

# Neo-Cartesian unified fluid and field theory: from Klein-Gordon equation to de Broglie's Lorentz invariant mechanics

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**Abstract.** A unified fluid and field theory containing all forces as special cases postulates that primordial fluid is formed by discrete, 3D-extended energy-like sagions obeying usual energy-momentum conservation laws. Contrary to Newtonian mechanics, mass and force are not primitive notions (hence, “Cartesian”), but the theory is atomistic (hence “neo”). Collective fluid behaviour obeys the homogeneous Klein-Gordon equation, or homogeneous 3D-wave equation, which besides usual harmonic periodic solutions has three families of quantized, Galilean-Lorentz isomorph, nonperiodic solutions discovered around 1995. Here, quantum mechanics is relativistic ab initio thus ensuring consistency with Einstein's general theory of relativity, which is extended to include effects upon gravity of chemical and nuclear composition, as indeed observed by Eötvös (1890s), and Majorana (1919). Planck constant is identified with the bit of minimum angular momentum in sagon-sagon interactions, thus introducing quantum features at the root of classical mechanics. Acceleration produced by successive pushes of a small projectile (say, a sagon, disagon, trisagon, 4-sagon) leads to a curve resembling Einstein's mass increase. This reinterprets one aspect of the restricted theory of relativity: rather than mass increase, Bertozzi experiment demonstrates inefficient transfer of linear momentum. The lower branch in our background universal acceleration curve is associated to static friction.

## 1. Introduction: on continuity, field, force and 3D-extension

An epigraph in proceedings of Vigier IX Symposium [1] quotes Einstein: “*I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In that case, nothing remains of my entire castle in the air, gravitation theory included, [and of] the rest of modern physics*”. Third quotation in same epigraph comes from Feynman's Nobel Prize lecture: “*Current fashion [is] field theory ... the chance is high that the truth lies in the fashionable direction. But, on the off chance that it is in another direction ... who will find it? Only some one who sacrifices himself ... from a peculiar and unusual point of view, one may have to invent for himself*” (underlining added).

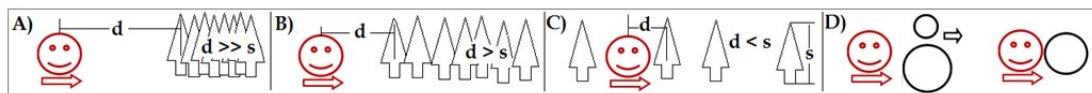
Against such background, present paper introduces a realistic fluid and field theory for discrete three-dimensionally extended entities. Our main contribution is, thus, discreteness and 3D-extension.

### 1.1 On continuity and discreteness

Regarding continuity, it may be noted at once that our natural world is formed by discrete objects, arranged in non-contiguous structures, so that, if the field concept implies continuity as claimed by Einstein, it never had the slightest chance of becoming the final theory of nature. In that context, any continuous mathematical equation is a mere first-order representation of the behavior of a large



collection of discrete objects. The apparent continuity in nature is a mere artifact of distance  $d$  from which a detector (say, the naked human eye) observes a group of objects, each of size  $s$ . For instance, from a twenty km distance an observer sees a forest as a “continuous” green area (say,  $d/s > 10^3$ ), discreteness becomes manifest as one approaches the forest (say  $10 < d/s < 100$ ), and at close range ( $d/s < 1$ ) observer sees the trees but not the forest (see figures 1A to 1C). This superficial or apparent dichotomy (tree-forest) also helps understand another long-standing controversy, the wave-particle issue: a wave, as in the sea, is the collective motion of myriads of individual particles (the water molecules). Stressing it once again, water seems continuous to a naked eye because  $d/s$  is very large.



**Figure 1.** Cartoon illustrating the distance effect upon the forest-tree dichotomy. Large arrow under observer represents linear momentum of his detecting device, which in the micro-world may be larger than linear momentum of particles being observed, as in panel D (see text).

Any process of measurement involves an exchange of linear momentum, as in billiard balls collisions (BBC for short). In the case of a forest, each tree reflects sunlight, and a photon beam eventually reaches the observer’s eye; linear momentum (qualitatively represented by arrows in figure 1) is usually negligible in macroscopic measurements. Contrariwise at the microscopic scale of panel D, the linear momentum associated with detection may be larger than the linear momentum of the particle being observed (the small arrow in panel D), leading to a large disruption in the system under observation. Even in commensurate situations  $d/s \sim 1$ , observed continuity may be a mere artifact too. An example is a dumbbell rotating with high frequency around an axis perpendicular to the rod. When observed by a human eye, which is a detector of low temporal resolution, the dumbbell appears as a continuous torus, but a high-speed camera brings out the discrete structure and features.

Quantum mechanics (QM) was formulated to explain phenomena at the atomic scale that were beyond classical physics at the turn of the 20<sup>th</sup> century. As a result, quantization is usually considered typical of the microscopic world: “*discontinuous changes characteristic of atomic processes*” [2], thus neglecting evidence existing at that time as the atomic-like structure of our solar system noted by Titius and Bode in the 18th century, and is of course contrary to recent quantum-like phenomena in fluids at laboratory scale [3,4,5]. Contrary to QM views, discontinuity is considered here as a manifestation of discreteness occurring at all scales of nature from clusters of galaxies down to fundamental particles, and is related to observation of a small number of objects from a near-range defined by small  $d/s$ .

Evidently, the behavior of the observed system is not the same after suffering the process of being observed, but it does not follow that it is impossible to predict the evolution of an identical unperturbed system. A limiting case involving two identical particles is illustrated in the far side of panel D: the observed particle is originally at rest, and completely exchanges the whole linear momentum with incoming detector; in BBC language, a ball at rest entirely absorbs linear momentum from the incoming ball and moves to the right carrying the detector’s original momentum, while the detector stays at rest. Nothing magic, the issue of distinguishability is not even mentioned.

### 1.2 On field, force and discreteness

Present paper introduces the reader to a classical unified fluid and field theory (UFF), applicable to all interactions in nature [6,7]. Our use of word “field” is not the same as Einstein’s; rather, “field” is a short-name or alias for the dynamic aspects of the collective behaviour of a primordial fluid formed by a large ensemble (several orders of magnitude larger than Avogadro’s number) of discrete individual energy-like objects called sagions, carriers of spin, energy, and linear momentum. An aspect of the small number case was tackled thirty years ago by this writer. It was found that static discrete

arrangements for a small number ( $N < 20$ ) of electric charges on the surface of a sphere exhibit inner structure and quantum-like properties [8,9], that are averaged out in standard continuous treatments. Thus, in few-body systems summation should be used instead of continuous integral representations.

Formation of highly symmetrical arrays is driven by minimization of potential energy; connection to a kinematic theory of matter is described in the companion paper at Vigier XI [10]. Foregoing ideas run counter to the received view. For instance, at the University of Chicago in 1929 Heisenberg noted *“the success of the classical mechanics in explaining the Wilson photographs ... Nevertheless, one can regard these achievements of classical theories only as a proof of the similarity of the classical and quantum theories, in the sense of the correspondence principle; for the answer to all quantitative questions an appeal must be made to the exact quantum theory”* [2], page 161, underlining added. We disagree. Regarding the “exactness” of QM, one should be aware that, right at the inception of QM, quantization was imposed from outside by Bohr as boundary conditions that he elevated to the status of a “quantization principle”, and at a later stage, the process was repeated by Hartree [2], page 164.

Rather than inventing something new as suggested by Feynman in the epigraph quoted in the first paragraph inhere, our theory is based on the completely unfashionable return to the roots of classical mechanics. Newton was a convinced atomist that believed in absolute time and three-dimensional space [11]. For the latter we go back another century to Ferrara for the twin notions of physical and mathematical absolute space propounded by Patrizi [12]. Both Patrizi and Newton considered that physical objects occupy a finite portion of absolute space  $\Sigma$ , so that there was a clear distinction between the homogeneous and continuous geometrical space  $\Sigma$ , and the discrete physical objects contained therein. Such distinction is notoriously absent in 20<sup>th</sup> century physics.

Shortcomings of Newtonian mechanics are well-known. Two of them are: (a) the faulty circular definition of mass in the first line of first page in the Principia [13, 14], and (b) the absence in the Principia of a mechanism for generation and propagation of gravitational force. From philosophical considerations on causality, the latter was immediately criticized by the followers of Descartes. About same time, Bentley, a dedicated defender of Newton, misinterpreted him on two accounts: gravity is inherent to matter, and gravity acts at a distance; both of them were strongly resisted by Newton, as in his third letter to Bentley: *“That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent is material or immaterial I have left to the consideration of the readers”* (underlining added) [15]. Misconceptions never disappeared. On the contrary, around 1860 Boscovich [16], who also was a convinced Newtonian, based his physics on the existence of a unique force emanating from punctual centers of force, and acting at a distance. Both misconceptions are still alive in 20th century textbooks, and many of us were exposed in our physics courses to such wrong views. Thus, if distance between two point masses tends to zero, both force and potential energy blow up to infinity. Present writer tried to fix the contradiction [17], but our approach would had been different if back in 1971 present writer had known, or at least suspected, that gravity is not inherent to matter.

Paying attention to Newton’s words, and to the ensuing developments by Fatio and Lesage [18], we postulated an energy-like primordial fluid (PF) as main pillar for a unified fluid theory, and developed its properties along the lines suggested by Lesage [19,20]. In Newton’s words, our choice is then for an “immaterial agent” as both the propagator and the final cause of non-contact forces.

Returning to Newtonian mass, if it is ill-defined, then Newton’s second law becomes also suspect. Instead of Newton’s three laws, classical mechanics may be derived from the principle of conservation of linear momentum. Such approach, rooted in Cartesian thinking, is more economical on fundamental principles [21-23]. A self-consistent scale for mass is operationally constructed by observing collisions of bodies against a reference mass, as in [24]. Such procedure implements the kinematic definition of mass proposed by Barré de Saint Venant in 1845 [25,26].

Both, in Einstein's general theory of relativity and in Cartesian mechanics, force is not a primitive notion. In apparently action-at-a-distance situations, force merely is a convenient name for the average exchange of linear momentum in a contact collision [27], involving discrete micro-scale objects from the PF. In interactions involving macroscopic contact, force may be viewed as a current or a flow of linear momentum [28,29].

### 1.3 On extension, atomism and discreteness

Present theory is also Cartesian in the sense of a close scrutiny to received views, and a rigorous adherence to logical rules, including the principle of continuity proposed by Leibniz in the 17<sup>th</sup> century [30] (page 293) and defended by Boscovich in the 18<sup>th</sup> century [31] (page 390). However, this writer disagrees with Descartes on two issues. Firstly, helical or swirl structures, similar to hurricanes and tornadoes, often form in any fluid, but they are not unique, and many other temporary and permanent structures also exist. Secondly, far more important, Descartes was not an atomist. His ether was a plenum, indefinitely divisible [32]. On the contrary, as Newton, present writer is a convinced atomist. This is why present theory is neo-Cartesian, rather than merely Cartesian.

Daily experience shows that matter is soft and deformable, has inner structure and occupies a finite volume in  $\Sigma$ . Properties of matter at scales bigger or smaller than the human scale seem to be the same. Invoking the universality principle, the smallest bit of matter must a fortiori occupy a finite 3D-volume, and must have internal structure capable of deformation. A huge bonus on logical consistency automatically ensues: since matter is deformable at all scales, a collision between material objects never violates Leibnitz continuity.

Since the smallest bit of matter has parts, it may be divided into components, which cannot be material by definition, for otherwise the smallest bit of matter would not be the smallest. This is why sagions are energy-like rather than particle-like. Sagions are indestructible and described by four main properties, and the corresponding parameters in script letters: (a) Permanent motion relative to  $\Sigma$  at high speed  $\mathcal{C}$ , same order of magnitude as speed of light  $c$ . (b) No rest mass relative to  $\Sigma$ , carriers of linear momentum  $\mathcal{P}$ . (c) 3D-extension represented by a homogeneous sphere of radius  $\mathcal{R}$ . (d) Inherent spin  $\mathcal{S}$ , with positive (negative) sign according to counter-clockwise (CCW) rotation or clockwise (CW) rotation. Neither force, nor charge are inherent properties of sagions.

Right since the early development of quantum theory in the 1920s, particles were assumed to be geometrical points endowed with physical properties as mass and spin —the latter was claimed to be a unique quantum feature. Even today, the electron is still considered in mainstream physics as a punctual object without structure. Such naive model led to physical inconsistencies (as infinite mass density), disguised under the neutral name of singularity, and “removed” by the highly questionable procedure of renormalization. On the contrary, in present theory matter is extended and has structure at all scales, so that spin of earth, spin of a rotating top, and spin of an electron, them all have the same classical explanation and origin.

In his Principia Newton proved that for calculations involving translation, mass of an extended 3D-body may be treated *as if* it were concentrated at its center of mass (CM). Likewise, the gravitational attraction exerted by a homogeneous spherical body of mass  $M$  and radius  $R$  at distance  $r > R$  may be calculated as if a punctual mass  $M$  were placed at the center of the sphere. The latter theorem was demonstrated by Newton in 1685, and it provided the mathematical support for Newton's treatment of extended bodies as punctual particles. The importance of this result led Kuhn to conjecture that Newton delayed publication of the Principia until he had demonstrated the CM theorem to his entire satisfaction [33]. Unfortunately, the creators of QM forgot the Newtonian “as if” part, and also forgot that the extended object was still there occupying a 3D-volume around the CM, and additionally forgot that to calculate spin one actually requires the details of the geometrically extended configuration. Singularities automatically disappear by re-introducing extension in physical theory.

### 1.4 Further fundamental controversial questions

This lengthy introduction closes with a recollection of some crucial events and forks in this writer's

fifty-year personal voyage. In summer 1971 while attending a colloquium at the International Centre for Theoretical Physics in Trieste, I became aware for the first time of the problem of limits when objects are discrete. This was tackled as conservation of energy in special relativity [17]. Regarding the significant weaknesses at the roots of QM, a relevant preliminary question was: is probability a physical object? No! The only species in nature that consciously conjectures about the future is humankind. Thus, probability is a mere construct of human mind, and QM cannot possibly be a theory about the natural world, but merely a model about how some human beings perceive nature. My view on probability is causal as in Laplace [34], and is close to Feynman's path integrals, but it evolved in risk studies for nuclear power plants as a manner to quantify the possibility of occurrence of non-repetitive large-scale events [35,36].

In second half 20<sup>th</sup> century there were claims that empirical evidence indicated that the Copenhagen interpretation of QM was right; hence, classical physics was dead. Bell's theorem was hurriedly checked, but I was not convinced on the validity of that theorem [37]. Then, the only acceptable version of QM is the stochastic interpretation [38], still providing a useful methodology to calculate, but it cannot possibly be a final theory of nature. As Chebotarev poses it: *"in the 70 years following the advent of quantum theory ... it has not been possible to achieve a satisfactory understanding of the fundamental physics underlying the mathematical scheme of quantum mechanics ... nor is there a satisfactory answer to the question of the physical nature of the wave function"* [38], page 1.

### 1.5 The Michelson-Morley experiment and absolute space

Around 1986 my next question was: what is the evidence against aether and absolute space? On the experimental side the Michelson and Morley experiment (MMX), and on the theoretical side Einstein's special theory of relativity (STR). Einstein reintroduced aether and/or absolute space in the context of his general theory of relativity (GTR) [39], and during the 20<sup>th</sup> century aether revived disguised as dark matter, dark energy, "physical vacuum", and zero-point field energy. However, there are still some brave efforts to clear Einstein of any guilt in the 1905 dismissal of aether: *"Quite undeservedly, the ether has acquired a bad name. There is a myth, repeated in many popular presentations and textbooks, that Albert Einstein swept it into the dustbin of history"* [40]. This may be contrasted with the perception that Dirac and Heisenberg had about the very same subject ninety years before. For instance, Heisenberg father of quantum mechanical uncertainty wrote that *"Faraday and Maxwell explained electromagnetic phenomena as the stresses and strains of an ether, but with the advent of the relativity theory, this ether was dematerialized; the electromagnetic field could still be represented as a set of vectors in space-time"* (underlining added) [2], page 63.

Since final arbiter in natural science is experiment, rather than theory, present writer checked all MMX from 1881 to 1930, period of inception of both QM and STR. Overall, experiments yielded positive results, but speed of laboratory motion was lower than expected [41]. So it was decided to repeat MMX using laser light, a stationary interferometer (i.e., one turn of apparatus per sidereal day), automatic data gathering and recording; software to convert interference photographs into digital information was developed in-house. Using modern values of solar velocity, the number of fringe-shifts expected in a 24-hour apparatus rotation was calculated assuming that light travels with constant speed in a frame attached to fixed stars [42]. Our two-year experiment was consistent with predictions [43-47]. The overall design, protocols, and execution of our experiment is superior to the pioneer MMX. The weakest part in our experiment was lack of environmental control, which nonetheless was still marginally superior to the original MMX. Additionally, we implemented stochastic corrections for pressure, humidity and temperature effects, while MMX did not. So, both absolute space and aether may be reinstated [48], thus returning to the Greek, Cartesian and Newtonian roots of logic, atomism, and classical physics.

This writer has slowly wandered over decades along the unfashionable path of discreteness and non-contiguity. Our "new" contribution is to explicitly recognize 3D-extension as an overall property of Nature, from clusters of galaxies down to the smallest entities: disagion, and energy-like sagion.

## 2. Reinstatement of primordial fluid

In 1938 Einstein, Infeld, and Hoffmann [49] revisited the link between GTR and the classical equations of motion. In Jammer's words: "*the nonlinear character of the field equations in general relativity ... made it possible to deduce the dynamical law from the field equations ... the approximation method employed by Einstein, Infeld, and Hoffman applies only to the case of slowly varying fields ... a unified field theory that subjects electromagnetic and possibly also nuclear forces to a similar treatment as gravitation, then it would lead us to a final stage in the history of the concept of force ... classical mechanics still admitted, tolerantly ... force as a methodological intermediate, the theory of fields would have to banish it even from this humble position*" [26] (emphases added).

Note that Einstein and co-authors were talking, of course, about classical Newtonian mechanics, not about Cartesian mechanics (where force does not exist ab initio, recall section 1). The remainder of the quotation is a good summary of the intentions of present paper.

### 2.1 Primordial fluid as a classical fluid equation

Absolute space  $\Sigma$  is populated by primordial fluid (PF) formed by discrete energy-like sagions. In intergalactic space and in regions devoid of matter, collective fluid obeys conservation of both total energy and linear momentum according to the homogeneous Klein-Gordon equation

$$\square\Pi = 0, \square\Pi = 0, \square \equiv \frac{\partial^2}{\partial w^2} - \nabla^2, \quad w \equiv Ct \quad (1)$$

In the scalar equation,  $\Pi$  is content of energy in the PF per unit three-dimensional volume in  $\Sigma$ , and in the vector equation,  $\Pi$  is flow of linear momentum carried by sagions per unit area. D'Alembertian operator is  $\square$ , and time  $t$  is given as a length  $w$ , where  $C$  is the average local speed of sagions. In the presence of matter the right-hand side is non-zero due to inelastic interactions of sagions with matter, and to sagon-matter inter-conversion. The classical equations of fluids are often formulated as vector equations [50], rather than the wave equations (1), but they are equivalent [6,7]. Relevant remarks are:

#### 2.1.1 Second-order equations.

The classical fluid equations (1) are second-order on both time and space, consistent in principle with GTR for an isolated system in vacuum.

#### 2.1.2 Realistic scalar and vector potentials.

The purely human, and hence subjective, concept of probability does not appear in our field equations. The pair of field variables  $(\Pi, \Pi)$  represent physical properties of a realistic fluid formed by discrete sagions.

#### 2.1.3 Continuity.

Continuous equations (1) represent collective behavior of zillions of sagions, and are not appropriate to represent local interactions of a small number of sagions, which must be treated using the methods of discrete classical mechanics [6, 8-10], as in the acceleration process in section 5.

#### 2.1.4 Causality and inherent uncertainty.

Present theory is causal in the strong classical sense of Laplace probability [34], but it does not follow that the future may be easily predicted. Uncertainty, and apparent randomness, arise because initial conditions are not accurately known, or because the process of measurement disturbs the observed system (section 1 above). Additionally, there is a novel inherent uncertainty, resulting from the logical and physical impossibility of measuring lengths with accuracy better than the smaller ruler: the size of sagon  $\mathcal{R}$ . For, if there existed rulers smaller than  $\mathcal{R}$ , then the sagon would not be the smallest object in nature. Likewise, time cannot be measured with accuracy better than  $\mathcal{T}$ , defined in equations (2).

### 2.1.5 Physical derivatives.

Usually, derivatives are defined in the mathematical sense of indefinitely divisible magnitudes. Contrariwise, in present theory derivatives with respect to space and time are physical, and have a lower limiting size [6]. For any physical magnitude  $W$ ,

$$\left(\frac{dW}{ds}\right)_{phys} \equiv \lim_{\Delta s \rightarrow 2\mathfrak{L}} \left(\frac{\Delta W}{\Delta s}\right), s = x, y, z, r; \left(\frac{dW}{dt}\right)_{phys} \equiv \lim_{\Delta t \rightarrow \mathfrak{T}} \left(\frac{\Delta W}{\Delta t}\right), \mathfrak{T} \equiv 2\mathfrak{L}/c \quad (2)$$

### 2.1.6 Lorentz invariance.

Poincaré demonstrated Lorentz invariance of equations (1) independently and before Einstein's STR [51-54]. Therefore, the velocity in Poincaré's transformations is relative to absolute space  $\Sigma$ , and is completely compatible with Newtonian aether, our PF, and equations (1).

The one-dimensional version of equation (1) is the well-known travelling wave equation. In the 19<sup>th</sup> century propagation of sound was described by the classical wave equation with  $C$  being the speed of sound. For time-independent, static and stationary problems equation (1) reduces to Laplace equation, that he used in 1787 to study the rings of Saturn. Thus, all physical problems described by  $1/r$ -static potentials (both in gravity and electromagnetism) are consistent with the PF propounded here. A related case is Yukawa's static potential, derived from the non-homogeneous wave equation [55]. Other connections of equations (1) with gravity, electrodynamics, and QM are listed next.

### 2.1.7 Classical EM theory and fluid equations

Maxwell formulated his electromagnetic (EM) theory guided by the transport of fluids [56,57]. An interesting, but seemingly unexplored path, is a representation of the electromagnetic field of electron by two scalar potentials ( $F, G$ ), both obeying equation (1) [58].

Maxwell's equations may be reduced to non-homogeneous wave equations in terms of ( $\mathbf{E}, \mathbf{B}$ ), or as a pair of scalar and vector potentials ( $\phi, \mathbf{A}$ ); the standard procedure imposes the Coulomb or transversal gauge  $\nabla \cdot \mathbf{A} = 0$  as an additional constraint [59]. In vacuum, electric charge density and electric current density are zero, thus reducing to the homogeneous wave equations (1). Present writer reached similar wave equations from his symmetrical Maxwell's equations, without imposing the transversal gauge (see [60], page 2095), which suggests that transversal propagation is not an intrinsic trait of Maxwell's equations, as conventionally believed. Indeed, direct substitution verifies the mathematical existence of elementary longitudinal solutions compatible with Maxwell's equations [61]. Additionally, there is some redundancy built-in the four Maxwell's equations, as attested in our derivation of wave equations using our symmetrized ( $\mathbf{P}, \mathbf{N}$ ) version [60]. This suggests that pair ( $\mathbf{E}, \mathbf{B}$ ) contains redundant information, and points towards simpler, more fundamental underlying physical processes. It is our contention that such physical reality is the PF described by equations (1).

The opposite approach is to explicitly derive Maxwell's equations from the wave equations (1) of primordial fluid. This was done in France in 1926 by Henri Malet [62], who defined

$$\phi = \rho C^2, \mathbf{A} = \rho C \mathbf{V} \quad (3)$$

The scalar potential  $\phi$  is the average kinetic energy density carried by the fluid, and the vector potential  $\mathbf{A}$  is the linear momentum density transported by the convection of a fluid moving with velocity  $\mathbf{V}$ . These definitions are similar to the meaning attached to ( $\Pi, \mathbf{\Pi}$ ) in equations (1). Unaware of Malet's prior work, present writer re-discovered a similar derivation of Maxwell's equations [6,7,63,64] from the vector fluid equations for the particular case of incompressible and constant density aether [63]. By the same epoch, at least three other authors independently rediscovered Malet's ideas [65-67]. Since Maxwell's equations are a special case of fluid theory, a richer electrodynamics may be based on the pair of potentials ( $\phi, \mathbf{A}$ ) = ( $\Pi, \mathbf{\Pi}$ ).

However, it is not usually realized that Maxwell's equations (ME) only apply to absolute space, or

to systems at rest therein. This explains the notorious absence in ME of laboratory speed relative to  $\Sigma$ , and justifies (at least in part) Maxwell's use of partial derivatives rather than total derivatives as it should be in a complete theory. Both before and after Maxwell, other electrodynamic theories were proposed in France and Germany: Ampère (1823) [68], Gauss (1835), Grassman (1845), Neumann (1845), Weber (1846), Helmholtz (1873), and Riemann (1875); for references see [69]. Some theories are based on the Newtonian concept of force which some authors [69,70] consider superior to the field concept (the latter implicit in Maxwell, Einstein, and present theory). Velocity of laboratory or detector explicitly appears in Weber's force revived by Assis [71], and in the neo-Hertzian field theory revived by Phipps [72,73], which is Galilean invariant, and may be derived from a fluid equation similar to equation (1), only difference is time that refers to detector's proper time [72], page 78.

By mid-20<sup>th</sup> century Jefimenko formulated a self-consistent causal theory applicable to both EM and gravity [74,75], and noted that a "sourceless" homogeneous wave equation cannot originate EM fields [75]. There is no contradiction with equation (1) above describing transport of linear momentum and energy in the primordial fluid, rather than "force" which indeed requires the presence of matter on the right-hand side of the wave equation, and leads to a force density, proportional to the gradient of field [6], page 253, or [63], page 73. Recall that in Cartesian theory, "force" is exchange of linear momentum, that in this case is a sagion-matter interaction.

Finally, Maxwell's theory cannot explain all EM phenomena. Examples are experiments by Tesla at end of 19<sup>th</sup> century, and by Graneau in the 1980s, see references in [70]. The observed apparent "excess energy" may be related to the unaccounted motion of earth relative to  $\Sigma$ .

#### 2.1.8 Gravity and the fluid equations

Similitude of Newton's and Coulomb's laws led Maxwell in 1865 to search for connections of gravity and aether, lead that he abandoned because undisturbed aether had "*an enormous intrinsic energy ... the presence of dense bodies influences the medium so as to diminish this energy wherever there is a resultant attraction*" [57], pages 492-3. This apparent extraction of energy from aether is similar to energy transferred from aether to matter in Le Sage's attenuation of ultramundane particles [18, 19], which manifests as gravity.

Twenty years before Einstein's GTR, Oliver Heaviside described in 1893 a gravito-magnetic field [76], which explained precession of Mercury's perihelion. Heaviside's equations were analogous to Maxwell's equations, and thus equivalent to our equations (1). Jefimenko's causal theory for EM and for gravity [75] is similar to Heaviside's.

In 1903 Whittaker considered "*gravitation and electrostatic attraction explained as modes of wave-disturbance ... gravitational force in each constituent field will be perpendicular to the wave-front, i.e. the waves will be longitudinal ... this undulatory theory of gravity ... propagated with a finite velocity,... need not be the same as that of light, and may be enormously greater*" (emphasis added) [77], page 355. Such notions are contrary to usual views that Maxwell's equations may only have transversal solutions; present author [61] agrees with Whittaker.

A connection between Einstein's GTR and the fluid equations is established in [6,7], but GTR is additionally constrained by the equivalence principle —made by Einstein a postulate of his theory.

In contrast, for Newton the equivalence between gravitational and inertial mass was empirical, amenable to experimental testing. Electromagnetic, atomic and nuclear interactions depend on chemical and nuclear composition of matter, according to the atomic number  $Z$  of its constituents [55], but Einstein postulated otherwise. Rather curious that Einstein never noted that his equivalence principle was a serious impediment towards his unification dream! Then, next question: Is Einstein's equivalence principle right? The answer is no! Newton was right, it as a mere empirical question. Two experiments violating such principle are briefly described in sections 3 and 4.

#### 2.2 Quantum theory and fluid equations

Old quantum theory began in 1913 when Bohr imposed an ad hoc external condition at the micro-world scale: "*an atomic system can exist in particular stationary or quantized states, each of which*



corresponds to a definite energy of the system. Transitions from one stationary state to another are accompanied by the gain or loss, as the case may be, of an amount of energy equal to the energy difference between the two states" [78].

Schiff noted that first candidate for a quantum equation was "the most familiar one-dimensional wave equation, that which describes the motion of transverse waves on a string or plane sound waves in a gas" [78], which Schrödinger discarded for superposition considerations, and opted instead for a differential equation with a first-order derivative with respect to time. Such equation does not guarantee conservation laws, and requires that electrostatic, gravitational, and nuclear forces be included by hand as derivable from a real potential function.

A connection between Schrödinger's equation and Madelung's fluid is known since 1926. Several papers reprinted in Vigier's 80<sup>th</sup> birthday commemorative book [38] indicate that both de Broglie and Vigier gave some consideration to Madelung ideas. Dirac used a Klein-Gordon equation for particles, and a homogeneous classical wave equation for aether [79], equations 21. The scalar homogeneous Klein-Gordon equation is the usual representation for a zero spin particle [80].

In our opinion, Schrödinger's choice was the crucial fork that led physics into current blind alley: gravity is not inherently-quantized, and quantum theory is not inherently-Lorentz-invariant. We propose a return to Schrödinger's fork to pursue the other branch. Quantum phenomena thus become the straightforward evolution of a primordial fluid described by a pair of scalar and vector Lorentz-invariant homogeneous classical wave equations (1).

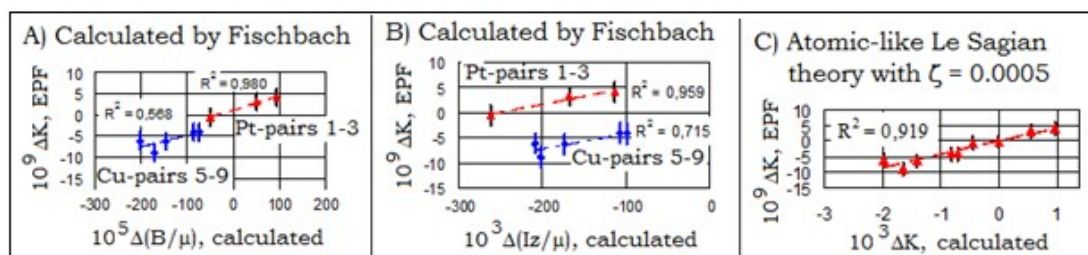
### 2.3 New solutions for the classical wave equation

Harmonic solutions for equations (1) were given by Poisson around 1820, Kirchhoff by 1883, and Whittaker in 1903 [77]; harmonic solutions to Laplace equation are particular solutions for equations (1). A new result on this subject is our discovery of novel nonperiodic solutions for the homogeneous classical wave equation [6,81,82,83], inherently exhibiting quantum-like structure, without Bohr's ad hoc quantum principle.

Actually, this constitutes a new additive metric  $(r, q, \theta, \varphi) = (r, \theta, \varphi) + (q, \theta, \varphi)$  that splits the Lorentz invariant solutions on the 4-sphere in the left side, into two separate fields over 3-spheres on the right side [10,84]. The  $(r, \theta, \varphi)$  solutions constitute a time independent background field, that always is present, thus solving long-standing puzzling questions as action-at-a-distance and non-locality. The  $(q, \theta, \varphi)$  solutions constitute a novel inherently quantized field, that instead of Lorentz invariance exhibits isomorphism for all the usual distance-time transformations: Doppler, Lorentz, Poincaré and Einstein. This novel property is sketched in equations (5) of [84].

### 3. The Eötvös, Pekár and Fekete (EPF) experiment

In Hungary in last decade of 19<sup>th</sup> century, EPF used torsion balances to compare earth's attraction on materials of different  $Z$ ; two series were carried out with copper Cu ( $Z=29$ ) and platinum Pt ( $Z=78$ ) as reference materials [85,86]. Conventional interpretation is that EPF experiment supports Newtonian gravity independent of  $Z$ . Einstein converted that single result into a law of nature: the principle of equivalence for mass.



**Figure 2.** Eötvös experiment revisited. In 1986 Fischbach found dependence of gravity (Y-axis) upon

baryonic composition of matter (X-axis in panel A), and upon nuclear isospin (X-axis in panel B) [86]. Our atomic-like Le Sagian gravity shows a high correlation (92%) between atomic number  $Z$  (horizontal axis) and all 9 pairs of data in EPF experiment (Y-axis in panel C) [19].

But there is some recent (although as old) counter-evidence. Thirty years ago Fischbach and co-workers suggested a fifth-force. Searching for empirical support they revisited EPF experiment, and identified variations of Newton's gravity with nuclear structure of interacting bodies: baryons (figure 2A), and isospin (figure 2B). In the 1990s, many experimenters looked for a Yukawa-type fifth-force, similar to the exponential component in equations (6) and (8) below. However, not all experiments summarized in [86] paid due attention to the  $Z$ -dependence, which significantly affects attenuation and build-up in equations (6) and (8). At any rate, Fischbach did not find enough support for his form of fifth-force, and abandoned the search by the end of 20th century [86].

However, Nieto, Hughes and Goldman noted in 1989 that “*even though the original analysis of Fischbach et al. had been corrected and their proposed coupling to hyper charge is ruled out ..., the correlation with baryon number is present in the Eötvös data*” (emphasis in the original) [87]. The unavoidable conclusion is that when it was formulated a century ago, Einstein's equivalence principle did not have any empirical support from the Eötvös experiment!

Present writer also revisited EPF experiment by himself, and identified two groups of data, clearly associated with atomic number  $Z$  of the Cu and Pt counter-weights [19, 88] (figures 2A and 2B). Fischbach had a near-miss, he ignored orbital electrons, and attributed all observed effects to nuclear structure only [86]. Our analysis included orbital electrons, thus obtaining a high correlation coefficient (figure 2C).

In our atomic-like Lesagian model for generation and propagation of gravity [19, 88], the small mass of the  $Z$ -orbital electrons enters in the cross-section of the sagion-electron interaction  $\sigma_E$ , while the baryons in the nucleus interact according to the proton-sagion and neutron-sagion cross sections ( $\sigma_P$  and  $\sigma_N$ ). The atomic cross section  $\sigma(A, Z)$ , for a nuclide formed by  $Z$  protons and  $(A-Z)$  neutrons is

$$\sigma(A, Z) = Z\sigma_P + (A - Z)\sigma_N + Z\sigma_E, \quad \frac{\sigma(A, Z)}{\sigma_P} \cong A + \zeta Z, \quad \text{for } \sigma_P \approx \sigma_N, \zeta = \frac{\sigma_E}{\sigma_P} \quad (4)$$

Intuitively it was clear to us that cross-section ratio  $\zeta$  was related to the electron-proton mass ratio ( $1/1836$ ), but it was decided to treat ratio  $\zeta$  as a free parameter. Its value was varied to optimize correlation between EPF data (Y-axis in panel C) and our model based on equation (4), X-axis in panel C. Highest correlations attain for  $\zeta$  in range 0.0005 to 0.001 ( $R^2 = 0.919$  at  $\zeta = 0.0005$ , and  $R^2 = 0.924$  at  $\zeta = 0.001$ ), thus confirming our intuitive guess.

Therefore, EPF data support our Le Sagian gravity, which depends on  $Z$ . The good news in the quest for unification is the absence of fundamental differences between gravity and the other three “forces” in nature.

#### 4. Quirino Majorana and his experiments on gravity attenuation

Reports on the possible absorption of gravity appeared at least since 1897, which led Quirino Majorana to start in 1918 his own well-designed experiments at the Polytechnic Institute of Turin, continued after 1922 at the University of Bologna [89-92]. Contrary to Newton's warnings noted in section 1, Majorana treated gravity as an inherent property of matter such that “*the force of gravitation could be explained by a kind of energic flux, continually emanating from ponderable matter*” (underlining added) [83], page 489, and argued that a spherical body of radius  $r$  and density  $\delta$  has apparent mass  $M_A$  and true mass  $M$  (MV in his notation) related by equation (5), where  $h$  is a new constant of nature:

$$M_A = M \exp(-h\delta r) \quad (5)$$

According to Majorana, the mass to enter Newton's gravitational law should be  $M_A$  rather than  $M$  so that “*Newton's Law would only be exact in the first approximation*” [90], page 490. Although equation (6) does not appear in Majorana's papers [89-91], he was suggesting that

$$F = G \frac{M_{A,1} M_{A,2}}{r^2} \exp\left(-\int h \delta dr\right) = G \frac{M_1 M_2}{r^2} \exp\left(-\int h \delta_1 dr - \int h \delta_2 dr - \int h \delta dr\right) \quad (6)$$

Density inside interacting bodies 1 and 2 is  $\delta_1$ ,  $\delta_2$ , and density of external matter along line joining the centres of mass of bodies 1 and 2 is  $\delta$ . Integrals are taken over appropriate distances. In the context of this paper, the apparent mass  $M_A$  is gravitational mass, while true mass  $M$  is inertial mass. In his critical article, Russell wrote equation (6) (far right), and associated it “with the ‘law of progressive absorption’ which holds good for radiation” [93], page 334.

Majorana designed his experiment to test for attenuation of gravity by a high Z-material between  $M_1$  (the earth) and  $M_2$  (a test ball), according to first part of equation (6). Gravitational force  $F$  manifests as weight  $W$  measured by a laboratory balance, so that

$$\frac{W(Z)}{W(\text{air})} = \frac{F(Z)}{F(\text{air})} = \frac{\exp\left(-\int h(Z) \delta(Z) dr\right)}{\exp\left(-\int h(\text{air}) \delta(\text{air}) dr\right)}, \text{ where } \int h(\text{air}) \delta(\text{air}) dr \approx 0 \quad (7)$$

Expansion of exponential on right-hand side allows an easy calculation of  $h(Z)$ . Majorana used a modified Ruprecht laboratory balance in vacuum to weigh a small lead sphere (3 cm radius, 1.274 kg mass). In a given experimental session two alternating measurements were carried out: series 1, with test mass in free air, and series 2, with test mass surrounded by a high Z material. First experiment began in Turin in April 1918, and included the day of a general strike (July that year), which provided a particularly quiet environment, free of vibrations. Shielding material was a cylinder (height: 22 cm, diameter: 22 cm) with a full capacity of 109.6 kg of mercury Hg ( $Z = 80$ ). Majorana reported 104 kg, and that the centre of mass (CM) of the test ball coincided with the CM of Hg. Each series produced a curve of weight versus time of day. The two curves were parallel, with series 2 consistently underneath series 1 ([89], page 92, and [90], page 499). Thence, Majorana obtained  $h(\text{Hg}) = 6.73 \times 10^{-12} \text{ cm}^2/\text{g}$  [90], page 502.

A second experiment used same lead ball surrounded by a massive 9,603 kg lead cube, 95 cm side [91,92], page 28; handling of the heavy shielding was difficult. A third experiment used a lighter 180 kg lead attenuator [92], page 30. Error analysis for Hg experiment is given in [82,83]; but analysis of experimental errors in the Pb experiments does not appear in [89-92]. Table 1 summarizes Majorana findings [89-92]; right side is our recalculation using his data and taking into account some details omitted by Majorana. Our  $h(Z)$  values are similar to Majorana’s, confirming two fundamental facts:  $h(Z) \neq 0$ , and  $h(\text{Hg}) \neq h(\text{Pb})$ .

**Table 1.** Gravity attenuation coefficients for mercury (Hg) and lead (Pb).

	Majorana calculations			Our calculations			Ref No.	Pages
	$r$ cm	$\Delta W$ $\mu\text{g}$	$h(Z)$ $\text{cm}^2/\text{g}$	$r$ cm	$\Delta W$ $\mu\text{g}$	$h(Z)$ $\text{cm}^2/\text{g}$		
Experiment 1/Hg	8.4 <sup>a</sup>	0.97	6.73 E-12	6.54 <sup>d,e</sup>	0.97	8.59 E-12	[83]	498,502
Experiment 2/Pb	47.5 <sup>b</sup>	1.72	2.50 E-12	43.5 <sup>d</sup>	~2	3.18 E-12	[84]	478
Experiment 2Pb	47.5 <sup>b</sup>	1.57	2.28 E-12				[85]	28
Experiment 3/Pb <sup>c</sup>	12.5 <sup>b</sup>	0.51	2.80 E-12	8.5 <sup>d</sup>	0.51	4.12 E-12	[85]	30
Average Pb experiments			2.50 E-12			3.65 E-12		

<sup>a</sup> Hg cylinder treated as sphere (12.35 cm radius) minus ball cavity (3.95 cm radius).

<sup>b</sup> Majorana assumed a punctual Pb-ball for the numerical calculations.

<sup>c</sup> Assuming a cubic arrangement of Pb bricks.

<sup>d</sup> Subtracting ball cavity (radius: 3.95 cm).

<sup>e</sup> Unfilled Hg cylinder, and Pb-ball at CM of Hg shielding (104 kg).

For the experiments with lead as attenuator Majorana expected  $h = h(\text{Pb}) = h(\text{Hg}) = 6.73 \text{ E-12 cm}^2/\text{g}$ , and he was quite puzzled [91], page 478 by results in table 1. Majorana could not tell whether

there was an error in his first experiment, or whether there was a defect in his theory. His scientific attitude must be praised: “*the experimental research represents the real foundation of science, and its results are facts which in any case enrich our patrimony of scientific knowledge. As to my researches, one can leave aside the a priori theories I proposed*” underlining added, [92], page 28. Since Majorana was a good experimenter, the failure surely was in his theory as explained next.

Present writer has the benefit of looking in retrospect, and some familiarity with interaction of gamma and X-radiation with matter. Neither Majorana, nor Russell [93] had to know in the 1920s that the attenuation coefficient  $h(Z, E_0)$  for interaction of EM radiation with matter is formed by four main microscopic processes: photoelectric absorption, Rayleigh scattering, Compton scattering and absorption, and attenuation by pair-production (say, electron-positron pairs), the four mechanisms depending on both  $Z$  and the energy of the photon  $E_0$  [55,94].

In the context of unification, and reasoning by similitude, it may be expected that gravity interactions are similar to electromagnetic interactions, at least regarding absorption and scattering. So, there is no reason for Majorana’s coefficient  $h$  to be a constant of nature, independent of  $Z$ ; also, gravitational build-up  $B(Z, \dots)$  might be present. This phenomenon manifests when thick attenuators are used —“thick” means several mean free-paths of radiation in that material. So, in general  $h(\text{Hg}) \neq h(\text{Pb})$ , as effectively observed.

Consider the ideal case of collimated mono-energetic flux  $I_0$  of energy  $E_0$  photons per unit area and unit time traversing a material slab of thickness  $r$ , density  $\delta$  and linear attenuation coefficient  $\mu$ . The photon flux  $I(E)$  at other side of slab is given by:

$$I(E) = B(Z, E_0, \mu r) I_0 \exp(-\mu r), \text{ when } r \rightarrow 0 : B(Z, E_0, \mu r) \rightarrow 1, I(E) \rightarrow I(E_0), \quad \mu = h\delta \quad (8)$$

Equation (8) is used for quick estimates in radiation protection, where  $B(\dots)$  is the build-up factor representing a mixture of undisturbed photons carrying energy  $E_0$ , and of scattered and re-emitted photons with energy  $E < E_0$  travelling in directions non-parallel to the incoming photons. Factor  $B(\dots)$  strongly depends on the thickness of material  $\delta r$ , geometry of attenuating material, and directional distribution of incoming photons [94]; precise calculations require MonteCarlo methods.

Table 1 and equations (8) yield a linear attenuation of gravity  $\mu(\text{Pb}) = 4.1\text{E-}11 \text{ cm}^{-1}$ , and the inverse mean free-path  $\lambda(\text{Pb}) = 2.4\text{E+}10 \text{ cm}$ , similar to the earth-moon distance ( $3.84\text{E+}10 \text{ cm}$ ). Thus, at laboratory scale gravitational build-up is negligible, while at the earth-moon scale (as in eclipses) gravitational build-up would be barely noticeable. Additionally,  $Z_{\text{ef}}$  of moon and earth are much smaller than  $Z = 80$  for mercury, so that any expected effect is even smaller (see table 2).

**Table 2.** Gravity attenuation coefficient  $h(Z)$  in  $\text{cm}^2/\text{g}$  for several power law dependences on  $Z$ .

$Z$	$Z_{\text{Hg}}/Z$	$(Z_{\text{Hg}}/Z)^2$	$(Z_{\text{Hg}}/Z)^3$	$(Z_{\text{Hg}}/Z)^4$	Substance	$Z_{\text{ef}}$	$Z_{\text{Hg}}/Z_{\text{ef}}$	$(Z_{\text{Hg}}/Z_{\text{ef}})^2$	$(Z_{\text{Hg}}/Z_{\text{ef}})^3$	$(Z_{\text{Hg}}/Z_{\text{ef}})^4$
H 1	1.1E-13	1.3E-15	1.7E-17	2.1E-19	Water	3.33	3.6E-13	1.5E-14	6.2E-16	2.6E-17
He 2	2.1E-13	5.4E-15	1.3E-16	3.4E-18	Earth crust	8.30	8.9E-13	9.2E-14	9.6E-15	1.0E-15
Li 3	3.2E-13	1.2E-14	4.5E-16	1.7E-17	Pb/10% Zn	68.46	7.4E-12	6.3E-12	5.4E-12	4.6E-12

If the analogy with gamma-ray scattering is correct, one expects  $h(\text{Hg}) < h(\text{Pb})$ , while experimental values in table 1 indicate otherwise. A quite plausible explanation is chemical purity of lead used to build Majorana’s lead bricks. Since Majorana was not testing for effects caused by atomic number  $Z$ , he surely used commercial grade lead, that may contain significant amounts of antimony (Sb) or zinc (Zn), thus lowering the effective atomic number  $Z_{\text{ef}}$ . For a substance formed by  $J$  nuclides of atomic number  $Z_j$ , mass number  $A_j$ , and weight percentage  $w_j$ , the value of  $Z_{\text{ef}}$  is given by

$$Z_{eff} = \sum_{j=1}^J \frac{w_j Z_j}{A_j} \left( \sum_{j=1}^J \frac{w_j}{A_j} \right)^{-1} \quad (9)$$

For instance, if Majorana's bricks contained 6% Sb ( $Z = 51$ ) and 94% Pb ( $Z = 82$ ), then  $Z_{ef} = 79.96$ ; and if composition was 10% Zn ( $Z = 30$ ) and 90% Pb, then  $Z_{ef} = 68.46$ . From tables 1 and 2 it seems that Majorana's lead contained Zn.

Table 2 shows gravity attenuation coefficients  $h(Z)$  estimated from  $h(\text{Hg}) = 8.59 \text{ E-12 cm}^2/\text{g}$  (table 1) for several power law dependences on  $Z$ . As a guidance, photon absorption dependence on  $Z$  is between  $Z^3$  and  $Z^4$ , while Compton scattering is linear on  $Z$  [55,94].

Majorana used  $h = h(\text{Hg}) = 6.7 \text{ E-12 cm}^2/\text{g}$  as a constant to calculate gravitational self-absorption in the sun. Since the main component of sun is hydrogen, Majorana significantly overestimated solar self-attenuation coefficient, which should be instead somewhere from  $1.1\text{E-13}$  to  $2.1\text{E-19}$  (left side of first row in table 2).

Russell used Majorana's  $h = h(\text{Hg}) = 6.7 \text{ E-12 cm}^2/\text{g}$  to evaluate self-absorption in solar planets and moons, and concluded that stability of our solar system requires  $h$  to be in a range from  $1.3 \text{ E-14}$  to  $6.7 \text{ E-16 cm}^2/\text{g}$  depending on the planetary bodies involved ([93], page 339). For planets with chemical composition similar to earth, these values coincide with values shown in last two columns of table 2. Thus, contrary to Russell's claim, if one acknowledges the  $Z$ -dependence, Majorana's empirical value for attenuation of gravity by Hg (see table 1) is compatible with the stability of our solar system.

Although Russell criticized Majorana, he fairly wondered "*what then becomes of Professor Majorana's long and careful series of experiments?*" ([93], page 342) and ended his paper stating that "*further evidence regarding the reality of the experimental effect appears to be urgently called for*" ([93], p 346). In April 1957 in his farewell papers Majorana was still pleading for an independent repetition of his experiments ([95], pp 397 and 402); he passed away on July 31/1957.

Geologists became interested in Majorana's gravity attenuation at least since the solar eclipse of 30 June 1954 [96]. Afterwards various groups used sensitive gravimeters trying to observe a decrease of gravity coincident with maximum eclipse.

During solar eclipse of 9 March 1997 a nice experiment was carried out at an isolated geophysical station in northern China; two significant anomalous valleys (about 6 and 7  $\mu\text{Gal}$  deep) were automatically recorded thirty minutes before first contact C1, and just after last contact C4. The authors suggested a "*possible shielding effect of the Moon on the gravitational force of the Sun*" ([97], p 041101-3) without stating that it did not coincide with Majorana's expected absorption of gravity. They were immediately criticized in the same journal because "*the expected shape of the signal in any reasonable model of shielding would be a bell shaped curve*" (underlining in the original) ([98], p 062002-2). Those critics are correct regarding absorption of parallel radiation by a thin piece of matter; but neglected the huge technical literature on scattering of radiation, of which reference [94] is just the tip of the iceberg. The Chinese group reconsidered their initial interpretation [99], invoking a "*rapid air mass movement for the bulk of the atmosphere ... as a sufficient explanation ... of the anomaly*" ([100], p 022002-1). However, the validity of the latter explanation is quite controversial since it "*presumes that air streams in from the surrounding area with speeds on the order of several hundred meter per second (imposing a hazard to airplanes flying at cruising altitudes during solar eclipses which has never been reported)*" ([101], p 271). Nonetheless, the related changes of pressure and temperature at totality are often invoked by other writers as sufficient explanation for eclipse gravity anomalies [102]. To avoid steep changes of temperature and pressure during totality, two possible locations to carry out gravity experiments during solar eclipses are: (1) Outside the totality band, as far as one to two thousand kilometer away, and (2) The anti-eclipse band, antipodal to the optical shadow.

Assuming that gravity and photons exhibit similar interactions, it is our claims that in scattering-dominated attenuation, the residual gravity curve during an eclipse may exhibit two lateral valleys, as effectively observed in at least six solar eclipses from 1954 to 1999 [103,104]. On the contrary, the mythical bell-shaped curve associated with pure gravitational absorption has never been observed in solar eclipses [96,98,102]. However, eclipses are not the most efficient way to (dis)confirm Majorana's discovery of gravity attenuation in 1918. It is best to have controlled conditions in a laboratory, and repeat his experiment with modern technology using attenuators of different Z, from Z=2 (liquid deuterium) up to Hg and Pb (do not use radioactive attenuators, as Th and U).

### 5. Acceleration in neo-Cartesian mechanics

In the Principia Newton postulated in his second law that change in the quantity of motion of a body is caused by action of an agent during a finite duration of time [11]. This integral-like mechanism is close to the Cartesian idea that acceleration arises from exchanges of linear momentum in collisions. Unfortunately, nowadays elementary textbooks formulate Newton's second law as mass multiplied by acceleration. This differential-like version of second law, due to Euler, only applies when mass is constant, and does not hint at what is the origin force. In contrast, in neo-Cartesian mechanics there is a simple microscopic mechanism to generate the apparently non-contact forces that seemingly act at a distance. It was independently proposed long ago by two Swiss citizens: Fatio and Le Sage [18].

#### 5.1 Acceleration by a succession of pushes and the universal acceleration curve

Consider a collinear elastic collision of an extended three-dimensional projectile or bullet B of mass  $m$ , speed  $C$  and linear momentum  $P_B = mC$  moving in the positive direction of X-axis and a target of mass  $M$ , initially at rest,  $b = M/m \geq 1$ . At the most fundamental scale the bullet is a sagion, and the target is an array of  $b$  coalesced sagions. At end of first collision target moves forward along X-axis with speed  $V_1$ ; a second identical bullet hitting target from behind transfers an additional amount of linear momentum, smaller than the first one, and so on. Each collision pushes the target forward. The  $n^{\text{th}}$  push transfers impulse  $I_n$ :

$$I_n = \Delta P_T = \frac{2(bP_B - P_T)}{b+1}, b \geq 1 \Rightarrow \Delta V_n = V_n - V_{n-1} = \frac{2(C - V_{n-1})}{b+1} \quad (10)$$

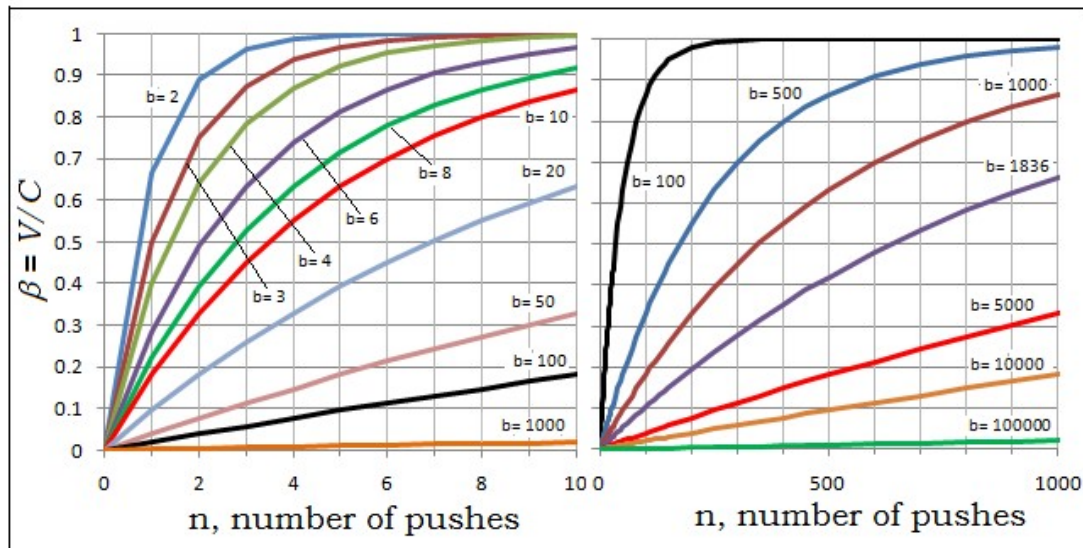
After some elementary algebra equation (10) becomes

$$\beta_n \equiv \frac{V_n}{C} = 1 - \left( \frac{b-1}{b+1} \right)^n, \quad b \geq 1, \quad n = 1, 2, 3, \dots \quad (11)$$

For  $b > 1$ , figure 3 shows that speed  $V_n$  of target approaches  $C$  after a finite number of pushes  $n$ ; note that as  $V_n$  increases, the impulse  $I_n$  transferred in the  $n^{\text{th}}$  push decreases. If a sagion is the bullet, then the sagion local speed  $C$  is a limiting speed, without invoking Einstein's STR. This simple physical process is not related to relativistic increases of target's mass, but rather to the obvious fact that if target moves with speed  $C$ , a bullet with the very same speed  $C$  will never catch up from behind! In a Newtonian context this would lead to a phenomenological model with force as function of velocity.

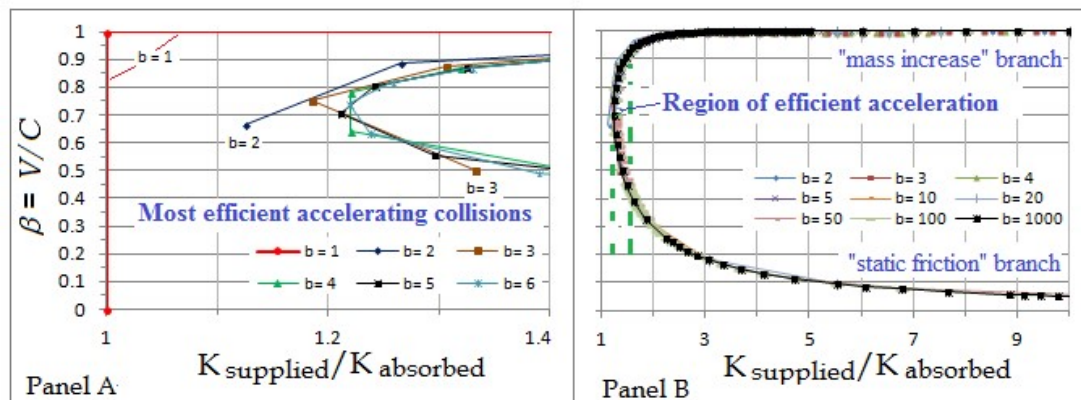
Since  $I_n$  decreases with  $n$ , kinetic energy transferred to a target, or absorbed by it ( $K_{\text{ABS}}$ ), decreases relative to the kinetic energy carried by the projectile. From the view point of cumulative kinetic energy carried by  $n$  bullets ( $K_{\text{SUP}}$ ), the average efficiency of energy  $K_{\text{ABS}}/K_{\text{SUP}}$  transferred per collision decreases as  $n$  increases:

$$\frac{K_{\text{ABS}}}{K_{\text{SUP}}} = \frac{b\beta_n^2}{n}, \text{ or } \frac{K_{\text{SUP}}}{K_{\text{ABS}}} = \frac{n}{b\beta_n^2}, \text{ or } \beta_n^2 = \frac{n}{b} \frac{K_{\text{ABS}}}{K_{\text{SUP}}} \quad (12)$$



**Figure 3.** Number of pushes required to accelerate target  $b$  to speed  $C$ . Target  $b = 4$  requires about ten pushes (left panel), for  $b = 100$  about 400 pushes are required (right panel). A neutron pushed by an electron,  $b = 1836$ , requires 5,000 pushes to attain  $C$ .

Figure 4 shows the increase in speed of target given by equation (12) as function of the cumulative kinetic energy carried by the  $n$  successive bullets impacting the target from behind; this is the total energy supplied ( $K_{\text{SUP}}$ ), or entering the acceleration region. In STR it is assumed that all energy entering the spatial where collisions take place is absorbed by the target, and becomes mass of the target. However, to our knowledge, the mass of the target has never been measured directly! For some details in Bertozzi experiment see below.



**Figure 4.** Increase of target speed as function of cumulative kinetic energy carried by  $n$  bullets ( $K_{\text{SUP}}$ ). Most efficient use of energy (1.2 to 1.4  $K_{\text{SUP}}/K_{\text{ABS}}$ ) is at intermediate speeds  $0.4 < \beta < 0.9$ . Panel A, light targets  $b = 1$  to 6: different discrete curves over the first initial pushes. Panel B ( $b < 1,000$ ): inefficient acceleration with speed limit  $C$  (as in STR) in upper branch, and static friction in lower branch, connected by region of efficient acceleration. Discrete structure not visible at this scale.



For  $b = 1$ , energy of projectile is transferred to target in first collision, which moves with speed  $C$  at end of this first and unique collision (vertical line along Y-axis in panel A); additional bullets entering the collision region can not transfer more energy to this target, thus energy of second and successive projectiles is completely wasted (horizontal line at  $\beta=1$  in panel A). Panel B shows curves from  $b = 2$  up to 1000, the individual curves are superimposed, and appear to the eye as a continuous curve with a finite width; details due to discreteness are not visible at the horizontal scale used in graph B, but it is clearly visible in panel A. The superposition of curves in panel B is called the universal acceleration curve, and it provides a straightforward classical mechanism for the long-term acceleration of massive cosmic rays in the primordial fluid.

Panel B shows that, from the view point of total energy entering the region, acceleration of bodies is inefficient both at low and at high speeds relative to  $\Sigma$ . The low end appears in macroscopic observations in a variety of guises: “static friction”, “hysteresis”, “inertial resistance”. The high end of figure 4 provides a novel interpretation for Kaufmann’s 1902 experiments [25], for Einstein’s 1905 special theory of relativity, and gives an alternative fresh view to the controversy as to whether mass is variable, and whether it is of electromagnetic or mechanical origin.

### 5.2 Bertozzi 1964 experiment

Bertozzi experiment is conventionally exhibited as proof of the mass increase predicted by Einstein’s STR [105]. The experiment directly measured time of flight of electrons over a distance of 8.4 m, and their kinetic energy  $K$  by stopping them against an aluminum target to measure temperature increase by calorimetry. Results appear in columns 1-4 of table 3. As mentioned earlier, mass of electrons was not measured in this experiment.

**Table 3.** Bertozzi experiment [105].

Experimental data				STR calculations		
Run	$\beta = V/C$	$K(\beta)$ MeV	$K(\beta)/E(0)$ Bertozzi <sup>a</sup>	$K(\beta)/E(0)$ here <sup>a</sup>	$K_{STR}/E(0)$ eq. (13)	$K(\beta)/K_{STR}$ obs./calc.
a	0.867	0.5	1	0.978	1.007	0.972
b	0.910	1.0	2	1.957	1.412	1.386
c	0.960	1.5	3	2.935	2.571	1.142
d	0.987	4.5	9	8.806	5.222	1.686
e	0.9995 <sup>b</sup>	15	30	29.354	29.433	0.997

<sup>a</sup> Bertozzi used  $E(0) = mC^2 = 0.5$  MeV as electron mass energy. We used  $E(0) = 0.511$  MeV.

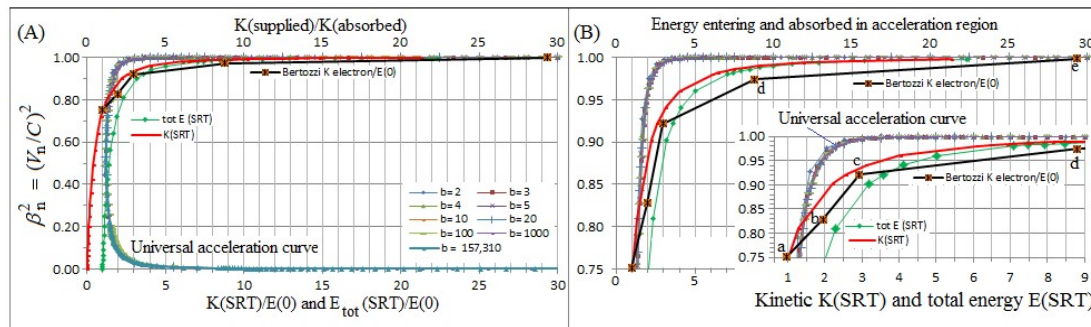
<sup>b</sup> Bertozzi rounded up to 1. To obtain observed  $K_{STR}$  from equation (13) we used 0.9995.

In STR the kinetic energy  $K_{STR}$  of a body moving with speed  $\beta$  is the difference between total energy  $E(\beta)$  and rest mass energy  $E(0)$ :

$$\frac{K_{STR}}{E(0)} = \frac{E(\beta) - E(0)}{E(0)} = (1 - \beta^2)^{-1/2} - 1 \quad (13)$$

Using the speed measured by Bertozzi (column 2 in table 3), the corresponding kinetic energy according to STR is given by equation (13) and appears in column 6. It is surprising that values for runs “b”, “c”, and “d” are significantly lower than the kinetic energy measured by Bertozzi (column 5, table 3). As shown in last column, observed kinetic energy is larger by 39%, 14% and 69% for runs “b”, “c” and “d” respectively. Also note that Bertozzi did not report error bars.





**Figure 5.** Bertozzi experiment and “mass increase” predictions. Panel A: full curves over whole speed interval for  $\beta$  from 0 to 1. Panel B shows that observed Bertozzi curve is below and to the right of the predicted  $K(\text{SRT})$ ; only two points out of five are on the curve.

Data in table 3 is plotted in figure 5 in the same way as figure 3 in the original paper ([105], page 553) namely,  $\beta^2$  versus  $K(\text{STR})/E(0)$ . Panel A shows that  $K(\text{STR})$  and  $E(\text{STR})$  are qualitatively similar to neo-Cartesian universal acceleration curve, although slope of the latter towards saturation in the upper branch is steeper than in STR. This means that the exchange of linear momentum is more efficient in our discrete description, while it is softened or averaged-out in the continuous mathematics underlying the derivation of STR equations.

Panel B zooms into the details of Bertozzi experiment, showing two regions with all five runs “a” to “e”, and four runs “a” to “d”. These details came as a surprise to this writer because they are not visible at the scale of figure 3 in the original paper [105]. From figure 5 and last column in table 3, it is difficult to understand in what sense Bertozzi experiment supports the validity of equation (13), and the associated interpretation that mass of particles increases with speed.

Nonetheless, from Bertozzi experiment it is quite evident that there is an unaccounted increase in the kinetic energy of the electrons, that is NOT explained by STR’s equation (13), and that is also larger than acceleration produced by the background primordial fluid (i.e. by the universal acceleration curve). This suggests that the acceleration processes in Bertozzi experiment, and modern everyday repetitions at CERN and the other large laboratories involve bullets with complex structure. This constitutes empirical input for our kinematic theory of matter currently under development [10].

## 6. Concluding remarks: empirical evidence is on our side

Four crucial laboratory experiments were briefly reviewed, and implications for our unified fluid and field (UFF) theory were noted. They were carried out by Michelson and Morley (1887), by Eötvös, Pekár and Fekete (1890s), by Quirino Majorana (1918), and by Bertozzi (1964). Observation of curved light paths by Eddington during solar eclipse of 1919 is consistent with a Le Sagian attenuation by sun of primordial fluid field around its vicinity. Of course, similar weaker modifications arise at larger scales, thus providing a classical mechanism for Einstein’s cosmological constant  $\Lambda$ .

Companion paper [10] constitutes the prolegomena towards a kinematic theory of matter currently under development. We have already obtained numerical values for main properties of sagion (to be reported elsewhere); they arise from identification of Planck constant with the bit of minimum angular momentum in sagion-sagion interactions, thus introducing quantum features at the root of classical mechanics.

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