmological constant to zero, the scalar auxiliary vev is

$$\langle M^* \rangle = -3m_{3/2}, \tag{4.42}$$

analogous to Eq. (4.21).

After performing the field redefinitions, the pertinent components of $\boldsymbol{C}$ are

$$\begin{aligned} e^{K/12}\mathcal{D}_\alpha \boldsymbol{C}| &= \frac{16}{3\sqrt{2}}(3T_G - T_R)K_i\chi_\alpha^i - \frac{32}{3\sqrt{2}}T_R\langle K_i\rangle \chi_\alpha^i + \frac{32}{\sqrt{2}}\frac{T_R}{d_R}\left\langle (\log\det K|_R^{''})_i \right\rangle \chi_\alpha^i \\ &\quad - \frac{32}{3\Box}(3T_G - T_R)(i\sigma^\mu \bar{\sigma}^{\nu\rho}\mathcal{D}_\mu \mathcal{D}_\nu \psi_\rho^\dagger)_\alpha + \cdots, \end{aligned} \tag{4.43}$$

$$\begin{aligned} e^{K/6}\mathcal{D}^2\boldsymbol{C}| &= \frac{32}{3}(3T_G - T_R)(-3m_{3/2} - F^i K_i) + \frac{64}{3}T_R K_i F^i \\ &\quad - 64\frac{T_R}{d_R}(\log\det K|_R^{''})_i F^i + \cdots. \end{aligned} \tag{4.44}$$

Here, it is understood that we have shifted all fields such that their vevs are zero and any expressions contained in angle brackets above are purely c-numbers. The ellipses represent omitted terms that do not correspond to any local terms in the resultant Lagrangian, but are necessary to maintain SUSY in the 1PI action. The gravitino coupling in the last term of Eq. (4.43) will be important in Sec. 4.4.2 below.

The Θ^2 component of $\boldsymbol{C}$ yields the gaugino soft mass, and the Θ component of $\boldsymbol{C}$ yields

[17] Indeed, the improved gauge fixing of Ref. [38] was designed to avoid having to perform such transformations.

the gauge boson-gaugino-goldstino coupling. We can now derive Eq. (4.1), after identifying the goldstino mode through Eq. (4.16), and we recover the same answer as Eq. (4.30):

$$\mathcal{L} \supset -\frac{1}{2}\left(m_{\text{AdS}} + c_\lambda\right)\lambda_a\lambda^a + \frac{c_\lambda}{\sqrt{2}F_{\text{eff}}}\lambda_a\sigma^{\mu\nu}\widetilde{G}_L F^a_{\mu\nu}, \tag{4.45}$$

with

$$m_{\text{AdS}} = \frac{g^2}{16\pi^2}\left(m_{3/2}(3T_G - T_R)\right), \tag{4.46}$$

$$c_\lambda = \frac{g^2}{16\pi^2}\left(\frac{K_i F^i}{3}(3T_G - 3T_R) + 2\frac{T_R}{d_R}(\log\det K|''_R)_{,i}F^i\right). \tag{4.47}$$

Because of the gravitino shift in Eq. (4.36) and the auxiliary field shift in Eq. (4.37), the super-Weyl anomaly contributes to both gravitino mediation and Kähler mediation. Again, we see that gravitino mediation is physically distinct from Kähler mediation by the absence of goldstino couplings.

4.4.2 Supercurrent Conservation and Goldstino Equivalence

The fact that gravitino mediation gives rise to gaugino soft masses without corresponding goldstino couplings is perhaps confusing from the point of view of the goldstino equivalence theorem [34, 33]. However, we will see that this is necessitated by conservation of the AdS supercurrent.

The goldstino equivalence theorem states that at energies well above the gravitino mass $m_{3/2}$, the couplings of longitudinal gravitinos can be described by the (eaten) goldstino mode. In global SUSY, linear couplings of the goldstino are fixed by conservation of the (flat space) supercurrent

$$\mathcal{L} = \frac{1}{\sqrt{2}F_{\text{eff}}}\partial_\mu\widetilde{G}_L j^\mu_{\text{flat}}. \tag{4.48}$$

The part of the supercurrent that depends on the gauge boson and gaugino is

$$j^\mu_{\text{flat}} \supset -\frac{1}{2}\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^\dagger_a F^a_{\nu\rho}. \tag{4.49}$$

Using the gaugino equation of motion (assuming a massless gauge boson for simplicity), this gives rise to the interaction

$$\text{Flat space:} \quad \frac{i m_\lambda}{\sqrt{2} F_{\text{eff}}} \widetilde{G}_L \sigma^{\mu\nu} \lambda_a F^a_{\mu\nu}, \tag{4.50}$$

where m_λ is the physical gaugino mass. In flat space, therefore, c_λ must equal m_λ, and there can be no contribution from gravitino mediation.

The resolution to this apparent paradox is that the gravitino mediation arises from uplifting an AdS SUSY vacuum to SUSY-breaking Minkowski space, so we should really be testing the goldstino equivalence theorem for (rigid) AdS space [104, 156, 94, 95].[18] Indeed, the last term in Eq. (4.43) contains an additional coupling to the gravitino, which contributes to the (AdS) supercurrent.[19] In principle, it should be possible to derive the one-loop AdS supercurrent directly from the SUSY algebra in AdS space, but we know of no such derivation in the literature. Instead, we can simply extract the one-loop contribution to the supercurrent by recalling that the gravitino couples linearly to the (AdS) supercurrent as

$$\mathcal{L} = -\frac{1}{2M_{\text{Pl}}} \psi_\mu j^\mu_{\text{AdS}} \to \frac{1}{\sqrt{2} F_{\text{eff}}} \partial_\mu \widetilde{G}_L j^\mu_{\text{AdS}}. \tag{4.51}$$

In this last step, we have identified the goldstino direction via [154]

$$\psi_\mu \to -\sqrt{\frac{2}{3}} m_{3/2}^{-1} \partial_\mu \widetilde{G}_L - \frac{i}{\sqrt{6}} \sigma_\mu \widetilde{G}_L^\dagger, \qquad m_{3/2} = \frac{F_{\text{eff}}}{\sqrt{3} M_{\text{Pl}}}, \tag{4.52}$$

and dropped the term proportional to $\widetilde{G}_L^\dagger \, \overline{\sigma}_\mu j^\mu_{\text{AdS}} / M_{\text{Pl}}$ since it does not contain a gauge boson-gaugino-goldstino coupling.

We see that Eq. (4.43) contains a linear (non-local) coupling to the gravitino, and thus an additional (local) coupling to the goldstino

$$-\left(\frac{g^2}{16\pi^2} m_{3/2} (3T_G - T_R) \right) \frac{i}{\sqrt{2} F_{\text{eff}}} \widetilde{G}_L \sigma^{\mu\nu} \lambda_a F^a_{\mu\nu}. \tag{4.53}$$

We recognize the term in parentheses as $-m_{\text{AdS}}$ from Eq. (4.31). Combining with Eq. (4.50),

[18] Rigid AdS corresponds to the limit $M_{\text{Pl}} \to \infty$ leaving the AdS curvature fixed. This limit maintains couplings proportional to $m_{3/2}$ despite the fact that the gravitino itself is decoupled.

[19] Strictly speaking, this term contributes only to the bulk AdS_4 supercurrent, as there is an additional boundary term that compensates to allow massless gauginos in SUSY AdS_4 [87]. After lifting AdS space to flat space, this boundary term becomes irrelevant.

the full goldstino coupling in SUGRA is

$$\text{AdS space:} \quad \frac{m_\lambda - m_{\text{AdS}}}{\sqrt{2}F_{\text{eff}}} i\widetilde{G}_L \sigma^{\mu\nu}\lambda F_{\mu\nu}, \tag{4.54}$$

in perfect agreement with Eq. (4.30). Thus, the goldstino equivalence theorem holds even in the presence of gravitino mediation, albeit with the AdS supercurrent. This is as we anticipated, since particles and sparticles have SUSY mass splittings in the bulk of AdS space, so "soft masses" arising from m_{AdS} should not have an associated goldstino coupling. We could alternatively derive the same effect in unitary gauge for the gravitino by realizing that the last term in Eq. (4.43) modifies longitudinal gravitino interactions by an amount proportional to $m_{\text{AdS}}/m_{3/2}$.

4.5 Discussion

In this chapter, we have shown that anomaly mediation consists of two physically distinct phenomena, which can be distinguished by their associated goldstino couplings. Gravitino mediation (i.e. traditional anomaly mediation) is familiar from the phenomenology literature, but it has the counter-intuitive feature that it has no associated goldstino coupling. Indeed, the difference $m_\lambda - c_\lambda = m_{\text{AdS}}$ is a physical way to measure gravitino mediation, and c_λ characterizes the degree of sequestering between the visible sector and the goldstino. Kähler mediation simply arises from linear couplings of SUSY-breaking fields in the Kähler potential, and appears in both global and local SUSY. The soft masses and goldstino couplings from Kähler mediation satisfy the (flat space) goldstino equivalence theorem.

While these two faces of anomaly mediation can be understood directly in SUGRA component fields as in Sec. 4.4, the physics is more transparent using the improved gauge fixing of Ref. [38]. In this gauge, it is obvious why soft masses proportional to $m_{3/2}$ do not have any associated goldstino couplings, since the conformal compensator $\mathbf{\Phi}$ contains a piece $(1 + \theta^2 m_{3/2})$ with no fermionic component. It is also obvious that the super-Weyl anomaly contributes both to gravitino mediation and to Kähler mediation. For deriving Kähler mediation in SUGRA, it is convenient that the Kähler and sigma-model anomalies are tied together into a single $\mathbf{\Omega}$ function.

As previewed in the introduction, the case of scalar soft masses is more subtle, and we leave a detailed study to Ch. 5. For gravitino mediation, conservation of the AdS_4

supercurrent must hold to all loop orders, such that any soft mass proportional to the AdS curvature will have no associated goldstino coupling. However, tree-level tachyonic scalars masses given by $-2m_{3/2}^2$ must be compensated by SUSY breaking to have a stable theory in flat space. This tachyonic piece is in addition to the well-known two-loop anomaly-mediated soft masses, so even in sequestered theories, there will be irreducible (but unambiguous) couplings between matter multiplets and the goldstino. Since anomaly mediation can be alternatively derived using Pauli-Villars regulating fields [84, 78, 77], we should find that the soft masses and goldstino couplings of the regulators are precisely those necessary to maintain the gravitino/Kähler mediation distinction in the regulated theory.

We have emphasized the fact that a gaugino soft mass can appear with no associated goldstino couplings in the case of strict sequestering, which yields pure gravitino mediation. Interestingly, there are also reversed cases where a goldstino coupling is present with no associated gaugino mass. Famously, anomaly mediation is absent in no-scale SUSY breaking (and suppressed in almost-no-scale models) [120]. Also, theories with extra-dimensional warping can have suppressed anomaly mediation [119]. However, these arise from a cancellation between gravitino mediation and Kähler mediation (through moduli F-components), and thus goldstino couplings are still present even when there are no anomaly-mediated soft masses. This bizarre result is nevertheless required by conservation of the AdS supercurrent, and emphasizes the fact that the underlying symmetry structure of our universe is not just SUSY, but SUSY in AdS space.

Chapter 5

Anomaly Mediation from Unbroken Supergravity

5.1 Introduction

As we discussed in Ch. 2, spontaneously broken SUSY yields a positive contribution to the cosmological constant, so in order to achieve the nearly zero cosmological constant we see today, the underlying symmetry structure of our universe must be SUSY in anti-de Sitter (AdS) space. In the context of supergravity (SUGRA), the inverse AdS radius $\lambda_{\text{AdS}}^{-1}$ is equal to the gravitino mass $m_{3/2}$. Thus, because of the underlying AdS SUSY algebra, there will be effects on the supersymmetric standard model (SSM) proportional to $m_{3/2}$. These would appear as "SUSY-breaking" effects from the point of view of the flat space SUSY algebra, but are actually SUSY-preserving effects when viewed from AdS_4 space.

Famously, anomaly mediation [135, 84] yields gaugino masses proportional to $m_{3/2}$. As we showed in Ch. 4, these gaugino masses do not break AdS SUSY, and are in fact necessary for conservation of the AdS supercurrent. We called this phenomenon "gravitino mediation" to separate this $m_{3/2}$ effect from other anomaly-mediated effects which have nothing to do with the AdS SUSY algebra.[1] Throughout this chapter, we will use the more familiar (but less accurate) name "anomaly mediation" to refer to all effects proportional to $m_{3/2}$ (i.e. gravitino mediation; see Refs. [35, 16, 17, 60, 87, 99, 44, 140] for additional

[1]These other effects were dubbed "Kähler mediation" since they arise from linear couplings of SUSY breaking to visible sector fields in the Kähler potential. Full anomaly mediation is simply the sum of Kähler mediation and gravitino mediation. See Ch. 5 for details. There is also a (usually subleading) anomaly-mediated effect noted in Ref. [44] if there are direct couplings of SUSY breaking to the gauginos at tree-level.

theoretical perspectives). Unlike usual SUSY-breaking effects, anomaly mediation generates gaugino masses without accompanying goldstino couplings, further emphasizing that this is a SUSY-preserving effect.

The goal of this chapter is twofold. First, we wish to extend the analysis of Ch. 5 to the case of sfermions. It is well known that anomaly mediation yields two-loop scalar mass-squareds proportional to $m_{3/2}^2$, but we will show that from the point of view of AdS$_4$ space, anomaly mediation already yields scalar masses at tree level. Following the strategy of Ch. 5, we will use goldstino couplings as a guide to determine which effects preserve AdS SUSY, allowing us to distinguish between SUSY-preserving effects that are genuinely proportional to $m_{3/2}$ versus SUSY-breaking effects that are only proportional to $m_{3/2}$ because of the need to fine tune the cosmological constant to zero. Second, we wish to counter recent claims by de Alwis that anomaly mediation does not exist [46, 47]. In contrast, we will use the same logical starting point as de Alwis (which is based on the analysis of Kaplunovsky and Louis [103]) but come to the conclusion that anomaly mediation not only exists, but is necessary for the preservation of AdS SUSY.

Along the way, we will encounter a number of surprises, all ultimately having to do with the structure of AdS SUSY:

- **Tree-Level Tachyons and Sequestering**. Already at tree-level in AdS space, the components of a chiral multiplet get SUSY mass splittings proportional to $m_{3/2}$. For example, if the fermionic component is massless, then its scalar partner has a negative mass-squared $-2m_{3/2}^2$, satisfying the Breitenlohner-Freedman bound [25].[2] In order to have a stable theory after AdS SUSY is lifted to flat space via SUSY breaking, this negative mass-squared must also be lifted. Since such a lifting must break AdS SUSY, this requires irreducible couplings between the SUSY-breaking sector ("hidden sector") and the SSM ("visible sector"), even in theories where the hidden and visible sectors are sequestered [135]. For a chiral multiplet with components $\{\phi, \chi, F\}$ there is necessarily a coupling to the goldstino $\widetilde{G}_L$ when the sfermion soft mass is zero in flat space:

$$\mathcal{L} \supset \frac{2m_{3/2}^2}{F_{\text{eff}}} \widetilde{G}_L \chi \phi^*, \tag{5.1}$$

where F_{eff} is the scale of SUSY breaking. Intriguingly, this coupling is renormalization-

[2] A fermion with mass $\pm\frac{1}{2}m_{3/2}$ will have one scalar partner with mass-squared $-\frac{9}{4}m_{3/2}^2$, exactly saturating the bound.

group invariant, and effectively defines what it means to sequester the hidden and visible sectors.[3]

- **Giudice-Masiero in AdS Space**. In flat space, the harmonic part of the Kähler potential (i.e. the chiral plus anti-chiral part) is unphysical. This is not the case in AdS space, and the Giudice-Masiero mechanism [82] is a way to generate μ and B_μ terms via $\boldsymbol{K} \supset \boldsymbol{H}_u \boldsymbol{H}_d + \text{h.c.}$ While the generated μ term preserves AdS SUSY, the B_μ term actually breaks AdS SUSY, since it secretly involves direct couplings between Higgs multiplets and the goldstino. When written in a more natural basis, it becomes clear that Giudice-Masiero arises from a combination of a SUSY-preserving and SUSY-breaking effect.

- **Anomaly Mediation and Super-Weyl Invariance**. As emphasized in Ref. [60], anomaly mediation is not due to any anomaly of SUSY itself,[4] but is rather due to the need to add local counterterms to preserve SUSY of the 1PI effective action. A related story presented in Ref. [87] is that bulk counterterms are needed to counteract otherwise SUSY-breaking effects due to the boundary of AdS_4. Here, we will follow the logic of de Alwis [46, 47] (based on the analysis of Kaplunovsky and Louis [103]) to show how anomaly mediation arises from preserving super-Weyl invariance of a UV-regulated SUGRA theory. While de Alwis (erroneously) concluded that anomaly mediation cannot exist in such a situation, we find that there is residual gauge dependence in de Alwis' calculation (and a similar issue implicit in Kaplunovsky and Louis). In the langauge of the Weyl compensator, anomaly mediation depends not just on the F_C component of the compensator (which can be gauge-fixed to zero), but on the super-Weyl-invariant combination

$$F_{\text{SW}} \equiv F_C - \frac{1}{3} M^*, \tag{5.2}$$

where M is the scalar auxiliary field. Accounting for the fact that $\langle F_{\text{SW}} \rangle$ depends on $m_{3/2}$, we reproduce the familiar anomaly-mediated spectrum.

[3] In Ch. 5, we (erroneously) advocated that the absence of goldstino couplings could be used as a physical definition of sequestering. Because of this tree-level tachyon subtlety, though, this goldstino coupling is needed to have a stable theory.

[4] Of course, the name "anomaly mediation" is still justified since it generates effects proportional to beta function coefficients.

- **Supertraces Resolve Spectrum Ambiguities.** We will use an ansatz for the SUGRA-invariant 1PI effective action to extract sfermion soft masses and goldstino couplings. Because there are many such ansätze consistent with SUGRA, there is an ambiguity in the resulting sfermion spectrum. For example, there are three terms that show up at $\mathcal{O}(m_{3/2}^2)$ in the 1PI effective action:

$$\mathcal{L}_{\text{soft mass}} = -\mathcal{C}_s \phi^* \phi - \mathcal{C}_a F^* \Box^{-1} F + i\mathcal{C}_f \chi^\dagger \overline{\sigma}^\mu \mathcal{D}_\mu \Box^{-1} \chi, \tag{5.3}$$

where $\Box$ is the d'Alembertian appropriate to curved space. The first term is the familiar sfermion soft mass-squared term, but the two non-local terms necessarily appear as m^2/p^2 corrections to the self-energies. We will find that while the coefficients $\mathcal{C}_i$ are indeed ambiguous (since they depend the precise form of the ansatz), the supertrace

$$\mathcal{S} = \mathcal{C}_s + \mathcal{C}_a - 2\mathcal{C}_f \tag{5.4}$$

is unambiguous and gives a useful measure of the "soft mass-squared" for a sfermion (see Ref. [12] for a related story). Not surprisingly, a similar supertrace is needed to define unambiguous "goldstino couplings".

- **SUSY-Breaking in the SUGRA Multiplet.** The key confusion surrounding anomaly mediation is that there are two different order parameters in SUGRA—one which sets the underlying AdS curvature and one which accounts for SUSY breaking—which are only related to each other after tuning the cosmological constant to zero. In particular, a non-vanishing vacuum expectation value (vev) for M^* (containing the term $-3m_{3/2}$ in SUGRA frame) does not break SUSY. Instead, the SUSY-breaking order parameter in SUGRA comes from the F-component of the chiral curvature superfield $\boldsymbol{R}$:

$$F_R \equiv \frac{1}{12}\mathcal{R} - m_{3/2}^2. \tag{5.5}$$

After using the Einstein equation, F_R vanishes for unbroken SUSY in AdS, but takes on the value $-m_{3/2}^2$ once the cosmological constant has been tuned to zero. Thus in flat space, we will find both SUSY-breaking and SUSY-preserving effects proportional

		Tree-Level	One-Loop	Two-Loop
SUSY AdS$_4$ ($\mathcal{R} = 12m_{3/2}^2$)	Soft Mass-Squared	$-2m_{3/2}^2$	$\gamma m_{3/2}^2$	$-\frac{1}{4}\dot{\gamma}m_{3/2}^2$
	Goldstino Coupling	—	—	
Curved Space (broken SUSY)	Soft Mass-Squared	$-\frac{1}{6}\mathcal{R}$	$\frac{1}{12}\gamma\mathcal{R}$	$-\frac{1}{4}\dot{\gamma}m_{3/2}^2$
	Goldstino Coupling	$-2(m_{3/2}^2 - \frac{1}{12}\mathcal{R})$	$\gamma(m_{3/2}^2 - \frac{1}{12}\mathcal{R})$	—
Flat Space (broken SUSY)	Soft Mass-Squared	—	—	$-\frac{1}{4}\dot{\gamma}m_{3/2}^2$
	Goldstino Coupling	$-2m_{3/2}^2$	$\gamma m_{3/2}^2$	—

Table 5.1: Sfermion soft masses and goldstino couplings from minimal anomaly mediation (i.e. "gravitino mediation" in the language of Ch. 5, so $\langle K_i \rangle = 0$). Here, γ is the anomalous dimension of the chiral multiplet and $\dot{\gamma} \equiv d\gamma/d\log\mu$. Starting with unbroken SUSY in AdS$_4$ with Ricci curvature $\mathcal{R} = 12\lambda_{\text{AdS}}^{-2} = 12m_{3/2}^2$, we show how the spectrum evolves as SUSY breaking is tuned to achieve flat space with $\mathcal{R} \to 0$. In this table, "soft mass-squared" and "goldstino coupling" refer to the supertraces in Eqs. (5.87) and (5.90), and the loop level refers to the order at which the effect starts. Minimal anomaly mediation also yields A-terms and B-terms, which are described in Sec. 5.4.5. This table only includes the contributions from bulk terms and not from one- and two-loop boundary terms (analogous to Ref. [87]) necessary to preserve the SUSY algebra in AdS$_4$; these boundary terms are irrelevant in flat space.

to $m_{3/2}^2$, and we will have to tease these two effects apart by carefully considering AdS SUSY. We will also find corresponding goldstino couplings proportional to F_R, arising from terms in the SUGRA multiplet proportional to the gravitino equations of motion.

- **Two-Loop Soft Masses and One-Loop Goldstino Couplings.** Using an ansatz for the all-orders SUGRA-invariant 1PI effective action, we will recover the familiar two-loop soft masses from anomaly mediation. But in addition, we will find one-loop goldstino couplings proportional to anomalous dimensions (on top of the tree-level goldstino coupling from Eq. (5.1)). As a cross check of our calculation, both the two-loop soft mass and the one-loop goldstino coupling are renormalization-group (RG) invariant quantities, as expected from the general analysis of Refs. [98, 97, 133, 12]. The complete sfermion spectrum is summarized in Table 5.1.

The remainder of this chapter is organized as follows. In Sec. 5.2, we review the structure of SUGRA at tree-level, and show how the underlying AdS algebra gives rise to SUSY-preserving mass splittings between fermions and sfermions. In Sec. 5.3, we discuss super-

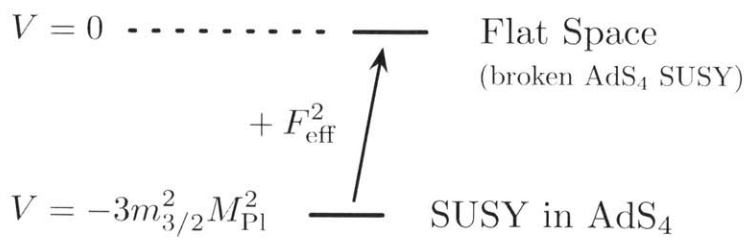

Figure 5-1: Fine-tuning of the cosmological constant, adapted from Ref. [21]. Starting with the underlying AdS radius $\lambda^{-1}_{\text{AdS}} = m_{3/2}$, SUSY-breaking effects lead to flat space with broken (AdS) SUSY.

Weyl invariance in UV-regulated SUGRA theories at one loop, and show how anomaly mediation arises as a super-Weyl-preserving and SUSY-preserving effect. In Sec. 5.4, we discuss anomaly mediation for sfermions up to two-loop order, completing the analysis of goldstino couplings that was initiated in Ch. 5. We conclude in Sec. 5.5.

5.2 Invitation: Anomaly Mediation at Tree Level

It is well known that rigid AdS SUSY requires mass splittings between particles and sparticles [25, 132]. Less well known is that those mass splittings have an impact on the phenomenology of SUGRA, even if the geometry (after SUSY breaking) is that of flat space. In particular, the couplings of the goldstino (eaten to form the longitudinal components of the gravitino) can be used to track which effects break SUSY and which effects preserve SUSY. Crucially, these couplings depends on $m_{3/2}$, which in turn depends on the underlying AdS radius $\lambda^{-1}_{\text{AdS}} = m_{3/2}$ *prior* to SUSY breaking. The fine-tuning of the cosmological constant to achieve flat space is summarized in Fig. 5-1.

Considering only chiral multiplets, we can write the fermion and sfermion masses and sfermion-fermion-goldstino couplings as

$$\mathcal{L} \supset -m^2_{\bar{i}j}\phi^{*\bar{i}}\phi^j - \frac{1}{2}B_{ij}\phi^i\phi^j - \frac{1}{2}M_{ij}\chi^i\chi^j + \frac{a_{\bar{i}j}}{F_{\text{eff}}}\phi^{*\bar{i}}\chi^j\widetilde{G}_L + \frac{b_{ij}}{F_{\text{eff}}}\phi^i\chi^j\widetilde{G}_L + \text{h.c.}, \tag{5.6}$$

where ϕ_i is a sfermion, ψ_i is its fermion partner, $\widetilde{G}_L$ is the goldstino, and F_{eff} is the scale of SUSY breaking. Assuming the flat space SUSY algebra, one can show that

$$a^{\text{flat}}_{\bar{i}j} = m^2_{\bar{i}j} - M_{\bar{i}}{}^{k}M_{kj}, \qquad b^{\text{flat}}_{ij} = B_{ij}, \tag{5.7}$$

which emphasizes that goldstino couplings arise when sfermions and fermions have non-zero mass splittings (i.e. when flat space SUSY is broken). In AdS space at tree-level, however, we will show that

$$a_{\bar{i}j}^{\text{AdS}} = m_{\bar{i}j}^2 - M_i{}^k M_{kj} + 2m_{3/2}^2 \delta_{\bar{i}j}, \qquad b_{ij}^{\text{AdS}} = B_{ij} + m_{3/2} M_{ij}, \tag{5.8}$$

which shows that one can have $m_{3/2}$-dependent mass splittings between multiplets without corresponding goldstino couplings (i.e. without breaking AdS SUSY).

In this section, we give two different derivations of Eq. (5.8), with a third derivation using the conformal compensator given in App. C.1. We then discuss the phenomenological implications of these goldstino couplings for sequestering, Giudice-Masiero terms, and regulator fields. Though the goldstino is eaten by the gravitino in SUGRA, the couplings of the goldstino are still physically relevant. Indeed, in the goldstino equivalence theorem regime with energies $E \gg m_{3/2}$, the interactions of the longitudinal components of the gravitino are captured by the goldstino couplings in Eq. (5.8) (plus modifications to those goldstino couplings that appear at higher-loop order).

5.2.1 Derivation from the SUGRA Lagrangian

The first way to derive Eq. (5.8) is to consider the SUGRA Lagrangian directly. The scalar potential for SUGRA is [154]

$$V = e^G (G^k G_k - 3), \tag{5.9}$$

where the Kähler-invariant potential G is given by[5]

$$G \equiv K + \log W + \log W^\dagger. \tag{5.10}$$

Throughout the text, we use the conventions of Ref. [154]. Here, subscripts represent derivatives with respect to scalar fields ($G_k = \partial G/\partial \phi^k$), and indices are raised and lowered with the Kähler metric $G_{i\bar{j}}$ and its inverse. The gravitino mass is given by

$$m_{3/2} = \left\langle e^{G/2} \right\rangle, \tag{5.11}$$

[5]The Kähler anomaly [16, 17] implies a physical difference between the Kähler potential and the superpotential, but it does not enter at tree level.

and the quadratic fermion interactions in SUGRA are

$$\mathcal{L} \supset -iG_{i\bar{j}}\chi^{\dagger\bar{j}}\bar{\sigma}^\mu\mathcal{D}_\mu\chi^i - \frac{1}{2}e^{G/2}(\nabla_i G_j + G_i G_j)\chi^i\chi^j + \text{h.c.} \tag{5.12}$$

where $\mathcal{D}_\mu$ and ∇_i are the Kähler-covariant derivatives with respect to spacetime and scalar fields, respectively.

If SUGRA is unbroken ($\langle G_i \rangle = 0$), then we have a negative cosmological constant ($\langle V \rangle = -3m_{3/2}^2 M_{\text{Pl}}^2$), so the spacetime background is AdS, with curvature $\lambda_{\text{AdS}}^{-1} = m_{3/2}$. The fermion mass matrix is

$$M_{ij} = m_{3/2}\,\langle \nabla_i G_j \rangle \qquad \text{(unbroken SUGRA)}, \tag{5.13}$$

and at the extremum of the potential ($\langle V_i \rangle = 0$), the scalar mass-squared and holomorphic mass can be expressed in terms of M_{ij} as

$$m_{i\bar{j}}^2 = M_{ik}M^k{}_{\bar{j}} - 2m_{3/2}^2\delta_{i\bar{j}}, \tag{5.14}$$

$$B_{ij} = -m_{3/2}M_{ij} \qquad \text{(unbroken SUGRA)}. \tag{5.15}$$

These are the same as the results we found in for rigid AdS SUSY in Sec. 2.7, as expected. Note that inserting these mass values into Eq. (5.8) yields no goldstino couplings, as is to be expected since there is no goldstino when SUGRA is unbroken.

If SUGRA is broken, then there are a few important effects. Defining the SUSY-breaking scale as

$$F_{\text{eff}} \equiv \sqrt{e^G G^k G_k}, \tag{5.16}$$

we find the the cosmological constant is modified to be

$$\langle V \rangle = F_{\text{eff}}^2 - 3m_{3/2}^2 M_{\text{Pl}}^2, \tag{5.17}$$

where we have restored factors of the Planck constant M_{Pl}. As shown in Fig. 5-1, it is possible to fine-tune $V = 0$ by choosing

$$F_{\text{eff}} = \sqrt{3}m_{3/2}M_{\text{Pl}}. \tag{5.18}$$

In addition, SUSY breaking gives rise to a goldstino, which (assuming no D-terms for the gauge multiplets for simplicity) points in the direction

$$\widetilde{G}_L = -\frac{1}{\sqrt{3}} G^i \chi_i. \tag{5.19}$$

The fermion and sfermion mass matrices are generically deformed due to the presence of SUSY breaking, and their form is well-known for $\langle V \rangle = 0$ and $\langle V_i \rangle = 0$ [154]:[6]

$$M_{ij} = m_{3/2} \langle \nabla_i G_j + G_i G_j \rangle , \tag{5.20}$$

$$m^2_{i\bar{j}} = m^2_{3/2} \left\langle \nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G_{i\bar{j}} \right\rangle , \tag{5.21}$$

$$m^2_{ij} = m^2_{3/2} \left\langle G^k \nabla_i \nabla_j G_k + 2 \nabla_i G_j \right\rangle , \tag{5.22}$$

where $R_{i\bar{j}kl}$ is the Kähler curvature tensor.[7]

The Yukawa couplings can similarly be extracted from Eq. (5.12):

$$\mathcal{L} \supset -\frac{1}{2} m_{3/2} \left\langle -R_{i\bar{j}k\bar{l}} G^{\bar{l}} + G_{i\bar{j}} G_k + G_i G_{k\bar{j}} \right\rangle \chi^i \chi^k \phi^{*\bar{j}} \tag{5.23}$$

$$- \frac{1}{2} m_{3/2} \langle \nabla_i \nabla_j G_k + G_i \nabla_j G_k + G_k \nabla_i G_j + G_j \nabla_k G_i + G_i G_j G_k \rangle \chi^i \chi^k \phi^j . \tag{5.24}$$

One can read off the couplings of the goldstino to visible-sector fields after picking out the goldstino direction:

$$a_{\bar{i}j} = m^2_{3/2} \left\langle -R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + 3 G_{i\bar{j}} \right\rangle , \tag{5.25}$$

$$b_{ij} = m^2_{3/2} \left\langle G^k \nabla_i \nabla_j G_k + 3 \nabla_i G_j \right\rangle , \tag{5.26}$$

recalling that $\langle G_i \rangle$ is negligible for visible-sector fields. This then yields the goldstino couplings anticipated in Eq. (5.8) (at least for the case of $\langle V \rangle = 0$).

Thus, despite the fact that SUGRA is broken and the cosmological constant is lifted to yield $\langle V \rangle = 0$, the goldstino couplings retain information about the structure of the underlying AdS SUSY, and not the structure of flat space SUSY.

[6] There is a typo in Ref. [154] which omits the first term in Eq. (5.22).

[7] Here, and throughout the text, we do not choose any gauge fixing for the gravitino, so there is also quadratic mixing between the goldstino and the gravitino. See Eq. (5.31) below.

5.2.2 Derivation from Supercurrent Conservation

An alternative derivation of Eq. (5.8) uses conservation of the AdS supercurrent. The supercurrent is the Noether current of (rigid) SUSY transformations, and in SUGRA, the linear couplings of the gravitino ψ_μ to matter are determined by the supercurrent alone:

$$\mathcal{L} = \epsilon^{\mu\nu\rho\tau}\psi^\dagger_\mu\overline{\sigma}_\nu\mathcal{D}_\rho\psi_\tau - m_{3/2}\psi^\dagger_\mu\overline{\sigma}^{\mu\nu}\psi^\dagger_\nu + \text{h.c.} - \frac{1}{2M_{\text{Pl}}}\psi^\dagger_\mu j^{\dagger\mu} + \text{h.c.} \tag{5.27}$$

Appropriate manipulation of the gravitino equation of motion (and the Einstein equation, given Eq. (5.17)) yields the relation

$$0 = \left(\mathcal{D}_\mu j^{\dagger\mu} + \frac{1}{2}im_{3/2}\overline{\sigma}^\mu j_\mu\right) - i\frac{F^2_{\text{eff}}}{M_{\text{Pl}}}\overline{\sigma}^\mu\psi_\mu. \tag{5.28}$$

This relation can be most naturally interpreted in the rigid limit ($M_{\text{Pl}} \to \infty$, $m_{3/2}$ and F_{eff} fixed), in which the last term vanishes and the spacetime background is AdS (with $\lambda^{-1}_{\text{AdS}} = m_{3/2}$). In the rigid limit, we see clearly that conservation of the supercurrent is different in flat space versus AdS space. In flat space, the fermionic SUSY transformation parameter ϵ satisfies the criteria $\partial_\mu\epsilon = 0$, whereas in AdS space

$$\mathcal{D}_\mu\epsilon = -\frac{i}{2}m_{3/2}\sigma_\mu\epsilon^\dagger, \tag{5.29}$$

where $\mathcal{D}_\mu$ is the (gravity) covariant derivative [4, 70]. Among other things, this implies that the goldstino in rigid AdS space has a mass of $2m_{3/2}$ [42, 39]. It also implies that the condition for conservation of the supercurrent is not $\partial_\mu j^\mu = 0$ but rather the rigid limit of Eq. (5.28), as Noether's theorem requires $\mathcal{D}_\mu(j^\mu\epsilon + j^{\dagger\mu}\epsilon^\dagger) = 0$.

When SUSY is broken, the supercurrent contains the goldstino

$$j^{\dagger\mu} = \sqrt{2}F_{\text{eff}}i\overline{\sigma}^\mu\widetilde{G}_L + \widetilde{j}^{\dagger\mu}, \tag{5.30}$$

where $\widetilde{j}_\mu$ is the remaining "matter" part of the supercurrent. Eq. (5.28) can then be interpreted as the goldstino equation of motion arising from the Lagrangian

$$\mathcal{L} = -i\widetilde{G}_L^\dagger \overline{\sigma}^\mu \nabla_\mu \widetilde{G}_L - \frac{1}{2}(2m_{3/2})\widetilde{G}_L\widetilde{G}_L + \text{h.c.} + \frac{i}{\sqrt{2}}\frac{F_{\text{eff}}}{M_{\text{Pl}}}\widetilde{G}_L^\dagger \overline{\sigma}^\mu \psi_\mu + \text{h.c.}$$
$$- \frac{1}{\sqrt{2}F_{\text{eff}}}\left(\mathcal{D}_\mu \widetilde{j}^\mu - \frac{1}{2} i m_{3/2}\widetilde{j}^{\dagger\mu}\overline{\sigma}_\mu\right)\widetilde{G}_L + \text{h.c.}, \tag{5.31}$$

where the last term is necessary for conservation of the AdS supercurrent.

In both flat space and AdS space, the supercurrent for chiral multiplets contains[8]

$$j^\mu \supset \sqrt{2} g_{i\bar{j}} \partial_\nu \phi^{*j} \chi^i \sigma^\mu \overline{\sigma}^\nu. \tag{5.32}$$

The other term proportional to $\chi^{\dagger i} D_{\bar{i}} W^* \chi^{\dagger \bar{i}} \overline{\sigma}^\mu$ is irrelevant for our discussions since it vanishes on the goldstino equation of motion. Using the equations of motion for the matter fields and the goldstino equation of motion, we find that Eq. (5.31) contains the goldstino couplings

$$a_{ij} = m_{i\bar{j}}^2 - M_{ik}M^k{}_{\bar{j}} + 2m_{3/2}^2 \delta_{i\bar{j}}, \tag{5.33}$$
$$b_{ij} = B_{ij} + m_{3/2} M_{ij}, \tag{5.34}$$

as expected from Eq. (5.8). Note that the terms proportional to $m_{3/2}$ arise from the additional goldstino mass and $\frac{1}{F_{\text{eff}}} i m_{3/2} \widetilde{j}^{\dagger\mu} \overline{\sigma}_\mu \widetilde{G}_L$ terms necessary for AdS supercurrent conservation.

5.2.3 Tachyonic Scalars and Sequestering

The fermions in the standard model are massless (prior to electroweak symmetry breaking), so in the absence of AdS SUSY breaking, the sfermions would be tachyonic, with a common mass-squared $-2m_{3/2}^2$ (see Eq. (5.14)). In order to have a (meta)stable vacuum after SUSY breaking, these tachyonic masses must be lifted, but from the a_{ij} term in Eq. (5.8), this implies an irreducible coupling between the goldstino and the matter fields.

This result is rather surprising from the point of view of strictly sequestered theories [135], where anomaly mediation is the only source of soft masses. As shown in Fig. 5-

[8] This assumes that the SUGRA action only contains a Kähler potential and a superpotential without additional higher-derivative interactions. The supercurrent is modified when loop effects are taken into account, giving rise to new effects detailed in Sec. 5.4.

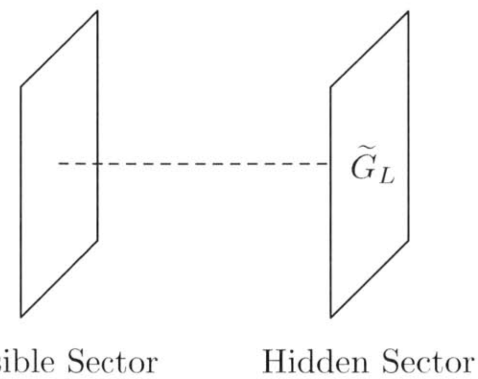

Figure 5-2: An extra-dimensional realization of the sequestered limit, where SUSY is broken only in a hidden sector. Naively, the goldstino is localized in the hidden sector and would not couple to visible sector fields. But due to mixing with the gravitino, there are irreducible couplings between the goldstino and chiral multiplets in the visible sector in order to have a stable tree-level theory in flat space after SUSY breaking.

2, one way to achieve the sequestered limit is to have the visible sector (i.e. the SSM) and the hidden sector (i.e. SUSY-breaking dynamics) live in different parts of an extra-dimensional space with no light degrees of freedom connecting the two apart from gravity. This implies a special sequestered form of the effective four-dimensional Kähler potential and superpotential:

$$-3e^{-K/3} = \Omega_{\text{vis}} + \Omega_{\text{hid}}, \qquad W = W_{\text{vis}} + W_{\text{hid}}. \tag{5.35}$$

Naively, one would think that the goldstino from SUSY-breaking must be localized in the hidden sector (assuming the SSM itself does not break SUSY [96, 20]), and therefore decoupled from the visible sector. But Eq. (5.8) shows that there are direct connections between the visible and hidden sectors necessary for stability of the theory. In particular, there is an irreducible coupling to the goldstino when the sfermion soft mass is zero in flat space:

$$\mathcal{L} \supset \frac{2m_{3/2}^2}{F_{\text{eff}}} \widetilde{G}_L \chi \phi^*. \tag{5.36}$$

There are two potential ways to interpret this result. One interpretation is to conclude that sequestering corresponds to a fine-tuned limit. After all, in the sequestered limit at tree-level, one has the underlying $-2m_{3/2}^2$ AdS tachyonic mass balanced against the $+2m_{3/2}^2$ SUSY-breaking mass to yield the physical tree-level sfermion mass of zero once the cosmological constant is tuned to zero. This interpretation is probably too pessimistic, though, since the tachyonic uplifting is an automatic consequence of adjusting the cosmological constant. Concretely, this uplifted mass arises from the scalar auxiliary field (and the corresponding goldstino couplings arise from mixing with the gravitino), so once you have the sequestered form of K and W, you necessarily obtain zero scalar masses but non-zero $a_{i\bar{j}}$ couplings.

A second, more optimistic, interpretation is that Eq. (5.36) gives a concrete definition of sequestering. While the extra-dimensional picture in Fig. 5-2 is a nice realization of sequestering, the sequestered limit can be achieved in more general theories. In four-dimensional models with conformal sequestering [123, 118, 141], the visible and hidden sectors effectively decouple under RG flow to the infrared, assuming all composite vector multiplets in the hidden sector have mass dimension greater than 2. As we explain in App. C.2, Eq. (5.36) is actually RG invariant, so one might conjecture that it corresponds to precisely the (attractive) IR fixed point needed to have a conformally sequestered theory. More generally, one can identify when a theory is sequestered if Eq. (5.36) (and corresponding loop corrections, see Sec. 5.4.5) is the only coupling between the visible and hidden sectors.[9]

Regardless of how one interprets this result, the irreducible goldstino coupling is an unavoidable consequence of AdS SUSY lifted to flat space, since something needs to lift the tachyonic scalars to have a stable theory in flat space. One might even hope to measure Eq. (5.36) experimentally as a way to gain access to the underlying AdS curvature.

5.2.4 Giudice-Masiero Terms

The Giudice-Masiero mechanism [82] is a way to generate a μ term and a B_μ term proportional to $m_{3/2}$ without (apparently) requiring couplings between the visible and hidden sectors. Via a holomorphic piece in the Kähler potential (written using boldface to emphasize that these are superfields)

$$-3e^{-3\boldsymbol{K}} \supset \epsilon \boldsymbol{H}_u \boldsymbol{H}_d + \text{h.c.}, \tag{5.37}$$

[9]As shown in Ch. 5, the sequestered limit implies that gaugino-gauge boson-goldstino couplings are zero.

one generates the fermion and scalar mass terms

$$\mathcal{L} \supset -\epsilon m_{3/2}\psi_u\psi_d - \epsilon m_{3/2}^2 h_u h_d + \text{h.c.} \quad \Rightarrow \quad \frac{B_\mu}{\mu} = +m_{3/2}. \tag{5.38}$$

The sign of B_μ here is crucial, since if instead one had the superpotential

$$\boldsymbol{W} \supset \mu \boldsymbol{H}_u \boldsymbol{H}_d, \tag{5.39}$$

the fermion and scalar mass terms would be

$$\mathcal{L} \supset -\mu\psi_u\psi_d + m_{3/2}\mu h_u h_d + \text{h.c.} \quad \Rightarrow \quad \frac{B_\mu}{\mu} = -m_{3/2}. \tag{5.40}$$

From Eq. (5.8), we see that the Giudice-Masiero mechanism actually does break SUSY (with $b_{ij} = 2m_{3/2}\mu$), while generating B_μ from the superpotential does not break SUSY (i.e. $b_{ij} = 0$). Written in this language, it is confusing how a goldstino coupling could appear in the Giudice-Masiero mechanism since there is no goldstino present in Eq. (5.37).

We can do a Kähler transformation to make the physics manifest. To model SUSY breaking, we use a non-linear goldstino multiplet [137, 116, 109, 42, 39]

$$\boldsymbol{X}_{\text{NL}} = F_X \left(\theta + \frac{1}{\sqrt{2}F_X}\widetilde{G}_L\right)^2 \tag{5.41}$$

that satisfies $\boldsymbol{X}_{\text{NL}}^2 = 0$. In a theory where the visible Higgs multiplets are sequestered from SUSY-breaking, the relevant pieces of the Kähler potential and superpotential are

$$-3e^{-\boldsymbol{K}/3} = -3 + \boldsymbol{X}_{\text{NL}}^\dagger \boldsymbol{X}_{\text{NL}} + \epsilon(\boldsymbol{H}_u\boldsymbol{H}_d + \text{h.c.}) + \ldots, \tag{5.42}$$

$$\boldsymbol{W} = m_{3/2} + f\boldsymbol{X}_{\text{NL}} + \ldots, \tag{5.43}$$

where the equations of motion set $F_X^* = -f$ and fine-tuning the cosmological constant to zero requires $f = \sqrt{3}m_{3/2}$. At tree-level, the physics is invariant to doing a Kähler transformation[10]

$$\boldsymbol{K} \to \boldsymbol{K} + \boldsymbol{\Omega} + \boldsymbol{\Omega}^\dagger, \qquad \boldsymbol{W} \to e^{-\boldsymbol{\Omega}}\boldsymbol{W}, \tag{5.44}$$

[10]At loop level, one must account for the Kähler anomaly [16].

so choosing $\boldsymbol{\Omega} = -\epsilon \boldsymbol{H}_u \boldsymbol{H}_d$, we have

$$-3e^{-\boldsymbol{K}/3} = -3 + \boldsymbol{X}_{\mathrm{NL}}^{\dagger} \boldsymbol{X}_{\mathrm{NL}} - \frac{\epsilon}{3} \boldsymbol{X}_{\mathrm{NL}}^{\dagger} \boldsymbol{X}_{\mathrm{NL}} (\boldsymbol{H}_u \boldsymbol{H}_d + \text{h.c.}) + \ldots, \tag{5.45}$$

$$\boldsymbol{W} = m_{3/2} + f \boldsymbol{X}_{\mathrm{NL}} + \epsilon m_{3/2} \boldsymbol{H}_u \boldsymbol{H}_d + \epsilon f \boldsymbol{X}_{\mathrm{NL}} \boldsymbol{H}_u \boldsymbol{H}_d + \ldots. \tag{5.46}$$

We see immediately that the Higgs multiplets have a SUSY-preserving $\mu = \epsilon m_{3/2}$, and a corresponding SUSY-preserving contribution to B_μ of $-\mu m_{3/2} = -\epsilon m_{3/2}^2$. But there are also SUSY-breaking B_μ terms from direct couplings to $\boldsymbol{X}_{\mathrm{NL}}$ in both the Kähler potential and superpotential. This yields a contribution to B_μ of $(-\frac{1}{3}+1)\epsilon |f|^2$, which equals $+2\epsilon m_{3/2}^2$ after tuning the cosmological constant to zero. Therefore, we have

$$B_\mu = -\epsilon m_{3/2}^2 + 2\epsilon m_{3/2}^2 = +\epsilon m_{3/2}^2, \qquad b_{ij} = 2\epsilon m_{3/2}^2, \tag{5.47}$$

as required by Eq. (5.8).

Despite the fact that Giudice-Masiero can be written in a sequestered form in Eq. (5.42), there is secretly a coupling between the visible sector Higgs multiplets and the hidden sector goldstino.[11] Thus, we conclude that the relation $B_\mu/\mu = +m_{3/2}$ is due to a partial cancellation between a SUSY-preserving and a SUSY-breaking effect, and corresponds to a tuning between (otherwise) independent parameters. In the strict sequestered limit where only irreducible goldstino couplings are allowed, Giudice-Masiero terms must be absent.

5.2.5 Mass Splittings for Regulators

In order to set the stage for talking about anomaly mediation at loop level in the next section, we want to discuss a bit about the physics that regulates logarithmic UV divergences in SUGRA. There are various ways to introduce an effective cut-off scale Λ_{UV} into SUGRA, for example by introducing Pauli-Villars regulators [75, 76] or higher-dimension operators that regulate the UV behavior [103]. However, already at tree-level, we can see the consequences of having a physical regulator in AdS SUSY.

Consider a Pauli-Villars chiral regulator field with a SUSY-preserving mass Λ_{UV}. If this regulator does not break AdS SUSY, then it must have an additional scalar negative mass-squared $-2m_{3/2}^2$ as well as a B-term of $-m_{3/2}\Lambda_{\mathrm{UV}}$, giving rise to SUSY-preserving

[11] Of course, the physics is invariant to Kähler transformations at tree-level; all we have done here is choose a convenient Kähler basis to make the physics more clear.

mass splittings between the Pauli-Villars fermions and scalars:

$$m_{\text{PV-scalar}}^2 = -2m_{3/2}^2 + \Lambda_{\text{UV}}^2 \pm m_{3/2}\Lambda_{\text{UV}}, \qquad m_{\text{PV-fermion}} = \Lambda_{\text{UV}}. \tag{5.48}$$

Any UV-divergent SUGRA calculation that properly includes the regulator modes will be affected by this mass splitting, and this fact is one way to understand the necessity of anomaly mediation.[12] We often say that anomaly mediation is "gauge mediation by the regulators", in the sense that the (SUSY-preserving) mass splitting at the threshold Λ_{UV} acts analogously to the (SUSY-breaking) messenger mass threshold of gauge mediation. Crucially, we will see that the mass splittings generated by anomaly mediation do not break AdS SUSY.

It is possible, however, to regulate SUGRA with a regulator multiplet whose scalar and fermionic components have a common mass Λ_{UV}, for example by appropriately coupling the regulators to the SUSY-breaking $\boldsymbol{X}_{\text{NL}}$. All this means is that the regulator multiplet must have corresponding goldstino couplings by conservation of the AdS supercurrent:

$$a_{\text{PV}} = 2m_{3/2}^2, \qquad b_{\text{PV}} = m_{3/2}\Lambda_{\text{UV}}. \tag{5.49}$$

Since there is no mass splitting among the regulators, no mass splittings are generated. However, we would instead get goldstino couplings from the regulator fields! One can of course consider an intermediate case with a combination of mass splittings and goldstino couplings. In either event, one can show that modifying regulator couplings in this fashion is phenomenologically equivalent to changing $\langle K_i F^i \rangle$ for the purposes of loop-level calculations,[13] so for simplicity we will assume regulators have no explicit coupling to SUSY breaking in the subsequent sections.[14]

[12] In Sec. 5.3.4, we will show how the regulators must be included to get super-Weyl-invariant gaugino masses.

[13] In the language of Sec. 5.3, coupling regulators in such a fashion is largely equivalent to making the replacement $\boldsymbol{C} \to \boldsymbol{C}(1 + \boldsymbol{X}_{\text{NL}}/\Lambda)$, with $\boldsymbol{C}$ the Weyl compensator.

[14] To avoid later confusion, we want to point out that there are two different types of ambiguities. The ambiguity discussed here is whether the regulators do or do not experience SUSY breaking, which is a physical effect that can be measured using goldstino couplings. There is a separate ambiguity in Sec. 5.4.4 having to do with how to write down a SUGRA-invariant 1PI effective action. This is (partially) resolved using supertraces to define the soft mass spectrum, up to a puzzling ambiguity in how the c_7 term affects $\mathcal{T}$.

5.3 Anomaly Mediation and Super-Weyl Invariance

In Ch. 5, we described one-loop anomaly-mediated gaugino masses using the conformal compensator formalism of SUGRA [143, 114, 80], which is a gauge fixing of super-conformal SUGRA. Here, we will instead use the super-Weyl invariant formulation of SUGRA, which will allow us to connect directly to the claims of de Alwis in Refs. [46, 47]. Starting with a review of the super-Weyl formalism, we will follow the logic of de Alwis (which itself follows the logic of Kaplunovsky and Louis [103]) to construct a Wilsonian effective action. After demonstrating the existence of anomaly mediation in the Wilsonian picture, we derive the same effect using a super-Weyl invariant and SUSY-preserving 1PI effective action. We will only consider gaugino masses in this section, leaving our main result on sfermion masses to Sec. 5.4.

5.3.1 Super-Weyl Formalism for SUGRA

The SUGRA Lagrangian can be derived from a gauge fixing of super-Weyl-invariant SUGRA. Super-Weyl transformations are the most general transformations that leave the torsion constraints of SUGRA unchanged, and they may be parameterized by a chiral superfield $\boldsymbol{\Sigma}$ (and its conjugate anti-chiral superfield $\boldsymbol{\Sigma}^\dagger$) [90, 154]. The components of the chiral superfield $\boldsymbol{\Sigma}$ correspond to different types of transformations which may be familiar from the superconformal algebra: Re $\boldsymbol{\Sigma}|$ corresponds to dilatations, Im $\boldsymbol{\Sigma}|$ to chiral $U(1)_R$ rotations, and $\mathcal{D}_\alpha \boldsymbol{\Sigma}|$ to conformal supersymmetry. The F_Σ component of $\boldsymbol{\Sigma}$ corresponds to a new symmetry which will play a key role in understanding anomaly mediation.[15]

The complete super-Weyl transformations are given in App. C.3. Crucially, the only field that transforms under F_Σ is the scalar auxiliary field M of supergravity [90, 154, 103]:

$$M^* \to M^* - 6F_\Sigma. \tag{5.50}$$

This auxiliary field appears in the determinant of the SUSY vielbein $\boldsymbol{E}$, the corresponding

[15]Super-Weyl transformations do not include special conformal transformations, and superconformal transformations do not include the symmetry generated by F_Σ, so neither super-Weyl transformations nor superconformal transformations are a subset of the other.

chiral density $2\boldsymbol{\mathcal{E}}$, and chiral curvature superfield $\boldsymbol{R}$:

$$\boldsymbol{E} \supset -\frac{1}{3}M^*\Theta^2 + \text{h.c.} + \frac{1}{9}|M|^2\Theta^4, \qquad 2\boldsymbol{\mathcal{E}} \supset -eM^*\Theta^2, \qquad \boldsymbol{R} \supset -\frac{1}{6}M - \frac{1}{9}|M|^2\Theta^2 + \ldots \tag{5.51}$$

We will often talk about the Weyl weights w of chiral superfields $\boldsymbol{Q}_w$ and vector superfields $\boldsymbol{V}_w$ which transform as [154]

$$\boldsymbol{Q}_w \to \boldsymbol{Q}_w e^{w\boldsymbol{\Sigma}}, \qquad \boldsymbol{V}_w \to \boldsymbol{V}_w\, e^{w(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)}. \tag{5.52}$$

Ordinary matter fields have Weyl weight 0, so the Kähler potential $\boldsymbol{K}$ and superpotential $\boldsymbol{W}$ also have Weyl weight 0. For a vector superfield of weight 0, the gauge-covariant superfield $\boldsymbol{W}_\alpha$ has Weyl weight -3. In the gravity multplet, $\boldsymbol{E}$ has Weyl weight 4 and $2\boldsymbol{\mathcal{E}}$ has Weyl weight 6.

The usual SUGRA action (e.g. in Ref. [154]) is not invariant under super-Weyl transformations, so one needs to introduce a super-Weyl compensator $\boldsymbol{C}$ with Weyl weight -2 (i.e. $\boldsymbol{C} \to e^{-2\boldsymbol{\Sigma}}\boldsymbol{C}$). In that case, the tree-level Lagrangian

$$\mathcal{L} = \int d^4\Theta\, \boldsymbol{E}\,\boldsymbol{C}^\dagger \boldsymbol{C}\,(-3e^{-\boldsymbol{K}/3}) + \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\,\boldsymbol{C}^3\boldsymbol{W} + \frac{1}{4}\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\,\boldsymbol{W}^\alpha\boldsymbol{W}_\alpha + \text{h.c.} \tag{5.53}$$

has Weyl weight 0 as desired. The components of the super-Weyl compensator are

$$\boldsymbol{C} = C\{1, \chi_C, F_C\}, \tag{5.54}$$

and due to the non-vanishing Weyl weight of $\boldsymbol{C}$, F_C transforms under F_Σ as

$$F_C \to F_C - 2F_\Sigma. \tag{5.55}$$

It should be stressed that this super-Weyl invariance (and the corresponding super-Weyl compensator) were introduced into Eq. (5.53) simply for calculational convenience, and physical results will not actually exhibit super-Weyl symmetry. After all, one can use the super-Weyl transformations to gauge-fix $\boldsymbol{C}$ in some convenient fashion, leaving a theory without spurious symmetries or degrees of freedom. Because F_Σ transformations are a gauge redundancy of the theory, though, physical observables will only depend on the combination[16]

$$F_{\rm SW} \equiv F_C - \frac{1}{3}M^*, \tag{5.56}$$

regardless of what gauge choice is ultimately made. As we will argue, this F_Σ-invariance is the key point missed in Refs. [46, 47] (and implicitly missed in Ref. [103]).

5.3.2 Choice of Gauge Fixing

To recover the familiar SUGRA Lagrangian from Eq. (5.53), one must gauge fix $\boldsymbol{C}$. The choice $\boldsymbol{C} = 1$ yields the Lagrangian in "SUGRA frame" (i.e. without performing any super-Weyl transformations). A more convenient choice is [103]

$$\log \boldsymbol{C} + \log \boldsymbol{C}^\dagger = \frac{1}{3}\boldsymbol{K}|_H, \tag{5.57}$$

with $\boldsymbol{K}|_H$ being the harmonic (i.e. chiral plus anti-chiral) part of the Kähler potential. This yields the Lagrangian in "Einstein frame" (i.e. after having performed appropriate super-Weyl transformations). Effectively, this gauge choice is the equivalent of going to Wess-Zumino gauge for the real superfield $\boldsymbol{K}$.[17] It must be stressed that Eq. (5.57) is not a supersymmetric relation amongst superfields, since $\boldsymbol{K}|_H$ is not a superfield itself. Instead, Eq. (5.57) should be thought of merely as a prescription for setting each component of $\boldsymbol{C}$ and $\boldsymbol{C}^\dagger$. Of course, other gauge-fixing prescriptions will give physically equivalent results, but Eq. (5.57) is particularly convenient since this choice for $\mathrm{Re}\, C$ yields canonically-normalized Einstein-Hilbert and Rarita-Schwinger terms and this choice for χ_C eliminates troublesome matter-gravitino mixings.

However, it is not so clear what is accomplished by gauge-fixing F_C. We can investigate this by examining the portion of Eq. (5.53) that depends on F_C and M^*, since these are the only two fields that are not inert under F_Σ transformations.

$$\begin{aligned} e^{-1}\mathcal{L} = C^*C\left(e^{-K/3}\right)\left(-3\left(F_C^* - \frac{1}{3}M\right)\left(F_C - \frac{1}{3}M^*\right) + K_i F^i\left(F_C^* - \frac{1}{3}M\right) + \text{h.c.}\right) \\ + 3C^3\left(F_C - \frac{1}{3}M^*\right)W + \text{h.c.} + \ldots \end{aligned} \tag{5.58}$$

[16]The superconformal formalism does not contain M^*, since that degree of freedom is contained in the F_Φ component of the conformal compensator (see App. C.1). In the super-Weyl case, the F_C component is a pure gauge degree of freedom.

[17]This gauge choice leaves still leaves $\arg C$ undetermined, though one can fix $\arg C$ by imposing that the gravitino mass parameter has no phase.

As expected from Eq. (5.56), F_C and M^* only appear in the F_Σ-invariant combination $F_{\text{SW}} \equiv F_C - \frac{1}{3}M^*$ which has the vacuum expectation value

$$\langle F_{\text{SW}} \rangle = m_{3/2} + \frac{1}{3}\left\langle K_i F^i \right\rangle. \tag{5.59}$$

Thus, different gauge-fixings for F_C only serve to shift the vev of M^*. After one solves the M^* equation of motion, physical observables do not (and cannot) depend on the gauge fixing of F_C.

5.3.3 Counterterms in the Wilsonian Effective Action

As emphasized in Ref. [46, 47], it is possible to regulate all UV-divergences in SUGRA in a way that preserves SUSY and super-Weyl invariance. This was shown in Ref. [103] using higher-derivative regulators in a version of Warr's regularization scheme [150, 151]. This implies that the super-Weyl symmetry discussed above is not anomalous, and consequently, any physical results we derive must be completely super-Weyl invariant. Indeed, we will see that anomaly mediation (despite its name) is necessary to preserve both SUSY and super-Weyl invariance.

The key observation of Ref. [103] is that to preserve super-Weyl invariance in a UV-regulated theory, the Wilsonian effective action must consist of Eq. (5.53) augmented with the counterterm

$$\Delta\mathcal{L} = \frac{3}{16\pi^2}(T_G - T_R)\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \log \boldsymbol{C}\, \boldsymbol{W}^\alpha_a \boldsymbol{W}^a_\alpha. \tag{5.60}$$

This term can be deduced from the requirement that the $U(1)_R$ part of the super-Weyl transformations remains non-anomalous. It is convenient to canonically normalize the matter fields $\boldsymbol{Q}^i$ by performing the (anomalous) rescaling $\boldsymbol{Q}^i \to \boldsymbol{Q}^i/\boldsymbol{C}$ such that the rescaled matter field have Weyl weight -2. Due to the Konishi anomaly [110, 43], this rescaling modifies Eq. (5.60) to become

$$\Delta\mathcal{L} = \frac{1}{16\pi^2}(3T_G - T_R)\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \log \boldsymbol{C}\, \boldsymbol{W}^\alpha_a \boldsymbol{W}^a_\alpha. \tag{5.61}$$

Immediately this presents a conundrum, since Eq. (5.61) contains a gaugino mass that depends only on F_C:

$$m_\lambda^{\text{ambiguous}} = -\frac{g^2}{16\pi^2}\,(3T_G - T_R)\,F_C. \tag{5.62}$$

Following the analysis of Ref. [103], Refs. [46, 47] claimed this was the complete formula for the gaugino mass, and by gauge-fixing $F_C = \frac{1}{3}K_iF^i$ as in Eq. (5.57), de Alwis found no contribution to m_λ proportional to the gravitino mass $m_{3/2}$, and hence no anomaly mediation.[18]

However, we see immediately that Eq. (5.62) cannot be the complete answer, since it is not invariant under F_Σ transformations. This is incompatible with the assertion that the physical predictions of this theory should be invariant under such super-Weyl transformations. By Eq. (5.56), the physics should depend on the combination $F_{\mathrm{SW}} \equiv F_C - \frac{1}{3}M^*$ (which does contain $m_{3/2}$). One could try to make the replacement

$$\log \boldsymbol{C} \to \log \boldsymbol{C} + \frac{1}{3}\log 2\boldsymbol{\mathcal{E}} \tag{5.63}$$

to make the dependence on F_{SW} manifest, but as emphasized emphatically (and correctly) in Refs. [46, 47], $2\boldsymbol{\mathcal{E}}$ is a chiral density and not a chiral superfield, and one cannot include arbitrary extra factors of a chiral density in a SUGRA-invariant action, just as one cannot include arbitrary extra factors of $\det e$ in a diffeomorphically-invariant action. Indeed, there is no local term that one can add to the Wilsonian action to make Eq. (5.61) manifestly super-Weyl invariant.[19]

5.3.4 Effect of the Regulators

The resolution to the above puzzle is that the Wilsonian effective action (as defined in Ref. [103]) *needs* to violate super-Weyl invariance in order for physical results to be super-Weyl invariant. This is familiar from Yang-Mills gauge theories with a hard Wilsonian cutoff, where the Wilsonian action must be non-gauge invariant in order compensate for the non-gauge invariance of the cutoff (see also Ref. [60]). In this case, the tree-level expression in Eq. (5.62) will combine with loops of the regulators to yield a super-Weyl invariant result.

To understand how this effect arises, consider a Pauli-Villars regulator, as anticipated in Sec. 5.2.5. Given a chiral superfield $\boldsymbol{Q}$ in some representation of a gauge group, one can regulate its contributions to loop diagrams by introducing two superfields, $\boldsymbol{L}$ and $\boldsymbol{S}$, with

[18] In the language of Ch. 5, de Alwis was only claiming the absence of gravitino mediation. The Kähler-mediated terms proportional to K_iF^i are not in dispute.

[19] We will see in Sec. 5.3.5 that one can write down a non-local 1PI effective action that depends only on F_{SW}.

$\boldsymbol{L}$ in the same representation of the gauge group and $\boldsymbol{S}$ in the conjugate representation:

$$\mathcal{L}_{\rm PV} = \int d^4\Theta\, \boldsymbol{E}\, \left[-\boldsymbol{L}^\dagger e^{\boldsymbol{V}} \boldsymbol{L} - \boldsymbol{S}^\dagger e^{\boldsymbol{V}} \boldsymbol{S} \right] + \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \Lambda_{\rm PV}\, \mathbf{L}\, \mathbf{S} + \text{h.c.} \qquad (5.64)$$

Gauge fields can be similarly regulated by introducing chiral superfield regulators in the adjoint representation. By using many such regulators and including appropriate couplings, all divergences of SUGRA can be removed [74, 75, 76]. The kinetic terms suggest that the regulator fields have Weyl weight -2, but since the Pauli-Villars mass term is $\Lambda_{\rm PV}$ instead of $\boldsymbol{C}\Lambda_{\rm PV}$, the Pauli-Villars fields break super-Weyl invariance. However, Ref. [103] showed that Eq. (5.61) is precisely the term needed to restore super-Weyl invariance of the action.

Now, because the Pauli-Villars regulators have a SUSY-preserving mass $\Lambda_{\rm PV}$, they exhibit boson/fermion mass splitting due to the Θ^2 component of $2\boldsymbol{\mathcal{E}}$. Expanding Eq. (5.64), we find

$$\mathcal{L}_{\rm PV} \supset -\frac{1}{3} \Lambda_{\rm UV} M^*\, L\, S\,, \qquad (5.65)$$

which is a B-term that is not super-Weyl invariant! Doing calculations with these regulators will yield an M^*-dependent gaugino mass at one loop. Adding this loop-level contribution to the tree-level contribution from Eq. (5.62), we have the super-Weyl invariant gaugino mass

$$m_\lambda^{\rm physical} = -\frac{g^2}{16\pi^2}\, (3T_G - T_R)\, F_{\rm SW} = -\frac{g^2}{16\pi^2}\, (3T_G - T_R) \left(m_{3/2} + \frac{1}{3} K_i F^i \right). \qquad (5.66)$$

This expression is manifestly super-Weyl invariant, and reproduces the familiar anomaly-mediated result. As discussed in Sec. 5.2.5, if the regulators couple to SUSY breaking in such a way to remove the $m_{3/2}$ dependence in the gaugino mass, this effect would show up as an $m_{3/2}$ dependence in the associated goldstino couplings.

One can avoid this subtlety of regulator contributions by making a gauge choice such that the vev $\langle M^* \rangle = 0$. In that gauge (and only for that gauge), there are no regulator B-terms, so Eq. (5.62) then yields the correct gaugino mass with $F_C = m_{3/2} + \frac{1}{3} K_i F^i$.[20] This is essentially the strategy used in Ch. 5 (since the superconformal framework automatically sets $M^* = 0$), and is effectively what was done in the original anomaly-mediated literature [135, 84] (though not in this language). For any other gauge—including the choice of

[20] It is worth noting here that $m_{3/2}$ here is really the vev of the superpotential W, which is allowed to appear in the gauge fixing of F_C.

Eq. (5.57) used by Refs. [103, 46, 47]—one cannot neglect contributions to the gaugino mass due to the UV regulators. Alternatively, one can regulate the theory with super-Weyl-invariant Pauli-Villars fields, in which case Eq. (5.61) is absent but the regulators have B-terms proportional to $F_{\rm SW}$, again reproducing Eq. (5.66).

5.3.5 1PI Effective Action and Goldstino Couplings

We argued above that there is no way to make super-Weyl invariance manifest in a Wilsonian effective action. However, the super-Weyl formalism is entirely valid at the quantum level, since there exists a variety of regularization schemes that preserve the super-Weyl symmetry (i.e. it is not anomalous). Therefore, we should be able to write down a 1PI effective action that exhibits all of the relevant symmetries of the theory (including super-Weyl invariance). Here, we will write down the relevant 1PI action to describe gauginos at one loop, and extend the logic to sfermions at two loops in Sec. 5.4.

One disadvantage of the 1PI action is that it will inevitably be non-local, since it involves integrating out light degrees of freedom. On the other hand, the 1PI action allows us to extract all anomaly-mediated effects from the action directly, without having to worry about the contributions of regulators explicitly as we did in Sec. 5.3.4. To avoid SUSY-breaking terms in the regulators as discussed in Sec. 5.2.5, we can study a 1PI effective action that does not have explicit dependence on $\boldsymbol{X}_{\rm NL}$. In general, the 1PI effective action will depend on $\boldsymbol{X}_{\rm NL}$, but this will just give extra soft masses and goldstino couplings in agreement with flat space intuition, whereas we are interested in isolating the anomaly-mediated effects.

At one-loop, the 1PI effective action for the gauge multiplet is

$$\mathcal{L} \supset \frac{1}{4}\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\boldsymbol{W}^\alpha \boldsymbol{S}(\widetilde{\Box})\boldsymbol{W}_\alpha, \tag{5.67}$$

The superfield $\boldsymbol{S}$ is a chiral superfield with the gauge coupling as its lowest component (see Ref. [12]). The running of the coupling with the momentum scale is encapsulated by the dependence of $\boldsymbol{S}$ on $\widetilde{\Box}$, an appropriately SUGRA-covariant, super-Weyl-covariant, and chiral version of the d'Alembertian. This 1PI action depends on the holomorphic gauge coupling, which is sufficient if we are only interested in one-loop expressions. To describe the canonical gauge coupling (including two-loop effects), one needs an alternative action described in App. C.4.

As we will discuss further in Sec. 5.4, the choice of $\widetilde{\Box}$ is in fact ambiguous. All choices are equivalent at $\mathcal{O}(m_{3/2})$, though, and we will choose to work with[21]

$$\widetilde{\Box}\boldsymbol{W}_\alpha \equiv -\frac{1}{8}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\mathcal{D}_\alpha \left[\frac{\mathcal{D}^\beta \boldsymbol{W}_\beta}{\boldsymbol{C}^\dagger \boldsymbol{C}}\right]. \tag{5.68}$$

It is then possible to expand out Eq. (5.67) and derive super-Weyl-invariant gaugino masses and goldstino couplings.[22] Note that $\widetilde{\Box}\boldsymbol{W}_\alpha$, like $\boldsymbol{W}_\alpha$, is chiral and has Weyl weight -3.

In practice, though, it is much more convenient to use the F_Σ gauge freedom to set $M^* = 0$. The remaining components of $\boldsymbol{C}$ can be fixed using the gauge choice in Eq. (5.57) such that (to linear order in fields)

$$\boldsymbol{C} = \left\{1, \frac{1}{3}K_i\chi^i, m_{3/2} + \frac{1}{3}K_iF^i\right\}. \tag{5.69}$$

Note that the fermionic component of $\boldsymbol{C}$ contains a goldstino if K_i attains a vev:

$$\chi_C = \frac{1}{3}\left\langle K_iF^i\right\rangle \frac{\widetilde{G}_L}{F_{\text{eff}}}. \tag{5.70}$$

In this gauge, the graviton and gravitino are canonically normalized and there are no gravitino-goldstino kinetic mixing terms to worry about. We can also drop the chiral curvature superfield $\boldsymbol{R}$ in Eq. (5.68) because it only contributes at $\mathcal{O}(m_{3/2}^2)$ in $M^* = 0$ gauge (and in fact gives no contribution in this gauge if the cosmological constant has been tuned to zero). Similarly, $-\frac{1}{8}\mathcal{D}^{\dagger 2}\mathcal{D}_\alpha\mathcal{D}^\beta\boldsymbol{W}_\beta$ equals the ordinary flat space d'Alembertian $\Box$ acting on $\boldsymbol{W}_\alpha$ at this order. So for the purposes of getting the $\mathcal{O}(m_{3/2})$ gaugino mass and goldstino couplings, we can simply make the replacement

$$\widetilde{\Box} \to \frac{1}{\boldsymbol{C}^\dagger\boldsymbol{C}}\Box + \frac{1}{2}i(\mathcal{D}^\dagger_{\dot{\alpha}}\boldsymbol{C}^\dagger)\bar{\sigma}^{\mu\dot{\alpha}\beta}\partial_\mu\mathcal{D}_\beta - \frac{1}{16}\left(\mathcal{D}^{\dagger 2}\boldsymbol{C}^\dagger\right)\mathcal{D}^2, \tag{5.71}$$

where we have dropped terms with superspace derivatives on multiple copies of $\boldsymbol{C}$ (they never contribute at $\mathcal{O}(m_{3/2})$) and terms with spacetime derivatives on $\boldsymbol{C}$ (they would only yield terms with derivatives on goldstinos, which can be ignored at this order in $m_{3/2}$ in the goldstino equivalence limit). The form of $\widetilde{\Box}$ in Eq. (5.71) is not as manifestly chiral as

[21] Ref. [103] never explicitly wrote down the form for $\widetilde{\Box}$ acting on $\boldsymbol{W}_\alpha$. This slightly complicated form is needed because $\boldsymbol{W}_\alpha$ has a spinor index.

[22] As written, this form of $\widetilde{\Box}$ is only gauge-invariant for an abelian gauge symmetry. It can be easily modified for non-abelian gauge symmetries by appropriate insertions of $e^{\pm \boldsymbol{V}}$.

in Eq. (5.68), but it can be verified to be chiral (up to terms that we have dropped at this order).

This gauge choice for $\boldsymbol{C}$ is equal to the gauge choice for the conformal compensator $\boldsymbol{\Phi}$ used in Ch. 5, and yields identical results. Plugging Eq. (5.71) into Eq. (5.67) yields the expected soft masses and goldstino couplings from traditional anomaly mediation:[23]

$$\mathcal{L} \supset -\frac{1}{2} m_\lambda \lambda_a \lambda^a + \frac{c_\lambda}{\sqrt{2} F_{\text{eff}}} \lambda_a \sigma^{\mu\nu} \widetilde{G}_L F^a_{\mu\nu}, \tag{5.72}$$

where

$$m_\lambda = -\frac{\beta_g}{g}\left(m_{3/2} + \frac{1}{3} K_i F^i\right), \qquad c_\lambda = -\frac{\beta_g}{g}\frac{1}{3} K_i F^i, \tag{5.73}$$

and β_g is the beta function for the relevant gauge group. Note that the piece of m_λ proportional to $m_{3/2}$ does not come with a goldstino coupling, which tells us that it is not an (AdS) SUSY breaking effect. Had we instead worked in a gauge where $M^* = -3m_{3/2}$ (as was the case in Refs. [46, 47]), then the gaugino mass proportional to $m_{3/2}$ would arise from the parts of $\widetilde{\Box}$ that depend on the lowest component of the chiral curvature superfield $\boldsymbol{R}$.

Thus, we have seen how anomaly mediation is a necessary consequence of SUSY invariance and super-Weyl invariance. Because of the underlying AdS SUSY algebra, terms proportional to $m_{3/2}$ necessarily appear in the regulated SUGRA action. Crucially, $m_{3/2}$ is not an order parameter for (AdS) SUSY breaking, so anomaly-mediated soft masses proportional to $m_{3/2}$ do not have associated goldstino couplings.

5.4 All-Orders Sfermion Spectrum from Anomaly Mediation

It is well-known that anomaly mediation yields sfermion soft mass-squareds at two loops proportional to $m_{3/2}^2$ [135]. In this section, we want to show that this effect can be understood as being a consequence of AdS SUSY. To do so, we will follow the logic of Sec. 5.3.5 and derive the sfermion spectrum by constructing a super-Weyl-invariant and SUSY-preserving 1PI effective action for chiral multiplets.

[23] Strictly speaking, this is only the piece of anomaly mediation related to the super-Weyl anomaly. See Refs. [16, 53] for how the Kähler and Sigma-Model anomalies contribute to the 1PI effective action.

The obvious choice for the 1PI effective action is

$$\mathcal{L} = \int d^4\Theta\, \boldsymbol{E} \boldsymbol{C}^\dagger \boldsymbol{C}\, \boldsymbol{Q}^\dagger \boldsymbol{Z}(\widetilde{\Box})\boldsymbol{Q}. \tag{5.74}$$

Here, $\boldsymbol{Q}$ is a chiral matter multiplet, $\boldsymbol{Z}$ is the superfield associated with wave function renormalization, and $\widetilde{\Box}$ is a super-Weyl invariant version of the d'Alembertian acting on chiral superfields. Our key task in this section is to figure out which pieces of Eq. (5.74) preserve SUSY and which pieces break SUSY. To do this, we first identify the order parameter F_R for SUSY breaking in the SUGRA multiplet, which is valid at order $\mathcal{O}(m_{3/2}^2)$. We then use F_R to help identify all places where the goldstino field can appear. Because $\widetilde{\Box}$ is in fact ambiguous at $\mathcal{O}(m_{3/2}^2)$, we will need to construct appropriate supertraces to extract unambiguous "soft mass-squareds" and "goldstino couplings". With these tools in hand, we can then use the 1PI effective action to derive the familiar two-loop scalar soft mass-squareds, as well as unfamiliar one-loop goldstino couplings.

5.4.1 The Order Parameter for SUSY Breaking

As already emphasized a number of times, the gravitino mass $m_{3/2}$ is not an order parameter for SUSY breaking but is simply a measure of the curvature of unbroken AdS space. With an appropriate gauge choice (see Eq. (5.79) below), we can extract $m_{3/2}$ from the lowest component of the chiral curvature superfield $\boldsymbol{R}$,

$$\boldsymbol{R}| = -\frac{1}{6}M^* = \frac{1}{2}m_{3/2}, \tag{5.75}$$

and effects proportional to $\boldsymbol{R}|$ will preserve (AdS) SUSY.

The SUGRA multiplet does contain a SUSY-breaking order parameter at order $\mathcal{O}(m_{3/2}^2)$, namely the highest component of $\boldsymbol{R}$:

$$-\frac{1}{4}\mathcal{D}^2\boldsymbol{R}| = \frac{1}{12}\mathcal{R} - \frac{1}{9}M^*M + \ldots, \tag{5.76}$$

where $\mathcal{R}$ is the Ricci scalar. Upon using the Einstein equation, this takes on the value

$$F_R \equiv \frac{1}{12}\mathcal{R} - m_{3/2}^2 = -\frac{F_{\rm eff}^2}{3M_{\rm Pl}^2}\,, \tag{5.77}$$

regardless of whether F_{eff} is tuned to yield flat space or not. Since F_{eff} is an order parameter for SUSY breaking, so is F_R for finite M_{Pl}. In an arbitrary gauge, we will define F_R in terms of Eq. (5.77) (instead of $-\frac{1}{4}\mathcal{D}^2\boldsymbol{R}|$).

As expected, $F_R = 0$ for unbroken AdS SUSY (i.e. $\frac{1}{12}\mathcal{R} = m_{3/2}^2$). When SUSY is broken and the cosmological constant is tuned to zero, then $F_R = -m_{3/2}^2$ (i.e. $\mathcal{R} = 0$). So while $m_{3/2}$ itself does not break SUSY, F_R can yield effects proportional to $m_{3/2}^2$ that do break SUSY. This distinction lies at the heart of the confusion surrounding anomaly mediation.

To better understand why F_R is an order parameter for SUSY-breaking, it is helpful to note that F_R controls the amount of gravitino-goldstino mixing in the super-Higgs mechanism. This can be seen by examining the various forms of the gravitino equation of motion one can obtain by plugging Eq. (5.30) into Eq. (5.27):

$$\begin{aligned}
\frac{1}{M_{\text{Pl}}}\epsilon^{\mu\nu\rho\tau}\overline{\sigma}_\nu \mathcal{D}_\rho \psi_\tau &= -\frac{3i}{\sqrt{2}}\frac{F_R}{F_{\text{eff}}}\overline{\sigma}^\mu \widetilde{G}_L + \dots, \\
\frac{1}{M_{\text{Pl}}}\sigma^{\mu\nu}\mathcal{D}_\mu \psi_\nu &= -\frac{3}{\sqrt{2}}\frac{F_R}{F_{\text{eff}}}\widetilde{G}_L + \dots, \\
-\frac{1}{M_{\text{Pl}}}i\overline{\sigma}^\mu \mathcal{D}_\mu \psi_\lambda &= \frac{3i}{\sqrt{2}}\frac{F_R}{F_{\text{eff}}}\sigma_\lambda \widetilde{G}_L + \dots,
\end{aligned} \tag{5.78}$$

where we have also used the Einstein equation from Eq. (5.77). Thus, gravitino couplings which look innocuous can secretly contain (SUSY-breaking) goldstino couplings when F_R is non-zero. This will be of great importance when we track goldstino couplings in the next subsection. The ellipses of Eq. (5.78) contain terms not relevant to our discussion. In particular, we can ignore any $m_{3/2}\psi_\mu$ terms since we only care about effects up to $\mathcal{O}(m_{3/2}^2)$. We can also ignore terms proportional to $\overline{\sigma}^\mu\psi_\mu$, since applying its equation of motion would only serve to reintroduce derivatives acting either on gravitinos or goldstinos.

5.4.2 Goldstinos in the SUGRA Multiplet

Since our ultimate goal is to compute the sfermion soft masses and goldstino couplings advertised in Table 5.1, it is crucial to identify all places where the goldstino field can appear.

The most straightforward case is when there are direct couplings between the visible sector fields and the SUSY-breaking superfield $\boldsymbol{X}_{\text{NL}}$ from Eq. (5.41), which has the goldstino as its fermionic component. This case is not interesting for our purposes since it generates soft masses and goldstino couplings in agreement with flat space intuition. We therefore take the wavefunction superfield $\boldsymbol{Z}$ to be independent of $\boldsymbol{X}_{\rm NL}$ for simplicity.

Somewhat less obviously, the Weyl compensator $\boldsymbol{C}$ itself can also contain a goldstino, and different (super-Weyl) gauge fixings give different goldstino dependence in $\boldsymbol{C}$. We find it convenient to work in the gauge where

$$\boldsymbol{C} = \left\{1, \frac{1}{3}\langle K_i\rangle \chi^i, \frac{1}{3}\langle K_i\rangle F^i\right\}. \tag{5.79}$$

This is effectively the gauge choice of Eq. (5.57) carried out to linear order in fields, which is the minimum necessary to have canonically-normalized Einstein-Hilbert and Rarita-Schwinger terms [38]. In this gauge $-\frac{1}{3}M^* = m_{3/2}$ (see Eq. (5.75)). Upon picking out the goldstino direction, neglecting other fermions, and dropping terms with multiple goldstinos,

$$\boldsymbol{C} = 1 + \frac{1}{3}\left\langle K_i F^i\right\rangle \left(\Theta + \frac{\widetilde{G}_L}{\sqrt{2}F_{\rm eff}}\right)^2 . \tag{5.80}$$

This gauge choice clearly shows that wherever $\langle F_C\rangle = \frac{1}{3}\left\langle K_i F^i\right\rangle$ appears in a soft SUSY-breaking term, it will have an associated goldstino coupling. Of course, F_C is always accompanied by $-\frac{1}{3}M^* = m_{3/2}$ by super-Weyl invariance, but effects proportional to M^* do not have associated goldstino couplings. After all, $\langle M^*\rangle \neq 0$ does not break AdS SUSY, whereas $\left\langle K_i F^i\right\rangle \neq 0$ does.

The most subtle case is to identify goldstino fields hiding in the SUGRA multiplet. These arise through the gravitino equations of motion shown in Eq. (5.78), which are necessarily SUSY invariant. The SUSY transformation of Eq. (5.78) then tells us any goldstino arising in such a fashion must be accompanied by an F_R, thus giving us an easy way to track such goldstinos. F_R only occurs (without derivatives acting on it) within the SUGRA superfields $\boldsymbol{R}$ and $\boldsymbol{G}_\mu$, and the components of these superfields can be extracted by the methods of Refs. [154, 19].[24]

Extensively using the gravitino equations of motion of Eq. (5.78), we find that $\boldsymbol{R}$ and

[24] There are also goldstinos lurking in $\boldsymbol{E}$, but these are most easily tracked by making the replacement

$$\int d^4\Theta\, \boldsymbol{E}\,\boldsymbol{\Omega} = \frac{1}{2}\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\left[-\frac{1}{4}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\boldsymbol{\Omega}\right] + \text{h.c.},$$

since $2\boldsymbol{\mathcal{E}}$ does not have hidden goldstinos.

$\boldsymbol{G}_\mu$ can be written as:

$$\boldsymbol{R} = -\frac{1}{6}M + F_R\left(\Theta + \frac{\widetilde{G}_L}{\sqrt{2}F_{\text{eff}}}\right)^2 + \ldots, \tag{5.81}$$

$$\boldsymbol{G}_\mu = \frac{1}{2}F_R\left(\Theta + \frac{\widetilde{G}_L}{\sqrt{2}F_{\text{eff}}}\right)\sigma_\mu\left(\Theta^\dagger + \frac{\widetilde{G}_L^\dagger}{\sqrt{2}F_{\text{eff}}}\right) + \ldots, \tag{5.82}$$

where the ellipses include terms containing $m_{3/2}\psi_\mu$, $\overline{\sigma}^\mu\psi_\mu$, b_μ, $\partial_\mu M$, $\partial_\mu F_R$,[25] or multiple gravitinos or goldstinos. For simplicity, we have assumed that the Ricci tensor is proportional to the metric, as it is in any homogeneous space.

Note that with this particularly convenient gauge choice, we can identify all of the goldstino couplings in $\boldsymbol{X}_{\text{NL}}$, $\boldsymbol{C}$, $\boldsymbol{R}$, and $\boldsymbol{G}_\mu$ by first finding the vevs of these fields, and then making the replacement

$$\Theta \to \Theta + \frac{\widetilde{G}_L}{\sqrt{2}F_{\text{eff}}}. \tag{5.83}$$

At the component level, this implies that any terms in the Lagrangian with coefficient F_X, K_iF^i, or F_R (but crucially *not* $m_{3/2}$) will have associated goldstino couplings. These can be found by making a global SUSY transformation of those terms[26] with infinitesimal SUSY parameter

$$\epsilon = -\frac{\widetilde{G}_L}{\sqrt{2}F_{\text{eff}}}. \tag{5.84}$$

This will allow us to identify goldstino couplings directly from the sfermion spectrum, without having to wrestle with complicated component manipulations.

The simplest application of this method for finding goldstino couplings is the tree-level analysis of Sec. 5.2. The tachyonic scalar masses are removed by a SUSY-breaking coupling $2F_R\phi^*\phi$ when uplifting from AdS to flat space. This indeed has a corresponding goldstino coupling in flat space proportional to $-2F_R/F_{\text{eff}} = 2m_{3/2}^2/F_{\text{eff}}$ (see Eq. (5.36)).[27]

[25] Terms containing $\partial_\mu F_R$ (which has vanishing vev) may have associated goldstino couplings, but they will always feature a derivative acting on the goldstino. Such terms will always be of $\mathcal{O}(m_{3/2}^3)$ in the goldstino equivalence regime, and can be ignored here.

[26] The situation is more subtle for terms with coefficients like $m_{3/2}K_iF^i$, a product of SUSY-breaking and SUSY-preserving effects. In such cases, one only makes half of the transformation of Eq. (5.84). This arises since for K_iF^i ($K_{\bar{i}}F^{*\bar{i}}$), one is really only making the replacement of Eq. (5.83) for Θ ($\Theta^\dagger$), not $\Theta^\dagger$ (Θ), recalling that we have a hermitian action.

[27] In practice, the use of gravitino equations of motion is less than transparent, which is the reason why we relied on the Einstein frame Lagrangian in Sec. 5.2.1. Finding the Einstein frame is more difficult beyond tree-level, however, which is why we choose to work in SUGRA frame in this section and exploit gravitino

5.4.3 Supertraces and the 1PI Effective Action

Now that we have identified our SUSY-breaking order parameters and how they are associated with goldstino couplings, we now need to consider what possible SUSY-breaking terms can arise from the 1PI effective action in Eq. (5.74). This action accounts for the quantum corrections coming from loop diagrams of massless particles. For this reason, one must be careful to include both local and non-local terms when considering SUSY-breaking in a 1PI effective action. For a chiral multiplet at quadratic order in fields, there are three terms at order m^2/p^2 (where m is some soft mass), corresponding to corrections to the field self-energies:

$$\mathcal{L}_{\text{SUSY-breaking}} = -\mathcal{C}_s \phi^* \phi - \mathcal{C}_a F^* \Box^{-1} F + i\mathcal{C}_f \chi^\dagger \overline{\sigma}^\mu \mathcal{D}_\mu \Box^{-1} \chi, \tag{5.85}$$

where the coefficients $\mathcal{C}_i$ are all $\mathcal{O}(m^2)$. In the context of anomaly mediation, these contributions are already $\mathcal{O}(m_{3/2}^2)$, so we can neglect any further SUGRA corrections. In particular, at this order the operator $\Box$ appearing in Eq. (5.85) can be thought as the d'Alembertian in flat space.

The non-local action in Eq. (5.85) does not break SUSY in the limiting case $\mathcal{C}_s = \mathcal{C}_a = \mathcal{C}_f$.[28] The simple field redefinition (or the appropriately super-Weyl- and SUGRA-covariant equivalent, see Ref. [103])

$$\boldsymbol{Q} \to \boldsymbol{Q} + \frac{\mathcal{C}}{2\Box}\boldsymbol{Q} \tag{5.86}$$

eliminates all three terms for $\mathcal{C}_i = \mathcal{C}$. Thus, a single coefficient $\mathcal{C}_i$ is not a good measure of SUSY-breaking by itself. On the other hand, the supertrace

$$\mathcal{S} = \mathcal{C}_s + \mathcal{C}_a - 2\mathcal{C}_f, \tag{5.87}$$

is invariant under the transformation of Eq. (5.86) and is an unambigous measure of SUSY-breaking. Ref. [12] considered a similar supertrace over the $\mathcal{O}(m^2)$ SUSY-breaking contributions to the self-energy for the components of vector superfields.

Of course, there is another independent combination of the $\mathcal{C}_i$ which is invariant under

equations of motion.

[28]Obviously, $\mathcal{C}_s$ also does not break SUSY if it arises in conjunction with a fermion mass term after an auxiliary field redefinition. We will therefore define $\mathcal{C}_s$ to exclude such contributions.

Eq. (5.86), which we take to be

$$\mathcal{T} = \mathcal{C}_a - \mathcal{C}_f. \tag{5.88}$$

This is the unique independent choice which vanishes for tree-level SUGRA (the tachyonic scalar mass in AdS discussed in Sec. 5.2 yields vanishing $\mathcal{T}$). A non-vanishing value of $\mathcal{T}$ is still a SUSY-breaking effect, and can be present even when the supertrace $\mathcal{S}$ vanishes. This can arise most notably from terms like

$$\mathcal{L} \supset \frac{1}{\Lambda^2} \int d^4\theta \, \frac{i}{2} \mathcal{D}^\dagger_{\dot{\alpha}} \boldsymbol{X}^\dagger_{\text{NL}} \overline{\sigma}^{\mu\dot{\alpha}\alpha} \mathcal{D}_\alpha \boldsymbol{X}_{\text{NL}} \, \boldsymbol{Q}^\dagger \Box^{-1} \mathcal{D}_\mu \boldsymbol{Q}. \tag{5.89}$$

which yields $\mathcal{S} = 0$ but $\mathcal{T} = F_X^2/\Lambda^2$. In the context of anomaly mediation, non-vanishing values for $\mathcal{T}$ frequently arise but they in general depend on how the theory is regulated. In contrast, we will find that the supertrace $\mathcal{S}$ from anomaly mediation is unambiguous and irreducible, so we will mainly focus on $\mathcal{S}$ in our explicit calculations.

Analogously to Eq. (5.85), there will be non-local goldstino couplings. In the case of global flat-space SUSY, one can simply transform the terms in Eq. (5.85) under SUSY, with infinitesimal parameter $\epsilon = -\frac{\widetilde{G}_L}{\sqrt{2}F_{\text{eff}}}$ (see Eq. (5.84)),

$$\mathcal{L}_{\text{goldstino}} = \frac{\mathcal{G}^{\mathcal{S}} - \mathcal{G}^{\mathcal{T}}}{F_{\text{eff}}} \widetilde{G}_L \chi \phi^* + \frac{\mathcal{G}^{\mathcal{T}}}{F_{\text{eff}}} i \widetilde{G}_L \sigma^\mu \mathcal{D}_\mu \chi^\dagger \Box^{-1} F. \tag{5.90}$$

For global flat-space SUSY, $\mathcal{G}^{\mathcal{S}} = \mathcal{S}$ and $\mathcal{G}^{\mathcal{T}} = \mathcal{T}$. This will not be the case, however, for AdS SUSY or for SUGRA, where there can be non-vanishing values of $\mathcal{S}$ or $\mathcal{T}$ that do not break SUSY. Such effects will always be proportional to the inverse AdS radius $\lambda_{\text{AdS}}^{-1} = m_{3/2}$. For example, at tree level in AdS SUSY, one would use the appropriate AdS SUSY transformations (which has terms proportional to $m_{3/2}$) on the full Lagrangian, which would yield $\mathcal{G}^{\mathcal{T}} = \mathcal{T}$ but $\mathcal{G}^{\mathcal{S}} = \mathcal{S} + 2m_{3/2}^2$. In the following subsections, we will find these relations to be modified, but always by terms proportional to $m_{3/2}$.

5.4.4 The Super-Weyl-Invariant d'Alembertian

The operator $\widetilde{\Box}$ appearing in Eq. (5.74) has not been yet defined. Its definition is the last ingredient we need to computing sfermion soft masses and goldstino couplings. We will see that while $\widetilde{\Box}$ is generically ambiguous, our final results for the supertrace $\mathcal{S}$ and corresponding goldstino coupling $\mathcal{G}^{\mathcal{S}}$ are not.[29]

The operator $\widetilde{\Box}$ is a super-Weyl-invariant version of the d'Alembertian acting on scalar superfields, which reduces to $\Box$ in the limit of global flat-space SUSY. Given a generic spinless superfield $\boldsymbol{U}$, there are a limited number of options (neglecting fractional powers of derivatives):

$$\begin{aligned}\widetilde{\Box}\boldsymbol{U} &= \mathcal{P}^\dagger\mathcal{P}\boldsymbol{U} + \mathcal{P}\mathcal{P}^\dagger\boldsymbol{U} - \frac{1}{8}\frac{1}{\boldsymbol{C}^\dagger\boldsymbol{C}}\mathcal{D}^\alpha(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\mathcal{D}_\alpha\boldsymbol{U} \\ &+ c_1(\boldsymbol{\mathcal{P}})\mathcal{P}^\dagger\boldsymbol{U} + c_1'(\boldsymbol{\mathcal{P}}^\dagger)\mathcal{P}\boldsymbol{U} + c_2(\boldsymbol{\mathcal{P}}^\dagger)\mathcal{P}^\dagger\boldsymbol{U} + c_2'(\boldsymbol{\mathcal{P}})\mathcal{P}\boldsymbol{U} \\ &+ c_3(\mathcal{P}^\dagger\boldsymbol{\mathcal{P}})\boldsymbol{U} + c_3'(\mathcal{P}\boldsymbol{\mathcal{P}}^\dagger)\boldsymbol{U} + c_4(\boldsymbol{\mathcal{P}}^\dagger)(\boldsymbol{\mathcal{P}})\boldsymbol{U} \\ &+ c_5(\boldsymbol{\mathcal{P}}^{\dagger 2})\boldsymbol{U} + c_5'(\boldsymbol{\mathcal{P}}^2)\boldsymbol{U} + c_6\mathcal{P}^\dagger((\boldsymbol{\mathcal{P}})\boldsymbol{U}) + c_6'\mathcal{P}((\boldsymbol{\mathcal{P}}^\dagger)\boldsymbol{U}) \\ &+ c_7\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}\boldsymbol{C}^{-1}\mathcal{D}^{\dagger\dot\alpha}\boldsymbol{C}^{\dagger -1}\mathcal{D}^\alpha\boldsymbol{U} - c_7'\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}\boldsymbol{C}^{\dagger -1}\mathcal{D}^\alpha\boldsymbol{C}^{-1}\mathcal{D}^{\dagger\dot\alpha}\boldsymbol{U} \\ &+ c_8\frac{1}{\boldsymbol{C}^\dagger\boldsymbol{C}}\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}\widetilde{\boldsymbol{G}}^{\alpha\dot\alpha}\boldsymbol{U}. \end{aligned} \tag{5.91}$$

The operators and superfields $\mathcal{P}$, $\boldsymbol{\mathcal{P}}$, and $\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}$ (and their hermitian conjugates) are super-Weyl covariant versions of $-\frac{1}{4}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})$, $2\boldsymbol{R}$, and $\boldsymbol{G}_{\alpha\dot\alpha}$, respectively, and are defined in App. C.3. For matter fields $\boldsymbol{Q}$ that are charged under a gauge group, the operators of Eq. (5.91) would need to be modified by appropriate insertions of $e^{\pm\boldsymbol{V}}$.[30]

Many of the terms in Eq. (5.91) vanish in the limit of global flat-space SUSY, so the associated coefficients c_i are left completely undetermined. We could impose certain desirable properties for $\widetilde{\Box}$, which would lead to constraints on the c_i. For example, requiring that $\widetilde{\Box}\boldsymbol{U}$ is chiral for chiral $\boldsymbol{U}$ and that $\widetilde{\Box}$ possesses a sensible analogue of integration by parts would set $c_6 = -1$ and all other $c_i = 0$. This is the choice made in Ref. [103] (which they denote $\triangle$), though it does not satisfy $\widetilde{\Box}1 = 0$.[31] In order to actually determine the c_i, one would have to explicitly take into account virtual effects to all orders in a specific regularization scheme, which is beyond the scope of this chapter. Because our final results for $\mathcal{S}$ and $\mathcal{G}^{\mathcal{S}}$ are independent of the c_i, we choose not to impose any constraint on them.

At this point, we could use the full machinery developed in Ref. [154] to extract the

[29]This ambiguity is a reflection of an ambiguity in how to write down a SUGRA-invariant 1PI effective action, which is in addition to the ambiguity discussed in Sec. 5.2.5 in whether the regulators feel SUSY breaking.

[30]There could also be additional possible operators proportional to the field strength $\boldsymbol{W}_\alpha$ which would not give any contributions to self-energy corrections or goldstino couplings at the desired order.

[31]Another obvious candidate is $\widetilde{\Box} = \mathcal{D}_a\mathcal{D}^a$ in the $\boldsymbol{C} = 1$ limit (corresponding to $c_i' = c_i$, $-c_1 = c_3 = c_4/2 = c_6 = c_7 = -1/2$, $c_2 = c_5 = c_8 = 0$), though it is not chiral.

components of $\widetilde{\square}\boldsymbol{U}$. We could then determine $\widetilde{\square}^n\boldsymbol{U}$ by recursion and find the component form of Eq. (5.74) by treating $\boldsymbol{Z}(\widetilde{\square})\boldsymbol{Q}$ as a Taylor expansion.[32] However, this procedure is overkill for our purposes, since we will ultimately use the trick in Sec. 5.4.2 to find goldstino couplings once we know the dependence of the supertrace on K_iF^i and F_R. By super-Weyl invariance, we know our results can only depend on two parameters:

$$F_{\rm SW} \equiv m_{3/2} + \frac{1}{3}K_iF^i \quad \text{and} \quad \frac{1}{12}\mathcal{R} \equiv m_{3/2}^2 + F_R. \tag{5.92}$$

Moreover, because $\mathcal{S}$ is dimension two, its only dependence on F_R can be linear,[33] so if we know the behavior of $\mathcal{S}$ for two different values of F_R, we can use interpolation to determine $\mathcal{S}$ for all F_R. Thus, it is sufficient to discuss two limiting cases where the behavior of $\widetilde{\square}\boldsymbol{U}$ simplifies.

The first limiting case is flat space but arbitrary $\langle K_i\rangle$. Here, one can use the gauge choice $F_C = F_{\rm SW}$ to set $M^* = 0$, and since $\mathcal{R} = 0$, one can use the global flat-space SUSY algebra to find the components of $\widetilde{\square}\boldsymbol{U}$, keeping careful track of all of the factors of $\boldsymbol{C}$ contained therein. In fact, one does not even need to be all that careful, by noting that

$$\widetilde{\square}_{\rm flat} = \frac{1}{\boldsymbol{C}^\dagger\boldsymbol{C}}\square + \Big(\text{terms with supercovariant derivatives on } \boldsymbol{C}, \boldsymbol{C}^\dagger\Big). \tag{5.93}$$

There is a limited set of the possible terms in the parentheses that can contribute to physics up to $\mathcal{O}(m_{3/2}^2)$. At $\mathcal{O}(m_{3/2})$, it can be shown explicitly that they have no effect (up to boundary terms). At $\mathcal{O}(m_{3/2}^2)$, the effects of all such terms can be eliminated by transformations like Eq. (5.86) or they take the form of Eq. (5.89) (with $\boldsymbol{C}$ in place of $\boldsymbol{X}_{\rm NL}$). In either case, they yield no contribution to the supertrace $\mathcal{S}$ of Eq. (5.87).[34] Therefore, for the purposes of finding $\mathcal{S}$ we need only consider the first term in Eq. (5.93), which is clearly independent of the c_i. Furthermore, this is exactly the term which is already considered in the anomaly mediation literature, so the results for $\mathcal{S}$ are well-known [135] (though they are usually stated as being the soft mass-squared and not the supertrace).

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[32] And we have.

[33] Fractional or negative powers of $m_{3/2}$ or $\mathcal{R}$ do not appear in the 1PI effective action.

[34] Terms of the latter form do contribute to the parameter $\mathcal{T}$ defined in Eq. (5.88), and contributions to $\mathcal{T}$ proportional to F_R should still be considered SUSY-breaking. It can be readily shown that non-zero values of $\mathcal{T}$ will only be induced by the first line of Eq. (5.91) or by the c_7 term (see Eq. (C.40)). This c_7 dependence implies that the value of $\mathcal{T}$ depends on exactly how one regulates the theory. In unbroken rigid AdS, this ambiguity does not arise; $\boldsymbol{G}_{\alpha\dot\alpha} = 0$ in rigid AdS, so the term associated with c_7 vanishes.

The second limiting case is unbroken SUSY in rigid AdS where $\langle K_i \rangle = 0$. Because a flat space analysis cannot distinguish between effects proportional to $m_{3/2}^2$ (which have no associated goldstino couplings) and those proportional to F_R (which do), we need a limiting case which captures terms proportional to the scalar curvature $\mathcal{R}$. Starting with unbroken SUSY in AdS, we can luckily consider the rigid ($M_{\text{Pl}} \to \infty$) limit without missing any physics. The rigid AdS SUSY algebra [4, 51, 104, 156, 95] is dramatically simpler than the SUGRA algebra, corresponding to the limit $\boldsymbol{C} = 1$, $\boldsymbol{R} = m_{3/2}/2$, $\boldsymbol{G}_{\alpha\dot{\alpha}} = \boldsymbol{W}_{\alpha\beta\gamma} = 0$ [70]. This reduces the number of independent operators in $\widetilde{\Box}$ to four:

$$\widetilde{\Box}_{\text{rigid AdS}} = \mathcal{D}_a \mathcal{D}^a - d_1 \frac{1}{4} \mathcal{D}^2 - d_1' \frac{1}{4} m_{3/2} \mathcal{D}^{\dagger 2} + d_2 m_{3/2}^2, \tag{5.94}$$

where the d_i coefficients are related to the c_i coefficients via

$$d_1 \equiv c_1 + c_2 + c_6, \quad d_1' \equiv c_1' + c_2' + c_6', \quad d_2 \equiv 2 + d_1 + d_1' + c_3 + c_3' + c_4 + c_5 + c_5'. \tag{5.95}$$

One can then use the AdS SUSY algebra to easily extract the components of $\widetilde{\Box}\boldsymbol{U}$ in AdS, find $\widetilde{\Box}^n \boldsymbol{U}$ by recursion, and $\boldsymbol{Z}(\widetilde{\Box})\boldsymbol{Q}$ by Taylor expansion.[35]

5.4.5 Soft Masses and Goldstino Couplings for Chiral Multiplets

We now have all of the ingredients to determine the soft masses and goldstino couplings which follow from Eq. (5.74).

Applying the procedure outlined in Sec. 5.4.4, we first find the behavior of $\boldsymbol{Z}(\widetilde{\Box})\boldsymbol{Q}$ at $\mathcal{O}(m_{3/2}^2)$ in the flat space and rigid AdS limits. Since we have argued that the final result (up to the transformation of Eq. (5.86)) will depend on no parameter in Eq. (5.91) except

[35] Alternatively, one could simply work with the component form of the AdS SUSY Lagrangian. In that case, $Z(\Box)$ does not commute with SUSY transformations due to Eq. (5.29), so one will find additional terms proportional to positive powers of $m_{3/2}$. This approach makes it clear that the results in AdS space must be completely independent of the c_i, up to the transformation Eq. (5.86).

for c_7, we will only present the answer for a choice of c_i such that $\boldsymbol{Z}(\widetilde{\Box})\boldsymbol{Q}$ is (nearly) chiral:[36]

$$\boldsymbol{Q} = \phi + \Theta\sqrt{2}\chi + \Theta^2 F, \tag{5.96}$$

$$\begin{aligned}\boldsymbol{Z}(\widetilde{\Box}_{\text{rigid AdS}}) &= Z(\Box)\Big[\Big(\phi - \tfrac{1}{2}m_{3/2}\gamma\Box^{-1}F + \tfrac{1}{8}m_{3/2}^2(\gamma^2 + \dot{\gamma} - 10\gamma)\Box^{-1}\phi\Big) + \Theta\sqrt{2}\chi \\ &\quad + \Theta^2\Big(F - \tfrac{1}{2}m_{3/2}\gamma\phi + \tfrac{1}{8}m_{3/2}^2(\gamma^2 + \dot{\gamma} + 2\gamma)\Box^{-1}F\Big)\Big],\end{aligned} \tag{5.97}$$

$$\begin{aligned}\boldsymbol{Z}(\widetilde{\Box}_{\text{flat}}) &= Z(\Box)\Big[(\phi - \tfrac{1}{2}F_{\text{SW}}\gamma\Box^{-1}F + \tfrac{1}{8}F_{\text{SW}}^2(\gamma^2 + \dot{\gamma} - 10\gamma)\Box^{-1}\phi) + \Theta\sqrt{2}\chi \\ &\quad + \Theta^2(F - \tfrac{1}{2}F_{\text{SW}}\gamma\phi + \tfrac{1}{8}F_{\text{SW}}^2(\gamma^2 + \dot{\gamma} + 2\gamma)\Box^{-1}F)\Big] \\ &\quad + Z(\Box)\tfrac{1}{2}F_{\text{SW}}^2\gamma\left[(1 - c_7)\Box^{-1}\phi + \Theta^2(1 + c_7)\Box^{-1}F\right],\end{aligned} \tag{5.98}$$

where the anomalous dimensions are defined as

$$\gamma \equiv 2\frac{d\log Z}{d\log\Box}, \qquad \dot{\gamma} \equiv 2\frac{d\gamma}{d\log\Box}. \tag{5.99}$$

While γ ($\dot{\gamma}$) is first non-zero at one-loop (two-loop) order, our results will hold to any loop order (at $\mathcal{O}(m_{3/2}^2)$). As outlined in Sec. 5.4.4, we can now find an appropriate super-Weyl invariant interpolation valid for any spacetime curvature,

$$\begin{aligned}\boldsymbol{Z}(\widetilde{\Box}) &= Z(\Box)\left(\widetilde{\phi} + \Theta\sqrt{2}\chi + \Theta^2\widetilde{F}\right), \\ \widetilde{\phi} &\equiv \phi - \tfrac{1}{2}F_{\text{SW}}\gamma\Box^{-1}F + \Big(\tfrac{1}{8}F_{\text{SW}}^2(\gamma^2 + \dot{\gamma} - (6 + 4c_7)\gamma) - \tfrac{1}{2}(m_{3/2}^2 + F_R)\gamma(1 - c_7)\Big)\Box^{-1}\phi, \\ \widetilde{F} &\equiv F - \tfrac{1}{2}F_{\text{SW}}\gamma\phi + \Big(\tfrac{1}{8}F_{\text{SW}}^2(\gamma^2 + \dot{\gamma} + (6 + 4c_7)\gamma) - \tfrac{1}{2}(m_{3/2}^2 + F_R)\gamma(1 + c_7)\Big)\Box^{-1}F,\end{aligned} \tag{5.100}$$

remembering that $F_{\text{SW}} = m_{3/2}$ in flat space, and $m_{3/2}^2 + F_R$ vanishes in flat space but is $m_{3/2}^2$ for unbroken SUSY in AdS.

It is now straightforward to expand the superspace action of Eq. (5.74) (dropping factors

[36] This choice corresponds $c_6 = -1 + c_7/2$, $c_1 = -c_7/2$, $c_3 = c_4 = -3/2$, and all other $c_i = 0$. This is not chiral outside of AdS space, but deviations from chirality only appear in terms with gravitinos, b_μ, or at $\mathcal{O}(m_{3/2}^3)$, so we neglect such terms in the following. This choice also has the appealing feature of automatically setting $\mathcal{C}_f = 0$.

of $Z(\Box)$ for clarity):

$$\begin{aligned}\mathcal{L} = \phi^*\Box\phi - i\chi^\dagger\bar{\sigma}^\mu\mathcal{D}_\mu\chi + \left|F + \frac{1}{2}(2-\gamma)F_{\rm SW}\phi\right|^2 + 2(F_R + m_{3/2}^2)\phi^*\phi \\ + \frac{1}{8}F_{\rm SW}^2(-\gamma^2 + \dot{\gamma} - (2+4c_7)\gamma)\phi^*\phi + \frac{1}{8}F_{\rm SW}^2(\gamma^2 + \dot{\gamma} + (2+4c_7)\gamma)F^*\Box^{-1}F \\ - \frac{1-c_7}{2}(m_{3/2}^2 + F_R)\gamma\phi^*\phi - \frac{1+c_7}{2}(m_{3/2}^2 + F_R)\gamma F^*\Box^{-1}F. \end{aligned} \tag{5.101}$$

To extract the sfermion spectrum, is it helpful to perform the shift

$$F \to F - \frac{1}{2}(2-\gamma)F_{\rm SW}\phi, \tag{5.102}$$

which renders the F equation of motion trivial, but induces non-zero B- and A-terms at $\mathcal{O}(m_{3/2})$ if there are superpotential terms. Generalizing to multiple fields $\boldsymbol{Q}^i$ with anomalous dimensions γ_i and a superpotential

$$\boldsymbol{W} = \frac{1}{2}\mu_{ij}\boldsymbol{Q}^i\boldsymbol{Q}^j + \frac{1}{6}\lambda_{ijk}\boldsymbol{Q}^i\boldsymbol{Q}^j\boldsymbol{Q}^k, \tag{5.103}$$

the associated scalar potential terms are

$$V \supset \frac{1}{2}B_{ij}\phi^i\phi^j + \frac{1}{6}A_{ijk}\phi^i\phi^j\phi^k + \text{h.c.}, \tag{5.104}$$

$$B_{ij} = \frac{1}{2}\mu_{ij}\left(-2 + \gamma_i + \gamma_j\right)\left(m_{3/2} + \frac{1}{3}K_kF^k\right), \tag{5.105}$$

$$A_{ijk} = \frac{1}{2}\lambda_{ijk}(\gamma_i + \gamma_j + \gamma_k)\left(m_{3/2} + \frac{1}{3}K_\ell F^\ell\right), \tag{5.106}$$

where we have expanded $F_{\rm SW} = m_{3/2} + \frac{1}{3}K_iF^i$. These are the familiar one-loop anomaly-mediated results that can be found in Ref. [135, 84].

These B- and A-terms will have corresponding goldstino couplings proportional only to K_iF^i but not to $m_{3/2}$. Because the result in Eq. (5.105) is super-Weyl invariant, we are free to choose the gauge of Eq. (5.79) and use the trick in Sec. 5.4.2 to extract goldstino couplings. For example, the B-term has a corresponding goldstino coupling b_{ij} defined in

Eq. (5.6). Performing the shift in Eq. (5.83), we find[37]

$$b_{ij} = \frac{1}{6}\mu_{ij}\left(-2 + \gamma_i + \gamma_j\right) K_k F^k. \tag{5.107}$$

At $\mathcal{O}(m_{3/2})$, this goldstino coupling is independent of tuning the cosmological constant. The difference between the B-term and the goldstino coupling is proportional to $m_{3/2}$

$$B_{ij} - b_{ij} = \frac{1}{6}\mu_{ij}\left(-2 + \gamma_i + \gamma_j\right) m_{3/2}, \tag{5.108}$$

emphasizing the role of AdS SUSY.

The key result of this chapter is the sfermion supertrace $\mathcal{S}$ defined in Eq. (5.87). After performing the auxiliary field shift of Eq. (5.102), we can read off the value at $\mathcal{O}(m_{3/2}^2)$:

$$\mathcal{S}_i = -\frac{1}{4}\dot{\gamma}_i \left| m_{3/2} + \frac{1}{3}K_k F^k \right|^2 - (2 - \gamma_i)(m_{3/2}^2 + F_R). \tag{5.109}$$

The first term is the usual two-loop anomaly-mediated result for $\mathcal{S}$ expected from Ref. [135]. The second term is the tree-level mass splitting in AdS discussed in Sec. 5.2, modified starting at one-loop order to include the anomalous dimension. The fact that we have a contribution to $\mathcal{S}$ proportional to $(2 - \gamma)$ could have been anticipated, since anomaly mediation effectively tracks scale-breaking effects, and $(2 - \gamma)$ is the true scaling dimension of the operator $\boldsymbol{Q}^\dagger \boldsymbol{Q}$.[38] Because $m_{3/2}^2 + F_R = \frac{1}{12}\mathcal{R}$, this second term vanishes in flat space, which is why it does not appear in the original literature.[39] As discussed further in App. C.2, this whole expression is RG-stable, as it must be since it comes from a 1PI effective action. The $\dot{\gamma}_i$ and γ_i terms are known to be RG-stable from the general arguments in Refs. [98, 97, 133, 12], while we argue in App. C.2 that the tree-level result is RG-stable once one accounts for goldstino-gravitino mixing.

We can again use the trick in Sec. 5.4.2 to extract the goldstino coupling $\mathcal{G}^S$ defined in

[37] Note that the result in Eq. (5.107) is still invariant under the super-Weyl F_Σ transformations. The $K_k F^k$ factor arises by isolating the goldstino direction out of the fermion in Eq. (5.79), not from F_C.

[38] The same factor appeared in the auxiliary field shift of Eq. (5.102) for related reasons.

[39] For any negative curvature, one expects the γ_i and $\dot{\gamma}_i$ terms to be partially cancelled off by AdS boundary effects, as in Ref. [87]. While we have not computed them explicitly, such boundary terms are necessary for the structure of the AdS SUSY algebra to be maintained in the unbroken limit.

Eq. (5.90):[40]

$$\mathcal{G}_i^{\mathcal{S}} = -\frac{1}{12}\dot{\gamma}_i K_k F^k \left(m_{3/2} + \frac{1}{3} K_\ell F^\ell \right) - (2 - \gamma_i) F_R, \tag{5.110}$$

As advertised, there are no goldstino couplings proportional to $m_{3/2}^2$. Like $\mathcal{S}_i$, this associated goldstino coupling is RG-stable. The tree-level and one-loop goldstino couplings arise because there are SUSY-preserving scalar masses in the bulk of AdS, which are then lifted by an amount proportional to the SUSY-breaking order parameter F_R. For $\langle K_i \rangle = 0$, the two-loop anomaly-mediated masses familiar from Ref. [135] have no corresponding goldstino coupling, as such masses are also present in the bulk of AdS when SUSY is unbroken. Curiously, such two-loop goldstino couplings also vanish in the no-scale limit (where $F_{\mathrm{SW}} = 0$) [115] and will be suppressed for almost no-scale models [120]. The difference between $\mathcal{S}_i$ and $\mathcal{G}^{\mathcal{S}}$ is

$$\mathcal{S}_i - \mathcal{G}_i^{\mathcal{S}} = -\frac{1}{4}\dot{\gamma}_i m_{3/2} \left(m_{3/2} + \frac{1}{3} K_k F^k \right) - m_{3/2}^2 \left(2 - \gamma_i \right). \tag{5.111}$$

which is independent of the curvature $\mathcal{R}$. As anticipated, this difference vanishes with vanishing $m_{3/2}$, as it is intimately related to SUSY-preserving anomaly mediation effects in AdS SUSY. Whereas the second term proportional to $m_{3/2}^2$ arises purely from the structure of unbroken AdS SUSY, the first term proportional to $m_{3/2} F_{\mathrm{SW}}$ is a cross term between a SUSY-preserving and a SUSY-breaking effect and vanishes in the no-scale limit.

Results for $\mathcal{S}_i$ and $\mathcal{G}_i^{\mathcal{S}}$ are shown in Table 5.1 for various values of the curvature. The answer is particularly striking when $\langle K_i \rangle = 0$ in the flat space limit with $F_R = -m_{3/2}^2$:

$$\begin{aligned} \mathcal{S}_i &= -\frac{1}{4}\dot{\gamma}_i m_{3/2}^2, \\ \mathcal{G}_i^{\mathcal{S}} &= (2 - \gamma_i) m_{3/2}^2, \qquad\qquad (\text{flat space, } \langle K_i \rangle = 0) \\ \mathcal{S}_i - \mathcal{G}_i^{\mathcal{S}} &= -m_{3/2}^2 \left(2 - \gamma_i + \frac{1}{4}\dot{\gamma}_i \right). \end{aligned} \tag{5.112}$$

While anomaly-mediated sfermion soft mass-squareds are colloquially described as a two-loop effect, this expression makes it clear that this is an artifact of tuning the cosmological constant to zero, since anomaly mediation has important tree-level and one-loop effects on

[40]As in footnote 37, the result in Eq. (5.110) is invariant under F_Σ transformations. F_R (arising here from the gravitino equations of motion of Eq. (5.78)) does not implicitly contain $M^* M$.

the goldstino couplings. Indeed, the difference $\mathcal{S}_i - \mathcal{G}_i^{\mathcal{S}}$ has important effects at all orders.

For completeness, we give results for the parameter $\mathcal{T}$ defined in Eq. (5.88) and the associated goldstino coupling $\mathcal{G}^{\mathcal{T}}$:

$$\mathcal{T}_i = -\frac{1}{8}\left(\gamma_i^2 + \dot{\gamma}_i + (2+4c_7)\gamma_i\right)\left|m_{3/2} + \frac{1}{3}K_k F^k\right|^2 + \frac{1+c_7}{2}\gamma_i(m_{3/2}^2 + F_R). \quad (5.113)$$

$$\mathcal{G}_i^{\mathcal{T}} = -\frac{1}{24}\left(\gamma_i^2 + \dot{\gamma}_i + (2+4c_7)\gamma_i\right)K_k F^k\left(m_{3/2} + \frac{1}{3}K_k F^k\right) + \frac{1+c_7}{2}\gamma_i F_R. \quad (5.114)$$

$$\mathcal{T}_i - \mathcal{G}_i^{\mathcal{T}} = -\frac{1}{8}m_{3/2}\left(m_{3/2} + \frac{1}{3}K_k F^k\right)\left(\gamma_i^2 + \dot{\gamma}_i + (2+4c_7)\gamma_i\right) + \frac{1+c_7}{2}m_{3/2}^2\gamma_i \quad (5.115)$$

As expected, the difference $\mathcal{T} - \mathcal{G}^{\mathcal{T}}$ is always proportional to $m_{3/2}$, arising as it does from the structure of AdS SUSY. However, these results are harder to interpret, since $\mathcal{T}$ has residual dependence on the parameter c_7 defined in Eq. (5.91). This indicates that the value of $\mathcal{T}$ depends on exactly how one regulates the theory (i.e. on the correct choice of $\widetilde{\Box}$ for a given regularization scheme). Note that if $\langle K_i \rangle = 0$ then $\mathcal{T} - \mathcal{G}^{\mathcal{T}}$ is independent of c_7. Furthermore, in unbroken AdS SUSY ($F_R = \langle K_i \rangle = 0$), all c_7 dependence vanishes since $\boldsymbol{G}_{\alpha\dot{\alpha}} = 0$ in rigid AdS SUSY.

5.5 Conclusions

This chapter completes the task originally started in Ch. 5 to understand anomaly mediation as being a SUSY-preserving effect in AdS space. For the R-violating terms (gaugino masses, A-terms, and B-terms), anomaly mediation generates soft masses proportional to $m_{3/2}$ without corresponding goldstino couplings, making it clear that these are SUSY-preserving effects.[41] For the sfermion soft mass-squareds, the situation is far more interesting, since there are SUSY-preserving effects proportional to $m_{3/2}^2$ and SUSY-breaking effects proportional to F_R, but these two effects are difficult to disentangle because F_R happens to equal $-m_{3/2}^2$ after tuning for flat space. Having successfully isolated these two effects, we see that the familiar two-loop anomaly-mediated sfermion soft mass-squareds are accompanied by tree-level and one-loop goldstino couplings, and all three terms are needed to preserve the underlying AdS SUSY structure.

Along the way, we have learned a number of lessons about AdS SUSY and SUGRA.

[41] Strictly speaking, we have not carried out the calculation of gaugino masses beyond one-loop order. We sketch how to do this in App. C.4.

First, the peculiar behavior of anomaly mediation is already evident at tree-level, and the irreducible goldstino coupling in Eq. (5.36) offers strong evidence that AdS SUSY (and not flat space SUSY) is the correct underlying symmetry structure for SUGRA theories. Second, to incorporate quantum effects, one has to work with a regulated SUGRA action. Unfortunately, it is impossible to write down a Wilsonian action that captures the full effects of anomaly mediation at tree-level, since there are important effects of the regulator fields at loop-level. Instead, we used a 1PI effective action to make super-Weyl invariance manifest, countering the (gauge-dependent) claims in Refs. [46, 47] (and implicit in Ref. [103]) about the non-existence of anomaly mediation. Third, even with a SUSY-preserving, super-Weyl-invariant 1PI effective action in hand, there is residual ambiguity starting at $\mathcal{O}(m_{3/2}^2)$ in how to write down a SUGRA-invariant theory. Luckily, the supertrace $\mathcal{S}$ is unambiguous, yielding the same soft mass-squareds known in the literature.

This chapter has focused on formal aspects of anomaly mediation, and therefore has not addressed a number of important phenomenological questions. First, anomaly mediation was motivated in part by the possibility of sequestering, and one would like to know whether the sequestered limit is physically obtainable without fine-tuning. To that end, it would be useful to know whether the irreducible goldstino coupling in Eq. (5.36) is indeed an attractive IR fixed point, as one would expect in conformally sequestered theories. Second, we have used goldstino couplings as a probe of which effect preserve SUSY and which effects break SUSY. Ideally, one would want to find an experimental context where these goldstino couplings could be measured, since this would give an experimental handle on the underlying AdS curvature. Measuring such a coupling to two-loop precision would even probe the value of F_{SW}, though the physical significance of that dependence is not clear to us. Third, in addition to the supertrace $\mathcal{S}$, we identified the independent trace $\mathcal{T}$ which is perhaps known to SUSY aficionados but is unfamiliar to us. Even in global flat space SUSY, it would be helpful to know what effects a non-zero value of $\mathcal{T}$ can have on phenomenology. Finally, the big question facing particle physics in 2013 is whether (weak scale) SUSY is in fact realized in nature. We of course have no insight into this broader question, but we can say that if (AdS) SUSY and SUGRA do exist, then anomaly mediation will yield irreducible physical effects proportional to $m_{3/2}$.

Chapter 6

A Photon Line from Decaying Goldstini Dark Matter

6.1 Introduction

As we discussed in Ch. 2, SUSY-breaking generically must occur in some hidden sector, whose couplings to the supersymmetric Standard Model (SSM) are suppressed by an inverse power of F, the SUSY-breaking scale and order parameter. The particles in the hidden sector generally have little phenomenological relevance with the one exception of the light goldstino, the goldstone fermion of SUSY. In supergravity (SUGRA), the goldstino is eaten to become the longitudinal components of the gravitino (the superpartner of the graviton) via the super-Higgs mechanism.

If SUSY is broken in multiple hidden sectors, there will generically be multiple goldstini [42], one linear combination will be eaten in the super-Higgs mechanism [55, 149, 71, 72], while the other, uneaten goldstini remain in the theory. One of the uneaten goldstini can easily be the predominant component of the dark matter in the universe [42]. It is not absolutely stable, however, as it is generically heavier than the gravitino, to which it can decay on cosmological timescales [42, 37]. The products of these decays can potentially be seen by indirect detection experiments such as FERMI-LAT, PAMELA, or AMS-02.

In this chapter, we discuss a hitherto-unexplored decay channel of the uneaten goldstino a two-body decay to a photon and the gravitino, which occurs via the small mass mixing between the uneaten goldstino and the electroweak gauginos induced by electroweak sym-

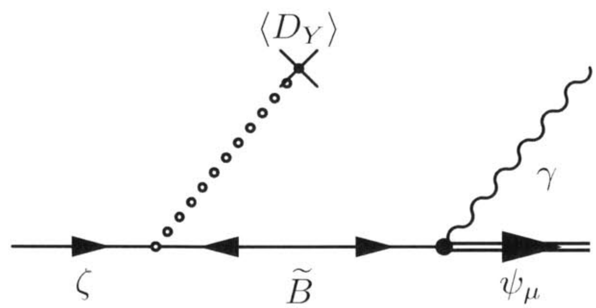

Figure 6-1: One of the leading Feynman diagrams mediating the decay of the uneaten goldstino dark matter ζ to a gravitino ψ_μ and a photon. The goldstino and bino have a mass mixing proportional to the hypercharge D-term after electroweak symmetry breaking, allowing the goldstino to decay to a gravitino and photon.

metry breaking. One of the dominant Feynman diagrams mediating this decay is given in Fig. 6-1. The resulting production of monochromatic gamma rays would be a striking signature at experiments such as the FERMI-LAT, being relatively easier to distinguish from background and harder to fake from astrophysical sources, allowing such experiments to probe longer dark matter lifetimes. Unlike other sources of gamma ray lines, this decay mode is not loop-suppressed compared to other modes, and so can naturally be the discovery channel for indirect detection of dark matter.

The usual obstacle to considering dark matter decays featuring a monochromatic photon is that they typically occur much too quickly; the dark matter would have either decayed away early in the lifetime of the universe, or we would be awash in the resulting gamma rays in the present era. This happens generically for decays induced by transition magnetic dipoles, the lowest dimension operator that would allow such a decay consistent with the observed electric neutrality of dark matter:

$$\mathcal{O}_{\rm MDM} = \frac{c}{M}\eta\sigma^{\mu\nu}\chi F_{\mu\nu}, \tag{6.1}$$

$$\Gamma_{\chi\to\gamma\eta} \sim \frac{c^2 m_\chi^3}{8\pi M^2}. \tag{6.2}$$

Even for $M \sim M_{\rm Pl}$, $c \lesssim 10^{-11}$ is required for consistency with observation. In the framework of multiple SUSY-breaking, however, an astrophysically-reasonable lifetime can arise quite naturally without the need to set any parameter to artificially small values. The leading operator mediating this decay is

$$\mathcal{O}_{\zeta\to\psi_\mu} = \frac{c}{M_{\rm Pl}F_2}\frac{m_{\rm soft,2}}{m_{\rm soft,1}}\psi_\mu\sigma_\nu\zeta^\dagger F^{\mu\nu}H^\dagger H, \tag{6.3}$$

with F_2 the smaller of the two SUSY-breaking scales and $m_{{\rm soft},i}$ the contribution of that sector to SUSY-breaking soft masses in the visible sector. Note that the decay requires electroweak symmetry breaking to proceed, and involves the couplings of each hidden sector to the SSM, thus providing a naturally small decay rate scaling as

$$\Gamma_{\zeta\to\gamma\psi_\mu} \sim \frac{c^2 M_Z^4 m_\zeta^5}{8\pi F_1^2 F_2^2}\frac{m^2_{\rm soft,2}}{m^2_{\rm soft,1}}. \tag{6.4}$$

In the next section, we review the framework of goldstini. We then discuss the major two- and three-body decay modes of the uneaten goldstino in Sec. 6.3. In Sec. 6.4, we discuss the prospects for indirect detection of goldstini dark matter via this decay mode. In Sec. 6.5, we discuss the renormalization group running of goldstino couplings, which is necessary to understand the branching ratios of the goldstino decay modes in the benchmark scenarios we present in Sec. 6.6. In Sec. 6.7, we discuss some non-minimal models that can enhance the photon mode's branching ratio without depending on electroweak symmetry breaking, by introducing additional mass mixing between the bino and other gauge singlet fermions, and we conclude in Sec. 6.8.

6.2 Review of Goldstini

As in Ch. 3, we consider two sequestered sectors, each of which spontaneously breaks SUSY—though many of our results can be easily generalized to the case with three or more SUSY-breaking sectors. Each sector has an associated goldstino (η_1 and η_2, respectively), and we characterize the size of SUSY breaking via the goldstino decay constants (F_1 and F_2, respectively). Each SUSY breaking sector can be parametrized in terms of a non-linear goldstino multiplet [109, 42][1]

$$\boldsymbol{X}_a = \frac{\eta_a^2}{2F_a} + \sqrt{2}\theta\eta_a + \theta^2 F_a, \tag{6.5}$$

[1]Throughout, we use boldface to denote a superfield, with regular typeface denoting its lowest component.

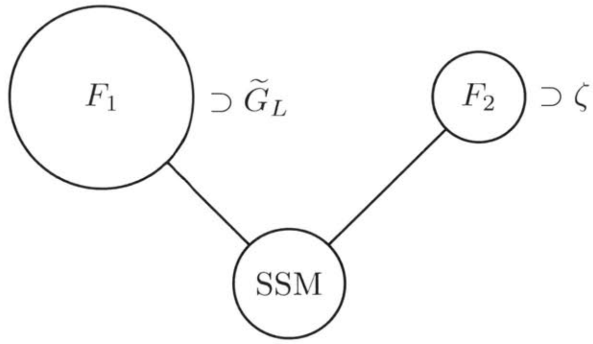

Figure 6-2: In our framework, SUSY is broken in two sectors sequestered from each other, with the strength of SUSY-breaking parameterized by $F_1 \geq F_2$. The true goldstino $\widetilde{G}_L$, eaten by the gravitino, resides mainly in sector 1, while the uneaten goldstino ζ resides mainly in sector 2. Each sector contributes to SUSY-breaking terms in the visible SSM sector, which are accompanied by corresponding goldstino couplings.

for $a = 1, 2$. We define the quantities

$$F_{\text{eff}} \equiv \sqrt{F_1^2 + F_2^2}, \qquad \tan\theta \equiv \frac{F_2}{F_1}, \qquad F_\perp = \frac{F_1 F_2}{F_{\text{eff}}}, \tag{6.6}$$

and we take $\tan\theta \leq 1$ ($F_1 \geq F_2$) without loss of generality.

The combination $\widetilde{G}_L = \sin\theta\, \eta_1 + \cos\theta\, \eta_2$ is eaten by the gravitino to become its longitudinal components via the super-Higgs mechanism, but the orthogonal goldstino $\zeta = \cos\theta\, \eta_1 - \sin\theta\, \eta_2$ remains uneaten and will be the focus of our study. Due to SUGRA effects, ζ receives a mass of $2m_{3/2}$ in the minimal goldstini scenario [42]. In addition, variations in the SUSY-breaking dynamics [45] or induced couplings between the two sectors [42, 10] can modify the mass term for ζ.[2]

Supersymmetry breaking is communicated from the two hidden sectors to the visible sector by means of a non-trivial Kähler potential and gauge kinetic function (presumably coming from integrating out heavy messenger fields). This is depicted schematically in

[2]At minimum, one expects loops of SM fields to generate $m_\zeta \simeq m_{\text{soft}}/(16\pi^2)^n$ [42], where n depends on the number of loops necessary to effectively connect sectors 1 and 2 and transmit the needed $U(1)_R$ breaking. The uneaten goldstino will also obtain a tree-level mass due to mixing with the neutralinos, but this is of order $1/F^2$ and is comparatively negligible.

Fig. 6-2. Some representative terms contributing to the SSM soft masses are[3]

$$\boldsymbol{K} = \boldsymbol{\Phi}^i\boldsymbol{\Phi}^{\dagger\bar{j}}\sum_a \frac{m^2_{i\bar{j},a}}{F_a^2}\boldsymbol{X}_a^\dagger\boldsymbol{X}_a, \tag{6.7}$$

$$\boldsymbol{f}_{AB} = \frac{1}{g_A^2}\delta_{AB}\left(1-\sum_a \frac{2M_{A,a}}{F_a}\boldsymbol{X}_a\right), \tag{6.8}$$

$$\boldsymbol{W} = \frac{1}{2}\mu_{ij}\boldsymbol{\Phi}^i\boldsymbol{\Phi}^j + \frac{1}{2}\boldsymbol{\Phi}^i\boldsymbol{\Phi}^j\sum_a \frac{B_{ij,a}}{2F_a}\boldsymbol{X}_a, \tag{6.9}$$

where $i = 1, 2$, and $\boldsymbol{\Phi}$ stands for a general SSM multiplet.[4] These yield the following terms in the lagrangian up to order $1/F$ [154]:

$$\begin{aligned}\mathcal{L} = &-\sum_a m^2_{i\bar{j},a}\phi^{*\bar{j}}\phi^i + \sum_a \frac{m^2_{i\bar{j},a}}{F_a}\eta_a\chi^i\phi^{*\bar{j}} + \sum_a \frac{2m^2_{3/2}\delta_{i\bar{j}}}{F_a}\frac{F_a^2}{F_{\text{eff}}^2}\eta_a\chi^i\phi^{*\bar{j}} \\ &-\frac{1}{2}\left(\sum_a B_{ij,a} - m_{3/2}\mu_{ij}\right)\phi^i\phi^j + \sum_a \frac{B_{ij,a}}{F_a}\eta_a\chi^i\phi^j + \text{h.c.} \qquad (6.10)\\ &-\frac{1}{2}\sum_a M_{A,a}\lambda^A\lambda^A - \sum_a \frac{iM_{A,a}}{\sqrt{2}F_a}\eta_a\sigma^{\mu\nu}\lambda^A F^A_{\mu\nu} + \sum_a \frac{M_{A,a}}{\sqrt{2}F_a}\eta_a\lambda^A D^A \qquad (6.11)\end{aligned}$$

Thus, the parameters $m^2_{i\bar{j},a}$, $M_{A,a}$, and $B_{ij,a}$ are the contributions to the SUSY-breaking scalar mass-squareds, gaugino masses, and B-terms, respectively, from the sector a. Note that they are intrinsically related to the coupling of the SSM fields to the goldstini. The final term on the first line of Eq. (6.10) is a universal supergravity effect, arising from the fact that for unbroken supergravity in anti de-Sitter space, scalar masses are $-2m^2_{3/2}$; this is a generalization to multiple goldstini of the term discussed in Ch. 5. Similarly, the second term on the second line, proportional to $m_{3/2}$ and the fermion mass matrix μ_{ij}, is a tree-level anomaly-mediated contribution to B-terms that is *not* truly a SUSY-breaking effect (as it is present in unbroken AdS SUSY), and therefore arises from neither sector [54].

Rotating to the $\widetilde{G}_L$–ζ basis yields similar interaction terms for the eaten goldstino $\widetilde{G}_L$

[3] This is not an exhaustive list of terms that contribute to the desired soft terms, especially in the Kähler potential. However, the omission of such terms here does not affect the final result, at least at tree level. We also do not include A-terms here, though they can have important RG effects, as we will discuss in Sec. 6.5.

[4] Throughout, we use the conventions of Ref. [154], except we have redefined their gauginos by a factor of i: $\lambda_{WB} \to i\lambda$.

and the uneaten goldstino ζ,

$$\mathcal{L}_{\widetilde{G}_L} = \frac{m^2_{i\bar{\jmath}} + 2m^2_{3/2}\delta_{i\bar{\jmath}}}{F_{\rm eff}}\widetilde{G}_L\chi^i\phi^{*\bar{\jmath}} + \frac{B_{ij} + m_{3/2}\mu_{ij}}{F_{\rm eff}}\widetilde{G}_L\chi^i\phi^{*j}$$
$$- \frac{iM_A}{\sqrt{2}F_{\rm eff}}\widetilde{G}_L\sigma^{\mu\nu}\lambda^A F^A_{\mu\nu} + \frac{M_A}{\sqrt{2}F_{\rm eff}}\widetilde{G}_L\lambda^A D^A, \tag{6.12}$$

$$\mathcal{L}_\zeta = \frac{\widetilde{m}^2_{i\bar{\jmath}}}{F_\perp}\zeta\chi^i\phi^{*\bar{\jmath}} + \frac{\widetilde{B}_{ij}}{F_\perp}\widetilde{G}_L\chi^i\phi^{*\bar{\jmath}}$$
$$- \frac{i\widetilde{M}_A}{\sqrt{2}F_\perp}\zeta\sigma^{\mu\nu}\lambda^A F^A_{\mu\nu} + \frac{\widetilde{M}_A}{\sqrt{2}F_\perp}\zeta\lambda^a D^a, \tag{6.13}$$

where the untilded mass parameters are defined as[5]

$$\widetilde{M}_A \quad = \quad M_{A,2}\cos^2\theta - M_{A,1}\sin^2\theta, \tag{6.14}$$

and analogously for scalar mass-squareds and B-terms. The parameters $m^2_{i\bar{\jmath}}$, M_A, and B_{ij} are here the physical scalar mass-squareds, gaugino masses, and B-terms. Generally, they are just equal to the sum of the contributions from each sector (i.e. $M_A = M_{A,1} + M_{A,2}$).[6]

6.3 Decays of the Goldstino

In the presence of electroweak symmetry breaking, there is a mass mixing between the goldstinos and the neutralinos of the MSSM, as can be seen directly from the Lagrangian of Eqs. (6.12) and (6.13). For the goldstino that is eaten by the gravitino, this is fundamentally due to the fact that electroweak symmetry breaking induces a small amount of SUSY-breaking in the visible sector, as the Higgs F-terms and electroweak D-terms obtain vevs. The amount of mixing between the eaten goldstino and the neutralinos can be directly read off the eaten goldstino direction, given by

$$\widetilde{G}_L = \frac{1}{F_{\rm eff}}\left(\langle F_i\rangle\chi^i + \frac{1}{\sqrt{2}}\langle D_A\rangle\lambda^A\right). \tag{6.15}$$

[5]Note that this differs by a factor of $\sin\theta\cos\theta$ from the equivalent definition in Ch. 3.

[6]This is not true for the B-terms at tree level, due to the anomaly-mediated contribution discussed above. A similar story will hold for loop-level anomaly-mediated contributions, so all soft terms m appearing in Eq. (6.12) should really be replaced by $m - m_{\rm AMSB}$. As $m > m_{3/2}$ in the scenario we consider (as the gravitino is the LSP), the effects of loop-level anomaly mediation on these coefficients are of little phenomenological relevance here. Also at loop level, the $2m^2_{3/2}$ term should have corrections starting at one loop proportional to anomalous dimensions, as in Ch. 5.

For the uneaten goldstino, the cancellations that ensure Eq. (6.15) do not occur, and the mixings can be parametrically larger (even after accounting for a possible hierarchy in the hidden sector F^a), arising from the terms

$$\mathcal{L} \supset \frac{\widetilde{B}_{ij}}{F_\perp}\zeta\chi^i \langle \phi^j \rangle + \frac{\widetilde{m}^2_{ij}}{F_\perp}\zeta\chi^i \langle \phi^{*j} \rangle + \frac{\widetilde{M}_A}{\sqrt{2}F_\perp}\eta_A\lambda^A \langle D^A \rangle + \text{h.c.} \tag{6.16}$$

Due to this mass mixing, an uneaten goldstino can undergo a two-body decay to a gravitino and a photon, Higgs, or Z boson through its neutralino components. Such a decay process has not previously been fully considered in the literature.[7]

Since the decay of a goldstino of mass $2m_{3/2}$ (in the minimal goldstini scenario) to a gravitino of mass $m_{3/2}$ is somewhat outside of the regime of the goldstino equivalence theorem [64, 34, 33], in order to calculate the decay rate, one must consider the explicit coupling of the spin-3/2 gravitino. While this presents its own technical challenges (see App. D.2), it does allow us to factorize the calculation in a convenient way. We will choose to work in unitarity gauge for the gravitino; in this gauge, the eaten goldstino is removed from the theory entirely (i.e. it is made infinitely massive). Therefore, terms in the Lagrangian that contain the fermion bilinear $\widetilde{G}_L\zeta$ coupled to Standard Model particles can be safely ignored. Furthermore, the gravitino, unlike the goldstini, no longer has any mass mixings with any other fermions after gauge fixing.

The goldstino to gravitino decay calculation can therefore be factorized into two separate problems. The first is finding the bino, wino, and Higgsino fractions of the uneaten goldstino, which derive solely from the neutralino-goldstino mass matrix. Then, one can calculate the decay rate of a hypothetical bino, wino, or Higgsino of physical mass $2m_{3/2}$ to a gravitino and a photon, Higgs, or Z boson. Including the mixing angles in the latter calculation then yields the goldstino to gravitino decay rates. We will consider each of these calculations in turn.

[7]Ref. [37] did consider the possibility of two-body decays to the Higgs or a longitudinal Z, though not a photon or transverse Z.

6.3.1 Goldstino-Neutralino Mass Matrix

The neutralino-goldstino mass matrix can be parameterized in the $\{\widetilde{B}, \widetilde{W}^3, \widetilde{H}_d, \widetilde{H}_u, \zeta, \widetilde{G}_L\}$ basis as follows [146]:

$$M = \begin{pmatrix} M_\chi & \widetilde{\rho} & \rho \\ \widetilde{\rho}^T & m_\zeta & 0 \\ \rho^T & 0 & 2m_{3/2} \end{pmatrix}, \tag{6.17}$$

$$\widetilde{\rho} = -\frac{v}{\sqrt{2}F_\perp} \begin{pmatrix} \frac{1}{2}M_Z\widetilde{M}_1 s_W c_{2\beta} \\ -\frac{1}{2}M_Z\widetilde{M}_2 c_W c_{2\beta} \\ \widetilde{m}^2_{H_d}c_\beta + \widetilde{B}_\mu s_\beta \\ \widetilde{m}^2_{H_u}s_\beta + \widetilde{B}_\mu c_\beta \end{pmatrix}, \tag{6.18}$$

$$\rho = -\frac{v}{\sqrt{2}F_{\text{eff}}} \begin{pmatrix} \frac{1}{2}M_Z M_1 s_W c_{2\beta} \\ -\frac{1}{2}M_Z M_2 c_W c_{2\beta} \\ (m^2_{H_d} + 2m^2_{3/2})c_\beta + (B_\mu - m_{3/2}\mu)s_\beta \\ (m^2_{H_u} + 2m^2_{3/2})s_\beta + (B_\mu - m_{3/2}\mu)c_\beta \end{pmatrix}, \tag{6.19}$$

where M_χ is the usual 4×4 neutralino mass matrix (see e.g. Ref. [124]),[8] $s_W = \sin\theta_W$, $c_\beta = \cos\beta$, $v \approx 246$ GeV. The uneaten goldstino has mass m_ζ, which takes the value $2m_{3/2}$ at tree level but may receive substantial radiative corrections [10].

Throughout, we will work in unitarity gauge for the gravitino, in which the eaten goldstino is removed from the theory. This may be done easily at $\mathcal{O}(1/F)$, and amounts to just considering the upper 5×5 block of Eq. (6.17).

[8]Note especially that the Higgsino mass term appearing in M_χ is $-\mu$, hence the apparent sign difference in the $m_{3/2}\mu$ term as compared to Eq. (6.12). Note also that our convention for B_μ differs by a sign from that in Ref. [124], and μ is taken to be real.

The overlap between the uneaten goldstino and a given neutralino is given by the vector

$$\widetilde{\theta} \equiv (m_\zeta \mathbb{1} - M_\chi)^{-1} \widetilde{\rho}. \tag{6.20}$$

For goldstino decay into a gravitino and a given boson, the mixing angle we care about is the one between the uneaten goldstino and the linear combination of the neutralinos that forms the superpartner of that boson (in the interaction basis):

$$\Theta_i \equiv P_i^T \widetilde{\theta}, \tag{6.21}$$

$$\begin{aligned} P_\gamma &= \{\cos\theta_W, \sin\theta_W, 0, 0\}, & P_h &= \{0, 0, -\sin\alpha, \cos\alpha\}, \\ P_{Z_T} &= \{-\sin\theta_W, \cos\theta_W, 0, 0\}, & P_{Z_L} &= \{0, 0, \cos\beta, -\sin\beta\}. \end{aligned} \tag{6.22}$$

We will assume throughout that the uneaten goldstino is lighter than the heavy Higgs states A^0 and H^0, so that they are not produced in decays. In such a Higgs decoupling limit, $\alpha \approx \beta - \pi/2$, so $P_h \approx \{0, 0, \sin\beta, \cos\beta\}$.

6.3.2 Goldstino Couplings

The couplings of a single gravitino to visible sector fields are entirely determined by the supercurrent [22, 154][9]:

$$\mathcal{L} \supset -\frac{1}{2M_{\rm Pl}} \psi_\mu j^\mu + \text{h.c.} \tag{6.23}$$

$$\supset -\frac{1}{\sqrt{2}M_{\rm Pl}} g_{i\bar{j}} \mathcal{D}_\nu \phi^{*\bar{j}} \chi^i \sigma^\mu \bar{\sigma}^\nu \psi_\mu + \frac{1}{2M_{\rm Pl}} \psi_\mu \sigma^{\nu\rho} \sigma^\mu \lambda^{\dagger A} F^A_{\nu\rho} + \text{h.c.} \tag{6.24}$$

We have dropped terms in the supercurrent proportional to $\sigma^\mu \xi^\dagger$ (for $\xi^\dagger$ any elementary or composite fermion), as the associated gravitino coupling terms will vanish on the unitarity gauge gravitino equations of motion.

As long as the uneaten goldstino ζ is the dark matter, it is necessarily lighter than all R-parity odd states in the SSM. As a result, we can simply integrate out all such particles to yield an effective field theory of goldstino decay, organized by powers of $m_\zeta / m_{\rm SUSY}$. We

[9] Note that this automatically includes all the tree-level effects proportional to $m_{3/2}$, such as those discussed in Eq. (6.10).

also assume that the goldstino is much lighter than the heavy Higgs scalars, so that we can integrate them out and take $\alpha = \beta - \pi/2$, $H_u = \sin\beta H$, and $H_d = \cos\beta\epsilon H^*$ at leading order. The first operators appear at dimension 6 after integrating out a single Higgsino at tree level:

$$\mathcal{L}_6 = -\frac{2\widetilde{B}_\mu \sin^2\beta + \widetilde{m}^2_{H_d}\sin 2\beta}{\sqrt{2}\mu F_\perp M_{\rm Pl}}\zeta\psi_\mu \mathcal{D}^\mu H^\dagger H - \frac{2\widetilde{B}_\mu \cos^2\beta + \widetilde{m}^2_{H_u}\sin 2\beta}{\sqrt{2}\mu F_\perp M_{\rm Pl}}\zeta\psi_\mu H^\dagger \mathcal{D}_\mu H. \tag{6.25}$$

These operators give the leading contributions to the two-body decays to Higgs and longitudinal Z discussed in the previous section, and can mediate three body decays to $hh\psi_\mu$ or $hZ_L\psi_\mu$ that we will discuss in the next section.

There are many operators at dimension 5 and 6 allowed by gauge symmetries that do not appear in Eq. (6.25). Many of these simply vanish on the unitary gauge equations of motion of the gravitino (most notably, $\overline{\sigma}^\mu\psi_\mu = 0$ and $\mathcal{D}^\mu\psi_\mu = 0$) and can be safely omitted. Even so, there is still one operator at dimension 5 and several at dimension 6 that cannot be so neglected a priori:

$$\mathcal{O}_5 \stackrel{?}{=} \psi_\mu\sigma_\nu\zeta^\dagger F^{\mu\nu}, \qquad\qquad \mathcal{O}_6^i \stackrel{?}{=} \psi_\mu\zeta\chi^{\dagger i}\overline{\sigma}^\mu\chi^i. \tag{6.26}$$

To explain why these do not appear, we need to remember that the couplings of the gravitino are strongly constrained by the fact that it needs to couple to the supercurrent. While the effective supercurrent can be heavily modified when one begins integrating out superpartners, it is still clear that an operator like $\mathcal{O}_5$ would only be allowed if ζ were at least partially the superpartner of the B boson in the absence of electroweak symmetry breaking. In models with a minimal SSM, it is not, but we will return to this suggestive possibility in Sec. 6.7.[10] The four-fermi operator $\mathcal{O}_6$ can be generated if SM fermions and the uneaten sgoldstino (which obtains a vev) are both charged under a gauge symmetry broken at a high scale, after integrating out the massive vector boson. However, one generally expects the scale of any such hidden sector dynamics to be much larger than the masses of the superpartners we integrated out to obtain Eq. (6.25), so we will neglect this possibility henceforth.

[10]Similar arguments forbid other operators (starting at dimension 6) that leverage the $U(1)$ nature of hypercharge in minimal models.

There are many additional operators at dimension 7:

$$\begin{aligned}\mathcal{L}_7 = & -\sum_A \frac{\sqrt{2}\widetilde{M}_A}{M_A F_\perp M_{\rm Pl}}\psi_\mu^\dagger \sigma_\nu \zeta F^{A\mu\rho}F^{A\nu}{}_\rho - \sum_i \frac{\sqrt{2}\widetilde{m}^2_{\phi^i}}{m^2_{\phi^i}F_\perp M_{\rm Pl}}\zeta^\dagger \chi^{\dagger i}\mathcal{D}^\mu \chi^i \psi_\mu \\ & + \frac{ig'[(\widetilde{M}_1 - 2\widetilde{B}_\mu/\mu)\cos 2\beta - (\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d})\mu^{-1}\sin 2\beta]}{2\sqrt{2}M_1 F_\perp M_{\rm Pl}}\psi_\mu^\dagger \sigma_\nu \zeta F^{\mu\nu} H^\dagger H \\ & + \frac{ig[(\widetilde{M}_2 - 2\widetilde{B}_\mu/\mu)\cos 2\beta - (\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d})\mu^{-1}\sin 2\beta]}{\sqrt{2}M_2 F_\perp M_{\rm Pl}}\psi_\mu^\dagger \sigma_\nu \zeta F^{A\mu\nu} H^\dagger T^A H \\ & - i\frac{2\widetilde{m}^2_{H_d}\cos^2\beta + \widetilde{B}_\mu \sin 2\beta}{\sqrt{2}\mu^2 F_\perp M_{\rm Pl}} H^\dagger \zeta^\dagger \overline{\sigma}_\mu \mathcal{D}^\mu(\mathcal{D}^\nu H \psi_\nu) \\ & - i\frac{2\widetilde{m}^2_{H_u}\sin^2\beta + \widetilde{B}_\mu \sin 2\beta}{\sqrt{2}\mu^2 F_\perp M_{\rm Pl}} \zeta^\dagger \overline{\sigma}_\mu \mathcal{D}^\mu(\mathcal{D}^\nu H^\dagger \psi_\nu) H + \text{h.c.} \end{aligned} \tag{6.27}$$

The operators on the first line mediate three-body decays to two gauge bosons or two fermions, respectively. The next two operators give the leading contributions to single photon and Z emission, while the last two give sub-leading contributions to decays to Higgs and Zs.

6.3.3 Two-Body Decays of the Goldstino

The decay rate of the uneaten goldstino ζ (with mixings with the neutralinos to be specified in the next section) to a single boson and a gravitino can be extracted from these couplings after some modest calculational effort, given the Θ_i from Eq. (6.21). The decay to the Higgs is mediated by the first term in Eq. (6.24), the decay to a photon is mediated by the second, while the decay to a Z is mediated by both after electroweak symmetry breaking.

The decay rates for the photon [66] and Higgs modes are given by

$$\Gamma_{\zeta\to\gamma+\psi_\mu} = \frac{m_\zeta^5 \Theta_\gamma^2}{16\pi F_{\rm eff}^2}\left(1 - \frac{m^2_{3/2}}{m^2_\zeta}\right)^3 \left(1 + \frac{3m^2_{3/2}}{m^2_\zeta}\right), \tag{6.28}$$

$$\Gamma_{\zeta\to h\psi_\mu} = \frac{m_\zeta^5 \Theta_h^2}{32\pi F_{\rm eff}^2}\left(\left(1 - \frac{m_{3/2}}{m_\zeta}\right)^2 - \frac{m_h^2}{m_\zeta^2}\right)^{3/2} \left(\left(1 + \frac{m_{3/2}}{m_\zeta}\right)^2 - \frac{m_h^2}{m_\zeta^2}\right)^{5/2}. \tag{6.29}$$

The decay rate for the Z mode for general $m_{3/2}$ for arbitrary m_ζ is too complicated to give here, and is left for App. D.3.

These decay rates are more tractable in certain limits. When the goldstino gets most of its mass from loop-level effects so that $m_\zeta \gg m_{3/2}$, the decay rates simplify considerably, and follow from the goldstino equivalence theorem:

$$\begin{aligned}
\Gamma_{\zeta\to\gamma\psi_\mu} &= \frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\Theta_\gamma^2, \\
\Gamma_{\zeta\to h\psi_\mu} &= \frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\left(1-\frac{m_h^2}{m_\zeta^2}\right)^4 \tfrac{1}{2}\Theta_h^2, \qquad\qquad \boxed{m_\zeta \gg m_{3/2}} \\
\Gamma_{\zeta\to Z\psi_\mu} &= \frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\left(1-\frac{M_Z^2}{m_\zeta^2}\right)^4 \left(\Theta_{Z_T}^2+\tfrac{1}{2}\Theta_{Z_L}^2\right),
\end{aligned} \tag{6.30}$$

where the angles Θ_i are the mixing angles of the uneaten goldstino with the superpartner of the corresponding boson, defined in Eq. (6.21).

When the goldstino gets most of its mass from tree-level SUGRA effects so that $m_\zeta = 2m_{3/2}$, the goldstino equivalence theorem is no longer a good approximation and the spin-3/2 nature of the gravitino cannot be ignored. The part of the calculation that may be the least familiar to the reader is the sum over final-state gravitino spinors; for convenience, these are given for our two-component fermion notation in App. D.2. The resulting decay rates of an uneaten goldstino to a photon, Higgs, or Z are given by

$$\begin{aligned}
\Gamma_{\zeta\to\gamma\psi_\mu} &= \frac{189}{256}\frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\Theta_\gamma^2, \\
\Gamma_{\zeta\to h\psi_\mu} &= \frac{189}{256}\frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\frac{9}{14}f_h\left[\frac{4m_h^2}{m_\zeta^2}\right]\Theta_h^2 \qquad\qquad \boxed{m_\zeta = 2m_{3/2}} \\
\Gamma_{\zeta\to Z\psi_\mu} &= \frac{189}{256}\frac{m_\zeta^5}{16\pi F_{\text{eff}}^2}\left(f_{Z_T}\left[\frac{4M_Z^2}{m_\zeta^2}\right]\Theta_{Z_T}^2+\frac{1}{14}f_{Z_L}\left[\frac{4M_Z^2}{m_\zeta^2}\right]\Theta_{Z_L}^2\right. \\
&\qquad\left. -\frac{32}{63}\frac{M_Z^3}{m_\zeta^3}f_{Z_X}\left[\frac{4M_Z^2}{m_\zeta^2}\right]\Theta_{Z_T}\Theta_{Z_L}\right),
\end{aligned} \tag{6.31}$$

One could trade F_{eff} for M_{Pl} in the above by using the relation $F_{\text{eff}} = \sqrt{3}m_{3/2}M_{\text{Pl}}$. The kinematic functions f are given by

$$f_h[x] \equiv (1-x)^{3/2}(1-x/9)^{5/2} \tag{6.32}$$

$$f_{Z_T}[x] \equiv (1-x)^{1/2}(1-x/9)^{1/2}(1-7x/9+11x^2/63-x^3/63) \tag{6.33}$$

$$f_{Z_L}[x] \equiv (1-x)^{3/2}(1-x/9)^{1/2}(1+2x/9+x^2/9) \tag{6.34}$$

$$f_{Z_X}[x] \equiv (1-x)^{3/2}(1-x/9)^{1/2}. \tag{6.35}$$

Note that $f[1] = 0$ (the decays shut off when they are no longer kinematically allowed), and we have chosen $f[0] = 1$ (corresponding to the $m_\zeta \gg m_h$ limit). Note in particular that the longitudinal Z mode is a factor of 7 weaker compared to the transverse Z mode than it was in the goldstino equivalence regime.

6.3.4 Three-Body Decays of the Goldstino

All the two-body decays only occur in the presence of electroweak symmetry breaking, as otherwise the goldstinos do not mix with the neutralinos. Therefore, all the two-body decay rates feature at least one factor of v^2, and may become dominated by three-body modes, despite the latter's smaller phase space, in the limit $m_\zeta \gg v$.

The same operators of Eq. (6.25) mediate both the $\zeta \to h\psi\mu$ and $\zeta \to hh\psi_\mu$; the same is true for $\zeta \to Z_L\psi_\mu$ and $\zeta \to Z_L h\psi_\mu$. As a result, the two- and three-body decay widths are intimately connected—to leading order, they are parameterized by the same mixing angles Θ_h and Θ_{Z_L}, respectively:

$$\begin{aligned}\Gamma_{\zeta\to hh\psi_\mu} &= \frac{\Theta_h^2 m_\zeta^7}{5120\pi^3 v^2 F^2} F_{hh}\left[\frac{m_{3/2}}{m_\zeta}\right] \\ \Gamma_{\zeta\to hZ_L\psi_\mu} &= \frac{\Theta_{Z_L}^2 m_\zeta^7}{1920\pi^3 v^2 F^2} F_{hZ_L}\left[\frac{m_{3/2}}{m_\zeta}\right]\end{aligned}$$

where we are working in the limit $m_\zeta \gg m_h$ where these modes may be competitive. The kinematic functions F_{hh} and F_{hZ_L} are discussed in the appendices, but take on the values $F_{hh}(0) = F_{hZ_L}(0) = 1$, $F_{hh}(1/2) = 0.44$, $F_{hZ_L}(1/2) = 0.04$. The hZ_L mode experiences destructive interference at non-zero $m_{3/2}$ due to the pseudoscalar nature of Z_L, hence explaining the disparity between the two functions at $m_\zeta = 2m_{3/2}$.

Since it is the same mixing angles Θ_h, Θ_{Z_L} that appear in the two- and three-body decay modes to Higgs and Z_L, ratios of these decay rates are largely model-independent to leading order in m_ζ/m_{SUSY}:

$$\frac{\Gamma_{\zeta\to hh\psi_\mu}}{\Gamma_{\zeta\to h\psi_\mu}} = \frac{m_\zeta^2}{160\pi^2 v^2} F_{hh/h}\left[\frac{m_{3/2}}{m_\zeta}\right] \tag{6.36}$$

$$\frac{\Gamma_{\zeta\to hZ_L\psi_\mu}}{\Gamma_{\zeta\to Z_L\psi_\mu}} = \frac{m_\zeta^2}{60\pi^2 v^2} F_{hZ_L/Z_L}\left[\frac{m_{3/2}}{m_\zeta}\right] \tag{6.37}$$

where the kinematic functions $F_{hh/h}$ and F_{hZ_L/Z_L} are given in the appendix, but take on the values $F_{hh/h}(0) = F_{hZ_L/Z_L}(0) = 1$, $F_{hh/h}(1/2) \approx 0.46$, $F_{hZ_L/Z_L} \approx 0.39$. We can use this to easily determine when exactly the two Higgs mode becomes dominant compared to the single Higgs mode: $m_\zeta \approx 10$ TeV for $m_\zeta \gg m_{3/2}$, and $m_\zeta \approx 14$ TeV for $m_\zeta \approx 2m_{3/2}$. These are both outside the sensitivity of the HESS experiment (whose present results only reach up to 5 TeV gamma rays), so we will neglect this possibility when considering the limits on photon lines in Sec. 6.4. The situation is slightly less dire for the hZ_L mode, which takes over from the Z_L mode for $m_\zeta \approx 6$ TeV for $m_\zeta \gg m_{3/2}$ or 10 TeV for $m_\zeta \approx 2m_{3/2}$. Although this mode may have an impact at the upper range of the HESS data for $\Theta_Z \gtrsim \Theta_h$, we will find that the limits on single Higgs production tend to preclude the possibility of observing the gamma ray signal by themselves, as the gamma ray mode is quite suppressed compared to the single Higgs mode at such high energies.

All other three-body decay modes are mediated by operators of at least dimension 7 (such as those in Eq. (6.27)), and will be subdominant by a factor of at least $m_\zeta^2/m_{\text{soft}}^2$. This may be compensated for by the much sheer number of modes allowed by dimension 7 operators, especially the difermion channel, so we consider them briefly here.

The decay width to a single chiral fermion species is given by

$$\Gamma_{\zeta \to \tilde{G}_L f \bar{f}} = \frac{N_c m_\zeta^9}{15360\pi^3 F_{\text{eff}}^2 F_\perp^2} \frac{\widetilde{m}_{\tilde{f}}^4}{m_{\tilde{f}}^4} F_f\left(\frac{m_{3/2}}{m_\zeta}\right), \tag{6.38}$$

with $F_f(0) = 1$, $F_f(1/2) \approx 1/8$, and the full expression for F_f given in App. D.3.

In the low mass regime $m_\zeta - m_{3/2} < M_Z$, the difermion and di-gauge boson modes are once again important as the only possible competition to the single photon mode. However, they are suppressed both by phase space and by the smallness of m_ζ compared to the weak scale. Barring cancellations, the three-body modes very rarely amount to more than a handful of percent of goldstino decays even for $m_\zeta - m_{3/2} = M_Z$, and can often be even more subdominant, as in split SUSY scenarios (see Sec. 6.6) or for the $m_\zeta = 2m_{3/2}$ case (see App. D.3). In such a regime, the single photon mode would likely be the only possible signal for the foreseeable future.

6.4 Indirect Detection

If the uneaten goldstino comprises some or all of the dark matter in the universe, its decay products can be observed by experiments, allowing indirect detection of goldstino dark matter. The decay $\zeta \to \gamma\psi_\mu$ would be particularly striking, as galactic sources would constitute a monochromatic source of photons. Such a photon line could stand out clearly from the diffuse photon background, and would be difficult to fake from astrophysical sources. In fact, there has been some excitement recently from tentative signals in gamma rays at about 130 GeV in gamma rays from the galactic center [26, 153, 145], and at 3.55 keV in X-rays from surveys of galactic clusters [29, 24]. Before discussing goldstino interpretations of these anomalies we will first review current constraints on goldstino decays.

The largest source of gamma rays originating from goldstino decays would be the galactic center, which also has the largest backgrounds of astrophysical gamma rays. However, for decaying dark matter the galactic center is not as prominent a source as it would be in the case of dark matter self-annihilation, as the flux scales only linearly (not quadratically) with dark matter number density. As a result, the spatially uniform extragalactic sources, with more manageable backgrounds, can be of comparable importance.

Both the FERMI-LAT and HESS collaborations have performed searches for monochromatic sources of gamma rays [3, 1], allowing limits to be determined on the lifetime $\tau_{\gamma\nu}$ for the decay of fermionic dark matter to a $\gamma\nu$ final state [3, 91]. The identity of the produced fermion is unimportant, as long as it can be treated as massless. Therefore, they apply equally to goldstino decays which also produce monochromatic gamma lines and may be very easily adapted for non-negligible final-state fermion mass $m_{3/2}$.

Of course, if the two-body decay featuring a photon is not the dominant decay mode, other goldstino decay modes may produce a detectable signal first. As we saw in the previous sections, the main competing modes at lower energies are the single Higgs and, to a lesser extent (up to phase space), the single Z. The ultimate decay products of the Higgs or Z will include all stable Standard Model particles—electrons and positrons, neutrinos, (anti)protons, and a soft photon spectrum. The astrophysical backgrounds are modest and well-understood [91]. Unfortunately, due to the effects of galactic electromagnetic fields, experiments can extract no information about their original energy and source location. Furthermore, these same effects mean that the expected flux of antiprotons given a particular dark matter mass and lifetime is highly dependent on the particular model of antiproton propagation through the galaxy.

Antiproton fluxes still provide the best limits on the lifetime of fermionic dark matter in the channels $h\nu$ and $Z\nu$, as has been studied extensively in Refs. [52, 79] using data from PAMELA [5, 6]. As with the photon mode, these limits also apply exactly to goldstino dark matter in the $m_{3/2} \ll m_\zeta$ regime.[11]

In adapting any of these limits it should be remembered that the number density (and thus the number of decays) scales as m_ζ^{-1}. Accounting for this and the appropriate values of m_ζ and $m_{3/2}$, this allows us to map known limits from standard decaying dark matter searches $\chi \to B\nu$. If these limits are defined at a specific DM mass m_χ as a limit on the lifetime $\tau(m_\chi)$, for the final states $B \equiv \gamma, Z, h$, then these limits may be adapted to goldstino decays $\zeta \to B\psi_\mu$ with the following mapping

$$\tau(m_\zeta, m_{3/2}, m_B) = \tau\left(m_\chi(m_\zeta, m_{3/2}, m_B)\right) \frac{m_\chi(m_\zeta, m_{3/2}, m_B)}{m_\zeta} \times BR(\zeta \to B\psi_\mu) \times F_\zeta, \tag{6.39}$$

where

$$m_\chi(m_\zeta, m_{3/2}, m_B) \equiv \frac{m_\zeta^2 + m_B^2 - m_{3/2}^2 + \sqrt{(m_\zeta^2 - (m_B + m_{3/2})^2)(m_\zeta^2 - (m_B - m_{3/2})^2)}}{2m_\zeta}, \tag{6.40}$$

and $BR(\zeta \to B\psi_\mu)$ is the branching ratio to that final state and F_ζ is the fraction of DM made up by the goldstino $F_\zeta = \Omega_\zeta/\Omega_\chi$.

Combining the gamma ray line and antiproton limits, we can find the minimum branching fraction needed for the photon mode $\zeta \to \gamma\psi_\mu$ to be currently observable while the existing antiproton limits on $\zeta \to Z\psi_\mu$ and $\zeta \to h\psi_\mu$ do not to exclude it. These are shown for both $m_{3/2}$ regimes in Fig. 6-4.

[11] This is only strictly true at tree level, as electroweak radative corrections can have a substantial impact on the number of produced antiprotons at higher dark matter masses when the fermion produced in the decay is a Standard Model neutrino [52]. Ref. [52] does not take these corrections (which do not exist in our case) into account explicitly in their analysis, however.

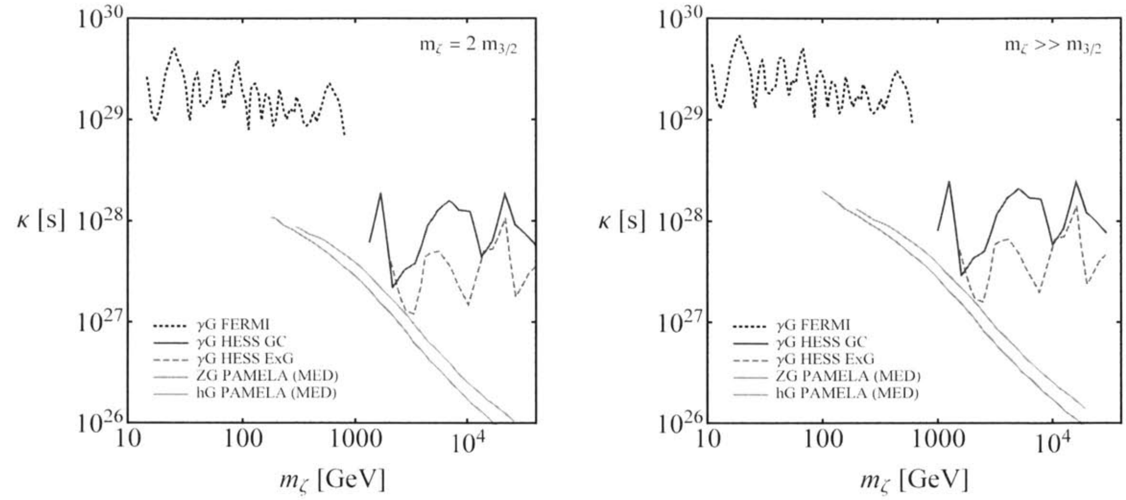

Figure 6-3: Constraints on the effective goldstino lifetime $\kappa = \tau/(BR(\zeta \to B\psi_\mu) \times F_\zeta)$ where $B = \gamma, Z, h$ calculated from Refs. [91, 79]. Constraints are shown for the MED propogation parameter choice in Ref. [79] for $B = Z, h$. Different propagation models lead to constraints that may beweaker or stronger by a factor $\mathcal{O}$(few). The left panel shows the scenario where the goldstino mass is dominated by tree-level mSUGRA effects, and the right panel the scenario where the goldstino picks up large one-loop corrections to the tree-level mass. $\zeta \to \gamma\psi_\mu$ lead to the strongest constraints however for $m_\zeta - m_{3/2} > m_Z, m_h$ the branching ratio to these final states may be larger.

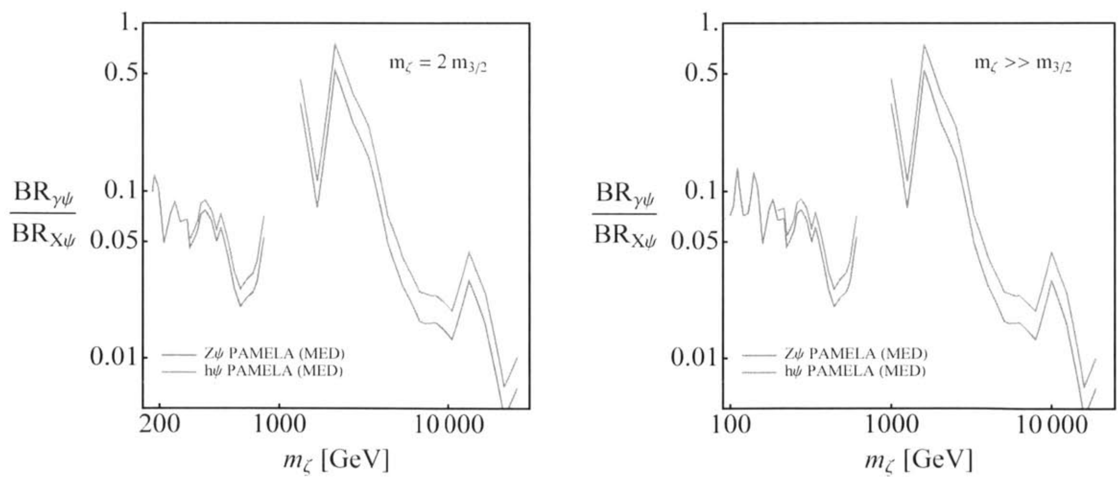

Figure 6-4: Minimum branching ratio required for a putative observed decay $\zeta \to \gamma\psi_\mu$ with effective lifetime κ just beyond current bounds to not be in conflict with constraints on $\zeta \to Z\psi_\mu$ and $\zeta \to h\psi_\mu$ from PAMELA. The plots begin at the lowest mass goldstino such that the decays are kinematically accessible. As in Fig. 6-3, different propagation models will result in minimum branching ratios that may be larger or smaller by a factor $\mathcal{O}$(few).

6.4.1 The 130 GeV Line

There has been some excitement in the last two years over the tentative observation of a gamma ray line at approximately 130 GeV originating very near the galactic center. Such a signal can arise from the decay of dark matter [26, 153, 145], although its spatial distribution is more suggestive of annihilation at present [28]. Rather than discussing the robustness of a DM interpretation of this anomaly we simply consider whether or not such a spectral feature could arise from goldstino decays in accordance with other limits and generic expectations in models of goldstino decays.

For such an interpretation an effective lifetime of $\kappa = \tau/(BR(\zeta \to B\psi_\mu)\times F_\zeta) \sim \mathcal{O}(10^{29}\text{s})$ is required. Furthermore, Fig. 6-4 demonstrates that the branching ratio to $\gamma\psi_\mu$ must be at least $\mathcal{O}(10\%)$ for the model to be consistent with antiproton bounds.

For the loop-dominated goldstino mass $m_\zeta \gg m_{3/2}$ a 130 GeV line would require $m_\zeta =$ 260 GeV. Using Eq. (6.30) we find that

$$\frac{BR_{\gamma\psi}}{BR_{Z\psi}} \approx \frac{1.7\Theta_\gamma^2}{\Theta_{Z_T}^2 + \frac{1}{2}\Theta_{Z_L}^2}, \tag{6.41}$$

thus for typical parameters if $\Theta_\gamma \sim \Theta_{Z_{T,L}}$ then the branching ratio constraints are satisfied. The more detailed analysis of Sec. 6.6 will show that $\Theta_\gamma/\Theta_{Z_L}$ scales roughly as M_Z/M_1, so the branching ratio constraints will be satisfied so long as the bino is not too far above the weak scale.

Obtaining the required lifetime is also quite feasible. Again using Eq. (6.30) we find

$$\tau_{\zeta\to\gamma\psi_\mu} \approx (3\times 10^{29}\text{ s})\left(\frac{\sqrt{F_{\text{eff}}}}{1\times 10^9\text{ GeV}}\right)^4 \left(\frac{\Theta_\gamma}{1\times 10^{-14}}\right)^{-2} . \tag{6.42}$$

For this value of F_{eff} we have $m_{3/2} \approx 24$ MeV, satisfying the criteria $m_\zeta \gg m_{3/2}$. Such a small value of Θ_γ is to be expected, since $\Theta_\gamma \propto (M_Z^2/F_2)(\widetilde{M}_{\text{SUSY}}/M_{\text{SUSY}})$ where M_{SUSY} is near the weak scale and $\widetilde{M}_{\text{SUSY}}$ is the typical soft-term mediated by sector 2 to the visible sector. For $\sqrt{F_{\text{eff}}} \approx 10^9$ GeV, one expects $\Theta_\gamma \sim 8\times 10^{-15}$ if the two sectors have a common messenger mass.

For the SUGRA-dominated goldstino mass $m_\zeta = 2m_{3/2}$ a 130 GeV line corresponds to a 346 GeV goldstino and 173 GeV gravitino. Phase space and kinematics further disfavor the Z mode here, so the branching ratio constraints are easier to satisfy:

$$\frac{BR_{\gamma\psi}}{BR_{Z\psi}} \approx \frac{1.5\Theta_\gamma^2}{\Theta_{Z_T}^2 + \frac{1}{14}\Theta_{Z_L}^2 - \frac{1}{120}\Theta_{Z_T}\Theta_{Z_L}}. \tag{6.43}$$

As the gravitino mass is fixed, we know $F_{\text{eff}} = \sqrt{3}m_{3/2}M_{\text{Pl}} = (2.70 \times 10^{10}\text{ GeV})^2$ and we have one fewer free parameter in the lifetime:

$$\tau_{\zeta\to\gamma\psi_\mu} \approx (1 \times 10^{29}\text{ s}) \left(\frac{\Theta_\gamma}{6 \times 10^{-12}}\right)^{-2} . \tag{6.44}$$

6.4.2 The 3.5 keV Line

There has also been excitement very recently regarding a 3.5 keV line in X-rays tentatively observed in various galactic clusters [29, 24]. The goldstini framework has many features that make it a plausible candidate for such observations. It can produce a monochromatic photon line without producing other features—the only other kinematically allowed modes are multiphotons and neutrinos, which are strongly phase-space suppressed at these energies (and the latter are effectively invisible). The morphology of the signal is also very suggestive of a dark matter decay, corresponding to a lifetime of $2 \times 10^{27} - 1 \times 10^{28}$ s.

However, it can be difficult to accommodate such a large X-ray signal in the goldstini framework. For the scenario $m_\zeta = 2m_{3/2}$ (and thus $\sqrt{F} \approx 4.5 \times 10^6$ GeV), it is almost impossible, as

$$\tau_{\zeta\to\gamma\psi_\mu} = \frac{6\pi M_{\text{Pl}}^2}{7E_\gamma^3\Theta_\gamma^2} \approx (7 \times 10^{27}\text{ s}) \left(\frac{\Theta_\gamma}{6}\right)^{-2} . \tag{6.45}$$

One cannot obtain a signal large enough to correspond to the observations of Refs. [29, 24] in the limit that goldstino-neutralino couplings are a small perturbation of the neutralino mass matrix. The same conclusion holds true to a somewhat greater degree for the scenario $m_{3/2} < m_\zeta < 2m_{3/2}$.

The prospects are less dire for $m_\zeta \gg m_{3/2}$ (as F, m_ζ, and M_{Pl} are no longer directly related), but it still requires a small SUSY-breaking scale and a relatively large Θ_γ:

$$\tau_{\zeta\to\gamma\psi_\mu} = \frac{16\pi F^2}{m_\zeta^5\Theta_\gamma^2} \approx (2 \times 10^{27}\text{ s}) \left(\frac{\sqrt{F}}{10\text{ TeV}}\right)^4 \left(\frac{\Theta_\gamma}{10^{-4}}\right)^{-2} \tag{6.46}$$

For this scenario to be feasible, the corrections to the goldstino mass need to be keV-scale. At tree level, these corrections arise at $\mathcal{O}(1/F^2)$, mainly from the direct couplings of the goldstino bilinears in the lowest components of $\boldsymbol{X}_i$[12]. These corrections scale roughly as

$$m_\zeta \sim \frac{\widetilde{B}_\mu M_Z^2 \mu}{F_\perp^2}, \frac{\widetilde{m}^2_{H_{u,d}} M_Z^2 \mu}{F_\perp^2}, \frac{\widetilde{M}_{1,2} M_Z^4}{F_\perp^2} \tag{6.47}$$

It can be quite difficult to make m_ζ be keV scale while simultaneously making Θ_γ large enough (and F small enough) for this decay to have the necessary lifetime; it is typically only feasible in minimal models for light binos ($M_1 < 20$ GeV).

Radiative corrections will also generally induce the operator

$$\frac{1}{(16\pi^2)^n} \int d^4\Theta\, \boldsymbol{E}\, \frac{1}{M} \boldsymbol{X}_1^\dagger \boldsymbol{X}_1 \boldsymbol{X}_2, \tag{6.48}$$

with M some effective messenger scale and n the number of loops required for cross-talk between the two sectors. This yields an uneaten goldstino mass of [42]

$$\delta m_\zeta \approx \frac{1}{(16\pi^2)^n} \frac{F_1}{F_2} \frac{F_1}{M}. \tag{6.49}$$

To make sure that the goldstino mass induced by such an operator is no larger than keV will typically require n to be relatively large or for there to be small couplings.

For the non-minimal models discussed in Sec. 6.7, the prospects can be much improved. For example, for the model of Sec. 6.7.1 dominated by kinetic mixing between the bino and uneaten goldstino has $\Theta_\gamma \approx \epsilon \cos\theta_W$, which can work for large enough ϵ, assuming that radiative corrections to the goldstino mass are well-controlled.[13]

Scenarios with three or more SUSY-breaking sectors can also more feasibly produce such an X-ray signal, arising from transitions between different uneaten goldstini. As there is no gravitino involved in such a decay, cancellations that occur for the true goldstino do not occur for the uneaten goldstino, and the decay rate is only proportional to the third (as opposed to the fifth) power of the 3.55 keV photon energy, allowing for a significant enhancement. Such decays are beyond the scope of this thesis, but are covered in more

[12]There are also contributions from the mass mixing with the neutralinos, but these are suppressed by powers of the neutralino masses assuming that soft SUSY-breaking arises mainly from sector 1.

[13]Note that the uneaten goldstino does *not* receive $\mathcal{O}(\epsilon^2)$ tree-level corrections to its mass in this scenario.

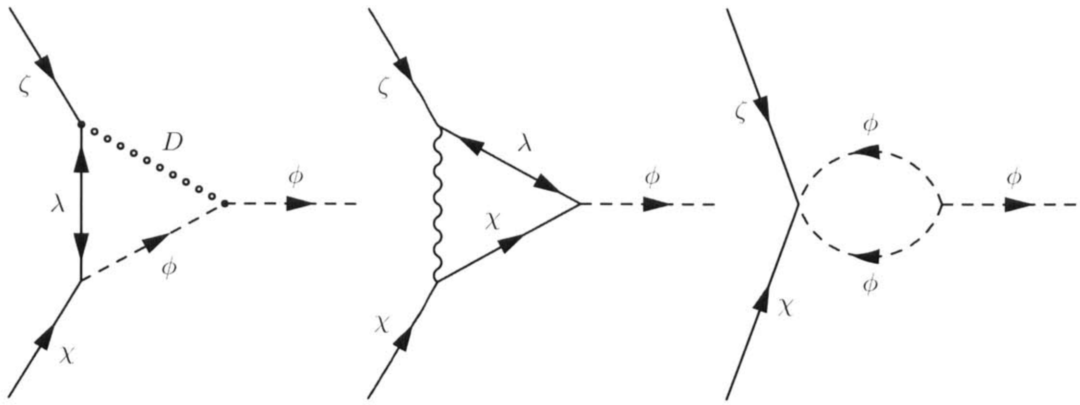

Figure 6-5: One-loop diagrams that serve to renormalize the uneaten goldstino mass-squared style couplings. Note that the resulting effects will be proportional to the gaugino-mass style coupling multiplied by the *physical* gaugino mass for the left and middle diagrams, and similarly for A-terms for the right diagram. If sector 1 provides the dominant contribution to soft terms, RG effects can greatly enhance the mass-squared style couplings of the uneaten goldstino.

detail in Ref. [126]. The same decay, with weak-scale goldstini, was considered on the vastly shorter timescales of the LHC in Ref. [69].

6.5 Renormalization Group Evolution of Goldstino Couplings

The couplings in Eq. (6.13) will help mediate goldstino decays, so in order to fully assess the relative importance of the possible decay modes, we will need to understand the possible relative strengths of these couplings. In general, we will have to take into account the RG flow of these couplings. One might not expect such effects to be extremely important, especially since we do not have definite expectations of how these couplings compare at the messenger scale. However, we will find that in scenarios in which the contributions of sector 2 to soft SUSY-breaking parameters are much smaller than those of sector 1, these RG effects can very quickly become the dominant source of mass-squared goldstino couplings.

The fundamental reason for this is that *physical* gaugino masses, A-terms, and fermion masses feed into the RG evolution of scalar mass-squared goldstino couplings, in addition to their corresponding goldstino couplings. This can be seen on the level of Feynman diagrams in Fig. 6-5. The one-loop RG equations for the mass-squared style couplings are (following the notation of Ref. [124])

$$\beta_{\widetilde{m}^2_{ij}} = \frac{1}{16\pi^2}\left[A^*_{ikl}\widetilde{A}_{jkl} - 8g_a^2 C_a(i) M_a^* \widetilde{M}_a \delta_{ij}\right] + \mathcal{O}(\widetilde{m}^2). \tag{6.50}$$

This looks extremely similar to the RG equations for the (untiled) soft scalar mass-squareds, and for good reason—removing the tildes from Eq. (6.50) yields the RG equations for the *eaten* goldstino couplings, which must be identical for those for the soft scalar mass-squareds by supercurrent conservation.

As a result, the RG equations for the other goldstino couplings are identical to those for their corresponding soft terms, after adding tildes to all soft parameters, as none of those RG equations contain terms proportional to products of soft terms. In particular, this implies the following RG equations for the B-terms:

$$\beta_{\widetilde{B}_{ij}} = \frac{1}{16\pi^2}\left[M^{ip}y^*_{pmn}\widetilde{A}^{mnj} + g_a^2 C_a(i) 4\widetilde{M}_a M^{ij}\right] + \mathcal{O}(\widetilde{B}_{ij}). \tag{6.51}$$

Note that these terms are unsurprisingly proportional to the *physical* SUSY-respecting fermion mass. Similar terms can arise at the messenger scale from Kähler potential terms like $\boldsymbol{X}_2\boldsymbol{H}^\dagger_{u/d}\boldsymbol{H}_{u/d}$, as well, of course.

This RG running behavior can have very important consequences for the uneaten goldstino decays. In the limit that the (messenger-scale) contributions to soft masses from sector 2 are smaller than weak scale, one might naively expect that one could neglect the dimension two couplings compared to the dimension 1 couplings. As discussed above, this can already be false at the messenger scale for B-terms. As one runs below the messenger scales, the dominant contributions to scalar mass-style couplings (and B-terms if they vanish at the messenger scale) will come from this RG-running, and they can no longer be neglected.

6.6 Benchmark Scenarios in the MSSM

In order to understand these decay rates, we need to understand the mixing angles Θ_i defined in Eq. (6.21), and more generally the uneaten goldstino direction $\widetilde{\theta}$ defined in Eq. (6.20), both of which are quite complicated in general. In this section, we will consider certain limits in which $\widetilde{\theta}$ simplifies, and some more general benchmark scenarios which can help to give a sense of the variety of branching ratios and decay rates of the uneaten goldstino.

6.6.1 The Ultra-Aligned Limit

The uneaten goldstino direction $\widetilde{\theta}$ defined in Eq. (6.20) will simplify dramatically it aligns precisely with the eaten goldstino direction, which we know to be exactly $\{F^i, D^a/\sqrt{2}\}$. This 'ultra-aligned' limit will occur when $m_\zeta = 2m_{3/2}$ and $\widetilde{\rho}$ is a multiple of ρ; that is, when

$$\widetilde{M}_{1,2} = \kappa M_{1,2}, \qquad \widetilde{m}^2_{H_{d,u}} = \kappa(m^2_{H_{d,u}} + 2m^2_{3/2}), \qquad \widetilde{B}_\mu = \kappa(B_\mu - m_{3/2}\mu), \tag{6.52}$$

for some constant κ. Leveraging the fact that we know the goldstino direction (or extensively using conditions arising from the minimization of the Higgs potential), we find

$$\widetilde{\theta} = \frac{v\kappa}{\sqrt{2}F_\perp}\begin{pmatrix} \frac{1}{2}M_Z \sin\theta_W \cos 2\beta \\ -\frac{1}{2}M_Z \cos\theta_W \cos 2\beta \\ \mu\sin\beta - m_{3/2}\cos\beta \\ \mu\cos\beta - m_{3/2}\sin\beta \end{pmatrix} = \frac{\kappa}{F_\perp}\begin{pmatrix} D_Y/\sqrt{2} \\ D_3/\sqrt{2} \\ F_{H_d} \\ F_{H_u} \end{pmatrix}. \tag{6.53}$$

Note that in this limit, Θ_γ is 0 and the goldstino does not decay to a photon and gravitino, as the effective D-term for electromagnetism vanishes in the absence of Fayet-Iliopoulos terms. This criterion provides an excellent check on all of our calculations, and is especially useful in the non-minimal scenarios discussed in Sec. 6.7. In this 'ultra-aligned' limit, the other mixing angles become:

$$\Theta_\gamma = 0 \qquad\qquad \Theta_h = \frac{\kappa v(\mu\cos(\alpha+\beta) + m_{3/2}\sin(\alpha-\beta))}{\sqrt{2}F_\perp} \tag{6.54}$$

$$\Theta_{Z_T} = -\frac{\kappa M_Z v\cos 2\beta}{2\sqrt{2}F_\perp} \qquad\qquad \Theta_{Z_L} = -\frac{\kappa m_{3/2} v\cos 2\beta}{\sqrt{2}F_\perp} \tag{6.55}$$

This 'ultra-aligned' limit is somewhat artificial, however, due to the presence of the $2m^2_{3/2}$ term in Eq. (6.52).[14] It can be readily seen from Eq. (6.50) that this limit is not RG-stable; even if it were imposed as a boundary condition at some scale, it would not hold true at any other scale.

[14]There is no similar issue with the $m_{3/2}\mu$ term; see Footnote 15.

6.6.2 The Aligned Limit

The more physically reasonable 'aligned' limit, in which $m^2_{i\bar{j},1}/m^2_{i\bar{j},2} = M_{A,1}/M_{A,2} = B_{ij,1}/B_{ij,2}$ for all soft terms, is RG-stable, and the $2m^2_{3/2}$ term in Eq. (6.52) would be omitted.[15] In turn, this implies the cancellation for Θ_γ would not be complete in the 'aligned' limit:

$$\Theta_\gamma = \frac{\sqrt{2}\kappa(M_1 - M_2)m^3_{3/2}M_Z v\cos 2\beta \sin 2\theta_W}{M_1 M_2 \mu^2 F_\perp} \qquad \text{('aligned' limit)}, \tag{6.56}$$

at lowest order in M_Z and $m_{3/2}$, while the other Θ_i would receive $\mathcal{O}(m^2_{3/2}/m^2_{\text{SUSY}})$ corrections. Although the photon mode is non-vanishing in this limit, it is generally quite suppressed as $2m_{3/2} < M_1, M_2, \mu$ in the regime of interest.[16] It is interesting to note that in this aligned limit, the tree level goldstino to photon decay channel is entirely due to the $2m^2_{3/2}$ coupling discussed in Ch. 5. Plots of branching ratios in this regime are given in Fig. 6-6 and Fig. 6-7, with the latter exploring some regions of parameter space in which the single photon mode cannot be neglected.

The cancellation also fails whenever m_ζ deviates from $2m_{3/2}$. To lowest order in $m_{3/2}$, m_ζ, and M_Z, the photino mixing angle becomes

$$\Theta_\gamma = \frac{(M_1 - M_2)M_Z(m_\zeta - 2m_{3/2})v\kappa\cos 2\beta \sin 2\theta_W}{4\sqrt{2}F_\perp M_1 M_2}, \tag{6.57}$$

while the other mixing angles receive subdominant corrections. This effect can be substantial for gauge-mediation inspired models in which $m_{3/2}$ is negligible and m_ζ arises mainly from loop effects, as can be seen in Fig. 6-8.

6.6.3 Split SUSY Models; Gauge Mediation

In split SUSY scenarios with $M_1, M_2 \ll \mu$ the effects of scalar soft masses can largely be neglected. RG running effects induce $\widetilde{m}^2_{H_{u/d}}$ proportional to $\widetilde{M}_2 M_2$, but to find the uneaten goldstino decay rates, one has to integrate out a Higgsino of mass μ. Similar considerations

[15]Note that the $m_{3/2}\mu$ term in Eq. (6.52) is not modified, as that contribution to B terms arises from neither hidden sector (and is in fact not SUSY-breaking, see Ch. 5). However, since B terms do not feed into the RG running of any other soft parameters or goldstino couplings (we neglect the subtle issue of threshold corrections here), one can replace the $m_{3/2}$ with any other fixed scale Λ without affecting the RG stability of the limit. Picking $\Lambda = 0$ means the cancellation in Θ_γ is even less complete, with Θ_γ non-vanishing already at $\mathcal{O}(m_{3/2})$. Picking $\Lambda \gg \mu$ is equivalent to the B-term dominance models we will discuss in Sec. 6.6.4.

[16]If the Z or Higgs modes should be kinematically inaccessible, the three-body decay to fermions would dominate instead.

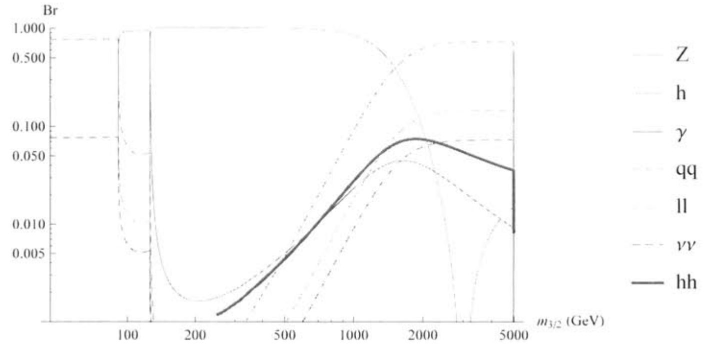

Figure 6-6: The branching ratios for goldstino decay in the aligned, $m_\zeta = 2m_{3/2}$ limit, as a function of $m_{3/2}$, for $M_1 = 10$ TeV, $M_2 = 20$ TeV, $\mu = 30$ TeV, $\tan\beta = 20$. Note that except for the photon mode and the upper limit on m_ζ, this plot is extremely insensitive to the values of M_1 and M_2. The vanishing of the single Higgs mode is due to a cancellation in Θ_h that occurs for for $m_{3/2} \approx \mu \sin 2\beta$ in the aligned limit. The same $\sin 2\beta$ suppression helps explain the relative subdominance of the two-Higgs mode at higher energies. The single photon mode is not visible on this plot, except for the case barely visible on the extreme right where m_ζ and M_1 are almost degenerate.

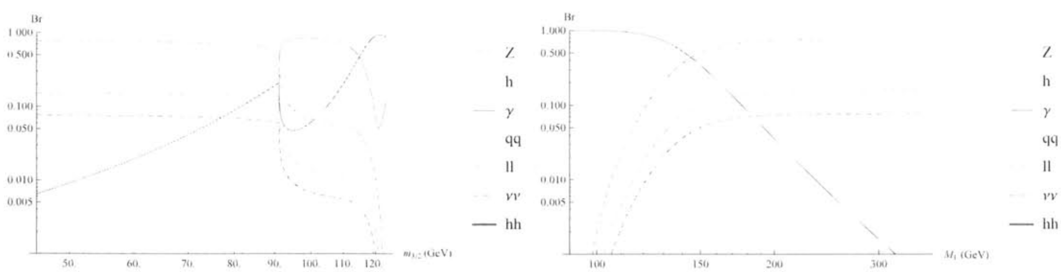

Figure 6-7: Left: the branching ratios for goldstino decay in the aligned, $m_\zeta = 2m_{3/2}$ limit, as a function of $m_{3/2}$, for $M_1 = 250$ GeV, $M_2 = 500$ GeV, $\mu = 750$ GeV, $\beta = \arctan 20$. Right: the branching ratios for goldstino decay in the same limit, as a function of M_1, for $m_{3/2} = 45$ GeV, $M_2 = 2M_1$, $\mu = 3M_1$. The photon mode can be non-negligible, even in this aligned limit, for modest neutralino masses.

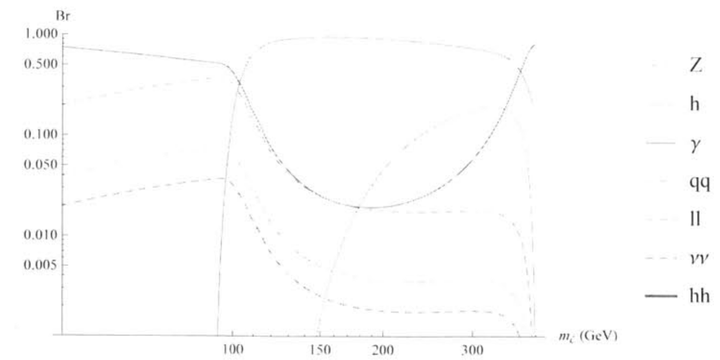

Figure 6-8: The branching ratios for goldstino decay in the aligned, $m_\zeta \gg 2m_{3/2}$ limit, as a function of m_ζ, for $M_1 = 400$ GeV, $M_2 = 800$ GeV, $\mu = 1200$ GeV, $\beta = \arctan 20$. Even when the couplings are aligned, the fact that $m_\zeta \neq 2m_{3/2}$ has a substantial effect, even for heavier neutralinos than in Fig. 6-7, with the photon mode remaining substantial over the whole range of m_ζ.

will suppress the difermion modes as well.[17] The same is not true of B-term style couplings, as the μ naturally occurring in $\widetilde{B}_\mu$ simply cancels the μ^{-1} from the Higgsino propagator; one cannot ignore such couplings even for ultra-heavy Higgsinos.

Working in this limit, with $\mu \gg M_2, M_1 \gg M_Z, m_\zeta$, and rewriting $\widetilde{B}_\mu = -\widetilde{b}\mu$,

$$\widetilde{\theta} = \frac{v}{\sqrt{2}F_\perp} \begin{pmatrix} \frac{1}{2}\frac{M_Z}{M_1} \sin\theta_W \cos 2\beta \left(\widetilde{M}_1 + 2\widetilde{b}\right) \\ -\frac{1}{2}\frac{M_Z}{M_2} \cos\theta_W \cos 2\beta \left(\widetilde{M}_2 + 2\widetilde{b}\right) \\ \widetilde{b}\cos\beta \\ \widetilde{b}\sin\beta \end{pmatrix}, \tag{6.58}$$

remembering that we are working in the $\{\widetilde{B}, \widetilde{W}, \widetilde{H}_d, \widetilde{H}_u\}$ basis. Note that if $\widetilde{M}_1/M_1 = \widetilde{M}_2/M_2$ (or even if they both vanish), the photon mode still occurs through the B-term coupling.

We see from Eq. (6.58) that if $\widetilde{b}$, $\widetilde{M}_1$ and $\widetilde{M}_2$ are comparable, the Higgs and longitudinal Z modes will tend to dominate in this regime. However, one can easily arrange for $\widetilde{b}$ to be smaller than $\widetilde{M}_1$ and $\widetilde{M}_2$—if it vanishes at a messenger scale, and there are few enough decades removed from m_ζ that Eq. (6.51) only has a modest effect. This in fact occurs if

[17]The difermion mode induced by scalar mass couplings in this regime will still dominate over those induced by A-terms, even for the $t\bar{t}$ mode.

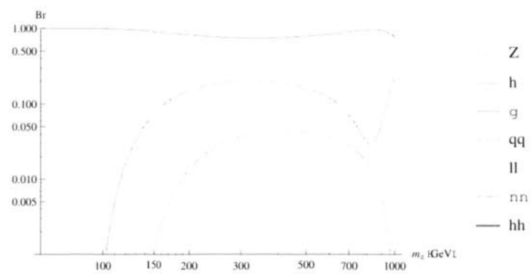

Figure 6-9: Branching ratios for goldstino decay as a function of m_ζ in a scenario inspired by gauge mediation, with $m_\zeta \gg m_{3/2}$, $M_1 = 1$ TeV, $M_2 = 2$ TeV, $\mu = 3$ TeV, $\tan\beta = 5$, $\widetilde{M}_1 = 1$ GeV, $\widetilde{M}_2 = -2$ GeV, $\widetilde{b} = 27$ MeV, $\widetilde{m}^2_{H_u}/\mu = 18$ MeV, and $\widetilde{m}^2_{H_d}/\mu = 10$ MeV.

sector 2 transmits SUSY-breaking to the SSM via minimal gauge mediation.

Fig. 6-9 shows plots of representative branching ratios for a scenario inspired by gauge mediation. Note that we have not picked a split SUSY benchmark here; all that is required is for RG running effects to be modest, B-terms from sector 2 to be suppressed at the messenger scale, and $\widetilde{M}_{\text{SUSY}} \ll M_{\text{SUSY}}$.

6.6.4 B-term Dominance

In a different extreme, one can have B-terms being the only non-vanishing contribution to soft masses from sector 2, as B-terms do not feed into the running of any other soft terms. In this limit (and only keeping terms to the lowest order in M_Z and m_ζ), the mixing angles become

$$\Theta_\gamma = \frac{(M_2 - M_1) M_Z v \widetilde{b} \cos 2\beta \sin 2\theta_W}{2\sqrt{2} F_\perp M_1 M_2} \qquad \Theta_h = \frac{v \widetilde{b} \sin(\beta - \alpha)}{\sqrt{2} F_\perp}$$
$$\Theta_{Z_T} = -\frac{M_Z v \widetilde{b} \cos 2\beta (M_1 \cos^2\theta_W + M_2 \sin^2\theta_W)}{\sqrt{2} F_\perp M_1 M_2} \qquad \Theta_{Z_L} = \frac{v \widetilde{b} \cos 2\beta}{\sqrt{2} F_\perp}. \tag{6.59}$$

Recall that in the Higgs decoupling limit, $\sin(\beta - \alpha) \approx 1$. Fig. 6-10 shows some characteristic branching ratios. As expected, once the higgs and Z modes are kinematically accessible, they become the dominant decay modes.

6.7 Non-Minimal Models

All two-body decay modes of the uneaten goldstino to a gravitino necessarily arise from a mass mixing of the uneaten goldstino with neutral visible-sector fermions. So far, we

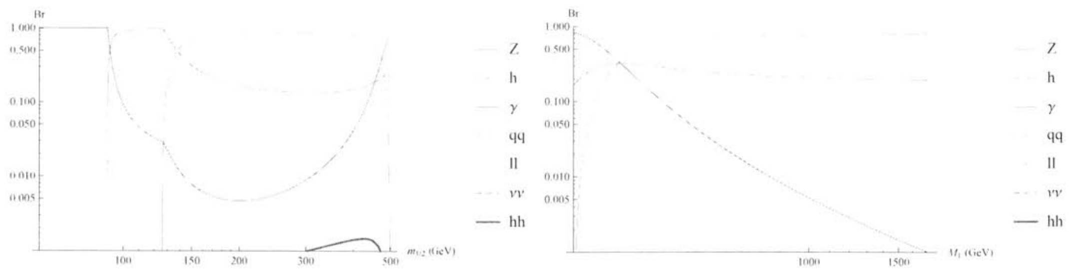

Figure 6-10: Branching ratios for goldstino decay in the limit of B-term dominance ($\widetilde{B}_\mu/\mu \gg \widetilde{M}_1, \widetilde{M}_2, \widetilde{m}^2_{H_{u/d}}/\mu$), with $m_\zeta = 2m_{3/2}$. Left: as a function of $m_{3/2}$ for $M_1 = 1$ TeV, $M_2 = 2$ TeV, $\mu = 3$ TeV, $\tan\beta = 5$. Right: as a function of M_1, for $m_{3/2} = 173$ GeV, $M_2 = 2M_1$, $\mu = 3M_1$, $\tan\beta = 5$.

have only considered mixings arising from electroweak symmetry breaking; as a result, the corresponding decay widths scale as at least v^2 (v^4 for the photon mode), and are thus subdominant for $m_\zeta \gtrsim 1$ TeV.

There can be mass mixing that does not depend on electroweak symmetry breaking, but only for the bino—the only gauge singlet fermion in the MSSM. In this chapter, we will discuss one way in which the bino can mix with the uneaten goldstino—kinetic mixing with the uneaten goldstino.[18] This mediates a decay of the uneaten goldstino to the gravitino and a B boson, the only accessible gauge singlet boson—in other words, a photon or a Z, with the latter mode suppressed by $\tan^2\theta_W$ (and phase space). We will find this can easily dominate over other contributions that depend on electroweak symmetry breaking, and that they can remain dominant over three-body modes even for $m_\zeta > 1$ TeV.

6.7.1 Bino-Uneaten Goldstino Kinetic Mixing

The uneaten goldstino can experience kinetic mixing with the bino via the operator:

$$\mathcal{L} \supset \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \frac{\epsilon}{2} \boldsymbol{W}'^{\alpha} \boldsymbol{W}^{Y}_{\alpha}, \tag{6.60}$$

with $\boldsymbol{W}'^{\alpha}$ defined as[19]

$$\boldsymbol{W}'^{\alpha} \equiv -\frac{1}{4F_2}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\mathcal{D}^{\alpha}[\boldsymbol{X}_2^{\dagger}\boldsymbol{X}_2], \tag{6.61}$$

[18] Other possibilities are discussed further in Ref. [126].

[19] Similar effects would arise from a mixing with sector 1, but they are comparatively suppressed assuming soft mass contributions come primarily from the first sector.

with ϵ a small parameter (i.e. we only work to $\mathcal{O}(\epsilon)$ throughout). Such an effect can be induced at loop level below the messenger scale if $\mathrm{Tr}\left[Y\widetilde{m}_{\phi}^2\right] \neq 0$, but can also arise more directly.

We can diagonalize the kinetic terms by the transformation

$$\boldsymbol{V} \rightarrow \boldsymbol{V} - \frac{\epsilon}{F_2}\boldsymbol{X}_2^\dagger \boldsymbol{X}_2 \tag{6.62}$$

on the hypercharge vector superfield. This induces new couplings of the visible and hidden sectors from the hypercharge gaugino mass terms and from the hypercharge gauge couplings to matter superfields. It should be stressed that it does *not* produce new couplings from the standard gauge kinetic terms themselves—or rather, any new couplings induced cancel completely against those in Eq. (6.60) by construction. In particular, this includes the gravitino coupling to the gauge kinetic part of the supercurrent, so the transformation of Eq. (6.62) does *not* induce a direct coupling of the gravitino to a photon and a goldstino.

The transformation of Eq. (6.62) acting on the matter-gauge coupling terms does produce an effective Fayet-Iliopoulos term for hypercharge (though note that D_Y itself does not obtain a vev):

$$\mathcal{L}_{FI} = -\frac{1}{2}g'\epsilon F_2 \sum_i Y_i \phi_i^* \phi^i. \tag{6.63}$$

If we do not want to break electromagnetism or color as a result, this gives a rough upper bound on ϵ of

$$\epsilon \lesssim \frac{m_{\mathrm{SUSY}}^2}{g' F_\perp}, \tag{6.64}$$

for m_{SUSY}^2 the scale of soft SUSY-breaking scalar mass-squareds.

When acting on the gaugino mass terms, the transformation of Eq. (6.62) also produces a mass mixing between the bino and the dark gaugino

$$\mathcal{L} \supset \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\, \frac{\epsilon M_{1,1}}{F_1} \boldsymbol{X}_1 \boldsymbol{W}'^{\alpha} \boldsymbol{W}_{\alpha}^{Y}, \tag{6.65}$$

where we have assumed for simplicity that no similar term appeared in the Lagrangian before the transformation of Eq. (6.62). Note that the equivalent term with $\boldsymbol{X}_2$ vanishes, as $\boldsymbol{X}_2\boldsymbol{W}'^{\alpha} = 0$ due to $\boldsymbol{X}_2^2 = 0$—this is an effect that only exists with multiple hidden sectors. This yields an uneaten goldstino-bino mass mixing:

$$\mathcal{L} \supset \epsilon M_{1,1}\frac{F_{\text{eff}}}{F_1}\zeta\lambda \approx \epsilon M_1\zeta\lambda, \tag{6.66}$$

where the approximation assumes that $F_1 \gg F_2$ and $M_{1,1} \gg M_{1,2}$. A careful examination of Eq. (6.65) reveals that there is no equivalent mixing for the *eaten* goldstino $\widetilde{G}_L$, as must be the case as this effect does not depend on any visible sector D obtaining a vev.

In the limit that this effect is the leading contribution to the uneaten goldstino decay and that $M_1 \gg m_\zeta$, the mixing angles take on the incredibly simple form

$$\widetilde{\theta} \approx (\epsilon, 0, 0, 0), \qquad \Theta_\gamma \approx \epsilon\cos\theta_W, \qquad \Theta_{Z_T} \approx -\epsilon\sin\theta_W. \tag{6.67}$$

For the case $m_\zeta \approx 2m_{3/2}$, this is an incredibly predictive framework, with only two free parameters:

$$\tau_{\zeta\to\gamma\psi_\mu} \approx (3\times 10^{29}\ \text{s})\left(\frac{100\ \text{GeV}}{m_{3/2}}\right)^3\left(\frac{10^{-11}}{\epsilon}\right)^2 \tag{6.68}$$

Naively, it seems like this reintroduces the problem we first discussed in the introduction—that a decay of this sort requires tuning a parameter to be incredibly small. However, in this case, the smallness of the parameter is required by the stability of the electromagnetic and color neutrality of the vacuum:

$$\epsilon \lesssim 10^{-10}\left(\frac{100\ \text{GeV}}{m_{3/2}}\right)\left(\frac{10^{-2}}{F_2/F_1}\right). \tag{6.69}$$

Furthermore, the parameter ϵ is generally expected to be the quite small

$$\epsilon \approx \frac{1}{(16\pi^2)^n}\frac{F_2}{M_{mess,2}^2}, \tag{6.70}$$

with $M_{mess,2}$ the effective messenger scale for sector 2 and n the number of loops required to induce such a coupling. This does not generically saturate the bound of Eq. (6.64).

6.8 Conclusions

In this chapter, we have argued that the goldstini framework provides a well-justified scenario which can feature a gamma ray line visible at indirect detection experiments, at energies ranging from a keV to a TeV. Such a line occurs from the decay of uneaten goldstino dark matter to a single photon and a gravitino, a process mediated by the small mass mixing between goldstini and electroweak gauginos after electroweak symmetry breaking. Since such a process proceeds through the suppressed couplings of both hidden sectors to the SSM, the goldstino lifetime can naturally be of cosmological timescales.

If such a line were to be observed, however, it would be difficult to determine that the decaying dark matter was truly comprised of goldstini. One striking piece of evidence would be the determination of the goldstino mass, which would take on the value $8/3E_\gamma$ in the minimal goldstino model with $m_\zeta = 2m_{3/2}$. While performing additional accurate spectroscopic measurements solely with indirect detection experiments would be all but impossible, a collider measurement could be quite promising. For example, a bino LOSP would have a lifetime of approximately:

$$c\tau_{\tilde{B}} \approx (1 \text{ m}) \left(\frac{\tau_{\zeta\to\gamma\psi_\mu}}{10^{30}\text{ s}}\right) \left(\frac{1 \text{ TeV}}{M_1}\right)^5 \left(\frac{m_{3/2}}{100 \text{ GeV}}\right)^3 \tag{6.71}$$

If binos were produced in cascade decays at the LHC, they could then decay, yielding a (very) displaced photon—a striking, if challenging signal at the LHC, yet excellent evidence that SUSY is broken in multiple hidden sectors.

Chapter 7

Conclusions

Supersymmetry is a well-motivated and extensively studied theory for physics beyond the Standard Model. In the coming years, as the LHC pushes to higher energies and dark matter experiments increase their reach, we are well-poised to discover SUSY particles at the TeV scale—or to rule out some of the more constrained supersymmetric models. As a result, it is more important than ever to consider non-minimal models of SUSY. In this thesis, I have discussed a number of results that explore some of the resulting unconventional and counterintuitive aspects of supersymmetric theory and phenomenology.

In Ch. 3, we found that if SUSY is broken in multiple hidden sectors, collider phenomenology can be drastically altered. Most notably, if one of those hidden sectors preserves an R-symmetry, a predominantly bino lightest observable-sector particle (LOSP) can decay predominantly to a Higgs and that hidden sector's goldstino, even in (in fact, especially in) the limit that the LOSP has vanishing Higgsino fraction. This would result in copious production of boosted Higgses in SUSY events at the LHC, providing a unique window into both the structure of the Higgs and SUSY-breaking sectors.

In Ch. 4 and Ch. 5, we stressed that the structure of unbroken supergravity (SUGRA) is SUSY in anti-de Sitter (AdS) space, which has a fundamentally different algebra than flat space SUSY, as we explored in Sec. 2.7. In particular, bosons and fermions in the same multiplet do not have the same mass for unbroken SUGRA in AdS. This underlying structure can be probed in the flat space in which we reside by considering goldstino couplings; there can be mass differences without associated goldstino couplings, and vice versa. This can arise already at tree level in the case of scalar masses and B-terms. At loop level, this manifests itself as (minimal) anomaly mediation, which we showed was not a SUSY-breaking effect, as it exists for unbroken SUSY in the bulk AdS. This work has provided a clearer perspective on and more rigorous grounding of anomaly mediation, which had been the object of some confusion in the literature.

In Ch. 6, we found that multiple SUSY breaking can also have profound implications for our understanding and observation of dark matter. A goldstino from one of the hidden sectors can decay to a photon and the gravitino (which mostly resides in another hidden sector), through each sector's communication of SUSY breaking to the SUSY Standard Model (SSM). The lifetime of such a decay can easily be on cosmological timescales, so if the goldstino comprises most of the dark matter of the universe, the resulting photon line could be a striking indirect detection signal of decaying goldstino dark matter at telescopes such as FERMI. The energy of such a line can quite plausibly be anywhere from a keV to hundreds of GeV (or even higher in non-minimal models), providing a well-motivated scenario for such a photon line without recourse to artificially small parameters. Furthermore, in the limit in which two SUSY-breaking sectors communicate SUSY breaking to the SSM in the same manner, this decay occurs via the (absence of) the goldstino couplings arising from AdS discussed in Ch. 5, providing a possible observational probe of the underlying AdS SUSY symmetry structure of the universe.

Appendix A

Goldstini Give the Higgs a Boost: Appendices

A.1 Tree-Level Higgs Potential

The MSSM tree-level Higgs potential for the neutral Higgs sector arises from a combination of F-terms, D-terms, and three soft SUSY-breaking terms:

$$\begin{aligned} V(H_u^0, H_d^0) &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 + B_\mu(H_u^0 H_d^0 + H_u^{0*} H_d^{0*}) \\ &+ \frac{g^2 + g'^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned} \tag{A.1}$$

Once we recall that

$$M_Z^2 = \frac{1}{2}(g^2 + g'^2)\left(\langle H_u^0\rangle^2 + \langle H_d^0\rangle^2\right), \tag{A.2}$$

$$\tan\beta \equiv \langle H_u^0\rangle / \langle H_d^0\rangle, \tag{A.3}$$

we can use the fact that the vacuum must minimize the Higgs potential to find relations among these parameters.

$$0 = m_{H_u}^2 + |\mu|^2 + B_\mu \cot\beta - M_Z^2 \frac{\cos 2\beta}{2}, \tag{A.4}$$

$$0 = m_{H_d}^2 + |\mu|^2 + B_\mu \tan\beta + M_Z^2 \frac{\cos 2\beta}{2}. \tag{A.5}$$

It is convenient to take linear combinations of these relations, one without $|\mu|^2$ and one without B_μ:

$$
\begin{aligned}
0 &= (m^2_{H_u} - m^2_{H_d})\sin 2\beta + 2B_\mu \cos 2\beta - M_Z^2 \frac{\sin 4\beta}{2}, &\text{(A.6)}\\
0 &= m^2_{H_u}\sin^2\beta - m^2_{H_d}\cos^2\beta - |\mu|^2\cos 2\beta - M_Z^2\frac{\cos 2\beta}{2}. &\text{(A.7)}
\end{aligned}
$$

In the Higgsino decoupling limit ($|\mu|^2, m^2_{A^0} \gg M_Z^2$), we may neglect the terms proportional to M_Z^2. Also in the same limit, the tree-level relation for the physical Higgs mixing angle α simplifies considerably:

$$
\tan 2\alpha = \tan 2\beta \, \frac{m^2_{A^0} + M_Z^2}{m^2_{A^0} - M_Z^2} \quad \Rightarrow \quad \alpha = \beta - \pi/2 + \mathcal{O}\left(\frac{M_Z^2}{m^2_{A^0}}\right). \tag{A.8}
$$

Once one applies Eq. (A.8), the relations Eqs. (A.6) and (A.7) are precisely those which cause the cancellation of the $\lambda \to h^0 + \widetilde{G}_L$ amplitude at the first two orders in μ/M_1 in Eqs. (3.54) and (3.55). Another linear combination of Eqs. (A.4) and (A.5) gives a (non-independent) relationship that can be useful for simplifying $C^6_{\text{net},Z}$,

$$
0 = |\mu|^2 + m^2_{H_u}\sin^2\beta + m^2_{H_d}\cos^2\beta + B_\mu \sin 2\beta + M_Z^2\frac{\cos^2 2\beta}{2}. \tag{A.9}
$$

A third relation, involving the pseudoscalar mass $m^2_{A^0}$, allows us to solve for all three soft mass parameters if desired:

$$
\begin{aligned}
B_\mu &= -\frac{1}{2} m^2_{A^0}\sin 2\beta, &\text{(A.10)}\\
m^2_{H_u} &= -|\mu|^2 + m^2_{A^0}\cos^2\beta + M_Z^2\frac{\cos 2\beta}{2}, &\text{(A.11)}\\
m^2_{H_d} &= -|\mu|^2 + m^2_{A^0}\sin^2\beta - M_Z^2\frac{\cos 2\beta}{2}. &\text{(A.12)}
\end{aligned}
$$

Of course, all of the above relations are valid only at tree-level, and one does expect corrections to these relations from the same loop effects needed to raise the physical Higgs mass to 126 GeV.

A.2 R-Symmetry Violating Decays

In the body of Ch. 3, we focused on the setup in Fig. 3-2 where sector 2 preserves an R-symmetry. If the sector 2 does not preserve an R-symmetry, then there are many more allowed operators that can mediate the decay of a bino LOSP to the uneaten goldstino. They are exactly those previously given for the gravitino in Sec. 3.5.1, except with the replacement of $\widetilde{G}_L$ with ζ and with all soft masses tilded.

A decay to photon at tree-level is now allowed through the usual operator

$$\mathcal{O}^5_{\not{R},B} = \frac{i\widetilde{M}_1}{\sqrt{2}F}\lambda\sigma^{\mu\nu}\zeta F_{\mu\nu}, \tag{A.13}$$

with resultant decay rate

$$\Gamma_\gamma = \frac{\widetilde{M}_1^2 m_\lambda^3 \cos^2\theta_W}{16\pi F^2}. \tag{A.14}$$

The couplings of the bino LOSP to the physical Higgs h^0 and any further couplings to the Z not already found in $\mathcal{O}^5_{\not{R},B}$ may be parametrized at the first two orders in m_λ/μ as

$$\mathcal{L} = -\frac{M_Z\mu\sin\theta_W}{\sqrt{2}F}\left[\left(C^5_{\text{net}} + \frac{m_\lambda}{\mu}C^6_{\text{net}}\right)\lambda\zeta h^0 - \frac{M_Z}{\mu}C^6_{\text{net},Z}\zeta^\dagger\bar{\sigma}^\mu\lambda Z_\mu\right], \tag{A.15}$$

with C_{net} representing the following linear combinations of Wilson coefficients:

$$\begin{aligned}
\frac{g'}{\sqrt{2}}C^5_{\text{net}} &= \left(C^5_R + C^5_{\not{R},H_u\cdot H_d}\right)\cos(\alpha+\beta) \\
&\quad - 2C^5_{\not{R},H_u}\sin\beta\cos\alpha + 2C^5_{\not{R},H_d}\cos\beta\sin\alpha, && \text{(A.16)}\\
-\frac{g'}{\sqrt{2}}C^6_{\text{net}} &= \left(C^6_{H_u,1} + C^6_{H_u,2}\right)\sin\beta\cos\alpha - \left(C^6_{H_d,1} + C^6_{H_d,2}\right)\cos\beta\sin\alpha \\
&\quad + \left(C^6_{\not{R},1} + C^6_{\not{R},3}\right)\sin\beta\sin\alpha - \left(C^6_{\not{R},2} + C^6_{\not{R},4}\right)\cos\beta\cos\alpha, && \text{(A.17)}\\
-\frac{g'}{\sqrt{2}}C^6_{\text{net},Z} &= -\left(C^6_{H_u,1} - C^6_{H_u,2}\right)\sin^2\beta + \left(C^6_{H_d,1} - C^6_{H_d,2}\right)\cos^2\beta \\
&\quad + \frac{1}{2}\left(C^6_{\not{R},1} - C^6_{\not{R},2} - C^6_{\not{R},3} + C^6_{\not{R},4}\right)\sin 2\beta. && \text{(A.18)}
\end{aligned}$$

Here, the factors of $g'/\sqrt{2}$ are inserted purely for convenience. In the R-symmetric limit, all the $C_{\not{R}}$ are of course zero.

For the decay to Higgs, the formula Eq. (3.38) for the decay rate still holds, but C^5_{net}

now has contributions proportional to $\widetilde{B}_\mu$ and $\widetilde{M}_1$, as it did in Eq. (3.54):[1]

$$C^5_{\text{net}} = \frac{(\widetilde{m}^2_{H_u} - \widetilde{m}^2_{H_d})\sin 2\beta + 2\widetilde{B}_\mu \cos 2\beta}{\mu^2} - \frac{\widetilde{M}_1}{\mu}\cos 2\beta. \tag{A.19}$$

For the decay to Z, $C^6_{\text{net},Z}$ obtains a term proportional to $\widetilde{B}_\mu$

$$C^6_{\text{net},Z} = -\frac{\widetilde{m}^2_{H_u}\sin^2\beta + \widetilde{m}^2_{H_d}\cos^2\beta + \widetilde{B}_\mu \sin 2\beta}{\mu^2}, \tag{A.20}$$

and we must also include the effects of $\widetilde{M}_1$ from $\mathcal{O}^5_{\not{\mu},B}$ to find the full decay rate:

$$\begin{aligned}\Gamma_Z &= \frac{m^3_\lambda \widetilde{M}^2_1 \sin^2\theta_W}{16\pi F^2}\left(1 - \frac{M^2_Z}{m^2_\lambda}\right)^2 \\ &\quad \times \left(1 + \frac{1}{2}\frac{M^2_Z}{m^2_\lambda} - \frac{3M^2_Z C^6_{\text{net},Z}}{\widetilde{M}_1 m_\lambda} + \left(\frac{M_Z C^6_{\text{net},Z}}{\sqrt{2}\widetilde{M}_1}\right)^2\left(1 + 2\frac{M^2_Z}{m^2_\lambda}\right)\right).\end{aligned} \tag{A.21}$$

For the gravitino, $C^6_{\text{net},Z}$ simplifies to unity at this order due to the tree-order relation Eq. (A.9), and the complicated expression in Eq. (A.21) simplifies to the same result we obtained from the supercurrent in Eq. (3.4), as it must. We demonstrated in Sec. 3.5 that the decay rate to Higgs bosons simplifies similarly and in fact completely cancels at this order. For an uneaten goldstino, however, such cancellations do not generically occur, unless the ratio $\tau_i \equiv \widetilde{M}_i/M_i$ is equal for all soft SUSY-breaking mass(-squared) terms M_i. It is precisely when all the τ_i are equal that one can make the field redefinition Eq. (3.57) to make the goldstino couple only derivatively to visible-sector fields. In this limit, it would couple in exactly the same way as the longitudinal gravitino, except with an enhancement factor of $\tau^2 \sim \cot^2\theta$. Of course, we should not expect such alignment to occur in general (if only due to loop corrections), so a generic uneaten goldstino will have branching ratios to photons, Zs, and Higgses of roughly the same order of magnitude, as suggested by Fig. 3-12.

A.3 All-Orders Tree-Level Calculation

The Higgsino decoupling limit studied in Sec. 3.4 is convenient for understanding the physical origin of the counterintuitive LOSP decays, but it is tedious in practice for moderate

[1] Again, we use the approximation $\alpha \approx \beta - \pi/2$ from Eq. (A.8), which is appropriate at this order in m_λ/μ. This eliminates a term proportional to $\widetilde{B}_\mu \cos(\beta - \alpha)$ in C^6_{net}.

values of μ. Instead of integrating out the Higgsinos and finding an arbitrarily long series of operators and associated Wilson coefficients, we may conduct the calculation with the original Lagrangian in the mass eigenstate basis. As long as one can explicitly diagonalize the 4×4 neutralino mass matrix (analytically or numerically), one can perform the full tree-level calculation to all orders in μ.

To do so, we parametrize the relevant interactions from Eq. (3.23) as follows:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}M_{ij}\chi_i\chi_j + \rho_i\zeta\chi_i - \frac{1}{2}Y_{ij}\chi_i\chi_j h^0 + y_i\zeta\chi_i h^0 \\ &\quad + G_{ij}\chi_i^\dagger\bar{\sigma}^\mu\chi_j Z_\mu - L_i\, i\zeta\sigma^{\mu\nu}\chi_i\partial_\mu Z_\nu.\end{aligned} \tag{A.22}$$

In the $\{\lambda_B,\lambda_3,\widetilde{H}_d^0,\widetilde{H}_u^0\}$ basis, the neutralino mass matrix is [124]

$$M = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \tag{A.23}$$

the linear mixing with the uneaten goldstino is

$$\rho = \frac{v}{\sqrt{2}F}\begin{pmatrix} \frac{1}{4}g' v\widetilde{M}_1\cos 2\beta \\ -\frac{1}{4}gv\widetilde{M}_2\cos 2\beta \\ \widetilde{m}_{H_d}^2 c_\beta + \widetilde{B}_\mu s_\beta \\ \widetilde{m}_{H_u}^2 s_\beta + \widetilde{B}_\mu c_\beta \end{pmatrix}, \tag{A.24}$$

the couplings to the physical Higgs boson are

$$Y = \frac{1}{2}\begin{pmatrix} 0 & 0 & g's_\alpha & g'c_\alpha \\ 0 & 0 & -gs_\alpha & -gc_\alpha \\ g's_\alpha & -gs_\alpha & 0 & 0 \\ g'c_\alpha & -gc_\alpha & 0 & 0 \end{pmatrix}, \qquad y = \frac{1}{\sqrt{2}F}\begin{pmatrix} -\widetilde{M}_1 M_Z s_W \sin(\alpha+\beta) \\ \widetilde{M}_2 M_Z c_W \sin(\alpha+\beta) \\ \widetilde{B}_\mu c_\alpha - \widetilde{m}^2_{H_d} s_\alpha \\ \widetilde{m}^2_{H_u} c_\alpha - \widetilde{B}_\mu s_\alpha \end{pmatrix}, \tag{A.25}$$

and the couplings to the Z boson are

$$G = \frac{g}{2c_W}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad L = \frac{\sqrt{2}}{F}\begin{pmatrix} \widetilde{M}_1 s_W \\ -\widetilde{M}_2 c_W \\ 0 \\ 0 \end{pmatrix}. \tag{A.26}$$

In the above matrixes, we have used the notation $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$, with W standing for the weak mixing angle θ_W.

To calculate the decays of the lightest neutralino, we go to the mass eigenstate basis:

$$M \to M' = P^T M P, \tag{A.27}$$

with P chosen to make M' diagonal. Note that we treat the linear mixing with the uneaten goldstino as an insertion, which is valid to leading order in $1/F$. The other matrices and vectors rotate as

$$\rho \to \rho' = P^T \rho, \qquad Y \to Y' = P^T Y P, \tag{A.28}$$

and so forth. The full tree-level amplitude for the decay of the lightest neutralino to a

Higgs/Z and a goldstino is thus:

$$\Gamma_{h^0} = \frac{m_\lambda}{16\pi}\left(y_1' - \sum_i \frac{Y_{1i}'\rho_i'}{m_{\chi_i^0}}\right)^2\left(1-\frac{m_{h^0}^2}{m_\lambda^2}\right)^2, \quad \text{(A.29)}$$

$$\Gamma_Z = \frac{m_\lambda^3}{16\pi}\left(1-\frac{M_Z^2}{m_\lambda^2}\right)^2\left(\frac{L_1'^2}{2}\left(1+\frac{1}{2}\frac{M_Z^2}{m_\lambda^2}\right) - 3\frac{L_1'K'}{m_\lambda} + \frac{K'^2}{M_Z^2}\left(1+2\frac{M_Z^2}{m_\lambda^2}\right)\right), \quad \text{(A.30)}$$

where the neutralino masses are labeled by $m_{\chi_i^0}$, the LOSP mass is $m_\lambda \equiv m_{\chi_1^0}$, and

$$K' \equiv \sum_i \frac{G_{1i}'\rho_i'}{m_{\chi_i^0}}. \quad \text{(A.31)}$$

A similar calculation for difermion production is beyond the scope of this work; it would in general need to include the effects of A-terms, finite fermion masses, and sfermion mixing for the third generation, as well as possible interference from difermions produced by off-shell Z bosons.

Appendix B

The Two Faces of Anomaly Mediation: Appendices

B.1 The Fourth Anomaly in Anomaly Mediation

As mentioned in Table 5.1 and footnote 4, there is a fourth anomaly which can contribute to the gaugino mass, though it is not so important for phenomenology since it requires direct couplings of SUSY breaking to the gauginos at tree-level. It was first pointed out in Ref. [44] in a string theory context. For completeness, we derive in this appendix the extra contribution within our framework, and we show that the associated goldstino coupling respects (flat space) supercurrent conservation.

Following the notation in Ref. [12], the Yang-Mills term in a SUSY gauge theory is

$$\mathcal{L} \supset \frac{1}{2}\int d^2\theta\, \boldsymbol{S}\, \boldsymbol{W}^{a\alpha}\boldsymbol{W}^{a}_{\alpha}, \tag{B.1}$$

where $\boldsymbol{S}$ is the holomorphic gauge coupling. The superfield $\boldsymbol{S}$ is chiral and does not run beyond one-loop in perturbation theory. However, the component fields of the gauge multiplet appearing in Eq. (B.1) are not canonically normalized. In order to go to a canonically-normalized basis, we need to perform an anomalous rescaling of the gauge multiplet. This will induce an additional anomaly-mediated contribution to the gaugino mass.

As shown in Ref. [12], the effects of this rescaling are encoded in the real vector superfield

$\boldsymbol{R}$ (not to be confused with the curvature superfield), given by[1]

$$\boldsymbol{R} \equiv \left(\boldsymbol{S} + \boldsymbol{S}^\dagger\right) + \frac{T_G}{8\pi^2} \log\left[\boldsymbol{S} + \boldsymbol{S}^\dagger\right] + \ldots . \tag{B.2}$$

The physical meaning of the components of $\boldsymbol{R}$ can be identified from the 1PI effective action

$$\mathcal{L}_{\rm 1PI} = \int d^4\theta\, \boldsymbol{R}\, \boldsymbol{W}^{a\alpha} \frac{D^2}{-8\Box} \boldsymbol{W}^a_\alpha + \text{h.c.} \tag{B.3}$$

The lowest component of $\boldsymbol{R}$ defines the canonical gauge coupling, and the θ^2 component is related to the physical gaugino mass, via

$$\frac{1}{g^2} = \boldsymbol{R}\big|_{\theta^0}, \qquad m_\lambda = \log \boldsymbol{R}\big|_{\theta^2}. \tag{B.4}$$

The physical gaugino-gauge boson vertex is determined by

$$\mathcal{L} \supset -\frac{i}{2}\lambda_a \sigma^{\mu\nu} F^a_{\mu\nu} \log \boldsymbol{R}\big|_\theta. \tag{B.5}$$

If $\boldsymbol{S}$ has a θ^2 component at tree-level, then there is an extra contribution to the gaugino mass and goldstino coupling from the second term in Eq. (B.2), in addition to the expected tree-level gaugino mass and goldstino coupling from the first term. This additional piece due to the anomalous rescaling of the gauge multiplet is

$$\Delta m_\lambda = \frac{g^2 T_G}{8\pi^2} F^i \partial_i \log S. \tag{B.6}$$

We can also read off the associated goldstino coupling from Eqs. (B.2) and (B.5), after identifying the goldstino direction through Eq. (4.16). This gives an additional goldstino coupling

$$\Delta c_\lambda = \Delta m_\lambda \tag{B.7}$$

in the notation of Eq. (4.1), consistent with (flat space) supercurrent conservation.

[1]The elided terms include the sigma-model anomaly term already contained in Eq. (4.14).

B.2 General Chiral Field Redefinitions

In order to derive Eq. (4.14), we want to find a field redefinition on our (charged) matter superfields of the form

$$\boldsymbol{Q}^i \to e^{\boldsymbol{\alpha}^i}\boldsymbol{Q}^i \tag{B.8}$$

that removes all chiral couplings of the $\boldsymbol{Q}^i$ to the SUSY-breaking fields $\boldsymbol{X}^i$, while preserving the canonical normalization of all kinetic terms. Explicitly, we want that after this field redefinition,

$$\langle \boldsymbol{K}_{i\bar{\jmath}}\rangle = \delta_{i\bar{\jmath}}, \quad \langle \boldsymbol{K}_{i\bar{\jmath}\ell}\rangle = 0, \tag{B.9}$$

where exactly one of the indices on the latter corresponds to a SUSY-breaking field.

Assuming we have shifted away all vevs of our scalar fields, the most general Kähler potential for charged matter can be written as

$$\boldsymbol{K} = \boldsymbol{Q}^i\boldsymbol{Q}^{\dagger \jmath}\delta_{i\bar{\jmath}} + A_{i\bar{\jmath}\ell}\boldsymbol{Q}^i\boldsymbol{Q}^{\dagger \jmath}\boldsymbol{X}^\ell + \text{h.c.} + \cdots , \tag{B.10}$$

where we have omitted any terms that have no impact on Eq. (B.9) and rotated and rescaled the matter fields to have canonical kinetic terms. The linear couplings to $\boldsymbol{X}^\ell$ can be removed by the field redefinition

$$\boldsymbol{Q}^i \to e^{-\frac{\overline{D}^2 D^2}{16\Box}(\log \boldsymbol{K}'')_{k\bar{\imath}}}\boldsymbol{Q}^k = (\delta_{k\bar{\imath}} - A_{k\bar{\imath}\ell}\boldsymbol{X}^\ell + \cdots)\boldsymbol{Q}^k \tag{B.11}$$

with $\boldsymbol{K}''$ being the Kähler metric. This redefinition induces the anomaly term

$$\begin{aligned}\delta\mathcal{L} &= \sum_i \int d^2\theta\, \frac{g^2}{16\pi^2}T_{R_i}\left(-\frac{\overline{D}^2 D^2}{16\Box}(\log \boldsymbol{K}'')_{i\bar{\imath}}\right)\boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha \\ &= \sum_R -\frac{g^2}{16\pi^2}\int d^2\theta\, \frac{T_R}{d_R}\frac{\overline{D}^2 D^2}{16\Box}\log\det \boldsymbol{K}|''_R\, \boldsymbol{W}^{a\alpha}\boldsymbol{W}^a_\alpha .\end{aligned}$$

The sum in the last line is now over the matter representations R.

B.3 Non-Local Anomaly Terms

The lowest component of the superfield $\boldsymbol{C}$ in Eq. (4.34) yields (non-local) terms in the Lagrangian that express the three anomalies of the theory. In supergravity frame, we have explicitly

$$\boldsymbol{C}| = \frac{1}{\Box}\left[\frac{8}{3}(T_R - 3T_G)\left(-\frac{1}{2}\mathcal{R} + i\partial_\mu b^\mu\right) - \frac{16}{3}T_R(K_i\Box A^i + K_{ij}\mathcal{D}_\mu A^i\mathcal{D}^\mu A^j)\right.$$
$$\left. +16\frac{T_R}{d_R}\left((\log\det K|''_R)_i\Box A^i + (\log\det K|''_R)_{ij}\mathcal{D}_\mu A^i\mathcal{D}^\mu A^j\right) + \cdots\right]. \qquad \text{(B.12)}$$

For example, the super-Weyl anomaly (or more accurately, the $U(1)_R$ anomaly [16]) is expressed via

$$\mathcal{L} \supset \frac{g^2}{96\pi^2}(3T_G - T_R)\frac{\partial_\rho b^\rho}{\Box}F_{\mu\nu}\widetilde{F}^{\mu\nu}, \qquad \text{(B.13)}$$

where b_ρ is the vector auxiliary field which shifts as $b_\rho \to b_\rho + \partial_\rho\alpha$ under a $U(1)_R$ transformation. Rearranging Eq. (B.12), the Kähler anomaly and sigma-model anomaly are similarly expressed via the Kähler connection and sigma-model connection [16]:

$$\mathcal{L} \supset -\frac{g^2}{96\pi^2}T_R\frac{\partial_\rho(iK_i\partial^\rho A^i - iK_{\bar{\imath}}\partial^\rho A^{*\bar{\imath}})}{\Box}F_{\mu\nu}\widetilde{F}^{\mu\nu}, \qquad \text{(B.14)}$$

$$\mathcal{L} \supset \frac{g^2}{32\pi^2}\frac{T_R}{d_R}\frac{\partial_\rho(i(\log\det K|''_R)_i\partial^\rho A^i - i(\log\det K|''_R)_{\bar{\imath}}\partial^\rho A^{*\bar{\imath}})}{\Box}F_{\mu\nu}\widetilde{F}^{\mu\nu}. \qquad \text{(B.15)}$$

Appendix C

Anomaly Mediation from Unbroken Supergravity: Appendices

C.1 Goldstino Couplings from the Conformal Compensator

In this appendix, we provide a third derivation of the goldstino couplings in Eq. (5.8), working in the conformal compensator formalism of SUGRA to connection to our previous analysis in Ref. [53].[1] Here, the extra gauge redundancies of conformal SUGRA are gauge fixed to recover minimal SUGRA [101, 100, 102, 147] via a conformal compensator $\boldsymbol{\Phi}$, a chiral field with conformal weight 1. We can use $\boldsymbol{\Phi}$ to build a superconformally invariant action at tree-level (dropping Yang-Mills terms for convenience)

$$\mathcal{L} = \int d^4\theta\, \boldsymbol{\Phi}^\dagger \boldsymbol{\Phi}\, \boldsymbol{\Omega} + \int d^2\theta\, \boldsymbol{\Phi}^3 \boldsymbol{W} + \text{h.c.} + \ldots, \qquad \boldsymbol{\Omega} \equiv -3e^{-\boldsymbol{K}/3}\,. \tag{C.1}$$

Here, we use global superspace variables to express only the matter parts of the action, and the ellipsis $(\ldots)$ represents the action for the gravity multiplet as well as couplings of the matter fields to the gravity multiplet (see, e.g., Refs. [114, 38]).

The gauge choice for $\boldsymbol{\Phi}$ proposed by Kugo and Uehara [113] allows us to use the "global superspace" terms of Eq. (C.1) to find the pertinent features of supergravity, including scalar

[1] For details on the conformal compensator formalism see Refs. [143, 80, 114]. This formalism is reviewed in Ref. [38] using two-component fermion notation.

masses and goldstino couplings in curved space, without having to worry about supergravity effects from the terms in the ellipsis.[2] This gauge is

$$\boldsymbol{\Phi} = e^{K/6-i/3\,\mathrm{Arg}\,W}\left\{1, \frac{1}{3}K_i\chi^i, F_\Phi\right\}, \tag{C.2}$$

where the field F_Φ is an auxiliary complex degree of freedom, corresponding to the complex auxiliary field M of supergravity. Unlike in the super-Weyl formalism, F_Φ is not a gauge degree of freedom.

The most general Kähler and superpotential for unbroken SUGRA in AdS (i.e. $\langle W_i\rangle = \langle K_i\rangle = 0$) is[3]

$$\boldsymbol{\Omega} = \boldsymbol{Q}^{\dagger\bar{i}}\boldsymbol{Q}^i + \frac{1}{2}\langle\Omega_{ij}\rangle\,\boldsymbol{Q}^i\boldsymbol{Q}^j + \text{h.c.} + \ldots, \tag{C.3}$$

$$\boldsymbol{W} = m_{3/2} + \frac{1}{2}\langle W_{ij}\rangle\,\boldsymbol{Q}^i\boldsymbol{Q}^j + \ldots, \tag{C.4}$$

where the ellipses represent higher-order terms. Inserting these expression into Eq. (C.1) and rescaling the fields $\boldsymbol{Q}^i \to \boldsymbol{Q}^i/\boldsymbol{\Phi}$, we can solve the F_Φ equation of motion to find $F_\Phi = m_{3/2} + \ldots$. The extra terms are suppressed by at least two powers of M_{Pl}, and thus irrelevant for our purposes. It is then simple to read off the cosmological constant, as well as the fermion and scalar mass matrices:

$$\langle V\rangle = -3m_{3/2}^2M_{\mathrm{Pl}}^2, \tag{C.5}$$

$$M_{ij} = \langle W_{ij}\rangle + m_{3/2}\langle\Omega_{ij}\rangle, \tag{C.6}$$

$$m_{i\bar{j}}^2 = M_{ik}M^k{}_{\bar{j}} - 2m_{3/2}^2\delta_{i\bar{j}}, \tag{C.7}$$

$$B_{ij} = -m_{3/2}\langle W_{ij}\rangle + m_{3/2}^2\langle\Omega_{ij}\rangle - 2m_{3/2}^2\langle\Omega_{ij}\rangle = -m_{3/2}M_{ij}. \tag{C.8}$$

Thus, we recover the universal tachyonic soft mass-squared in Eq. (5.14) for scalars in unbroken AdS SUGRA, as well as B-terms proportional to the fermion mass matrix.

SUSY breaking effects then lift AdS space up to flat space. We represent the source of

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[2]An alternative gauge fixing was proposed in Ref. [38], but it is only valid in flat space. Given this limitation, it would obfuscate the derivation of the sfermion spectrum in curved space.

[3]For simplicity, we assume none of the visible-sector fields are singlets. The physics does not appreciably change if there are singlets, as long as there is no SUSY breaking in the visible sector.

SUSY breaking in the hidden sector by a non-linear goldstino multiplet [137, 116, 109, 42, 39]

$$\boldsymbol{X}_{\mathrm{NL}} = F_X \left(\theta + \frac{1}{\sqrt{2} F_X} \widetilde{G}_L \right)^2 , \tag{C.9}$$

where $\widetilde{G}_L$ is the goldstino. Because of the constraint $\boldsymbol{X}_{\mathrm{NL}}^2 = 0$, the Kähler potential and superpotential terms involving the non-linear field $\boldsymbol{X}_{\mathrm{NL}}$ are strongly constrained

$$\boldsymbol{\Omega} \supset -3 + \langle \Omega_X \rangle \boldsymbol{X}_{\mathrm{NL}} + \langle \Omega_{\bar{X}} \rangle \boldsymbol{X}_{\mathrm{NL}}^\dagger + \langle \Omega_{X\bar{X}} \rangle \boldsymbol{X}_{\mathrm{NL}}^\dagger \boldsymbol{X}_{\mathrm{NL}}, \tag{C.10}$$

$$\boldsymbol{W} \supset m_{3/2} + \langle W_X \rangle \boldsymbol{X}_{\mathrm{NL}}. \tag{C.11}$$

The coefficients $\langle \Omega_X \rangle$ and $\langle W_X \rangle$ can be made real by using our freedom to rotate $\boldsymbol{X}_{\mathrm{NL}}$ and perform Kähler transformations. A canonically-normalized goldstino (i.e. $\boldsymbol{K} \supset \boldsymbol{X}_{\mathrm{NL}}^\dagger \boldsymbol{X}_{\mathrm{NL}}$) enforces the condition $\langle \Omega_{X\bar{X}} \rangle = 1 - \frac{1}{3} \langle \Omega_X \rangle^2$. Upon rescaling the non-linear field $\boldsymbol{X}_{\mathrm{NL}} \to \boldsymbol{X}_{\mathrm{NL}}/\boldsymbol{\Phi}$ and integrating out auxiliary fields, we find from Eq. (C.1):

$$\langle F_X \rangle = - \left\langle W_X - m_{3/2} \Omega_X \right\rangle , \tag{C.12}$$

$$\langle F_\Phi \rangle = m_{3/2} + \frac{1}{3} \left\langle \Omega_X F^X \right\rangle , \tag{C.13}$$

$$\langle V \rangle = \left\langle F_X^2 \right\rangle - 3 m_{3/2}^2 . \tag{C.14}$$

The amount of SUSY breaking to achieve flat space is thus $\langle F_X \rangle = \sqrt{3} m_{3/2}$. We also have a canonically-normalized goldstino with mass $2m_{3/2}$ [42, 39].

The Kähler potential and superpotential will also include direct couplings between visible matter fields and the SUSY breaking sector. For simplicity, we start our study of goldstino couplings for massless visible sector fermions (e.g. $\boldsymbol{Q}^i \boldsymbol{Q}^j$ is never a singlet under any of the gauge symmetries in the theory). In this simple case the operators we can add are

$$\boldsymbol{\Omega} \supset \left\langle \Omega_{i\bar{j}X\bar{X}} \right\rangle \boldsymbol{Q}^{\dagger\bar{j}} \boldsymbol{Q}^i \boldsymbol{X}_{\mathrm{NL}}^\dagger \boldsymbol{X}_{\mathrm{NL}}, \tag{C.15}$$

$$\boldsymbol{W} \supset \frac{1}{6} \langle W_{ijk} \rangle \boldsymbol{Q}^i \boldsymbol{Q}^j \boldsymbol{Q}^k + \frac{1}{6} \langle W_{ijkX} \rangle \boldsymbol{Q}^i \boldsymbol{Q}^j \boldsymbol{Q}^k \boldsymbol{X}_{\mathrm{NL}}, \tag{C.16}$$

where we have eliminated any possible $\boldsymbol{Q}^{\dagger\bar{j}} \boldsymbol{Q}^i \boldsymbol{X}_{\mathrm{NL}}$ terms by using our freedom to perform a transformation $\boldsymbol{Q}^i \to \boldsymbol{Q}^i + n^i{}_j \boldsymbol{Q}^j \boldsymbol{X}_{\mathrm{NL}}$ [53]. The scalar masses and A-terms can be easily read off from Eq. (C.1):

$$m_{i\bar{j}}^2 = -3m_{3/2}^2 \langle \Omega_{i\bar{j}X\bar{X}} \rangle , \tag{C.17}$$

$$A_{ijk} = \sqrt{3} m_{3/2} \langle W_{ijkX} \rangle . \tag{C.18}$$

The terms in Eq. (C.15) also yield goldstino couplings to visible sector fields from the fermionic component of $\boldsymbol{X}_{\rm NL}$; namely $a_{i\bar{j}} \supset m_{i\bar{j}}^2$. Less obvious is that there are additional goldstino couplings coming from $\boldsymbol{\Phi}$. In the gauge from Eq. (C.2), the fermionic component of $\boldsymbol{\Phi}$ contains visible sector fermions (coupled to its conjugate scalar):

$$\frac{1}{3} K_i \chi^i = \frac{1}{3} \langle \Omega_X \rangle \widetilde{G}_L + \frac{1}{3} \phi^{*\bar{i}} \chi^i + \ldots \tag{C.19}$$

This means that the $\langle W_X \rangle \boldsymbol{X}_{\rm NL}$ term in the superpotential of Eq. (C.11) (multiplied by $\boldsymbol{\Phi}^2$ after rescaling) gives an additional coupling $(2m_{3/2}^2/F_X)\, K_i \chi^i \widetilde{G}_L$ (i.e. the universal goldstino couplings from Eq. (5.36)). The full goldstino coupling reads

$$a_{i\bar{j}} = m_{i\bar{j}}^2 + 2m_{3/2}^2 \delta_{i\bar{j}} , \tag{C.20}$$

in agreement with Eq. (5.8) in the $M_{ij} = 0$ limit.

Finally, we consider superpotential and Giudice-Masiero mass terms for the fermions. This introduces a plethora of new possible terms:

$$\boldsymbol{\Omega} \supset \frac{1}{2} \boldsymbol{Q}^i \boldsymbol{Q}^j \left[\langle \Omega_{ij} \rangle + \langle \Omega_{ijX} \rangle \boldsymbol{X}_{\rm NL} + \langle \Omega_{ij\bar{X}} \rangle \boldsymbol{X}_{\rm NL}^\dagger + \langle \Omega_{ijX\bar{X}} \rangle \boldsymbol{X}_{\rm NL}^\dagger \boldsymbol{X}_{\rm NL} \right] , \tag{C.21}$$

$$\boldsymbol{W} \supset \frac{1}{2} \langle W_{ij} \rangle \boldsymbol{Q}^i \boldsymbol{Q}^j + \frac{1}{2} \langle W_{ijX} \rangle \boldsymbol{Q}^i \boldsymbol{Q}^j \boldsymbol{X}_{\rm NL}. \tag{C.22}$$

Fermion masses and B-terms can be easily extracted from this Lagrangian. Goldstino couplings are more difficult to read off. As already mentioned, the goldstino lives both in $\boldsymbol{\Phi}$ and $\boldsymbol{X}_{\rm NL}$, but in addition, the Kähler potential cubic terms $\boldsymbol{Q}^i \boldsymbol{Q}^j \boldsymbol{\Phi}^\dagger$ and $\boldsymbol{Q}^i \boldsymbol{Q}^j \boldsymbol{X}_{\rm NL}^\dagger$ contain derivative interactions with the goldstino. After using the equation of motion for the goldstino of mass $2m_{3/2}$

$$\phi^j \chi^i (-i\sigma^\mu \partial_\mu \widetilde{G}_L^\dagger) \to 2m_{3/2} \phi^j \chi^i \widetilde{G}_L, \tag{C.23}$$

these yield Yukawa interactions between matter fields and the goldstino.[4] The resulting goldstino couplings are exactly those of Eq. (5.8).

C.2 Renormalization Group Invariance of Irreducible Goldstino Couplings

In Sec. 5.2, we found a universal tree-level goldstino coupling to matter scalars and fermions proportional in $m_{3/2}^2$. In Sec. 5.4, we expanded this result to all loop orders, finding further couplings by carefully analyzing the SUGRA- and super-Weyl invariant 1PI effective action:

$$\mathcal{G}_i^S = 2m_{3/2}^2 - \gamma_i m_{3/2}^2 - \frac{1}{12}\dot{\gamma}_i K_j F^j \left(m_{3/2} + \frac{1}{3}K_j F^j\right) \qquad \text{(flat space).} \tag{C.24}$$

Since these results follow from a 1PI action, they have incorporated all quantum corrections and are thus completely RG stable—that is, their coefficients solve their own RG equations. For the terms proportional to γ_i and $\dot{\gamma}_i$, it has long been known in the literature [98, 97, 133, 12] that mass terms of such a form are RG stable. This is true for the γ_i term by itself, and is true for the $\dot{\gamma}_i$ term given corresponding A terms in the form of Eq. (5.106). The same logic for soft terms can be trivially extended to goldstino couplings, which makes it clear that the goldstino couplings proportional to γ_i and $\dot{\gamma}_i$ above are also RG stable.[5]

However, the tree-level term, proportional to a constant, is not so clearly RG stable. Naively, one would expect it to receive quantum corrections starting at one loop (separate from the term proportional to γ in Eq. (C.24)), just as a constant scalar mass would. This puzzle is resolved by remembering that the goldstino and gravitino mix in SUGRA, so quantum corrections to gravitino couplings feed into quantum corrections to goldstino couplings, making the tree-level goldstino coupling in Eq. (C.24) RG stable.

For clarity, we give an example of how this occurs in one concrete model: a sequestered theory (in the sense of Eq. (5.35)) in flat space with $\langle K_i \rangle = 0$ and a Wess-Zumino visible sector:

$$\boldsymbol{W}_{\text{vis}} = \frac{1}{6}\lambda \boldsymbol{Q}^3, \tag{C.25}$$

[4] The problematic cubic term $\boldsymbol{Q}^i\boldsymbol{Q}^j\boldsymbol{\Phi}^\dagger$ could have been eliminated by a redefinition of $\boldsymbol{\Phi}$, or equivalently choosing a different gauge fixing than the one in Eq. (C.2). The $\boldsymbol{Q}^i\boldsymbol{Q}^j\boldsymbol{X}_{\text{NL}}^\dagger$ term, however, cannot be eliminated by any redefinition that preserves $\boldsymbol{X}_{\text{NL}}^2 = 0$.

[5] This logic is less clearly applicable for the $\dot{\gamma}_i m_{3/2} K_j F^j$ crossterm, as the goldstino coupling corresponding to the A-terms of Eq. (5.106) is not expected to depend on $m_{3/2}$. Nevertheless, the logic still holds.

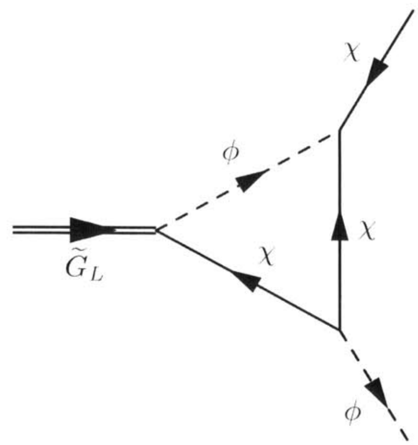

Figure C-1: One-loop diagram that renormalizes the goldstino coupling to visible-sector scalars and fermions in the Wess-Zumino theory from Eq. (C.25). The diagram has the same logarithmic divergence in both global SUSY and SUGRA, and would seem to renormalize the tree-level goldstino coupling $\mathcal{G}_i^{\mathcal{S}} \supset 2m_{3/2}^2$.

with $\boldsymbol{Q} = \{\phi, \chi, F\}$. The goldstino coupling seems to receive a correction from the logarithmically divergent diagram in Fig. C-1. Using a Pauli-Villars regulator, the divergent part of this diagram is

$$i\mathcal{M}_1 = i\frac{2m_{3/2}^2}{F_{\text{eff}}} x_{\widetilde{G}_L} y_\chi \left(-\frac{\lambda^2}{(4\pi)^2} \log \Lambda^2 \right) + \ldots, \tag{C.26}$$

with $x_{\widetilde{G}_L}$ and y_χ the external wave function spinors for the goldstino and the visible-sector fermion, respectively.[6] The presence of such a divergence would be fine if it could be completely absorbed by the wave-function renormalization of the visible sector fields. However, we know that it cannot be absorbed in the global SUSY case, which features the exact same diagram (up to a soft scalar mass that does not affect its divergent part). Explicitly, one can see this by noting that the divergent one-loop contribution to Z is

$$Z = \frac{1}{2}\frac{\lambda^2}{(4\pi)^2} \log \Lambda^2 + \ldots. \tag{C.27}$$

This differs by a factor of -2 from what would be needed to have the entire divergence in Eq. (C.26) explained by wave function renormalization. Thus, one would seem to find that the $\mathcal{G}_i^{\mathcal{S}} \supset 2m_{3/2}^2$ goldstino coupling runs at one-loop order, in conflict with the claims that $\mathcal{G}_i^{\mathcal{S}}$ arises from a valid 1PI effective action.

What we have not accounted for, however, is the mixing between the gravitino and the

[6] We use the methods of Ref. [61] for calculations here, but keep the sign and sigma matrix conventions of Ref. [154].

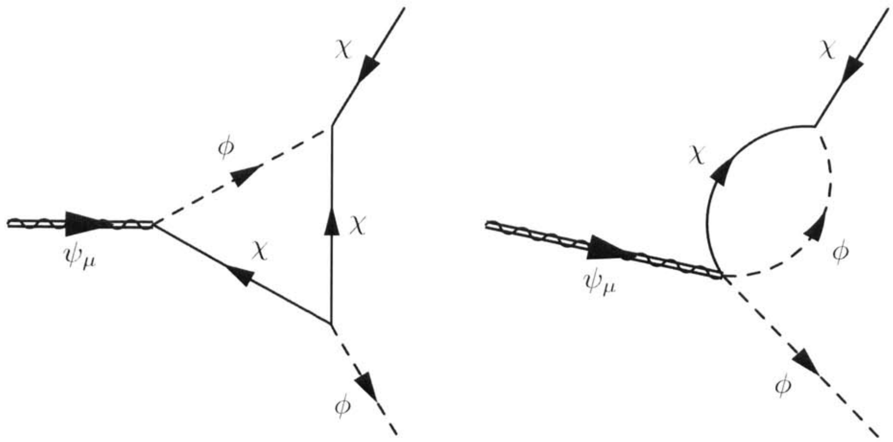

Figure C-2: These two diagrams yield logarithmically divergent corrections to the goldstino coupling after using the equation of motion in Eq. (C.28) for the gravitino. When combined with the diagram in Eq. (C-1), the goldstino coupling $\mathcal{G}_i^S \supset 2m_{3/2}^2$ is RG stable.

goldstino in SUGRA. Recall that the equation of motion of the gravitino in flat space is

$$\sigma^{\mu\nu}\mathcal{D}_\mu\psi_\nu = \sqrt{\frac{3}{2}}m_{3/2}\widetilde{G}_L + \frac{3}{4}im_{3/2}\sigma^\mu\psi_\mu^\dagger, \tag{C.28}$$

so diagrams with an external gravitino may yield corrections to the goldstino coupling after using this equation of motion (or making an appropriate field redefinition).[7] Effectively, by trading away couplings proportional to the left-hand side of Eq. (C.28), we are making sure that we are still in Einstein frame at one-loop order.

Using G defined in Eq. (5.10), the gravitino couples to visible-sector fields as [154]

$$\begin{aligned}\mathcal{L} &= -\frac{1}{\sqrt{2}M_{\text{Pl}}}g_{i\bar{j}}\partial_\nu\phi^{*\bar{j}}\chi^i\sigma^\mu\overline{\sigma}^\nu\psi_\mu - e^{\frac{G}{2M_{\text{Pl}}^2}}\frac{i}{\sqrt{2}}G_i\chi^i\sigma^\mu\psi_\mu^\dagger + \text{h.c.} &\text{(C.29)}\\ &= -\sqrt{\frac{3}{2}}\frac{m_{3/2}}{F}\psi_\mu\sigma^\nu\overline{\sigma}^\mu\chi\partial_\nu\phi^* + \frac{1}{2}i\lambda\sqrt{\frac{3}{2}}\frac{m_{3/2}}{F}\psi_\mu\sigma^\mu\chi^\dagger\phi^{*2} + \ldots + \text{h.c.}, &\text{(C.30)}\end{aligned}$$

where in the second line we have specialized to the theory in Eq. (C.25). The two diagrams featuring an external gravitino that can give contributions proportional to the left-hand side of Eq. (C.28) are shown in Fig. C-2. Each of these diagrams is logarithmically divergent,[8]

[7]One can of course pick a gauge for the Rarita-Schwinger gravitino field which removes the the quadratic mixing and changes this equation of motion. As in the text, we will only pick a gauge for the gravitino-goldstino system after computing quantum corrections to all orders in visible-sector couplings. This does not pose a problem as we never have to consider gravitinos or goldstinos (whose couplings are suppressed by M_{Pl}^{-1}) as internal legs when computing such quantum corrections.

[8]In fact, they are linearly divergent, but any ensuing subtleties will only affect the finite pieces, not the logarithmically divergent ones.

and they give equal corrections to the goldstino coupling. Combining these with Eq. (C.26), we find

$$i\mathcal{M}_{\text{total}} = i\frac{2m_{3/2}^2}{F_{\text{eff}}} x_{\widetilde{G}_L} y_\chi \left(\frac{1}{2}\frac{\lambda^2}{(4\pi)^2}\log\Lambda^2\right) + \ldots . \tag{C.31}$$

Comparing this to Eq. (C.27), we see this is precisely the logarithmic divergence that can be completely absorbed by the wave function renormalization of the visible-sector fields. At the one-loop level in this model, we confirm that the tree-level goldstino coupling does not run, as we knew had to be the case from our 1PI analysis in Sec. 5.4.

C.3 Super-Weyl Transformations

Super-Weyl transformations are the most general transformations that leave the torsion and chirality constraints of SUGRA unchanged. They may be completely parameterized by a chiral superfield $\boldsymbol{\Sigma}$ and its conjugate anti-chiral superfield $\boldsymbol{\Sigma}^\dagger$ [90, 154]. The super-Weyl transformations act infinitesimally on the gravity multiplet as [90, 154, 103]

$$\begin{aligned}
\delta E_M{}^a &= (\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)E_M{}^a, & \delta E_M{}^\alpha &= (2\boldsymbol{\Sigma}^\dagger-\boldsymbol{\Sigma})E_M{}^\alpha - \frac{i}{2}E_M{}^a(\mathcal{D}^\dagger_{\dot\alpha}\boldsymbol{\Sigma}^\dagger\bar\sigma_a^{\dot\alpha\alpha}),\\
\delta\mathcal{D}_\alpha &= (\boldsymbol{\Sigma}-2\boldsymbol{\Sigma}^\dagger)\mathcal{D}_\alpha - 2(\mathcal{D}^\beta\boldsymbol{\Sigma})L_{\alpha\beta}, & \delta\mathcal{D}^\dagger_{\dot\alpha} &= (\boldsymbol{\Sigma}^\dagger-2\boldsymbol{\Sigma})\mathcal{D}^\dagger_{\dot\alpha} - 2(\mathcal{D}^{\dagger\dot\beta}\boldsymbol{\Sigma}^\dagger)L_{\dot\alpha\dot\beta},\\
\delta\boldsymbol{E} &= 2(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)\boldsymbol{E}, & \delta(2\boldsymbol{\mathcal{E}}) &= 6\boldsymbol{\Sigma}(2\boldsymbol{\mathcal{E}}) + \ldots,\\
\delta\boldsymbol{R} &= 2(\boldsymbol{\Sigma}^\dagger-2\boldsymbol{\Sigma})\boldsymbol{R} - \frac{1}{4}\mathcal{D}^{\dagger 2}\boldsymbol{\Sigma}^\dagger, & \delta\boldsymbol{G}_{\alpha\dot\alpha} &= -(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)\boldsymbol{G}_{\alpha\dot\alpha} + i\mathcal{D}_{\alpha\dot\alpha}(\boldsymbol{\Sigma}^\dagger-\boldsymbol{\Sigma}),\\
\delta\boldsymbol{W}_{\alpha\beta\gamma} &= -3\boldsymbol{\Sigma}\boldsymbol{W}_{\alpha\beta\gamma},
\end{aligned} \tag{C.32}$$

where a is a local Lorentz spacetime index, $L_{\alpha\beta}$ are the Lorentz generators acting on spinors, $\boldsymbol{E}$ is the determinant of the supersymmetric vielbein, $2\boldsymbol{\mathcal{E}}$ is the corresponding chiral density, $\boldsymbol{R}$ is the chiral curvature superfield, and $\boldsymbol{G}_{\alpha\dot\alpha}$ is the real superfield having the vector auxiliary field of supergravity b_μ as its lowest component. The ellipsis in the transformation of the chiral vielbein are omitted terms irrelevant for the construction of a super-Weyl invariant action. The transformation of $\mathcal{D}_a$ is too complicated to include here, but $\mathcal{D}_a$ may always be expressed as some composition of the above objects. For example, when acting on a Lorentz scalar superfield $\boldsymbol{U}$,

$$\mathcal{D}_a\boldsymbol{U} = -\frac{1}{4}i\bar\sigma_a^{\dot\alpha\alpha}\{\mathcal{D}^\dagger_{\dot\alpha},\mathcal{D}_\alpha\}\boldsymbol{U}. \tag{C.33}$$

Chiral superfields $\boldsymbol{Q}$ and vector superfields $\boldsymbol{V}$ transform as [154]

$$\delta\boldsymbol{Q} = w\boldsymbol{\Sigma}\boldsymbol{Q}, \qquad\qquad \delta\boldsymbol{V} = w'(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)\boldsymbol{V}, \tag{C.34}$$

where w and w' are the Weyl weights of their respective superfield; for ordinary matter or gauge superfields, these weights vanish. Note that the higher components of matter superfields still transform, due to the non-trivial transformation of the $\mathcal{D}_\alpha$ used to project them out. For a vector superfield of weight 0, the superfield

$$\boldsymbol{W}_\alpha \equiv -\frac{1}{4}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\mathcal{D}_\alpha \boldsymbol{V} \tag{C.35}$$

transforms as a chiral superfield of Weyl weight -3.

The SUGRA action of Ref. [154] can be made super-Weyl invariant by including a super-Weyl compensator $\boldsymbol{C}$ of Weyl weight -2. The tree-level Lagrangian then reads

$$\mathcal{L} = \int d^4\Theta\, \boldsymbol{E}\,\boldsymbol{C}^\dagger\boldsymbol{C}\,(-3e^{-\boldsymbol{K}/3}) + \int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\,\boldsymbol{C}^3\boldsymbol{W} + \frac{1}{4}\int d^2\Theta\, 2\boldsymbol{\mathcal{E}}\,\boldsymbol{W}^\alpha\boldsymbol{W}_\alpha + \text{h.c.} \tag{C.36}$$

The super-Weyl compensator can also be used to build versions of $\boldsymbol{R}$ and $\boldsymbol{G}_{\alpha\dot\alpha}$ that transform homogeneously under super-Weyl transformations:

$$\boldsymbol{\mathcal{P}} \equiv -\frac{1}{4}\frac{1}{\boldsymbol{C}^2}(\mathcal{D}^{\dagger 2} - 8\boldsymbol{R})\boldsymbol{C}^\dagger, \tag{C.37}$$

$$\boldsymbol{\mathcal{P}}^\dagger \equiv -\frac{1}{4}\frac{1}{\boldsymbol{C}^{\dagger 2}}(\mathcal{D}^2 - 8\boldsymbol{R})\boldsymbol{C}, \tag{C.38}$$

$$\delta\boldsymbol{\mathcal{P}} = \delta\boldsymbol{\mathcal{P}}^\dagger = 0, \tag{C.39}$$

$$\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha} \equiv \boldsymbol{G}_{\alpha\dot\alpha} - \frac{1}{4\boldsymbol{C}^\dagger}\mathcal{D}_\alpha\mathcal{D}^\dagger_{\dot\alpha}\boldsymbol{C}^\dagger + \frac{1}{4\boldsymbol{C}}\mathcal{D}^\dagger_{\dot\alpha}\mathcal{D}_\alpha\boldsymbol{C} + \frac{1}{4\boldsymbol{C}^\dagger\boldsymbol{C}}(\mathcal{D}_\alpha\boldsymbol{C})(\mathcal{D}^\dagger_{\dot\alpha}\boldsymbol{C}^\dagger), \tag{C.40}$$

$$\delta\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha} = -(\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger)\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}. \tag{C.41}$$

These objects also obey appropriately-modified versions of the Bianchi identities:

$$\mathcal{D}^\dagger_{\dot\alpha}\boldsymbol{\mathcal{P}} = 0, \qquad\qquad \mathcal{D}_\alpha\boldsymbol{\mathcal{P}}^\dagger = 0, \tag{C.42}$$

$$\mathcal{D}^\alpha(\boldsymbol{C}\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}) = \frac{1}{2}\boldsymbol{C}^{\dagger 2}\mathcal{D}^\dagger_{\dot\alpha}\boldsymbol{\mathcal{P}}^\dagger, \qquad\qquad \mathcal{D}^{\dagger\dot\alpha}(\boldsymbol{C}^\dagger\widetilde{\boldsymbol{G}}_{\alpha\dot\alpha}) = \frac{1}{2}\boldsymbol{C}^2\mathcal{D}_\alpha\boldsymbol{\mathcal{P}}. \tag{C.43}$$

The superfield $\boldsymbol{\mathcal{P}}$ ($\boldsymbol{\mathcal{P}}^\dagger$) can also be serve as an operator, which we denote by the nonboldface $\mathcal{P}$ ($\mathcal{P}^\dagger$). When acting on a super-Weyl invariant spinless superfield, $\mathcal{P}$ ($\mathcal{P}^\dagger$) returns a super-Weyl invariant (anti-)chiral superfield [103]. The operator $\mathcal{P}$ ($\mathcal{P}^\dagger$) thus acts as an (anti-)chiral projector.

C.4 1PI Gaugino Masses

In Eq. (5.67), we used a 1PI effective action for the gauge multiplet built as an integral over chiral superspace. This is sufficient for extracting one-loop results, but in a general renormalization scheme, the 1PI action must instead be written as an integral of a non-local quantity over all of superspace. For the familiar case of global SUSY in flat space, we may write the 1PI action as [12, 84]

$$\mathcal{L} \supset \frac{1}{4}\int d^4\theta\, \widetilde{\boldsymbol{R}}(\Box)\boldsymbol{W}^\alpha \left[-\frac{1}{4}\mathcal{D}^2\right]\Box^{-1}\boldsymbol{W}_\alpha + \text{h.c.}, \tag{C.44}$$

or alternatively, remembering that $\frac{1}{16}\mathcal{D}^{\dagger 2}\mathcal{D}^2 = \Box$ when acting on chiral superfields,

$$\mathcal{L} \supset \frac{1}{4}\int d^4\theta\, \widetilde{\boldsymbol{R}}(\Box)\boldsymbol{W}^\alpha \left[-\frac{1}{4}\mathcal{D}^{\dagger 2}\right]^{-1}\boldsymbol{W}_\alpha + \text{h.c.} \tag{C.45}$$

The superfield $\widetilde{\boldsymbol{R}}$ (not to be confused with the chiral curvature superfield $\boldsymbol{R}$) is the real vector superfield with the 1PI gauge coupling as its lowest component. The dependence of $\widetilde{\boldsymbol{R}}$ on $\Box$ encapsulates the running of the coupling with the momentum scale (selected by $\Box$, which should be thought of as acting only on the first $\boldsymbol{W}^\alpha$). A non-vanishing θ^2 component for $\widetilde{\boldsymbol{R}}$ yields a gaugino mass. If $\widetilde{\boldsymbol{R}}$ only has a lowest component, it then follows trivially that Eq. (C.45) is equivalent, after integrating over half of superspace, to the usual expression for the gauge kinetic Lagrangian in chiral superspace (proportional to $\int d^2\theta\, \boldsymbol{W}^\alpha \boldsymbol{W}_\alpha$).

It is now a simple matter to generalize most of Eq. (C.45) to be SUGRA and super-Weyl covariant

$$\mathcal{L} \supset \frac{1}{4}\int d^4\Theta\, \boldsymbol{E}\,\boldsymbol{C}^\dagger\boldsymbol{C}\widetilde{\boldsymbol{R}}(\widetilde{\Box})\widetilde{\boldsymbol{W}}^\alpha\mathcal{P}^{-1}\widetilde{\boldsymbol{W}}_\alpha + \text{h.c.}, \tag{C.46}$$

where $\widetilde{\boldsymbol{W}}_\alpha = \boldsymbol{C}^{-\frac{3}{2}}\boldsymbol{W}_\alpha$ has vanishing Weyl weight, and $\mathcal{P}$ is the super-Weyl covariant chiral projector given in Eq. (C.37). It can be easily verified that when $\widetilde{\boldsymbol{R}}(\widetilde{\Box})\widetilde{\boldsymbol{W}}^\alpha$ is chiral, Eq. (C.46) reduces to Eq. (5.67), an integral of a local quantity over chiral superspace.

The only potentially ambiguous part of this equation is $\tilde{\Box}$, the appropriately super-Weyl covariant version of $\Box$ acting on a super-Weyl inert superfield with an undotted spinor index. If we only care about $\mathcal{O}(m_{3/2})$ effects such as gaugino masses, however, there are only two families of possible choices[9]

$$\begin{aligned}\tilde{\Box}\boldsymbol{U}_\alpha = &\frac{1}{2}\boldsymbol{C}^{\frac{1}{2}}\boldsymbol{\mathcal{P}}\boldsymbol{C}^{\dagger-1}\mathcal{D}_\alpha\frac{\mathcal{D}^\beta\boldsymbol{C}^{\frac{3}{2}}\boldsymbol{U}_\beta}{\boldsymbol{C}^\dagger\boldsymbol{C}} + \frac{1}{2}\boldsymbol{C}^{\frac{1}{2}}\boldsymbol{C}^{\dagger-1}\mathcal{D}_\alpha\frac{\mathcal{D}^\beta\boldsymbol{C}^{\frac{3}{2}}\boldsymbol{\mathcal{P}}\boldsymbol{U}_\beta}{\boldsymbol{C}^\dagger\boldsymbol{C}}\\ &+\frac{1}{4}\boldsymbol{C}^{-\frac{1}{2}}\boldsymbol{C}^{\dagger-1}\mathcal{D}^\dagger_{\dot\alpha}\mathcal{D}_\alpha\boldsymbol{C}^{-1}\mathcal{D}^\beta\boldsymbol{C}^{\frac{1}{2}}\mathcal{D}^{\dagger\dot\alpha}\boldsymbol{U}_\beta\\ &+a(\boldsymbol{\mathcal{P}}^\dagger)\boldsymbol{\mathcal{P}}\boldsymbol{U}_\alpha + \frac{1}{2}b\mathcal{D}_\alpha\frac{\mathcal{D}^\beta(\boldsymbol{C}^{3/2}(\boldsymbol{\mathcal{P}})\boldsymbol{U}_\beta)}{\boldsymbol{C}^\dagger\boldsymbol{C}},\end{aligned} \tag{C.47}$$

parameterized by arbitrary coefficients a and b.[10] Note that the choice $a = 0$, $b = -1$ is especially convenient, as $\tilde{\Box}\boldsymbol{U}_\alpha$ is chiral for $\boldsymbol{U}_\alpha$ chiral. This is precisely the choice used in Eq. (5.68), and allows us to write the 1PI action as an integral over chiral superspace. However, this choice is not necessary; regardless of the values of a and b chosen, a (more difficult) calculation shows that

$$m_\lambda = \frac{\beta_g}{g}m_{3/2}. \tag{C.48}$$

[9] For $\mathcal{O}(m_{3/2}^2)$ effects, such as non-local contributions to the self-energies of the particles in the vector multiplet (as considered in Ref. [12]), one would need to consider additional terms. Such effects, the equivalents of the $\mathcal{S}$ and $\mathcal{T}$ of Sec. 5.4.3 for vector multipcts, are beyond the scope of this work.

[10] This is only gauge invariant for an abelian gauge theory; appropriate factors of $e^{\pm \boldsymbol{V}}$ would need to be inserted for a non-abelian gauge theory.

Appendix D

A Photon Line from Decaying Goldstino Dark Matter: Appendices

D.1 Complete Goldstino-Neutralino Mixing Angles

We give here the complete set of mixing angles between the uneaten goldstino and the neutralinos (to lowest order in $1/F_{\perp}$), as defined in Eq. (6.20):

$$\widetilde{\theta} = \frac{v}{\sqrt{2}F_{\perp}} \frac{1}{2\det\left[M_{\chi} - m_{\zeta}\mathbb{1}\right]} \begin{pmatrix} C_Y \\ C_3 \\ C_d \\ C_u \end{pmatrix}, \tag{D.1}$$

with

$$
\begin{aligned}
d &\equiv 2\det\left[M_\chi - m_\zeta \mathbb{1}\right] \qquad\qquad \text{(D.2)}\\
&= -2(M_1 - m_\zeta)(M_2 - m_\zeta)(\mu^2 - m_\zeta^2)\\
&\quad + M_Z^2(\mu\sin 2\beta + m_\zeta)(M_1 + M_2 - 2m_\zeta + (M_1 - M_2)\cos 2\theta_W),\\
C_Y &= M_Z \sin\theta_W \Big((\widetilde{M}_1 - \widetilde{M}_2) M_Z^2 \cos 2\beta \cos^2\theta_W (\mu \sin 2\beta + m_\zeta)\\
&\quad + (M_2 - m_\zeta)\Big(-\widetilde{M}_1 \cos 2\beta(\mu^2 - m_\zeta^2) + 2\widetilde{B}_\mu \mu \cos 2\beta\\
&\quad - m_\zeta(\widetilde{m}_{H_u}^2 + \widetilde{m}_{H_d}^2)\cos 2\beta + (\widetilde{m}_{H_u}^2 - \widetilde{m}_{H_d}^2)(\mu\sin 2\beta + m_\zeta)\Big)\Big),\\
C_3 &= -M_Z \cos\theta_W \Big((\widetilde{M}_2 - \widetilde{M}_1) M_Z^2 \cos 2\beta \sin^2\theta_W (\mu \sin 2\beta + m_\zeta)\\
&\quad + (M_1 - m_\zeta)\Big(-\widetilde{M}_2 \cos 2\beta(\mu^2 - m_\zeta^2) + 2\widetilde{B}_\mu \mu \cos 2\beta\\
&\quad - m_\zeta(\widetilde{m}_{H_u}^2 + \widetilde{m}_{H_d}^2)\cos 2\beta + (\widetilde{m}_{H_u}^2 - \widetilde{m}_{H_d}^2)(\mu\sin 2\beta + m_\zeta)\Big)\Big),\\
C_d &= -M_Z^2 \cos 2\beta(\mu\sin\beta + m_\zeta\cos\beta)\Big(\widetilde{M}_1(M_2 - m_\zeta)\sin^2\theta_W + \widetilde{M}_2(M_1 - m_\zeta)\cos^2\theta_W\Big)\\
&\quad + 2(M_1 - m_\zeta)(M_2 - m_\zeta)(\widetilde{B}_\mu(\mu\cos\beta - m_\zeta\sin\beta) + \widetilde{m}_{H_u}^2\mu\sin\beta - m_\zeta \widetilde{m}_{H_d}^2\cos\beta)\\
&\quad - M_Z^2\sin\beta(M_1 + M_2 - 2m_\zeta + (M_1 - M_2)\cos 2\theta_W)(\widetilde{B}_\mu + (\widetilde{m}_{H_u}^2 + \widetilde{m}_{H_d}^2)\sin\beta\cos\beta),\\
C_u &= M_Z^2 \cos 2\beta(\mu\cos\beta + m_\zeta\sin\beta)\Big(\widetilde{M}_1(M_2 - m_\zeta)\sin^2\theta_W + \widetilde{M}_2(M_1 - m_\zeta)\cos^2\theta_W\Big)\\
&\quad + 2(M_1 - m_\zeta)(M_2 - m_\zeta)(\widetilde{B}_\mu(\mu\sin\beta - m_\zeta\cos\beta) + \widetilde{m}_{H_d}^2\mu\cos\beta - m_\zeta \widetilde{m}_{H_u}^2\sin\beta)\\
&\quad - M_Z^2\sin\beta(M_1 + M_2 - 2m_\zeta + (M_1 - M_2)\cos 2\theta_W)(\widetilde{B}_\mu + (\widetilde{m}_{H_u}^2 + \widetilde{m}_{H_d}^2)\sin\beta\cos\beta).
\end{aligned}
$$

D.2 Spin-3/2 Fermions in Two Component Notation

When calculating the decay rate of the uneaten goldstino ζ to the spin-3/2 gravitino ψ_μ, one cannot use the goldstino equivalence theorem [64, 34, 33], as the relevant energy, the mass of the uneaten goldstino at $2m_{3/2}$, is not much greater than the gravitino mass $m_{3/2}$. Therefore, one needs to use the vector-spinors of the fully spin-3/2 gravitino when performing calculations, including its transverse polarization modes.

The main obstacle in dealing with these vector-spinors is in the sums over them when summing over the gravitino polarization modes. For the convenience of the reader, we give these spin sums below for two-component fermion notation, as to our knowledge they do not appear elsewhere in the literature. The notation of these spin sums should be interpreted analogously to those in Ref. [61], though note that we use the sigma matrix and metric conventions of Ref. [154] throughout.[1]

$$\sum_s x^\mu_\alpha x^{\dagger\nu}_{\dot\alpha} = \left(\frac{2}{3}\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right) k^\rho - \frac{1}{3} i\epsilon^{\mu\nu\rho\tau} k_\tau\right) \sigma_{\rho\alpha\dot\alpha} \tag{D.3}$$

$$\sum_s y^\mu_\alpha y^{\dagger\nu}_{\dot\alpha} = \left(\frac{2}{3}\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right) k^\rho - \frac{1}{3} i\epsilon^{\mu\nu\rho\tau} k_\tau\right) \sigma_{\rho\alpha\dot\alpha} \tag{D.4}$$

$$\sum_s x^{\dagger\mu\dot\alpha} x^{\nu\alpha} = \left(\frac{2}{3}\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right) k^\rho + \frac{1}{3} i\epsilon^{\mu\nu\rho\tau} k_\tau\right) \overline{\sigma}_\rho^{\alpha\dot\alpha} \tag{D.5}$$

$$\sum_s y^{\dagger\mu\dot\alpha} y^{\nu\alpha} = \left(\frac{2}{3}\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right) k^\rho + \frac{1}{3} i\epsilon^{\mu\nu\rho\tau} k_\tau\right) \overline{\sigma}_\rho^{\alpha\dot\alpha} \tag{D.6}$$

$$\sum_s y^{\dagger\mu\dot\alpha} x^{\dagger\nu}_{\dot\beta} = \frac{2}{3} m_{3/2}\left[\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right)\mathbb{1} + \overline{\sigma}^{\mu\nu} + \frac{k^\nu k_\lambda}{m_{3/2}^2}\overline{\sigma}^{\mu\lambda} - \frac{k^\mu k_\lambda}{m_{3/2}^2}\overline{\sigma}^{\nu\lambda}\right]^{\dot\alpha}{}_{\dot\beta} \tag{D.7}$$

$$\sum_s x^\mu_\alpha y^{\nu\beta} = \frac{2}{3} m_{3/2}\left[\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right)\mathbb{1} + \sigma^{\mu\nu} + \frac{k^\nu k_\lambda}{m_{3/2}^2}\sigma^{\mu\lambda} - \frac{k^\mu k_\lambda}{m_{3/2}^2}\sigma^{\nu\lambda}\right]_\alpha{}^\beta \tag{D.8}$$

$$\sum_s y^\mu_\alpha x^{\nu\beta} = -\frac{2}{3} m_{3/2}\left[\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right)\mathbb{1} + \sigma^{\mu\nu} + \frac{k^\nu k_\lambda}{m_{3/2}^2}\sigma^{\mu\lambda} - \frac{k^\mu k_\lambda}{m_{3/2}^2}\sigma^{\nu\lambda}\right]_\alpha{}^\beta \tag{D.9}$$

$$\sum_s x^{\dagger\mu\dot\alpha} y^{\dagger\nu}_{\dot\beta} = -\frac{2}{3} m_{3/2}\left[\left(g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{3/2}^2}\right)\mathbb{1} + \overline{\sigma}^{\mu\nu} + \frac{k^\nu k_\lambda}{m_{3/2}^2}\overline{\sigma}^{\mu\lambda} - \frac{k^\mu k_\lambda}{m_{3/2}^2}\overline{\sigma}^{\nu\lambda}\right]^{\dot\alpha}{}_{\dot\beta} \tag{D.10}$$

These two-component spin sums can be obtained from their four component equivalents, such as those found in [152, 138]. They can be easily derived independently by considering the on-shell gravitino equations of motion (the Dirac equation, as well as the constraints $\overline{\sigma} \cdot x = \overline{\sigma} \cdot y = k \cdot x = k \cdot y = 0$), and goldstino equivalence can be used to fix the overall normalization.

D.3 Arbitrary Mass Goldstino Decays

The simplest case is the decay to a photon and a gravitino. The decay is mediated only by the last term in Eq. (6.24), and is simply given by [66]

$$\Gamma_{\zeta\to\gamma+\psi_\mu} = \frac{m_\zeta^5 \Theta_\gamma^2}{16\pi F_{\text{eff}}^2}\left(1 - \frac{m_{3/2}^2}{m_\zeta^2}\right)^3 \left(1 + \frac{3m_{3/2}^2}{m_\zeta^2}\right), \tag{D.11}$$

[1] To convert to the conventions of Ref. [61], for the purposes of these spin sums, send $\sigma_\rho \to -\sigma_\rho$, $\overline{\sigma}_\rho \to -\overline{\sigma}_\rho$, $\epsilon^{\mu\nu\rho\tau} \to -\epsilon^{\mu\nu\rho\tau}$, $\sigma^{\mu\nu} \to -i\sigma^{\mu\nu}$, and $\sigma^{\mu\nu} \to -i\sigma^{\mu\nu}$, then, following the prescriptions of Ref. [61], change metric conventions from mostly plus to mostly minus.

where Θ_γ is the ζ-photino mixing angle, which is defined in Eq. (6.21). If we had attempted to use the goldstino equivalence theorem for the gravitino here, we would have omitted the final factor, which can roughly be interpreted as the enhancement to the decay rate arising from the transverse gravitino modes.

The decay to a Higgs and gravitino is mediated solely by the first term in Eq. (6.24):

$$\Gamma_{\zeta\to h\psi_\mu} = \frac{m_\zeta^5\Theta_h^2}{32\pi F_{\text{eff}}^2}\left(\left(1-\frac{m_{3/2}}{m_\zeta}\right)^2-\frac{m_h^2}{m_\zeta^2}\right)^{3/2}\left(\left(1+\frac{m_{3/2}}{m_\zeta}\right)^2-\frac{m_h^2}{m_\zeta^2}\right)^{5/2}. \tag{D.12}$$

The angle Θ_h is the mixing angle between ζ and the superpartner of the physical Higgs (i.e. $\widetilde{H}_u\cos\alpha - \widetilde{H}_d\sin\alpha$).

The decay to a Z boson has contributions from both terms in Eq. (6.24), the first roughly corresponding to longitudinal Z couplings, and the second to transverse Z couplings. As a result, the decay rate is the complicated

$$\begin{aligned}\Gamma_{\zeta\to Z\psi_\mu} = \frac{1}{16\pi m_\zeta F_{\text{eff}}^2}&\sqrt{\left(1-\frac{M_Z^2-m_{3/2}^2}{m_\zeta^2}\right)^2-\frac{4m_{3/2}^2}{m_\zeta^2}}\times\\ &\Big(\Theta_{Z_T}^2\left((m_\zeta^2-M_Z^2)^3+m_{3/2}^2(m_\zeta^4-M_Z^4)-m_{3/2}^4(5m_\zeta^2+M_Z^2)+3m_{3/2}^6\right)\\ &\quad+\frac{1}{2}\Theta_{Z_L}^2\left((m_\zeta-m_{3/2})^2-M_Z^2\right)\left((m_\zeta^2-m_{3/2}^2-M_Z^2)^2+8M_Z^2m_{3/2}^2\right)\\ &\quad-2\Theta_{Z_T}\Theta_{Z_L}(2m_{3/2}M_Z)\left((m_\zeta-m_{3/2})^2-M_Z^2\right)\times\\ &\qquad\left(M_Z^2-(m_\zeta-2m_{3/2})(m_\zeta+m_{3/2})\right)\Big).\end{aligned} \tag{D.13}$$

D.3.1 Three-body Decay to Fermions

For $m_\zeta \gg M_Z$, three body modes become increasingly important compared to the two-body modes we mainly discussed in this paper. As discussed in Sec. 6.3.4, the leading decay modes in that limit will generically be $\zeta \to hh\psi_\mu$ and $\zeta \to hZ_L\psi_\mu$, dominating by a factor of m_ζ^2/m_{soft}^2. However, the sheer number of possible difermion modes may compensate for this in aggregate, so we consider them here.

Working in the limit in which A-terms and fermion masses can be neglected (a good approximation for all fermions but the top quark), and the limit $m_{\tilde{f}} \gg m_\zeta$, the decay rate of a goldstino to a pair of Standard Model fermions of a given handedness and a gravitino is given by

$$\Gamma_{\zeta \to \tilde{G}_L f \bar{f}} = \frac{N_c m_\zeta^9}{15360 \pi^3 F_{\text{eff}}^2 F_\perp^2} \frac{\widetilde{m}_{\tilde{f}}^4}{m_{\tilde{f}}^4} F_f \left(\frac{m_{3/2}}{m_\zeta} \right), \tag{D.14}$$

$$F_f(x) = 1 - 8x^2 + 30x^4 - 80x^6 + 35x^8 + 24x^{10} - 2x^{12} - 120x^8 \log x \tag{D.15}$$
$$+ x - 20x^3 - 220x^5 + 80x^7 + 155x^9 + 4x^{11} - 120x^5(2 + 4x^2 + x^4) \log x$$

For $m_\zeta = 2m_{3/2}$, $F_f(x) \approx 1/8$.[2]

D.3.2 Three-body Decay to Two Higgses

Using the effective field theory term of Eq. (6.25), we find the differential width in terms of the two energies of the produced higgses as

$$\frac{d\Gamma}{dE_1\, dE_2} = \frac{c^2}{F^2} m_\zeta (m_\zeta - E_1 + E_2) \left(m_\zeta^2 \left(2 \left(E_1^2 + E_2^2 \right) - 2m_\zeta (E_1 + E_2) + m_\zeta^2 \right) \right.$$
$$\left. + 2m_{3/2}^2 \left(m_\zeta (E_1 + E_2 - m_\zeta) - 2m_h^2 \right) + m_{3/2}^4 \right) \tag{D.16}$$

$$c \equiv \frac{2\widetilde{B}_\mu - (\widetilde{m}_{H_u}^2 + \widetilde{m}_{H_d}^2) \sin 2\beta}{2\sqrt{2}\mu F_\perp} \tag{D.17}$$

An analytic expression for $d\Gamma/dE_1$ exists, but not for Γ to the authors' knowledge, except in certain limits. For $m_\zeta - m_{3/2} \gg m_h$ (the regime in which this decay is most dominant):

$$\Gamma = \frac{c^2 m_\zeta^7}{7680 \pi^3 F^2} F_h \left[\frac{m_{3/2}}{m_\zeta} \right] \tag{D.18}$$

$$F_h(x) \equiv 1 - \frac{15}{4} x^2 - 10x^6 + 15x^8 - \frac{9}{4} x^{10} - 30x^6 \log x \tag{D.19}$$

Note that $F_h(1/2) \approx .29$.

[2] Note that this is *smaller* by a factor of about three than the equivalent calculation in Ref. [37], which used the goldstino equivalence theorem. In the $x \to 0$ limit, where the theorem is valid, our results agree, as expected.

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