# DVCS and the skewness effect at small $x$


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We analyze small-$x$ DVCS data using flexible GPD models and compare our outcome with the full Shuvaev transformation. We point out that the full Shuvaev transform is a ***model*** that is equivalent to a *conformal* GPD and a *minimalist* "dual" parameterization. Some mathematical subtleties of conformal representations are recalled.


Deeply virtual Compton scattering (DVCS), $\gamma^*(q_1)\, p(P_1) \to \gamma(q_2)\, p(P_2)$, is viewed as the cleanest process to access generalized parton distributions (GPDs), which encode a partonic description of the nucleon structure. In the kinematics of H1 and ZEUS collider experiments at HERA, the DVCS cross section is to a large extent dominated by the flavor singlet part of the helicity conserved Compton form factor (CFF), denoted as ${}^{\rm S}\mathcal{H}$:

$$\frac{d\sigma}{dt}(W, t, \mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\xi^2 \left|{}^{\rm S}\mathcal{H}\left(\xi, t = \Delta^2, \mathcal{Q}^2\right)\right|^2 . \tag{1}$$

Here, $W$ is the c.o.m. energy, $\Delta = P_2 - P_1$ is the momentum transfer, $-\mathcal{Q}^2 = q_1^2$ is the incident photon virtuality, and $\xi = \mathcal{Q}^2/(2W^2+\mathcal{Q}^2) \approx x_{\rm Bj}/2$ is a Bjorken-like scaling variable.

The CFF ${}^{\rm S}\mathcal{H}$ factorizes further into a convolution of the partonic, i.e., hard scattering, amplitude $\boldsymbol{C} = ({}^{\Sigma}C, {}^{G}C)$ and GPDs $\boldsymbol{H} = ({}^{\Sigma}H, {}^{G}H)$, ($\Sigma$=singlet quark, $G$=gluon),

$$ {}^{\rm S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int_{-1}^{1} {\rm d}x\, \boldsymbol{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \boldsymbol{H}(x, \eta = \xi, t, \mu^2)\, , \tag{2}$$

where the skewness parameter $\eta = -\Delta \cdot q/(P_1 + P_2) \cdot q$ is set equal to $\xi$. The factorization scale $\mu$ separates short- and long-distance dynamics and is often taken as $\mu = \mathcal{Q}$. The scale dependence is governed by evolution equations. Note that gluons do not directly enter the DVCS amplitude at leading order (LO), but rather drive the evolution of singlet quarks. Since the momentum fraction $x$ is integrated out in (2), GPDs cannot be directly revealed.

It is our objective to find flexible GPD models, which satisfy the known theoretical constraints, and which can be pinned down by fits to H1 and ZEUS DVCS data [2] at LO and beyond [5]. Thereby, we find it convenient to work with conformal GPD moments. For *integral* conformal spin $j + 2$ they are defined by convolution with Gegenbauer polynomials $C_j^{\nu}(x)$, e.g., for quarks:

$$H_j^q(\eta, t, \mu^2) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} {\rm d}x\, \eta^j\, C_j^{3/2}(x/\eta)\, H^q(x, \eta, t, \mu^2)\, , \tag{3}$$

where $q$ is the flavor index. In the forward limit $\Delta \to 0$ these moments simply reduce for $j = 0, 1, 2, \cdots$ to familiar $q_j(\mu^2)$ Mellin moments of PDFs. Conformal symmetry guarantees that they evolve autonomously under evolution at LO (except for the quark-gluon mixing). Now we require an appropriate behavior of the conformal moments (3) for $j \to \infty$ with

$|\arg(j)| \leq \pi/2$ [3]. Carlson's theorem states that their analytic continuation with respect to $j$ is unique and the GPD moments are used to calculate the corresponding CFF, cf. (2),

$$ {}^{\rm S}\mathcal{H}(\xi,t,\mathcal{Q}^2) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i+\tan\left(\frac{\pi j}{2}\right)\right] \boldsymbol{C}_j(\mathcal{Q}^2/\mu^2,\alpha_s(\mu))\boldsymbol{H}_j(\xi,t,\mu^2) \qquad (4) $$

within a Mellin-Barnes integral [3, 4, 5]. Here the singularities in the integrand, except for those in $\tan(\pi j/2)$ at $j = 1, 3, \cdots$, lie on the l.h.s. of the integration path.

To have flexible GPDs and CFF $\mathcal{H}$ and to make loosely contact with Regge terminology [4, 5], we expand their moments in terms of $t$-channel SO(3) partial waves (PWs) $\hat{d}^J_{0,0}$ [6],

$$ H^q_j(\eta,t,\mu_0^2) = \sum_{J=J^{\min}}^{j+1} \frac{h^J_j}{J-\alpha(t)}\, \beta^q_J(t)\, \eta^{j+1-J} \hat{d}^J_{0,0}(\eta)\,, \qquad (5) $$

which are labelled by the $t$-channel angular momentum $J$. These PWs can be expressed in terms of the familiar Legendre polynomials, where the cosine of the scattering angle $\theta$ may be approximated by $-1/\eta$ and which are normalized to one for $\eta \to 0$. We include an effective "pomeron" pole at $\alpha(t) = \alpha_0 + \alpha' t$ in the PW amplitudes and parameterize the residual $t$-dependence $\beta_J(t)$ by a dipole or exponential ansatz. The strengths $h^J_j$ of leading $J = j + 1$ PWs and the intercept $\alpha \equiv \alpha(t = 0)$ are obtained from fits to some DIS data, and the remaining parameter space is further reduced by either truncation of the sum (5) or its model dependent resummation, so-called $\Sigma$-PW model, and is constrained by DVCS fits, which turn out to have good quality $\chi^2/\text{d.o.f.} \approx 1$, see Fig. 1. (We take $\mu_0 = 2\,\text{GeV}$ as input scale.) One important partonic quantity that can be extracted from such fits is the skewness

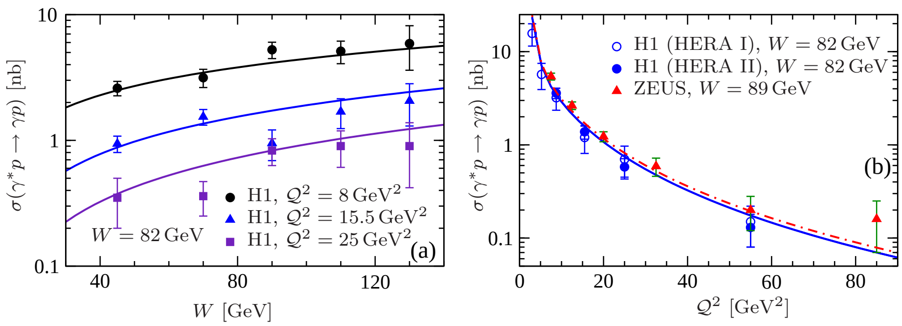


Figure 1: LO fit to the DVCS cross section from H1 and ZEUS versus $W$ (left) and $\mathcal{Q}^2$ (right).

ratio of GPDs, which is given by the value of the skewness function at $\vartheta = \eta/x = 1$:

$$ r(x,\vartheta,\mu^2) \equiv \frac{H(x,\eta=\vartheta x,t=0,\mu^2)}{H(x,\eta=0,t=0,\mu^2)}\,. \qquad (6) $$

Before we present results of our fits, some issues should be clarified. Let us recall that there are at least four different conformal GPD representations that are general and are used in phenomenology: Shuvaev-Noritzsch[a] transformation [7, 8], "dual" parameterization

[a]This integral transformation was proposed by A. G. Shuvaev in [7]. The term *(full) Shuvaev transformation* was used in Refs. [12, 13] for an integral transformation that results in a *restricted* GPD transform. This restriction was removed by J. Noritzsch in Ref. [8]. He was also the first who utilized this integral transformation to set up flexible GPD models, describing small-$x$ DVCS data at LO. Thus, — not only to avoid confusion — we name the *general* integral transformation Shuvaev-Noritzsch.

[9], and two versions of Mellin-Barnes representations [3, 10]. All of them start from an integral conformal SL(2 $|\mathbb{R}$) PW expansion of the (crossed) GPD in terms of Gegenbauer polynomials; however, in the end they provide different representations for GPDs. Since these specific conformal representations involve some mathematical subtleties, efforts to understand them have been undertaken in Refs. [8, 3, 10, 11, 5]. We consider the one-to-one correspondence among the various representations established already by construction, as sketched by the thick arrows in Fig. 2, although the inverse transformations are only partly known. Furthermore, if some mathematical subtleties are ignored, general conformal representations *degenerate* to *parameter-free* GPD models for the small-$x$ region.

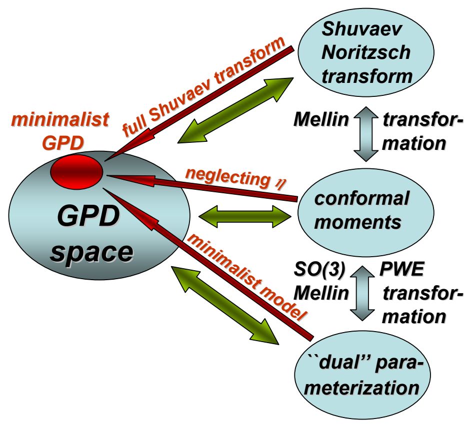


Figure 2: The one-to-one relation of various conformal GPD representations is indicated by thick arrows. The restriction of minimalist GPD models is sketched by thin arrows.

In the small-$x$ region the one-to-one correspondence among the different parameter-free models can be analytically shown by noting that they all have the conformal value of skewness function, e.g., for $t = 0$ and fixed $\mu^2$

$$r^{\rm con}(\vartheta, \mu^2) \simeq {}_2F_1\left(\begin{matrix}\alpha/2, (1+\alpha)/2\\ 3/2+\alpha\end{matrix}\middle|\vartheta^2\right). \tag{7}$$

Assuming, like in most standard PDF parameterizations, that *effective* Regge behavior is included in the intercept $\alpha \equiv \alpha(t = 0)$, one easily finds the skewness function (7) from the Mellin-Barnes representation [3, 10] of a conformal GPD model [4] by shifting the original integration path to the l.h.s. in the complex $j$-plane, cf. Eq. (4). Thereby, the hypergeometric function in (7) is nothing but the Clebsch-Gordan coefficient of the conformal PW expansion, taken at $j = \alpha - 1$. As long as the conformal moments behave smoothly in the vicinity of $\eta = 0$, a GPD model degenerates to a conformal one. For the full Shuvaev transformation model the skewness function (7) can be found in Ref. [13] as the standard integral representation of hypergeometric functions,

$$r^{\rm con}(\vartheta, \mu^2) \simeq \frac{2^{2\alpha+1}\Gamma(3/2+\alpha)}{\Gamma(1/2)\Gamma(1+\alpha)}\, x^\alpha \int_0^1 ds\, s^\alpha (1-s)^\alpha (x+\xi-2\xi\, s)^{-\alpha}\Big|_{\xi=\vartheta x}. \tag{8}$$

For an effective "pomeron" ($\alpha \sim 1$) and "Reggeon" ($\alpha \sim 1/2$) pole such analytic approximation works quite well for $x = \xi \lesssim 10^{-2}$. Further singularities ($0 < \alpha \lesssim 1$) might be taken into account, too. If the scale grows the effective "pomeron" intercept $\alpha$ increases, according to the well-known double-log approximation. If "Reggeon" contributions still play a role, the analytic approximation of a GPD will of course be a linear combination of conformal skewness functions (7).

As the reader realizes, for the same small-$x$ GPD model two different names are used, or even three if one includes the *minimalist* "dual" parameterization. In the remaining part of this presentation we simply adopt the terminology from the "dual" parameterization [9] and use instead of the notion parameter-free prediction the term minimalist GPD for both our conformal and the full Shuvaev transformation model. Let us now scrutinize some statements presented in Ref. [13], and used there to support the problematic idea that the minimalist GPD should be considered as more than just a restricted GPD model [14].

*i.* A naive Taylor expansion of integral GPD moments around $\eta = 0$ would imply that the Shuvaev-Noritzsch GPD transform reduces to a PDF at small $\eta$. However, it is well known that the operations of the small-$\eta$ expansion and of an analytic continuation of polynomials with respect to their order do not commute. The Sommerfeld-Watson transformation of a series, representing a GPD in terms of integral moments, is only possible if the assumptions of the Carlson theorem are satisfied. The theorem then states that the continuation of conformal moments is unique. If the integral GPD moments are first reduced to PDF ones, the Sommerfeld-Watson transformation is not applicable [3]. That the Shuvaev-Noritzsch GPD transform might not reduce to a PDF is known since Ref. [15, 8]. The analogue of this in the "dual" parameterization framework is the increase of singular behavior of non-leading forward-like functions [11].

*ii.* It was said in [13] that the absence of poles in the right half of complex $j$ plane is a physical requirement that is sufficient to guarantee that the full Shuvaev tranformation (minimalist) model is the unique GPD model at small $\eta$ to $O(\eta)$ accuracy. We first remark that such a requirement would be *physical* only for the angular momentum $J$. Moreover, counterexamples are known, such as resummed $\Sigma$-PW model used here and in [5] for DVCS fits.

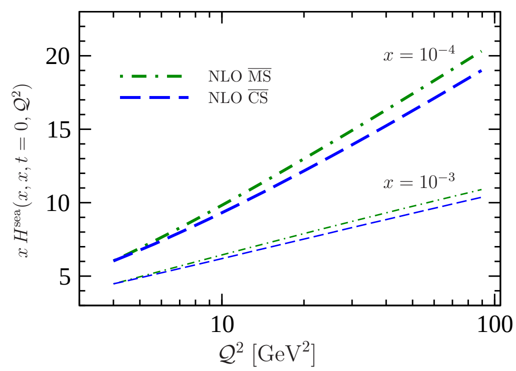


Figure 3: GPD on the cross-over line versus $\mathcal{Q}^2$ in the $\overline{\mathrm{MS}}$ scheme (dot-dashed) and the $\overline{\mathrm{CS}}$ one (dashed). The discrepancy is the error if a full Shuvaev transform is used in the $\overline{\mathrm{MS}}$ scheme.

*iii.* It was stated in [13] that up to order $O(\eta)$ the full Shuvaev transformation is compatible with the NLO evolution equation in the $\overline{\mathrm{MS}}$ scheme. However, this is not true, as spelled out, e.g., in [4], and demonstrated in Fig. 3, where we show the NLO evolution of a GPD on $\eta = x$ both in the $\overline{\mathrm{MS}}$ scheme and within the procedure [13], which actually corresponds to an evolution operator in $\overline{\mathrm{CS}}$ scheme, as dot-dashed and dashed curves, respectively. One realizes that the discrepancy increases with growing $\mathcal{Q}^2$ and decreasing $x$. Hence, it cannot be an $O(\eta)$ effect.

The phenomenological status of a minimalist GPD model was to a large extent discussed in Ref. [5]. It is shown in Fig. 4a that a flexible model (solid), pinned down by a good $\chi^2$/d.o.f. $\approx 1$ fit, is almost two times smaller than a minimalist GPD model, shown by the other curves, and so the latter model is ruled out at LO. This was also confirmed within the "dual" parameterization and standard PDFs [16]. (Small shape discrepancy of our minimalist model (dotted) and those of Ref. [13] (dot-dashed, dashed) comes from our neglecting of "Reggeon" contributions.) At NLO one might conclude from Fig. 4b that within experimental and theoretical uncertainties the minimalist models [13] (dot-dashed, dashed) are compatible with ours (solid, dot-dot-dashed), obtained from data. Still, a more detailed view shows that skewness effect, $t$-dependence, and scheme convention are interrelated. For instance, we found that in $\overline{\mathrm{MS}}$ scheme the $\Sigma$-PW model with dipole $t$-dependence (dot-dot-dashed) is somewhat away from a minimalist one (which provided only $\chi^2$/d.o.f. $\approx 1.5$). On the other hand in the $\overline{\mathrm{CS}}$ scheme at NLO (solid) and NNLO it turned out to be close to a minimalist one. We add that the error from the wrong evolution, see Fig. 3, is within the PDF uncertainties and that for the kinematical region of interest

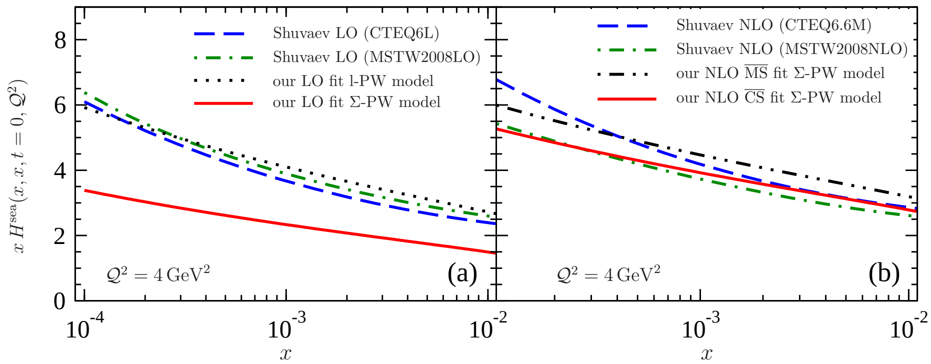


Figure 4: Sea quark GPDs on the cross-over line obtained from good fits in $\overline{\mathrm{CS}}$ (solid) and $\overline{\mathrm{MS}}$ (dot-dot-dashed) schemes are compared with minimalist GPD models used in our bad $\chi^2/\mathrm{d.o.f.} \approx 3$ LO fit (dotted) and from Ref. [13] (dashed, dot-dashed) at LO (left) and NLO (right).

even the forward evolution operator is not stable in perturbation theory. Note that the reparameterization effects for our quark GPD models, resulting from switching on NLO corrections, are much larger than for PDFs. We view this as a sign that in our modelling we have not yet reached control over evolution.

In conclusion, the full Shuvaev transform is a minimalist GPD model valid at small-$x$, which is also realized in other versions of conformal representations. So far we see no general theoretical arguments that support preference of this model over others. NLO evolution is performed in Ref. [13] in such a way that it is inconsistent with the $\overline{\mathrm{MS}}$ scheme. A minimalist model might describe DVCS data at NLO, however, not necessarily when precision level is reached. Finally, we recall that the skewness effect for gluons is in all popular GPD models approximately the same, namely, zero (i.e. $r^{\rm G}(\vartheta = 1) \approx 1$; what is often quoted as a large skewness effect is $2^{\alpha-1}$) [5]. Hence, the skewness model uncertainty, compared to the phenomenological PDF uncertainty, might presumably be considered as small at NLO. Fully flexible GPD models, allowing the control over evolution, should be invented and confronted with DVCS data in further studies. We hope that more DVCS HERA II run data will become available and so the statistical errors will be reduced. This might then allow to address the observables and partonic quantities, as discussed in Ref. [5].

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