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Tetraquarks, pentaquarks and dibaryons

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Summary. — The discovery of two pentaquarks by LHCb has reinforced the case of exotic hadrons, which have diquarks and antidiquarks as basic units. I review i) the cases studied until know, the so-called XYZ and pentaquark states, ii) the theoretical basis for this concept and iii) the implications for the existence of further states, in particular with baryon number equal to two (dibaryons).

Dedicated to Guido Altarelli, a friend for a lifetime.
We shared the privilege to see the unfolding of the Standard Theory.

1. – Introduction

For long, we lived with the simplest paradigm [1]

$$\text{mesons} = (q\bar{q}); \qquad \text{baryons} = (qqq). \tag{1}$$

This paradigm rested on the absence of $I = 2, \pi\pi$ resonances and of $S > 0$ baryons. The case had to be revisited because the lowest-lying octet of scalar mesons $f_0(980)$, $a_0(980)$, $\kappa(800)$ and $\sigma(600)$ does not fit in the picture. Later, in 2003, the $X(3872)$, a narrow width resonance, which decays into $J/\Psi + 2\pi/3\pi$, was discovered by Belle [2] and it was recognised that it does not fit into the charmonium sequence of states.

Since then, BaBar [3], CDF [4], D0 [5], CMS [6] and LHCb [7] have confirmed the $X(3872)$ and reported many other states that do not fit the charmonium picture, called $X(J^{PC} = 1^{++})$ and $Y(J^{PC} = 1^{--})$ states.

In 2007, Belle observed a charged charmonium [8], $Z^+(4430)$ decaying into $\psi(2S) + \pi$ that could not be interpreted as molecule, but later BaBar suggested [9] that it was simply a reflection of K* states. LHCb, in 2014, has confirmed [10] the $Z^+(4430)$ while other similar states, $Z^+(3900)$ and $Z^+(4020)$, have been discovered by BES III [11, 12]

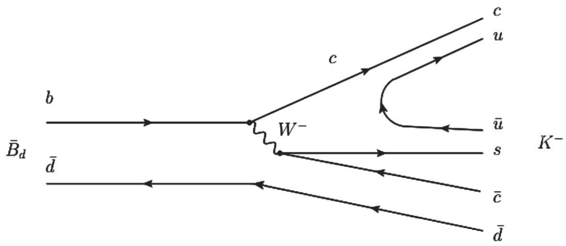

Fig. 1. – Quark diagram of the weak decay $B_d \to K^- + Z^+$ with a four valence quark $Z^+ = cu\bar{c}\bar{d}$.

and confirmed by BELLE [13] and by CLEO [14]. Two baryon resonances decaying in $\psi + p$, promptly called "pentaquarks", have been discovered by LHCb [15] in 2015.

Few words about the terminology of these exotic states. One distinguishes among X, Y and Z states as follows:

- X, *e.g.* $X(3872)$: neutral, typically seen in $J/\Psi +$ pions, $J^{PC} = 0^{++},\ 1^{++},\ 2^{++}$;
- Y, *e.g.* $Y(4260)$: neutral, seen in e^+e^- annihilation with Initial State Radiation, therefore $J^{PC} = 1^{--}$;
- Z, *e.g.* $Z(4430)$: charged/neutral, typically positive parity, four valence quarks manifest, mostly seen to decay in $J/\Psi + \pi$ and some in $h_c(1P) + \pi$; valence quarks: $c\,\bar{c}\,u\bar{d}$.

The existence of hadrons with a valence quark composition not fitting the paradigm (1) is by now established and the list of "unanticipated" hadrons is, for sure, bound to increase([1]). The theoretical interpretation is still unclear.

I will mainly restrict to the *compact tetraquark* model, explored in refs. [18-22]. For molecular and resonance models see refs. [23, 24]. Unlike tetraquarks, the latter speculations envisage effects due to the residual short-range forces generated by colorless meson exchange between color neutral objects.

Some authors propose X, Y, and Z structures to be only kinematic effects due to the opening of new channels, see *e.g.* [25]. However, it takes a lot of unconventional dynamics to produce the $X(3872)$ as a "cusp". Also, as we shall see, the phase of $Z(4430)$ seems to go at $90°$ at the peak, like a text-book Breit-Wigner resonance.

2. – Latest in exotics

The quark diagram in $\bar{B}_d = (b\bar{d})$ meson weak decay, to produce the four valence quark state $Z^+(4430)$ recoiling against a K^- is shown in fig. 1. The decay $Z^+(4430) \to \psi(2S)\pi^+$ should produce a line in the Dalitz plot of the energies of $\psi(2S)$, π^+ and K^-, corresponding to a peak in the invariant mass distribution $M_{\psi(2S)\pi}$. With high statistics, LHCb confirms BELLE's observation of a bump that cannot be built as the reflection of known K^* resonances.

([1]) A four valence quark resonance, $Z(5568) \to B_s + \pi$ with open beauty and strangeness has been reported by $D0$ [16], but later has not been confirmed by LHCb, with larger statistics; the issue of its existence is still pending. LHCb has just reported observation of several states with positive parity decaying into $J/\Psi\ \phi$ [17].

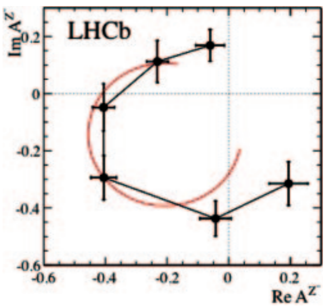

Fig. 2. – The Argand plot of the $Z(4430)$ amplitude, LHCb [10].

LHCb produces also the Argand plot of the phase of the resonant amplitude [10], shown in fig. 2: $Z(4430)$ looks definitely like a genuine resonance.

The molecular picture, proposed originally in [26], has been advanced to explain the nature of the $Z(4430)$ resonance. In this case, however, the loosely bound mechanism does not work as there are no open charm thresholds with $J^{PG} = 1^{++}$ quantum numbers at that mass. In [27] it is suggested that the $Z(4430)$ might be a $D^*(2010)\bar{D}_1(2420)$ bound state in S-wave, but this has $J^P = 0^-, 1^-, 2^-$, not consistent with the recent observations strongly suggesting $J^P = 1^+$. For the molecular picture see also [28, 29]. Other theoretical interpretations include $\Lambda_c\bar{\Sigma}_c$ baryonium [30], cusp effect [31], D_s radial excitation [32], as well as sum rules calculations based on the $D^*\bar{D}_1$ molecule [33, 34].

With a similar strategy, LHCb has focused on the weak decays of the baryon $\Lambda_b(5620)$, observing two pentaquark resonances in the mass distribution of the system $\psi(1S)\ p$, recoiling against a K^-.

The two observed resonances, $P_c^+(4380)$ and $P_c^+(4450)$, correspond to the pentaquark composition $[cu][ud]\bar{c}$ and parameters [15]

$$
\begin{aligned}
&P_c^+(4380) : M = 4380 \pm 8 \pm 29\,\text{MeV},\\
&\Gamma = 205 \pm 18 \pm 86\,\text{MeV}\ \left(\text{preferred fit: } J^P = \frac{3}{2}^-\right);\\
&P_c^+(4450) : M = 4449.8 \pm 1.7 \pm 2.5\,\text{MeV},\\
&\Gamma = 39 \pm 5 \pm 19\,\text{MeV}\ \left(\text{preferred fit: } J^P = \frac{5}{2}^+\right)
\end{aligned}
\tag{2}
$$

A clear resonant behaviour is observed for the narrow state, $P_c^+(4450)$, but more statistics will be needed to elucidate the other state.

3. – Attraction and repulsion in diquarks

In the non-relativistic limit, QCD forces and spin-spin interactions are attractive in the completely antisymmetric diquark

$$
[qq'] : SU(3)_{\text{color}} = \bar{\mathbf{3}},\ SU(3)_{\text{flavor}} = \bar{\mathbf{3}},\quad \text{spin} = 0.
\tag{3}
$$

This is the *good diquark* of Jaffe [35]. The result holds in perturbative QCD (one gluon exchange) and non-perturbative QCD (one instanton exchange). Such diquarks make a simple unit to form color singlet hadrons [36, 37].

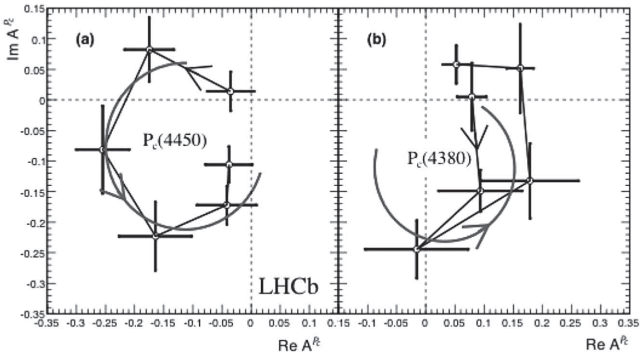

Fig. 3. – Argand plots of the $P_c^+(4380)$ and $P_c^+(4450)$ amplitudes, LHCb [15].

An argument for the relevance of the antysimmetric diquark in baryon spectroscopy goes back to Feynman [38]. In electron deep inelastic scattering at $x \to 1$ what recolis against the struck parton is the lowest mass configuration of two quarks. If one makes the hypothesis that this is a $[ud]$ diquark with $J = 0, I = 0$, the struck parton would be a d quark for the neutron and a u quark for the proton, and the ratio of the structure functions would simply be

$$\frac{F_n(x \to 1)}{F_p(x \to 1)} = \frac{Q_d}{Q_u}^2 = \frac{1}{4}, \tag{4}$$

in agreement with data. The lowest mass diquark of Feynman is indeed the *good* diquark (taken literally, Feynman's diquark conflicts with Fermi statistics, but this is solved by antisymmetry in color). An early discussion based on the Dolen-Horn-Schmidt duality [39] applied to baryon-(anti)baryon scattering and annihilation channels [40] led to the prediction of tetraquarks, called "baryonia", discussed in [41].

In the light hadrons, one may suppose that bad diquarks, flavor and spin symmetric, are dynamically unfavored (but they are needed to explain the difference between Σ and Λ baryons).

If we go to heavy-light diquarks such as $[cq]$, spin-spin interactions are reduced by the ratio m_q/m_c, and one may assume the spin 1, bad diquarks, and spin 0, good diquarks, to be on the same footing.

To form hadrons, the good or bad diquarks need to combine with other colored objects to form confined color singlets. The simplest possibilities are to combine:

- with one quark, to give a *baryon*, *e.g.* Λ;
- with an antidiquark, to give a *tetraquark*, *e.g.* $[\bar{c}\bar{u}][cd]$;
- with another diquark and an antiquark (both color $\bar{\mathbf{3}}$) to give a *pentaquark*, *e.g.* $\bar{c}[cu][ud]$;
- with two other diquarks to give a *dibaryon*, *e.g.* $[cd][ud][ud]$.

We expect many tetraquark states since the color string joining, *e.g.*, diquarks to antidiquarks may have radial and orbital excitations. In a relativisti picture, tetraquarks

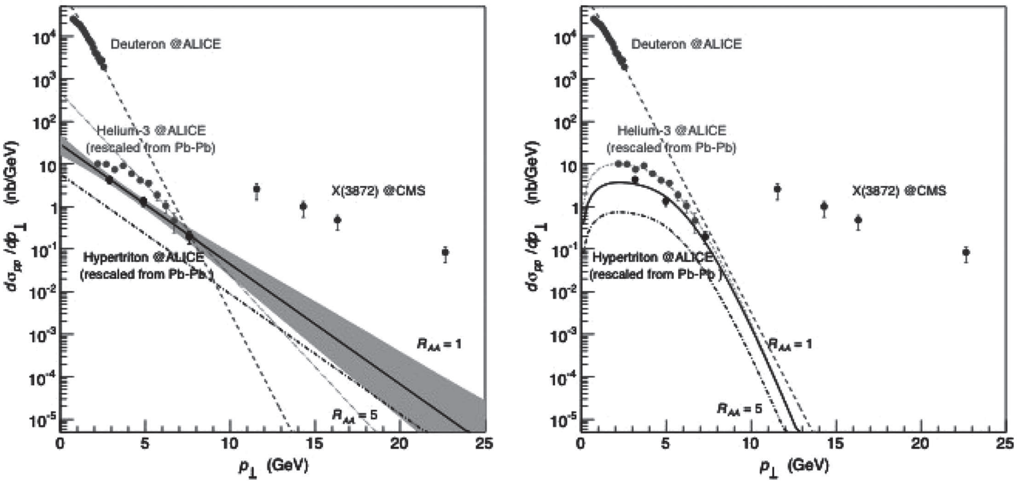

Fig. 4. – Proton-proton cross sections for the production of light nuclei, obtained by rescaling data in Pb-Pb collisions, compared to the $X(3872)$ proton-proton cross sections by CMS. Rescaling from Pb-Pb to proton-proton is done with: Glauber model (left panel) and blast-wave function (right panel) (R_{AA} or R_{CP} =1 assumed). Note that collective effects in Pb-Pb (*e.g.* quark-gluon plasma) that would give R_{AA}, $R_{CP} > 1$ would enhance nuclear cross sections and therefore reduce the cross sections of light nuclei rescaled to *p*-*p*.

would have to be on rising Regge trajectories, due to the confining nature of QCD forces [42].

We have already some evidence for the existence of orbital excitations in multiquark states, from the parities of the X, Y, Z mesons, compared to $q\bar{q}$ mesons, and from pentaquark parities, compared to those of the baryons.

The lowest-lying tetraquarks, X and Z, have positive parities, as expected for the S-wave diquark-antidiquark pair. Y states are higher in mass and have negative parities, as appropriate to diquark-antidiquarks in the P-wave. This is the opposite of what happens in normal mesons, where the lowest-lying $q\bar{q}$ S-wave states (π, ρ) have negative parity and the first orbital P-wave excitations (A_1, A_2, f_2, etc.) have positive parity.

Normal baryons in the S-wave have only quarks, hence positive parity, with the first excitations, *e.g.* $N(1520)$, with negative parity, corresponding to one unit of orbital momentum. On the other hand, the lowest pentaquark has negative parity, the signal of the antiquark, with the next state, presumably the first orbital excitation, with positive parity, the solution preferred by data.

Meson-meson molecules, on the other hand, are supposed to be bound by short-range forces, due to color singlet meson exchange. If bound at all, they should have a limited spectrum, in particular no orbital excitations.

4. – Light nuclei and antinuclei as candlesticks

Nuclei are obvious prototypes of hadronic molecules: color singlet protons and neutrons bound by short range forces, due to the exchange of color singlet pions and, to some extent, vector mesons. The recent ALICE observation [43, 44] of deuterons, tritons and hypertritons in high energy, high p_{perp}, Pb-Pb collisions opens the way to an interesting test of molecular *vs.* tetraquark theories of exotic hadrons, of the $X(3872)$ in particular.

In fig. 4, the light nuclei cross sections reported by ALICE [45] are rescaled to proton-proton cross section by two different methods (Glauber method, left panel, and blast-wave function method, right panel) and compared to the $X(3872)$ proton-proton cross section measured by CMS [46]. The difference in behavior of light nuclei *vs.* $X(3872)$ at large p_{perp} is quite spectacular, and a good indication that indeed $X(3872)$ is made of color parts confined by long-range color forces, similarly to what happens in normal mesons.

5. – X, Y, Z tetraquarks

We consider heavy-light diquarks bound to a similar antidiquark in the S-wave. Such states have $I = 1, 0$, positive parity, and angular momentum resulting from the composition on the diquark-antidiquark spins, $s, \bar{s} = 1, 0$. The neutral states are mixtures of isotriplet and isosinglet.

5·1. $L = 0$, $1S$ *states.* – In the $|s, \bar{s}\rangle_J$ basis we have the following states [18]:

$$J^P = 0^+ \quad C = + \quad X_0 = |0,0\rangle_0,\ X_0' = |1,1\rangle_0, \tag{5}$$

$$J^P = 1^+ \quad C = + \quad X_1 = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 + |0,1\rangle_1\right), \tag{6}$$

$$J^P = 1^+ \quad G = + \quad Z = \frac{1}{\sqrt{2}}\left(|1,0\rangle_1 - |0,1\rangle_1\right),\ Z' = |1,1\rangle_1, \tag{7}$$

$$J^P = 2^+ \quad C = + \quad X_2 = |1,1\rangle_2. \tag{8}$$

We identify

$$\begin{aligned} &X(3872) = X1,\\ &Z(3900), Z(4020) = \text{linear combinations of } Z, Z' \text{ that diagonalize } H,\\ &X(3940) = X2\ (??). \end{aligned} \tag{9}$$

In the non-relativistic constituent quark model, mass differences inside the multiplet are due to spin-spin interactions of the form

$$H = 2M_{diquark} + 2\sum_{i<j} \kappa_{ij}(\vec{s}_i \cdot \vec{s}_j). \tag{10}$$

A tentative mass spectrum for S-wave tetraquarks was derived in [18], assuming the strength of the spin-spin interactions in tetraquarks to be the same as those in S-wave mesons and baryons. However, if this was the case, light quark $\kappa_{q\bar{q}}$ would dominate, predicting one Z degenerate with $X(3872)$, which is good news, but also $M(Z') < M(Z)$, which does not agree with the level ordering of $X(3872)$, $Z(3900)$ and $Z(4020)$.

A new, simple ansatz can explain the observed spectrum, namely the hypothesis that the dominant interactions in tetraquarks are the spin-spin interactions between quarks (antiquarks) in the same diquark (antidiquark). In this case, the Hamiltonian (10) becomes

$$\begin{aligned} H_{new} &\approx 2M_{diquark} + 2\kappa_{qc}\ (\mathbf{s}_q \cdot \mathbf{s}_c + \mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{c}}) =\\ &= 2M_{diquark} + \kappa_{qc}\left[s(s+1) + \bar{s}(\bar{s}+1) - 3\right]. \end{aligned} \tag{11}$$

H_{new} is diagonal in the basis of eqs. (5) to (8) and it would simply count the number of spin=1 diquarks in the hadron. We see from (6) and (7) that $X(3872)$ and $Z(3900)$ have one spin-1 diquark, hence are degenerate, while $Z(4020)$ has two spin-1 diquarks, hence it is heavier. The multiplet mass spectrum is determined by just two numbers, in this approximation, the diquark mass and the spin-spin coupling κ_{cq}. From the masses of $X(3872)$, $Z(3900)$ and $Z(4020)$ we find

$$M_{cq} = 1980\,\text{MeV}; \quad \kappa_{cq} = 67\,\text{MeV}. \tag{12}$$

We note that κ_{cq} in the diquark is about three times larger than the corresponding spin-spin interaction obtained from a fit to the charmed baryon masses, given *e.g.* in [18]. Since κ is expected to be proportional to the overlap probability of the two quarks, $|\psi(0)|^2$, this could indicate a rather compact diquark, which would also go along with a small value of the quark-antiquark overlap probability, since in this case the pair would be made by particles in different bags.

In this approximation, X_0' and X_2 would be almost degenerate with $Z(4020)$, stretched but not impossible for the spin-2 candidate $X(3940)$, and the lightest scalar would be around 3800 MeV.

Needless to say, with only three masses available, one cannot go beyond this, crude, zeroth-order approximation. Determination of the other couplings inside tetraquarks will be possible when more experimental information will be available.

5˙2. $L = 0$, $2S$ *states*. – In 2007 we classified the $Z(4430)$ as the radial excitation of an S-wave tetraquark with $J^{PC} = 1^{+-}$, the almost degenerate companion of $X(3872)$ [19]. This was because its mass is about 530 MeV larger than the $X(3872)$ mass(²) and because of its preference to decay into $\psi(2S)+\pi$, see also [47] for theoretical arguments supporting the $2S$ assignment. We noted in [19] that *a crucial consequence of a* $Z(4430)$ *charged particle is that a charged state decaying into* $\psi(1S)\pi^{\pm}$ *or* $\eta_c\rho^{\pm}$ *should be found around* 3880 *MeV*, almost degenerate with $X(3872)$. The $Z_c(3900)$ has been seen by BES III few years later and confirmed by Belle and CLEO, with the anticipated decay

$$Z^{+}(3900) \to \psi(1S)\pi^{+}. \tag{13}$$

If the radial excitation hypothesisis of the $Z(4430)$ is correct, one should find nearby the radial excitation of $X(3872)$ and the the other 2S Z, X_0, X_0', X_2 states, about which we may advance now quite definite mass predictions.

5˙3. $J^{PC} = 1^{--}$, Y *states as the first orbital excitations*. – Several 1^{--} unexpected resonances have been reported in the literature. Our survey [21] includes six candidate states, some well established, like the $Y(4260)$, the first one to be discovered, and others still preliminary

- $Y(4660)$ and $Y(4360)$, decaying into $\psi(2S) + \pi$;
- $Y(4630)$ decaying into $\Lambda_c\bar{\Lambda}_c$;

(²) Radial excitation energies in S-wave quarkonia: $M[\psi(2S)] - M[\psi(1S)] = 539\,\text{MeV}$, $M[\Upsilon(2S)] - M[\Upsilon(1S)] = 563\,\text{MeV}$.

TABLE I. – *Spin composition of $L = 1$ tetraquarks with $J^{PC} = 1^{--}$. In columns 2 and 3 the probabilities of $c\bar{c}$ spin $= 1, 0$.*

	Spin composition:$\vert(s,\bar{s})_S, L >_J$	$P(s_{c\bar{c}} = 1)$	$P(s_{c\bar{c}} = 0)$	assign.
Y_1	$\vert(0,0)_0, 1\rangle_1$	0.75	0.25	$Y(4008)$
Y_2	$\frac{1}{\sqrt{2}}\{\vert(1,0)_1, 1\rangle_1 + \vert(0,1)_1, 1\rangle_1\}$	1	0	$Y(4260)$
Y_3	$\vert(1,1)_0, 1\rangle_1$	0.25	0.75	$Y(4230)$
Y_4	$\vert(1,1)_2, 1\rangle_1$	1	0	$Y(4630)$

- $Y(4220)$, narrow (and $Y(4290)$, wide ???), observed by BES III in $h_c(1P)+2\pi$ and, possibly, in $\chi_c(1P)+\omega$;
- $Y(4260)$ and $Y(4008)$ decaying into $J/\psi + \pi$.

Tetraquark states with $J^{PC} = 1^{--}$ can be obtained with odd values of the orbital angular momentum $L = 1,\ 3$ and diquark and antidiquark spins $s,\ \bar{s} = 0, 1$. We denote these states as

$$|s, \bar{s}, L = 1\rangle_{J=1} \tag{14}$$

Using charge conjugation invariance we get four states with $L = 1$, $J = 1$, as shown in table I.

We have left aside the $L = 3$ state, which is presumably too heavy.

To identify the Y resonances corresponding to $Y_{1\text{–}4}$, we proceed as follows.

- We discard $Y(4360)$ and $Y(4660)$, which are probably radial excitations of $Y(4008)$ and $Y(4260)$: they decay into $\psi(2S)$ and have mass differences $\Delta M \sim 350, 400$ MeV, in the range of ΔM of $L = 1$ charmonia and bottomonia([3]);
- the states Y_{1-4} are identified with the remaining $Y(4008)$, $Y(4260)$, $Y(4220)$ (the narrow structure in the h_c channel) and $Y(4630)$.

There would be no place for the wide structure $Y(4290)$, should it exist at all.

In the third and fourth column of table I we give the probabilities to find the $c\bar{c}$ pair in the spin states $s_{c\bar{c}} = 0, 1$. In the limit $m_c \to \infty$, the heavy-quark spin is conserved and the value of $s_{c\bar{c}}$ provides a relevant selection rule. The assignment $Y(4260) = Y_2$ is compatible with the observed decay in $J/\Psi + 2\pi$ and $Y(4230) = Y_3$ is compatible with the observation of both decays: $h_c\ \pi^+\pi^-$ [49] and $\chi_c\ \omega$ [50].

The identical spin structure implied in the model for $Y(4260)$ and $X(3872)$ suggests the observed decay [51]

$$Y(4260) \to X(3872) + \gamma \tag{15}$$

to be an unsuppressed $E1$ transition [52], with $\Delta L = 1$ and $\Delta\text{Spin} = 0$, similar to the observed transitions of P-wave χ states.

([3]) Radial excitation energies in P-wave quarkonia: $\chi_{cJ}(2P) - \chi_{cJ}(1P) \approx 437$ MeV, $\chi_{bJ}(2P) - \chi_{bJ}(1P) \approx 360$ MeV, see *e.g.* [48].

5˙4. *Mass formulae for* $L = 1$. – In the spirit of a first exploration, we add to the Hamiltonian of S-wave tetraquarks an orbital term proportional to $\mathbf{L}^2$ and a spin-orbit interaction proportional to $\mathbf{L} \cdot \mathbf{S}$. The restriction of the spin-spin couplings to the interaction within the same diquark, as discussed before, is more than justified here, due to the angular momentum barrier, and we leave open the possibility that the coupling may take a different value from the S-wave case.

We write

$$M = M_{00} + B_c \frac{\mathbf{L}^2}{2} - 2a\ \mathbf{L} \cdot \mathbf{S} + 2\kappa'_{qc}\ [(\mathbf{s}_q \cdot \mathbf{s}_c) + (\mathbf{s}_{\bar{q}} \cdot \mathbf{s}_{\bar{c}})]. \tag{16}$$

Signs are chosen so that, for B_c, a, κ positive, energy increases for increasing $\mathbf{L}^2$ and $\mathbf{S}^2$. With obvious manipulations, we obtain

$$\begin{aligned} M = M_{00} + B_c \frac{L(L+1)}{2} + a\ [L(L+1) + S(S+1) - 2] \\ + \kappa'_{qc}\ [s(s+1) + \bar{s}(\bar{s}+1) - 3]\,, \end{aligned} \tag{17}$$

namely

$$M = M_0 + \left(\frac{B_c}{2} + a\right) L(L+1) + a\ S(S+1) + \kappa'_{qc}\ [s(s+1) + \bar{s}(\bar{s}+1)], \tag{18}$$

where

$$M_0 = M_{00} - 2a - 3\kappa'_{qc}. \tag{19}$$

With four masses and three parameters, we find the relation

$$M_2 = \frac{3M_1 + M_3 + 2M_4}{6}. \tag{20}$$

The above formulae require $M_2 > M_1$ and $M_4 > M_3$, however the sign of the mass difference $M_3 - M_2$ can take either sign, as it is determined by the difference of two constants which are *a priori* of a similar size.

We keep fixed the assignments

$$Y(4260) = Y_2 = \frac{1}{\sqrt{2}}(|1, 0; 1, 1\rangle_1 + |0, 1; 1, 1\rangle_1) \tag{21}$$

and $Y_3 = Y(4220)$, see ref. [21] for more details.

From the mass relation above one obtains

$$(M_2)_{\rm th} = 4251\,\text{MeV} \tag{22}$$

and the value of the parameters

$$a = 73,\ \kappa'_{qc} = 53 \tag{23}$$

The value found for κ'_{qc} is close to the value in (12), supporting the difference between diquarks in tetraquarks and diquarks in baryons.

The spin structure of $Y(4260)$ and $X(3872)$ and their spin interactions being exactly the same, we may obtain the energy of the orbital excitation directly from their mass difference. Starting from eq. (17), neglecting the difference between κ_{qc} and κ'_{qc} and using eq. (19) we obtain

$$M(Y_2) = M(X) + B_c + 2a, \;\; i.e. \qquad B_c = 278\,\text{MeV}. \tag{24}$$

This value compares well with values found in normal hadrons.

Finally, a large separation between $Y_{L=3}$ and the states $Y_{1\text{–}4}$ is implied

$$M_{L=3} = M_2 + 5B_c + 14a \sim 6420 \text{ MeV}. \tag{25}$$

6. – A closer look at pentaquarks

The discovery of pentaquarks has prompted a considerable flux of theoretical papers, where ideas previously developed for X, Y and Z mesons have been extended to the new particles: compact diquark-diquark-antiquark [53], diquark-triquark [54], molecules and resonances [55], baryocharmonia [56].

The observed decay

$$\mathbf{P}^+ \to J/\Psi + p \tag{26}$$

indicates that the new particle features the valence quark composition

$$\mathbf{P}^+ = \bar{c}cuud. \tag{27}$$

Pentaquarks realizing the valence quark structure (27) are of two types,

$$\mathbf{P}_u = \epsilon^{\alpha\beta\gamma}\; \bar{c}_\alpha\, [cu]_{\beta,s=0,1}\, [ud]_{\gamma,s=0,1}, \tag{28}$$

$$\mathbf{P}_d = \epsilon^{\alpha\beta\gamma}\; \bar{c}_\alpha\, [cd]_{\beta,s=0,1}\, [uu]_{\gamma,s=1}, \tag{29}$$

where greek indices are for color, diquarks are in the color antisymmetric $\bar{\mathbf{3}}$ configuration and overall antisymmetry requires flavor symmetric light-light diquark to have $s = 1$.

There are two possible quark amplitudes leading to the pentaquark production in Λ_b decay, see fig. 5.

In (a), the b-quark spin is shared between the kaon and the $\bar{c}$ and $[cu]$ components. Barring angular momentum transfer due to gluon exchanges between the light diquark and light quarks from the vacuum, the final $[ud]$ diquark has to have spin zero, therefore $I = 0$.

In diagram b, the $[ud]$ diquark in $\mathbf{P}_u$ is formed from the original d quark and the u quark from the vacuum. Angular momentum is shared among all final components and the $[ud]$ diquark may well have spin one and $I = 1$. In addition, in diagram (b) a $[uu]_{s=1}$ diquark could also be formed, so both pentaquarks $\mathbf{P}_d$ and $\mathbf{P}_u$ can be produced in Λ_b decay.

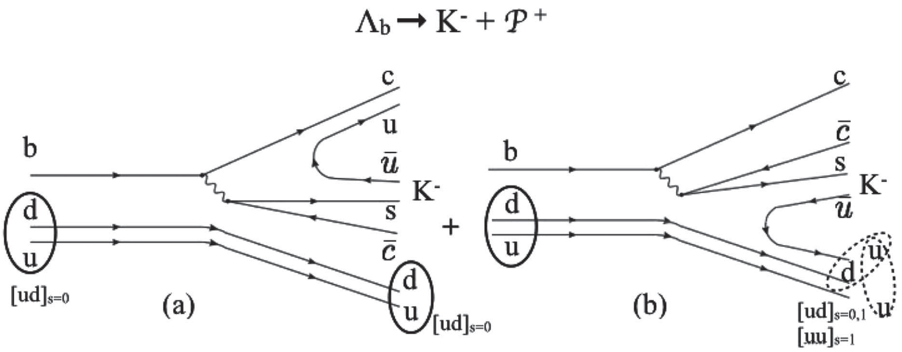

Fig. 5. – (a) The $[ud]$, spin-zero diquark in Λ_b is transmitted to the $\mathbf{P}_u$-type pentaquark; (b) the u quark from the vacuum participates in the formation of the light-light diquark: spin zero and one are permitted; amplitude (b) may produce a $[uu]_{s=1}$ diquark, giving rise to a $\mathbf{P}_d$ pentaquark.

6˙1. *The mass difference*. – The discussion about production amplitudes has a bearing on the issue of the mass difference between the two observed pentaquarks.

Positive parity indicates that the heavier pentaquark should be an $L = 1$ orbital excitation.

At first sight, the 70 MeV difference does not go well with the energy associated with orbital excitations. One orbital excitation in mesons and baryons carries an energy difference which is typically of order 300 MeV, as exemplified by the mass difference in $\Lambda(1405) - \Lambda(1116) \sim 290$ MeV. Mass formulae for the orbital excitation in X, Y, Z mesons, as discussed in [21], lead to $\Delta M(L = 0 \to 1) \sim 280$ MeV, see eq. (24).

However, the mass difference between light-light diquarks with spin $s = 1, 0$ estimated from charm and beauty baryon spectra [57], is of order 200 MeV, *e.g.* $\Sigma_c(2455) - \Lambda_c(2286) \simeq 170$ MeV, $\Sigma_b(5811) - \Lambda_b(5620) \simeq 190$ MeV.

If we assume the compositions

$$\begin{aligned} \mathbf{P}(3/2^-) &= \{\bar{c}\,[cq]_{s=1}[q'q'']_{s=1}, L = 0\}, \\ \mathbf{P}(5/2^+) &= \{\bar{c}\,[cq]_{s=1}[q'q'']_{s=0}, L = 1\}, \end{aligned} \tag{30}$$

the orbital gap is reduced to about 100 MeV, which brings back the mass difference in the range of the mass difference indicated in (2).

6˙2. *Flavor SU(3) structure*. – Extending to flavor $SU(3)$ the structure in (28) and (29), we generate two distinct series of pentaquarks according the light-light diquark symmetry

$$\begin{aligned} \mathbf{P}_A &= \epsilon^{\alpha\beta\gamma}\,\{\bar{c}_\alpha\,[cq]_{\beta,s=0,1}\,[q'q'']_{\gamma,s=0}, L\} \\ &= \mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}, \end{aligned} \tag{31}$$

$$\begin{aligned} \mathbf{P}_S &= \epsilon^{\alpha\beta\gamma}\,\{\bar{c}_\alpha\,[cq]_{\beta,s=0,1}\,[q'q'']_{\gamma,s=1}, L\} \\ &= \mathbf{3} \otimes \mathbf{6} = \mathbf{8} \oplus \mathbf{10}. \end{aligned} \tag{32}$$

For S-waves, the first and second series give the angular momenta

$$\mathbf{P}_A(L=0): \; J = 1/2\ (2), 3/2\ (1), \tag{33}$$

$$\mathbf{P}_S(L=0): \; J = 1/2\ (3), 3/2\ (3), 5/2\ (1) \tag{34}$$

(in parenthesis the multiplicity of each spin value). In consideration of (30), we propose to assign the $3/2^-$ and the $5/2^+$ states to the symmetric and antisymmetric series, respectively.

A discussion of the production of octet and decuplet pentaquarks in the decays of strange and doubly strange bottom baryons and of their possible decays into charmed baryons and mesons can be found in [53].

7. – Dibaryons

As discussed previously, color antisymmetric diquarks can replace anti-quarks in a color singlet hadron, to give conventional, but also unconventional new hadrons.

Starting from the pentaquark of the previous section, $\bar{c}[cq_1][q_2q_3]$, and making the substitution $\bar{c} \to [q_4q_5]$, we obtain a charm-one *dibaryon*, a $B = 2$ color bound alternative to the deuteron, and all its strange, doubly charmed, etc. variations. It seems a reasonable possibility that tetraquarks, pentaquarks and dibaryons make the next layer of hadron spectroscopy following the first layer made by the Gell-Mann–Zweig baryons and mesons.

Dibaryons were envisaged by Jaffe [58,59] to bind 6 quarks in a stable 0^+ flavor singlet at a mass of about 2000 MeV (called a H-dihyperon, later dibaryon). For a recent lattice QCD study of baryon-baryon interactions see [60].

Dibaryons at about 2 GeV have been considered in a number of papers, usually as 6-quark states in a MIT bag, see [61, 62]. Diquarks have been used by Jaffe and Wilczek [59] to describe complex hadron structures like the (later disproved) "old pentaquark".

The lightest charmed dibaryon may be observed in Λ_b decay, already a source of pentaquarks, see fig. 6. We start with the Cabibbo allowed decay, adding two light pairs from the vacuum

$$\Lambda_b(bud) \to cd\bar{u} + ud + (u\bar{u}d\bar{d})_{\rm vac},$$

that gives

$$\Lambda_b \to \bar{p} + [cd][ud][ud] = \bar{p} + \mathbf{D}_c^+, \tag{35}$$

$$M(\mathbf{D}_c^+) < 4682\,\text{MeV}. \tag{36}$$

The antiproton should be a good indicator to select the interesting events, and, judging from the cq diquark mass given in (12), the phase space limit in (36) should be amply satisfied for the single-charm dibaryon.

The decays of the dibaryon determine its visibility. The decay of the charmed dibaryon, $\mathcal{D}_c^+$, may take different routes, according to its mass and in relation to pentaquark masses. The preferred decay would be by string breaking, into a baryon plus a pentaquark. However it is possible that this route is forbidden by energy conservation, even for the lightest, spin-1/2 pentaquarks. Indeed, the known X, Y, Z, with the

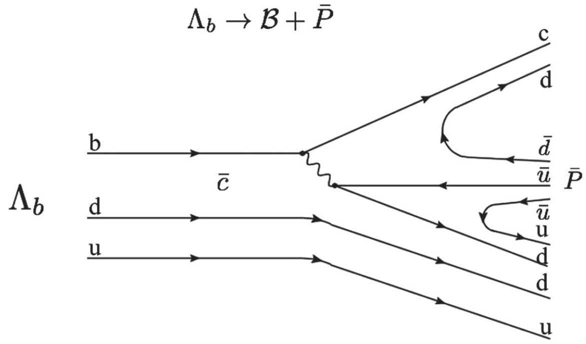

Fig. 6. – Quark diagram of the weak decay $\Lambda_d \to \bar{P} + \mathcal{D}^+$ with a six valence quark dibaryon $\mathcal{D}^+ = [cd][ud][ud]$.

exception of $Y(4630)$ [63], do not decay into baryon-antibaryon pairs (string breaking) but rather into charmonium plus meson (quark rearrangement). Similarly, the observed pentaquarks do not decay into the channels preferred by string breaking, such as $X(Y)$ plus proton, forbidden by energy conservation, but in the quark rearrangement channel, $J/\Psi + p$.

7˙1. *Quark rearrangement.* – For analogy with the observed tetraquark and pentaquark decay, we put in the first line the quark rearrangement decays

$$\begin{aligned} &\mathcal{D}_c^+ = [cd][ud][ud] \to \\ &\to p + \Sigma_c^0(\to p + \Lambda_c^+ + \pi^-), \quad \text{or } n + \Lambda_c^+. \end{aligned} \tag{37}$$

Note the occurrence of Σ_c^0 in the first decay, necessary if a proton is required in the final state, rather than a neutron, more difficult to see.

7˙2. *String breaking.* – Breaking one color string by a $u\bar{u}$ pair, a possible decay path is

$$\mathcal{D}_c^+ \to p + \mathcal{P}_c^0(\bar{u}[cd][ud]), \tag{38}$$

with the final charmed pentaquark decaying as

$$\mathcal{P}_c^0 \to \Lambda_c^+ + \pi^- \quad \text{or} \quad \mathcal{P}_c^0 \to n + D^0. \tag{39}$$

Another experimental signature is obtained with a $s\bar{s}$ pair from the vacuum, replacing step (38) by

$$\mathcal{D}_c^+ \to \Lambda + \mathcal{P}_{c\bar{s}}^+(\bar{s}[cd][ud]), \tag{40}$$

followed by

$$\mathcal{P}_{c\bar{s}}^+ \to K^0 + \Lambda_c^+. \tag{41}$$

7·3. *Overall* Λ_b *decay chains.* – Discarding decay channels with a neutron, the interesting Λ_b decay chain in (37) and (39) is

$$\Lambda_b \to \bar{p} + p + \Lambda_c^+ + \pi^-, \quad M(\mathcal{D}_c^+) > 3364\,\text{MeV}, \tag{42}$$

with (37) and (39) distinguished by the occurrence of a pentaquark resonance or of the Σ_c^0 in the $\Lambda_c^+\pi^-$ channel.

The case (40) leads to

$$\begin{aligned} &\Lambda_b \to \bar{p} + \Lambda + \Lambda_c^+ + K^0, \\ &M(\mathcal{D}_c^+) > M(\Lambda) + M(\mathcal{P}_{c\bar{s}}^+) > 3901\,\text{MeV}. \end{aligned} \tag{43}$$

7·4. *Semileptonic decays.* – For dibaryon mass below the limit in (42), the β-decay of the charm quark allows the dibaryon to transform into uncharmed baryon pairs with strangeness $S = -1, 0$ according to

$$\begin{aligned} &\mathcal{D}_c^+ \to e^+ + \nu_e + \Sigma^- + p, \\ &M(\mathcal{D}_c^+) > 2136\,\text{MeV (Cabibbo allowed)} \end{aligned} \tag{44}$$

or

$$\begin{aligned} &\mathcal{D}_c^+ \to e^+ + \nu_e + \Delta^- + p, \\ &M(\mathcal{D}_c^+) > 2174\,\text{MeV (Cabibbo forbidden)} \end{aligned} \tag{45}$$

For lower masses, the lightest charmed dibaryon is stable.

8. – Outlook

New data from electron positron and high-energy hadron colliders have brought new exotic resonances, more information and some clarification.

Coincidence of exotic hadrons with thresholds is less and less evident, in particular with $Z(4430)$ and pentaquarks. The simple molecular model is disfavoured by ALICE data on light-nuclei production in Pb-Pb, compared to $X(3872)$ production in proton-proton as reported by CMS.

Diquarks seem to be a useful organising principle to classify the structure of exotic mesons, pentaquarks and yet-to-be-discovered dibaryons;

Dibaryons can be searched for in Λ_b decays for a wide range of masses, from 4680 down to 2135 MeV; if found, dibaryons would complete a second layer of hadron spectroscopy, following the Gell-Mann–Zweig layer and complete the saturation possibilities of one and three QCD strings.

Much remains to be done, in theory and experiments.

Many states are lacking. The shopping list includes the charged counterpart of the $X(3272)$, the radial excitation of $X(3872)$, to be found near the $Z(4430)$, and the b-quark analog of $X(3872)$. To a finer level, Y resonances should be doubled, similarly to the $\omega^0 - \rho^0$ complex. We do not know yet if the present ignorance reflects a substantial failure of the model, or the action of some, still unidentified, selection rule [64] or simply the lack of more precise data.

The exotics seen until now all contain heavy-quark flavours: an experimental reexamination of the lack of existence of light exotic mesons ("bad" diquarks) and positive-strangeness baryons is in order.

The goal of these studies is ambitious: to provide new clues to the understanding of QCD in the fully non-perturbative regime. The continuing flux of new discoveries leads to think that exciting times for hadron spectroscopy are still ahead of us.

* * *

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REFERENCES

[1] Gell-Mann M., *Phys. Lett.*, **8** (1964) 214; Zweig G., *An SU(3) model for strong interaction symmetry and its breaking. Version 2,* in *Developments in the Quark Theory of Hadrons, Volume 1*, edited by Lichtenberg D. and Rosen S. (Hadromic Press, Nomantum, MA) 1980, pp. 22–101.

[2] Belle Collaboration (Choi S. K. *et al.*), *Phys. Rev. Lett.*, **91** (2003) 262001.

[3] BaBar Collaboration (Aubert B. *et al.*), *Phys. Rev. D*, **71** (2005) 071103.

[4] CDF Collaboration (Acosta D. *et al.*), *Phys. Rev. Lett.*, **93** (2004) 072001.

[5] D0 Collaboration (Abazov V. M. *et al.*), *Phys. Rev. Lett.*, **93** (2004) 162002.

[6] CMS Collaboration (Chatrchyan S. *et al.*), *JHEP*, **1304** (2013) 154.

[7] LHCb Collaboration (Aaij R. *et al.*), *Eur. Phys. J. C*, **72** (2012) 1972.

[8] Belle Collaboration (Choi S. K. *et al.*), *Phys. Rev. Lett.*, **100** (2008) 142001.

[9] BaBar Collaboration (Aubert B. *et al.*), *Phys. Rev. D*, **79** (2009) 11200.

[10] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **112** (2014) 222002.

[11] BESIII Collaboration (Ablikim M. *et al.*), *Phys. Rev. Lett.*, **110** (2013) 252001.

[12] BESIII Collaboration (Ablikim M. *et al.*), *Phys. Rev. Lett.*, **111** (2013) 242001.

[13] Belle Collaboration (Liu Z. Q. *et al.*), *Phys. Rev. Lett.*, **110** (2013) 252002.

[14] Xiao T., Dobbs S., Tomaradze A. and Seth K. K., *Phys. Lett. B*, **727** (2013) 366.

[15] LHCb Collaboration (Aaij R. *et al.*), *Phys. Rev. Lett.*, **115** (2015) 072001.

[16] D0 Collaboration (Abazov V. M. *et al.*), arXiv:1602.07588 [hep-ex].

[17] LHCb Collaboration (Aaij R. *et al.*), arXiv:1606.07895 [hep-ex].

[18] Maiani L., Piccinini F., Polosa A. D. and Riquer V., *Phys. Rev. D*, **71** (2005) 014028.

[19] Maiani L., Polosa A. D. and Riquer V., *New J. Phys.*, **10** (2008) 073004.

[20] Light diquarks were first considered as the building blocks of pentaquarks in Jaffe R. L. and Wilczek F., *Phys. Rev. Lett.*, **91** (2003) 232003; See also Jaffe R. and Wilczek F., *Eur. Phys. J. C*, **33** (2004) S38; Wilczek F., in *From Fields to Strings, Vol. 1*, edited by Shifman M. *et al.* (World Scientific, Singapore) 2005, pp. 77–93 arXiv: hep-ph/0409168; Selem A. and Wilczek F., arXiv:hep-ph/0602128.

[21] Maiani L., Piccinini F., Polosa A. D. and Riquer V., *Phys. Rev. D*, **89** (2014) 114010.

[22] Ali A., Hambrock C., Ahmed I. and Aslam M. J., *Phys. Lett. B*, **684** (2010) 28; Ali A., Hambrock C. and Aslam M. J., *Phys. Rev. Lett.*, **104** (2010) 162001; **107** (2011) 049903(E).

[23] One of the first contribuition on loosely bound molecules in this context can be found in Tornqvist N. A., *Z. Phys. C*, **61** (1994) 525; based on Tornqvist N. A., *Phys. Rev. Lett.*, **67** (1991) 556; Meson-meson molecules were also studied in Manohar A. V. and Wise M. B., *Nucl. Phys. B*, **399** (1993) 17; although there it is concluded that the meson-antimeson sector has annihilation channels not allowing stable (narrow) bound states. The heavy-quark spin structure in molecules is discussed in Bondar A. E., Garmash A., Milstein A. I., Mizuk R. and Voloshin M. B., *Phys. Rev. D*, **84** (2011) 054010 along with Guo F. K., Hanhart C. and Meissner U. G., *Phys. Rev. Lett.*, **102** (2009) 242004.

[24] See also Cleven M., Guo F. K., Hanhart C., Wang Q. and Zhao Q., *Phys. Rev. D*, **92** (2015) 014005.
[25] Swanson E. S., *Int. J. Mod. Phys. E*, **25** (2016) 1642010 and references therein.
[26] De Rujula A., Georgi H. and Glashow S. L., *Phys. Rev. Lett.*, **38** (1977) 317.
[27] Rosner J. L., *Phys. Rev. D*, **76** (2007) 114002.
[28] Meng C. and Chao K.-T., arXiv:0708.4222 [hep-ph]; Branz T., Gutsche T. and Lyubovitskij V. E., *Phys. Rev. D*, **82** (2010) 054025.
[29] Wang Q., Hanhart C. and Zhao Q., *Phys. Rev. Lett.*, **111** (2013) 132003; Wang Q., Hanhart C. and Zhao Q., *Phys. Lett. B*, **725** (2013) 106.
[30] Qiao C.-F., *J. Phys. G*, **35** (2008) 075008.
[31] Bugg D. V., *J. Phys. G*, **35** (2008) 075005.
[32] Matsuki T., Morii T. and Sudoh K., *Phys. Lett. B*, **669** (2008) 156.
[33] Lee S. H., Mihara A., Navarra F. S. and Nielsen M., *Phys. Lett. B*, **661** (2008) 28.
[34] Bracco M. E., Lee S. H., Nielsen M. and Rodrigues da Silva R., *Phys. Lett. B*, **671** (2009) 240.
[35] Jaffe R. L., *Phys. Rep.*, **409** (2005) 1.
[36] Rajagopal K. and Wilczek F., in *At the Frontier of Particle Physics*, Vol. 3, edited by Shifman M. (World Scientific, Sinagapore) 2001, pp. 2061–2151, arXiv:hep-ph/0011333.
[37] Rapp R., Schafer T., Shuryak E. V. and Velkovsky M., *Phys. Rev. Lett.*, **81** (1998) 53.
[38] Feynman R. P., *Phton-Hadron Interaction* (Advanced Book Classics, Westview, 1989).
[39] Dolen R., Horn D. and Schmid C., *Phys. Rev.*, **166** (1968) 1768.
[40] Rossi G. C. and Veneziano G., *Nucl. Phys. B*, **123** (1977) 507.
[41] Montanet L., Rossi G. C. and Veneziano G., *Phys. Rep.*, **63** (1980) 149.
[42] Sonnenschein J. and Weissman D., arXiv:1606.02732 [hep-ph].
[43] ALICE Collaboration (Adam J. *et al.*), *Phys. Lett. B*, **754** (2016) 360.
[44] ALICE Collaboration (Adam J. *et al.*), *Phys. Rev. C*, **93** (2016) 024917.
[45] Esposito A., Guerrieri A. L., Maiani L., Piccinini F., Pilloni A., Polosa A. D. and Riquer V., *Phys. Rev. D*, **92** (2015) 034028.
[46] CMS Collaboration (Chatrchyan S. *et al.*), *JHEP*, **1304** (2013) 154.
[47] Brodsky S. J., Hwang D. S. and Lebed R. F., *Phys. Rev. Lett.*, **113** (2014) 112001.
[48] He L. P., Chen D. Y., Liu X. and Matsuki T., *Eur. Phys. J. C*, **74** (2014) 3208.
[49] Yuan C. Z., *Chin. Phys. C*, **38** (2014) 043001, arXiv:1312.6399 [hep-ex].
[50] BESIII Collaboration (Ablikim M. *et al.*), *Phys. Rev. Lett.*, **114** (2015) 092003.
[51] Radiative transition
[52] Chen H.-X., Maiani L., Polosa A. D. and Riquer V., *Eur. Phys. J. C*, **75** (2015) 550.
[53] Maiani L., Polosa A. D. and Riquer V., *Phys. Lett. B*, **749** (2015) 289.
[54] Lebed R. F., *Phys. Lett. B*, **749** (2015) 454; Zhu R. and Qiao C. F., *Phys. Lett. B*, **756** (2016) 259.
[55] Guo F. K., Meiner U. G., Wang W. and Yang Z., *Phys. Rev. D*, **92** (2015) 071502; Liu X. H., Wang Q. and Zhao Q., *Phys. Lett. B*, **757** (2016) 231; Mikhasenko M., arXiv:1507.06552 [hep-ph]; Meiner U. G. and Oller J. A., *Phys. Lett. B*, **751** (2015) 59; Chen H. X., Chen W., Liu X., Steele T. G. and Zhu S. L., *Phys. Rev. Lett.*, **115** (2015) 172001; He J., *Phys. Lett. B*, **753** (2016) 547; Roca L., Nieves J. and Oset E., *Phys. Rev. D*, **92** (2015) 094003; Chen R., Liu X., Li X. Q. and Zhu S. L., *Phys. Rev. Lett.*, **115** (2015) 132002.
[56] Kubarovsky V. and Voloshin M. B., *Phys. Rev. D*, **92** (2015) 031502.
[57] De Rujula A., Georgi H. and Glashow S. L., *Phys. Rev. D*, **12** (1975) 147.
[58] Jaffe R. L., *Phys. Rev. Lett.*, **38** (1977) 195.
[59] See the papers by Jaffe and Jaffe and Wilczek quoted in ref. [20] and Jaffe R. L., *Phys. Rep.*, **409** (2005) 1, arXiv:hep-ph/0409065.
[60] HAL QCD Collaboration (Inoue T. *et al.*), *Phys. Rev. Lett.*, **106** (2011) 162002, arXiv:1012.5928 [hep-lat].
[61] Mulders P. J., Aerts A. T. M. and De Swart J. J., *Phys. Rev. D*, **21** (1980) 2653.

[62] Balachandran A. P., Barducci A., Lizzi F., Rodgers V. G. J. and Stern A., *Phys. Rev. Lett.*, **52** (1984) 887.
[63] Cotugno G., Faccini R., Polosa A. D. and Sabelli C., *Phys. Rev. Lett.*, **104** (2010) 132005.
[64] For a recent attempt in this direction, see Esposito A., Pilloni A. and Polosa A. D., *Phys. Lett. B*, **758** (2016) 292.