Evolution of spherical over-density in thawing dark energy models

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Abstract. In this paper, we have studied the general evolution of spherical over-densities for thawing class of scalar field dark energy (DE) models. We have considered the scalar fields having canonical as well as non-canonical kinetic energy (particularly the Born-Infeld form of kinectic term) with various type of potentials, and also investigated the situation where DE is homogeneous as well as the case where DE virializes together with matter (inhomogeneous). Our study has shown that models with linear potential, in particular, can have significant deviation from the ΛCDM model in terms of density contrast at the time of virialization, and further study of the cluster number counts has shown that the total cluster number counts of different DE models can have substantial deviation from ΛCDM, and this deviation is most significant for all the models we have considered.

1. Introduction

One of the most significant discoveries in cosmology in recent years is the fact that our Universe is currently going through an accelerated expansion phase. This late time acceleration of the Universe can be due to the presence of an exotic fluid with large negative pressure known as *dark energy* (DE) or due to the modification of gravity itself. Inclusion of cosmological constant as DE is a minimal way to explain this late time acceleration, and also allowed by all cosmological observations. But this ΛCDM model is plagued with fine tuning and cosmic coincidence problems. Scalar field models mimicking a variable Λ can alleviate the fine tuning and coincidence problems and provide an interesting alternative to cosmological constant. These scalar field models are broadly classified into two categories depending upon the form of their potentials: Fast roll and slow roll models termed as freezing and thawing models in the literature [1]. Among these, thawing scalar field models are particularly interesting as they can naturally mimic equation of state very close to $w \sim -1$, which is preferred by all the observational data.

On the other hand, from the recent studies of the large scale structure surveys, it is found that DE, not only affects background expansion rate and the distance-redshift relation, but also affects the growth of structure in the universe. Hence, DE is expected to have an impact on observables such as cluster number counts and lensing statistics. Therefore, the studies of galaxy clusters would provide a useful tool to constrain the model parameters and would help to infer the properties of dark energy by discriminating among the different dark energy models. In order to understand accurately the effect of DE on the clustering properties of matter, one has to perform N-body simulations, but here, we try to understand it by considering the semi-analytical approach, called a spherical collapsed model. This model (first developed by Gunn and Gott [2]) is the simplest and the most fundamental tool for understanding the non-linear clustering of matter. It describes how a small spherical over-density decouples from the background evolution, slows down, then eventually turns around reaching the maximum radius and begins to collapse. After that, it virializes forming a bound system. In this series, we have calculated the matter density contrast at virialization and the cluster number counts with the thawing type of scalar field DE models [1] and compare the results with the corresponding ΛCDM model.

2. Details

2.1. Background evolution

We have considered a flat, homogeneous and isotropic background universe driven by non-relativistic matter and DE of thawing type, i.e., $\Omega_\phi + \Omega_m = 1$. These thawing type DE models are characterized by the fact that in the early universe the scalar field is frozen by very large Hubble damping, and the scalar field starts evolving slowly down its potential at the later time. So, the equation of state $w(a) = p_\phi/\rho_\phi$ initially starts with $w = -1$ and slowly departs from it in the later time. We have considered both ordinary scalar field with canonical kinetic term as well as tachyon type scalar field having Born-Infeld type kinectic term, which are minimally coupled to the gravity sector. The equations of motion for the canonical scalar field and the tachyon field are given by:

$$\ddot{\phi} + 3H\phi + dV/d\phi = 0, \quad \text{and} \quad \ddot{\phi} + 3\mathrm{H}\dot{\phi}(1-\dot{\phi}^2) + \frac{\mathrm{dV/d}\phi}{\mathrm{V}}(1-\dot{\phi}^2) = 0 \tag{1}$$

respectively. (For details, see [3, 4].) H is the Hubble parameter, which describes the background expansion. Here, the energy density of DE is represented as $\rho_\phi = \rho_{\phi 0} f(a)$ with $f(a) = \exp\left[3\int_a^1 \left(\frac{1+w(u)}{u}\right) \mathrm{d}u\right]$. We have considered various types of potentials: e.g., $V = \phi$, $V = \phi^2$, $V = e^\phi$ and $V = \phi^{-2}$ as well as the Pseudo Nambu-Goldstone-Boson (PNGB) model, which is characterized by the potential $V(\phi) = m^4[\cos(\phi/f) + 1]$, with $f = 1$. The Hubble parameter is given by:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_\phi). \tag{2}$$

Here, $\rho_m = \rho_{m0} a^{-3}$ is the background matter density, and ρ_ϕ represents the DE density.

2.2. The spherical collapse model

The dynamics of a spherical region of radius $r(t)$ evolving in a cosmologically expanding background in the presence of DE is governed by the Raychaudury equation:

$$\frac{\ddot{r}}{r} = -4\pi G\left[\left(w(r) + \frac{1}{3}\right)\rho_{\phi c} + \frac{1}{3}\rho_{mc}\right], \tag{3}$$

where subscript "c" denotes inside the spherical over-densities. It is easier to solve the equations after normalizing at the turn-around point (denoted by subscript "t"). Therefore, with new variables $x = \frac{a}{a_t}$ and $y = \frac{r}{r_t}$, the equation of background evolution H, and that of the spherical perturbation, given in Eq. (2) become:

$$\dot{x}^2 = {H_t}^2\Omega_{m,t}[\Omega_m(x)x]^{-1}, \quad \text{and} \quad \ddot{\mathrm{y}} = -\frac{\mathrm{H_t^2}\Omega_{\mathrm{m,t}}}{2}\left[\frac{\zeta}{\mathrm{y}^2} + \frac{1-\Omega_{\mathrm{m,t}}}{\Omega_{\mathrm{m,t}}}\mathrm{yI(x,y)}\right], \tag{4}$$

where

$$I(x,y) = \begin{cases} [1+3w(r(y))]\,\frac{f(r(y))}{f(a_t)} & \text{Clustered DE} \\ [1+3w(x)]\,f(x) & \text{Homogeneous DE.} \end{cases} \tag{5}$$

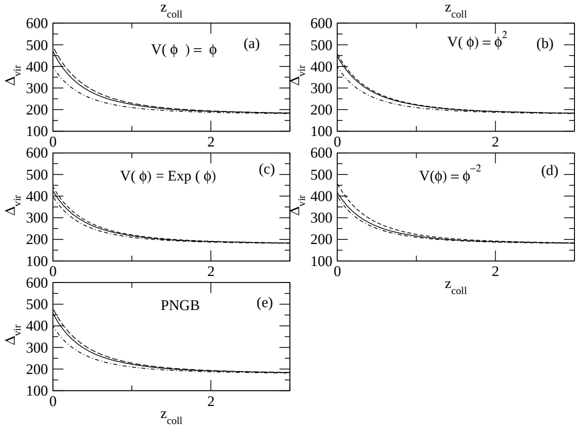

Figure 1. The non-linear density contrast at virialization vs. the collapse redshift (Z_{coll}) for $\Omega_{m0} = 0.25$. In each figure, solid, dashed and dotdashed represent inhomogeneous, homogeneous and ΛCDM models respectively. PNGB model is also compared with ΛCDM model in the last figure.

Here, ζ is the matter density contrast at turn-around, and we have calculated it from the above equations by applying the boundary conditions $(dy/dx)_{x=1} = 0$, $y_{x=0} = 0$, and $y_{x=1} = 1$ (See the details in [5]). Typically, the scalar fields have extremely small masses ($\sim H_0$ value in natural units), which is necessary for having nearly flat potentials at present day. Due to this, the scale of fluctuations for these scalar fields is extremely large, making it a smoothly distributed field within the horizon scale. So, it is safe to assume DE to be homogeneous. But it is still interesting to consider the case, where the DE clusters along with the dark matter and avoid the energy non-conservation problem examined in [6]. Hence, we have assumed both the cases.

In general, the spherical collapse formalism leads to a point singularity as the final state of the system. But physically, the objects go through a virialization process and stabilize to a finite size. Such process of virialization is not built in the spherical collpase formalism, but we have to put it by hand in order to ensure virialization. First of all, the total energy (TE) has to be conserved always, i.e., TE at the time of turn-around is equal to that at the time of virialization. At the turn-around, only the potential energy contributes, since $\dot{r} = 0$, whereas, at the virialization, the virial theorem holds. Further, we have investigated two specific cases:

- In inhomogeneous DE case, we have assumed that DE virializes together with the matter. i.e., The virial theorem holds for the total kinetic and potential energy of the system. Then, applying the energy conservation together with this, we have obtained a cubic equation of the ratio between the final (virial) r_f and the turn-around radius r_t, defined as the collapse factor $\lambda = \frac{r_f}{r_t}$ [5] as:
$$2n_1\lambda^3 - (2+n_2)\lambda + 1 = 0, \qquad (6)$$
where $n_1 = -(3w(a_f)+1)\frac{\Omega_{\phi 0} f(a_f)}{\zeta\Omega_{m0}a_t^{-3}}$, and $n_2 = -(3w(a_t)+1)\frac{\Omega_{\phi 0} f(a_t)}{\zeta\Omega_{m0}a_t^{-3}}$.
- In homogeneous DE case, we have assumed DE does not virialize inside the cluster. Its only effect is to contribute to the potential energy of the system. Similarly, applying the energy conservation, we have obtained λ for the homogeneous DE models.

After knowing ζ and λ, one can easily calculate the density contrast at virialization: $\Delta_{\rm vir} = \frac{\rho_{\rm mc,f}}{\rho_{\rm m,f}} = \frac{\zeta}{\lambda^3}\left(\frac{a_{\rm f}}{a_{\rm t}}\right)^3$, assuming that at the collapse point, the system has virialized fully. The relation between $a_{\rm f}$ and $a_{\rm t}$ is determined from the condition that the time needed to collapse is twice the turn-around time.

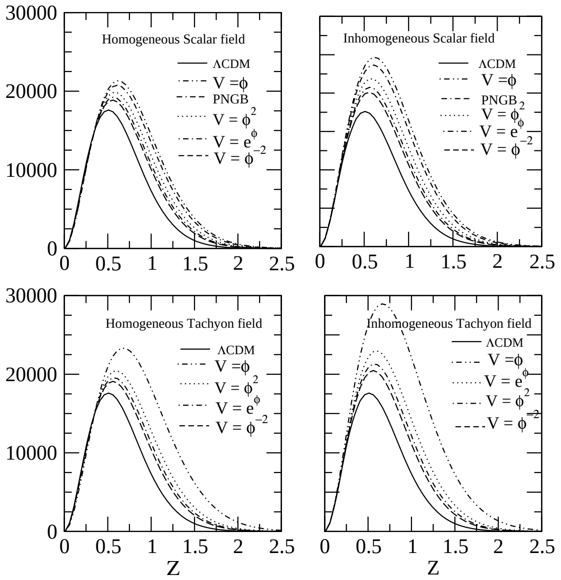

Figure 2. The total number counts N vs. Z, integrated over mass (from $M_{min} = 2 * 10^{14} M_{\odot}$) for a survey area of 10000 square degree. All the models are normalized to the same number density of haloes today. *Upper panel*: The canonical scalar field models with homogeneous (left) and inhomogeneous DE cases (right). *Lower panels*: The tachyon scalar field models with homogeneous and inhomogeneous dark energy cases in the left and right panels respectively. The different potentials correspond to the different line types. The concordance ΛCDM model (black solid line) is also plotted for comparison.

2.3. Number counts

The number of clusters in a redshift interval dz, above a given minimum (threshold) $M = M_{min}$ is obtained from $dn(M,z)/dM$ as:

$$\frac{dN}{dz}(M > M_{min}) = f_{sky}\frac{dV}{dz}\int_{M_{min}}^{\infty} dM \frac{dn}{dM}(M,z) = -f_{sky}\frac{dV}{dz}\int_{M_{min}}^{\infty} dM \frac{\rho_{m0}}{M}\frac{d\ln\sigma(M,z)}{dM} f(\sigma), \tag{7}$$

where f_{sky} is the fraction of the sky being observed and $\frac{dV}{dz}$ is the comoving volume element.

For numerical computation, the upper limit of integration in Eq. (7) is replaced by some finite mass value M_{max}. $f(\sigma(z))$ is the Sheth-Torman mass function [7]. $\sigma^2(R) = \frac{D(a)}{2\pi^2}\int_0^\infty k^3 P(k) W^2(kR)\frac{dk}{k}$ is the dispersion of the density field on a given comoving scale R, containing mass $M = 4\pi\rho_{m0}R(M)^3/3$. We have normalized the growth function such that $D(a) = 1$ at the present epoch. Assuming that the baryon density parameter $\Omega_{B0} \ll \Omega_{CDM,0}$, the CDM power spectrum can be approximated by $P(k) \propto kT^2(k)$, and we have used $T(k)$, Eisenstein and Hu's transfer function [8]. The effects of DE in the halo mass function come through the growth function $D(a)$ and the linear density contrast δ_c at the collapsed redshift. Using WMAP-7 data, we have normalized the power spectrum by setting $\sigma_8(\Lambda CDM) = 0.80$. We have set the other cosmological parameters as: $\Omega_{m0} = 0.25$, $\Omega_{\phi 0} = 0.75$, $h = 0.72$, $\Omega_{B0} = 0.0456$, and $n_s = 1$. (For details, see [9].)

3. Results and conclusion

In Figure 1, we have shown that although, all the models become indistinguishable for objects collapsing earlier, for objects collapsing around present time, some of the thawing models deviate significantly from the CDM model. The deviations are enhanced in the homogeneous DE case, where matter only virializes inside the cluster. This shows that inhomogeneous DE acts against the matter clustering. But, as the model is quite non-linear, it is difficult to predict the percentage of effects coming from the background and from DE clustering.

Thawing models with linear potential can have significant deviation from CDM model. One can see from Figure 2 that there exists significant difference in the number counts between different DE models considered, and the difference is most significant around $z \sim 0.5$ to 1. More specifically, seeing the top-left panel of Figure 2, the difference in number counts between ΛCDM and $V = \phi$ homogeneous scalar field model is ~ 2000 at $z \sim 0.7$, which is significantly larger than the statistical uncertainties (would be ~ 100 for the surveys like eROSITA and WFXT). Hence, the cluster number counts can be used for discriminating between different models. Although, this is a simplified approach to study the non-linear evolution of matter over-densities inside the cluster, and is not applicable to actual physical situation, it gives some interesting insight into the non-linear clustering of matter in the presence of thawing class of DE models. Given the fact that a large number of cluster surveys are currently ongoing as well as a number of future surveys are being planned, this can be a smoking gun to distinguish different dark energy models from ΛCDM. A proper analysis of how to constrain the models would involve error estimates of the parameters with a combination of different observational data sets, e.g., cluster counts, CMBR, BAO, etc.

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References
[1] Caldwell R R and Linder E V 2005 *Phys. Rev. Lett.* **95** 141301
[2] Gunn J E and Gott J R 1972 *Ap. J.* **176** 1
[3] Scherrer R J and Sen A A 2008a *Phys. Rev. D* **77** 083515
[4] Scherrer R J and Sen A A 2008b *Phys. Rev. D* **78** 067303
[5] Devi N C and Sen A A 2011 *MNRAS* **413** 2371
[6] Maor I and Lahav O 2005 *JCAP* **7** 3
[7] Sheth R K and Tormen G 1999 *MNRAS* **308** 119
[8] Eisenstein D J and Hu W 1998, *Ap. J.* **496** 605
[9] Devi N C, Choudhury T R and Sen A A 2011 *MNRAS* [arXiv:1112.0728]