



Thermodynamics of quantum-corrected Schwarzschild black hole surrounded by quintessence

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Abstract

Considering a quantum correction due to vacuum fluctuation in the Schwarzschild space–time with quintessence, the thermodynamic properties such as mass, temperature, heat capacity—as functions of entropy, are studied. Also the equation of state of the black hole is derived. Furthermore, the results of the quantum-corrected Schwarzschild black hole surrounded by quintessence field are compared with the conventional Schwarzschild black hole with quintessence and the quantum-corrected Schwarzschild black hole without quintessence.

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1. Introduction

Our universe is running on an accelerated expansion since the termination of the matter dominated deceleration epoch after the big bang, in contrast with the universal gravitational attractive force, thanks to the high precision measurements of cosmological observations providing convincing evidences in favor of it, like the studies on type Ia supernova [1–4], observations of the anisotropy of cosmic microwave background radiation [5–7], large scale structure [8,9], and galaxy cluster abundances [10,11]. In accordance with the belief of the scientists, the cosmic

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evolution of the expansion with acceleration is attributable to an eerie form of self-repulsive gravity—coined as ‘dark energy,’ which dominates almost 70% of the universe. Probably the most enigmatic property related to the ‘anonymous’ dark energy is its negative pressure. The cosmological constant comes as the simplest candidate for this elusive energy [12,13], but it has a potential setback, known as the fine-tuning problem [14]. That is, the experimental value is too small to be compared with the theoretical value. This is not the end of the tunnel, there are some other alternative models as dark energy candidates, most of which consist of dynamical scalar fields. A few of them are quintessence [15–21], k -essence [22–24], chameleon field [25], tachyon field [26], phantom [27–29], quintom [30,31], and dilaton dark energy [32]. A comprehensive review on the various models representing dark energy is provided in Ref. [33]. These models differ with one another in the value of the equation of state parameter which denotes the relation between the pressure and the energy density.

The quintessence is a time-evolving, spatially inhomogeneous scalar field coupled to gravity, and its equation of state parameter ω_q has the value in the range $-1 < \omega_q < -1/3$. The equation of state for the quintessence matter is $P = \omega_q \rho_q$, where P is the pressure, and ρ_q is the energy density. The pressure P is negative to cause the expansion. Quintessence has been investigated from different perspectives, and in the case of comparative study with other dynamical scalar fields of dark energy, the correspondence between quintessence and tachyon with constant equation of state parameter [34], spherically symmetric space–time singularity in quintessence/phantom [35], discussion on the possibility to differentiate quintessence from cosmological constant [36], dynamics of interacting phantom and quintessence [37], are to name a few. Quintessence model has been extended by coupling the scalar field to the Ricci scalar [38]. The effect of quintessence on the shift of gravitational frequency and the deflection of light has been analyzed [39]. Studying the black hole space–time in the presence of quintessence started when Kiselev first derived the metric of static spherically symmetric black hole surrounded by quintessence matter by solving Einstein field equations with the energy–momentum tensors satisfying the conditions of additivity and linearity as $T_t^t = T_r^r = \rho_q$, and $T_\theta^\theta = T_\phi^\phi = -\rho_q (3\omega_q + 1)/2$ [40]. The new solution depends on the quintessential state parameter ω_q . After that, quasinormal modes of black hole surrounded by quintessence [41–45], thermodynamic properties [46–49], and $P - V$ criticality [50] have been extensively explored. Particularly, in Ref. [51], the temperature and the heat capacity of the Schwarzschild black hole surrounded by quintessence have been derived and discussed for different equation of state parameter values of ω_q . And in Ref. [52], for a specific value of $\omega_q = -2/3$, those properties, along with the $P - V$ isothermal situation have been investigated and compared with the Reissner–Nordström quintessence black hole. In this paper, these studies are extended in the quantum-corrected Schwarzschild space–time with quintessence matter.

Taking into account the back-reaction of the space–time due to quantum fluctuations, in 1994, Kazakov and Solodukhin found a modified expression for the Schwarzschild metric [53]. The metric co-efficient of the quantum-corrected Schwarzschild black hole is $f(r) = -2M/r + \int^r U(\rho)d\rho/r$, where M is the black hole mass. For an empty space, $U(\rho) = 1$, and the metric can be reduced to the Schwarzschild black hole solution. Considering the quantum fluctuation of the vacuum, the quantity $U(\rho)$ transforms to $U(\rho) = e^{-\rho} [e^{-2\rho} - \frac{4}{\pi}G_R]^{-1/2}$ [54], where $G_R = G_N \ln(\mu/\mu_0)$, G_N is the Newton’s gravitational constant, μ is a scale parameter satisfying $t = \ln(\mu/\mu_0)$, while $\mu = \mu_0$ for $t = 0$, ρ is analog to the logarithmic of an inverse radius as $\rho = \ln(1/r)$, such that $U = r [r^2 - \frac{4}{\pi}G_R]^{-1/2}$. The metric function for the quantum-corrected Schwarzschild black hole can be written as $f(r) = [-2M + \sqrt{r^2 - a^2}]/r$, the quantum cor-

rection parameter a has the dimensionality of length on which the curvature of the space–time depends, and $a^2 = 4G_R/\pi$. The radial coordinate r is restricted to $r > a$. For $r \gg a$, the metric co-efficient becomes $f(r) \simeq 1 - 2M/r - a^2/2r^2$, which seems like the metric function of a charged body with mass M and imaginary charge $\pm ia/\sqrt{2}$ [55,56]. Thus to say, a acts like an imaginary charge added to the space–time metric.

In Sect. 2, the mass of the quantum-corrected Schwarzschild black hole surrounded by quintessence is evaluated in terms of entropy. The temperature and the heat capacity are calculated in Sect. 3, and the equation of state is derived in Sect. 4. Finally, in Sect. 5, concluding remarks are presented.

2. Black hole mass as function of entropy

The geometry of the quantum-corrected Schwarzschild black hole surrounded by quintessence is given by

$$ds^2 = -f(r)dt^2 + dr^2/f(r) + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

where

$$f(r) = -\frac{2M}{r} + \frac{\sqrt{r^2 - a^2}}{r} - \frac{c}{r^{3\omega_q + 1}},$$

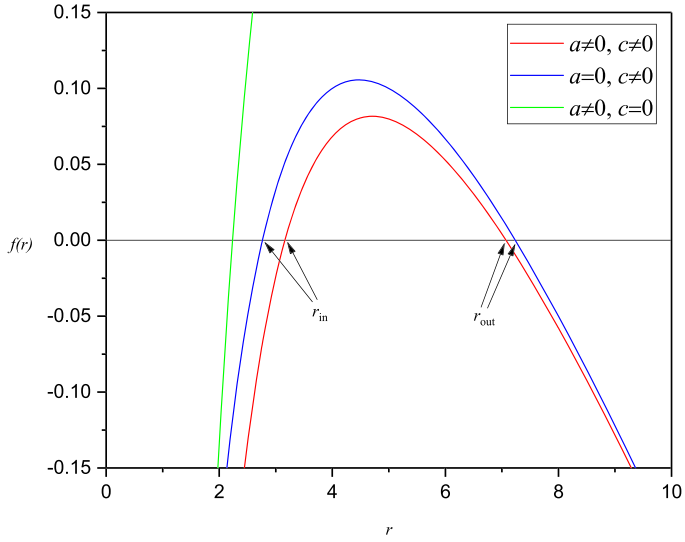
M is the mass of the quantum-corrected black hole, and c is the positive renormalization factor depending on the quintessence energy density by the relation $\rho_q = -\frac{c}{2} \frac{3\omega_q}{r^{3(\omega_q+1)}}$. The event horizon radius r_H can be found by setting $f(r)|_{r=r_H} = 0$. Note that the values of ω_q in the range $-1 < \omega_q < -1/3$ can lead to multiple horizons. Specifically, for $\omega_q = -2/3$, there are two horizons. One is the inner horizon r_{in} , and the other is the outer horizon r_{out} which is commonly called as the quintessence horizon—similar to the cosmological horizon in de Sitter space–time [57]. For $8Mc < 1 - 4a^2c^2$,

$$r_{in} = \frac{1}{\sqrt{2c}} \sqrt{-4Mc + 1 - \sqrt{1 - 4a^2c^2 - 8Mc}},$$

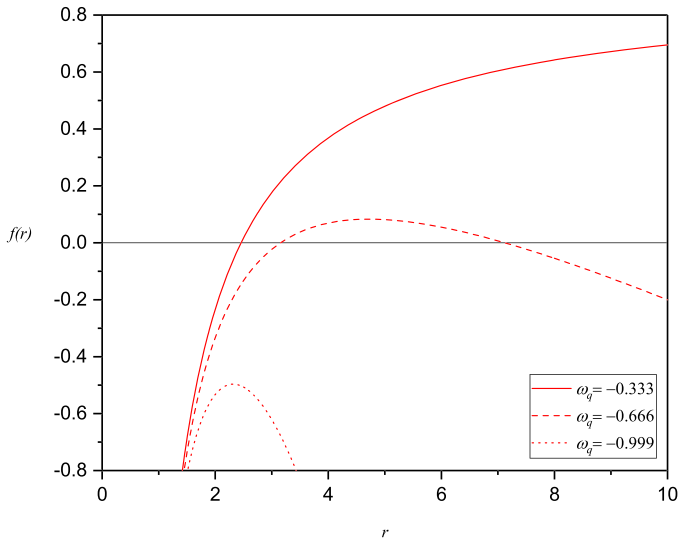
and

$$r_{out} = \frac{1}{\sqrt{2c}} \sqrt{-4Mc + 1 + \sqrt{1 - 4a^2c^2 - 8Mc}}.$$

In Fig. 1(a), the lapse function $f(r)$ is plotted in variation with r . The red, blue, and green solid lines are for the quantum-corrected Schwarzschild black hole with quintessence, the quintessence conventional Schwarzschild black hole, and the quantum-corrected Schwarzschild black hole without quintessence, respectively. In the presence of quintessence field, for $\omega_q = -2/3$, there are two horizons, r_{in} and r_{out} . The quantum correction, in the presence of quintessence matter, increases the inner horizon radius and decreases the outer horizon radius. Without quintessence, the quantum-corrected black hole (green solid line) has one horizon at the distance $r_H = [(2M)^2 + a^2]^{1/2}$. Fig. 1(b) shows the variation of $f(r)$ with respect to r for the quantum-corrected Schwarzschild black hole surrounded by quintessence for different values of ω_q . It is evident that for some central values of ω_q in the range $(-1, -1/3)$, there are two horizons, and when approaching to the boundary values in the ‘right’, there is a single horizon, and approaching to the ‘left’ leads to no horizon.



(a) $f(r)$ vs. r , setting $M = 1$, $a = 1$, $c = 0.1$, and $\omega_q = -2/3$.



(b) $f(r)$ vs. r , for different ω_q , setting $M = 1$, $a = 1$, and $c = 0.1$.

Fig. 1. Variation of $f(r)$ with respect to r . (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

The relation between the horizon radius r_H and the black hole mass M is

$$M = \frac{1}{2} \left[\sqrt{r_H^2 - a^2} - \frac{c}{r_H^{3\omega_q}} \right]. \tag{1}$$

Note that for $a = 0$, Eq. (1) stands as $M = \frac{1}{2} \left[r_H - c/r_H^{3\omega_q} \right]$, which is the relation for the Schwarzschild black hole with quintessence [51]. From the well-known area-entropy relation for black hole event horizon as $S = A/4$, where S is the entropy and A is the surface area of the horizon [58–60], one gets $S = \pi r_H^2$. This area-entropy relation is compatible with the present space–time under consideration (see Appendix A). Using this relation in Eq. (1), the mass of the quintessence quantum-corrected Schwarzschild black hole can be written in terms of entropy:

$$M = \frac{1}{2} \left[\left(\frac{S}{\pi} - a^2 \right)^{1/2} - c \left(\frac{\pi}{S} \right)^{3\omega_q/2} \right]. \quad (2)$$

It is evident from Eq. (2) that the quantum deformation imposes a lower bound on the entropy of the quantum-corrected Schwarzschild black hole, which is $S > \pi a^2$.

In Fig. 2(a), the mass variation with respect to entropy of quantum-corrected quintessence Schwarzschild black hole (red solid line), quintessence Schwarzschild black hole (blue solid line), and quantum-corrected Schwarzschild black hole (green solid line) is plotted for the quintessential equation of state parameter $\omega_q = -2/3$. The mass of quantum-corrected Schwarzschild black hole surrounded by quintessence with same entropy is smaller than the other two black holes. In Fig. 2(b), the mass of the quantum-corrected quintessence Schwarzschild black hole as a function of entropy is shown, for different ω_q . It can be found that as ω_q increases, mass of the black hole also increases, for a fixed entropy value.

3. Temperature and heat capacity

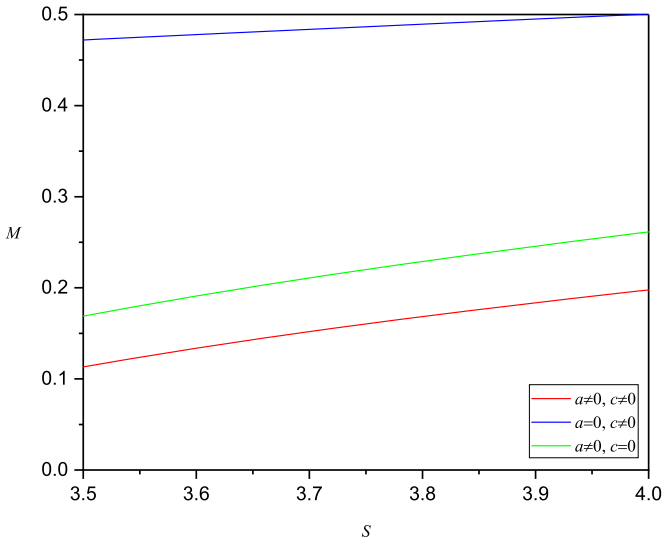
The first law of black hole thermodynamics is $dM = TdS + \Phi dQ$, where T , Φ , and Q are the temperature, the electric potential, and the electric charge of the black hole, respectively [59]. Therefore, for neutral black hole ($Q = 0$), the temperature is defined as $T = \partial M / \partial S$. Also the heat capacity is $C = T \partial S / \partial T$. Using the mass expression in Eq. (2), the temperature of the quantum-corrected Schwarzschild black hole can be derived as

$$T = \frac{1}{4} \left[\frac{3c\omega_q \pi^{3\omega_q/2}}{S^{(3\omega_q/2)+1}} + \frac{1}{\pi \left(\frac{S}{\pi} - a^2 \right)^{1/2}} \right]. \quad (3)$$

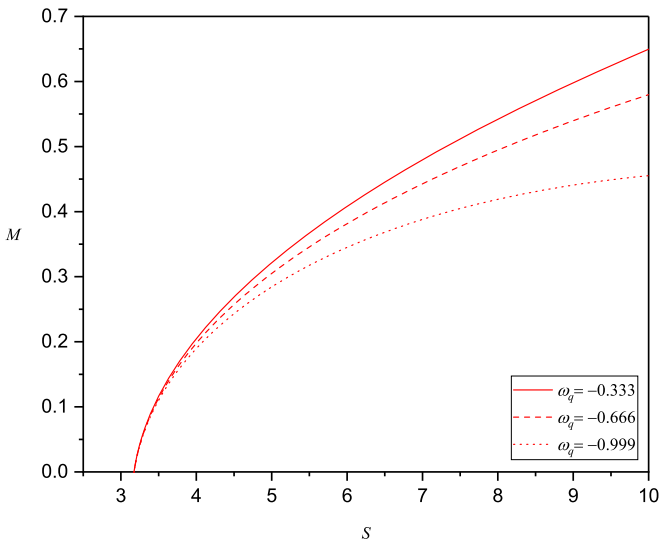
Temperature T as a function of entropy S is plotted in Fig. 3. The red solid line in Fig. 3(a) represents the temperature of the quintessence quantum-corrected Schwarzschild black hole, the blue solid line is for the quintessence Schwarzschild black hole, and the green solid line is for the quantum-corrected Schwarzschild black hole. The presence of quintessence matter decreases the temperature of the quantum-corrected Schwarzschild black hole. Of the three comparing black holes, for a fixed value of entropy, the temperature of the quantum-corrected Schwarzschild black hole is the largest. Fig. 3(b) shows the temperature of the quantum-corrected Schwarzschild black hole for varying ω_q . An increment in the value of the equation of state parameter of quintessence increases the temperature of the black hole.

The heat capacity C , in terms of entropy and quintessence parameter is

$$C = - \frac{2S(S - \pi a^2) \left[3c\omega_q \pi^{(3\omega_q+1)/2} (S - \pi a^2)^{1/2} + S^{(3\omega_q+2)/2} \right]}{6c\omega_q \pi^{(3\omega_q+1)/2} (S - \pi a^2)^{3/2} \left(\frac{3\omega_q}{2} + 1 \right) + S^{(3\omega_q+4)/2}}. \quad (4)$$



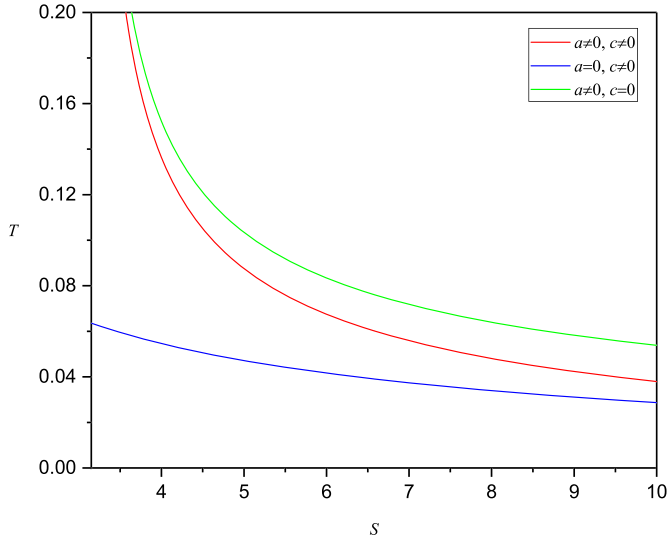
(a) M vs. S , setting $a = 1$, $c = 0.1$, and $\omega_q = -2/3$.



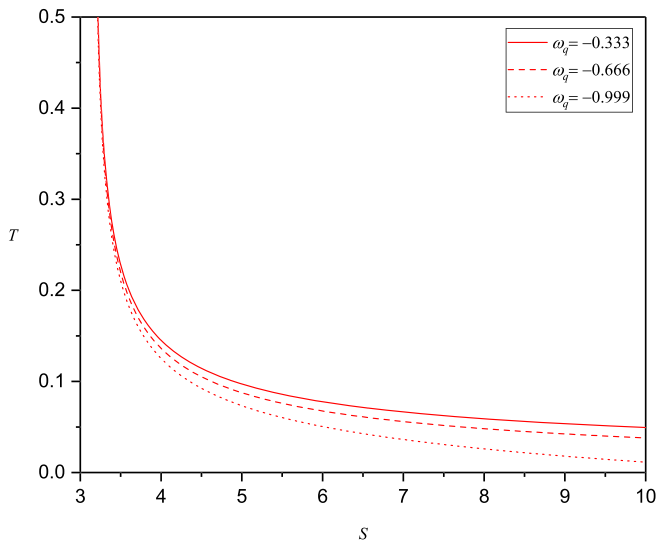
(b) M vs. S , for different ω_q , setting $a = 1$, and $c = 0.1$.

Fig. 2. Variation of M with respect to S .

In Fig. 4(a), the red solid line shows the variation in heat capacity with respect to the entropy of the quantum-corrected Schwarzschild black hole surrounded by quintessence. As mentioned earlier, the entropy of this black hole must satisfy the condition $S > \pi a^2$, the heat capacity is always negative. This fact is apparent from Eq. (4) also. In black hole thermodynamics, the sign of the heat capacity states the stability of a black hole. $C > 0$ indicates that a black hole is thermodynamically stable, while $C < 0$ denotes an instability. Therefore, a quantum-corrected



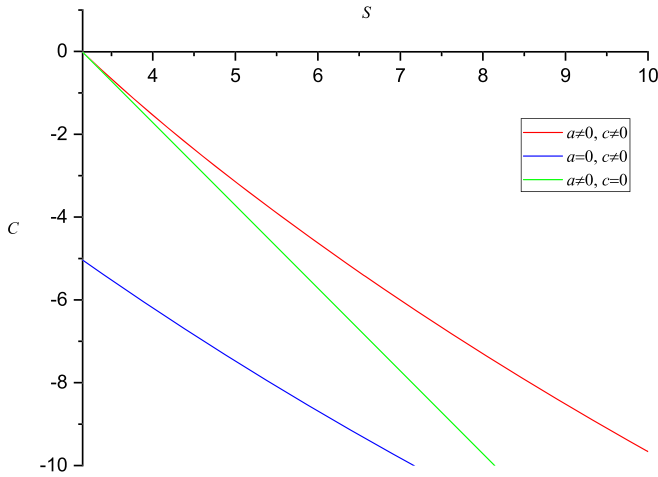
(a) T vs. S , setting $a = 1$, $c = 0.1$, and $\omega_q = -2/3$.



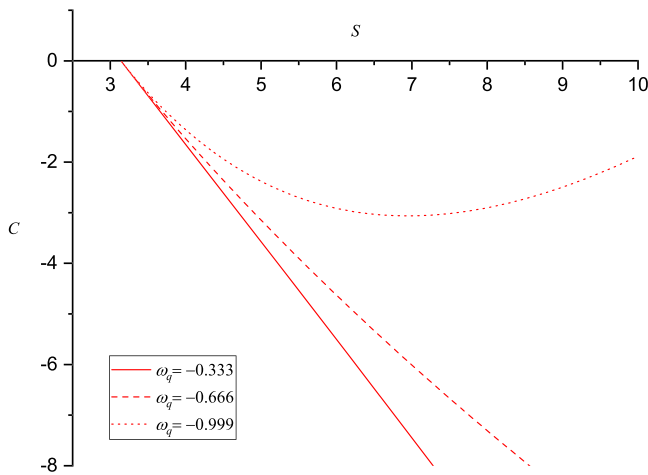
(b) T vs. S , for different ω_q , setting $a = 1$, and $c = 0.1$.

Fig. 3. Variation of T with respect to S .

Schwarzschild black hole with quintessence is unstable. The blue solid line indicates the heat capacity of quintessence Schwarzschild black hole, and the green solid line, the heat capacity of quantum-corrected Schwarzschild black hole. Of the three black holes, the heat capacity of quintessence quantum-corrected Schwarzschild black hole with the same entropy is higher than the other two. Fig. 4(b) shows the heat capacity of quantum-corrected Schwarzschild black hole



(a) C vs. S , setting $a = 1$, $c = 0.1$, and $\omega_q = -2/3$.



(b) C vs. S , for different ω_q , setting $a = 1$, and $c = 0.1$.

Fig. 4. Variation of C with respect to S .

in the presence of quintessence field for different equation of state parameter values. For a higher value of ω_q , the heat capacity of the black hole tends to decrease.

4. Equation of state

In black hole thermodynamics, volume is considered as a thermodynamic variable [61,62]. So, the equation of state can be derived by evaluating the volume. The natural variables for black hole enthalpy H are entropy and pressure, and as the mass is associated with the enthalpy, one can write H , in turn M , in terms of S and P , which is $M = H(S, P)$ [45]. Using the mass

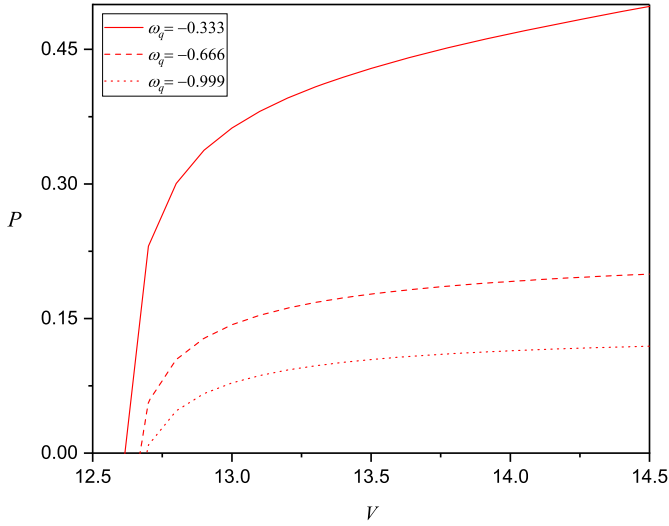


Fig. 5. Variation of P with respect to V for different ω_q , setting $T = 1$, and $a = 1$.

expression in Eq. (2), and the relation between pressure P and the parameter c , $P = -c/8\pi$, the enthalpy H of the quantum-corrected Schwarzschild black hole surrounded by quintessence can be written as

$$H(S, P) = \frac{1}{2} \left[\left(\frac{S}{\pi} - a^2 \right)^{1/2} + 8\pi P \left(\frac{\pi}{S} \right)^{3\omega_q/2} \right].$$

Then the volume can be found by Legendre transformation,

$$V = (\partial H / \partial P)_S = 4\pi / r^{3\omega_q}.$$

Therefore, the equation of state for the black hole can be derived as

$$T = \frac{1}{4\pi} \left[\left(\left(\frac{4\pi}{V} \right)^{\frac{2}{3\omega_q}} - a^2 \right)^{-1/2} - \frac{6\omega_q P}{(4\pi)^{\frac{2}{3\omega_q}}} V^{\left(\frac{2}{3\omega_q} + 1 \right)} \right]. \quad (5)$$

In this equation of state, $V > 4\pi a^{-3\omega_q}$. For different values of ω_q , $P - V$ isotherms are shown in Fig. 5. The pressure P with the same volume V is higher for higher value of the equation of state parameter ω_q .

5. Closing remarks

The thermodynamics of the Schwarzschild black hole along with the Reissner–Nordström black hole in the quintessence have been investigated recently [52]. And the thermodynamics with the Gross–Perry–Yaffe type phase transition had been investigated a while ago [54]. But the thermodynamics of the quantum-corrected Schwarzschild black hole surrounded by quintessence are missing in the literature. In this paper, the thermodynamic properties have been explored in the quantum-deformed quintessence Schwarzschild space–time. Due to vacuum fluctuation, Schwarzschild space–time undergoes a modification. The modified quantum-corrected

Schwarzschild space–time, in the presence of quintessence matter, has more than one horizon, like the de Sitter space–time. Using the area–entropy relation for the black hole horizon, the thermodynamic properties have been expressed in terms of entropy. The outcomes of the present analyses are summarized below.

- The length–dimensional quantum–correction term imposes a lower bound on the entropy and the volume of the quintessence quantum–deformed Schwarzschild black hole. Namely, the entropy $S > \pi a^2$, and the volume $V > 4\pi a^{-3\omega_q}$.
- The correction term stretches the inner horizon and shrinks the outer horizon in the quantum–deformed Schwarzschild black hole with quintessence. For $\omega_q = -2/3$, the horizons of the conventional Schwarzschild black hole surrounded by quintessence are $r_{\text{in}} = \frac{1-\sqrt{1-8Mc}}{2c}$ and $r_{\text{out}} = \frac{1+\sqrt{1-8Mc}}{2c}$, where $8Mc < 1$ [52]. Specifically, $\frac{\sqrt{-4Mc+1-\sqrt{1-4a^2c^2-8Mc}}}{\sqrt{2c}} > \frac{1-\sqrt{1-8Mc}}{2c}$, and $\frac{1+\sqrt{1-8Mc}}{2c} > \frac{\sqrt{-4Mc+1+\sqrt{1-4a^2c^2-8Mc}}}{\sqrt{2c}}$. Fig. 1(a) illustrates these facts.
- And the most important result is that the heat capacity of the quintessence quantum–corrected Schwarzschild black hole, under the entropy constraint, is negative, pointing towards the instability. So the black hole evaporates quickly, and vanishes eventually.

Acknowledgement

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Appendix A. Compatibility of Bekenstein–Hawking relation

In this section, it is shown that the entropy of the quantum–corrected Schwarzschild black hole surrounded by quintessence matter is one–fourth of the horizon area, following the Noether charge approach developed by Wald [63]. Before that, the derivation of the vacuum quantum–corrected Schwarzschild metric, shortly discussed in Sect. 1, needs some elaboration as in Ref. [54]. The starting point is the Lagrangian in the Einstein–Hilbert action

$$\mathcal{L} = \frac{R}{16\pi G_N} + L_{\text{matter}}. \quad (\text{A.1})$$

Neglecting, for now, the classical matter contribution to the Lagrangian \mathcal{L} , a spherically symmetric decomposition of the four dimensional metric into the two dimensional angular part can be expressed in terms of the diffeomorphic dilaton field ϕ as

$$ds_2^2 = \frac{2G_N}{\pi} e^{-2\phi} \left(d\theta^2 + \sin^2\theta d\varphi^2 \right),$$

with Lagrangian

$$\mathcal{L}_2 = \frac{1}{2\pi} \left[e^{-2\phi} \left(R + 2(\nabla\phi)^2 \right) + \frac{\pi}{G_N} \right].$$

Adding a generalized potential $U(\phi)$ into the last term of \mathcal{L}_2 for renormalization, and then introducing a two dimensional non–linear σ –model, the potential term stands as

$$U(\phi) = e^{-\phi} \left[e^{-2\phi} - 4G_R/\pi \right]^{-\frac{1}{2}}.$$

Then solving the equations of motion for the action with U -added Lagrangian \mathcal{L}_2 , the metric co-efficient of the quantum-corrected Schwarzschild black hole can be found as

$$f(r) = \frac{1}{r} \left[-2M + \int^r U(r) dr \right] = \frac{1}{r} \left[-2M + \sqrt{r^2 - a^2} \right].$$

The quantum-deformed Schwarzschild black hole characterized by the above lapse function has the event horizon slightly larger than the conventional Schwarzschild black hole. The space–time is not Ricci flat for the non-vanishing curvature $\frac{2a^4}{r^6}$, but the black hole is static spherically symmetric like the usual Schwarzschild case. As the event horizon of any static black hole is a Killing horizon, so is of the quantum-corrected Schwarzschild black hole. According to the zeroth law of black hole mechanics, the surface gravity is constant over a Killing horizon, and further, the horizon is of bifurcate type. It is worth noting that the surface gravity of the quantum-corrected Schwarzschild black hole horizon is $\left[4\pi \sqrt{r_H^2 - a^2} \right]^{-1} = \frac{1}{8\pi M}$, just like the regular Schwarzschild black hole.

In the well-known Noether charge approach, the first law of black hole mechanics can be written as [63]

$$\delta \int_{\Sigma} \mathbf{Q} = \delta \mathcal{E} - \Omega_H^{(\mu)} \mathcal{J}_{(\mu)},$$

where \mathbf{Q} is the Noether potential associated with the diffeomorphism on the manifold, Σ is the bifurcation $(n - 2)$ -surface of the black hole, \mathcal{E} is the canonical energy corresponding the ADM mass with additional contributions coming from matter field if present, Ω_H is the angular velocity of the horizon, and \mathcal{J} is the canonical angular momentum. If \mathbf{Q} can be expressible in terms of ‘local, geometric’ quantities of the space–time, then the entropy of the black hole is

$$S = \frac{2\pi}{\kappa} \int_{\Sigma} \mathbf{Q},$$

κ being the surface gravity of the black hole.

It has been demonstrated that the quantum deformation only alters the potential in the action. Therefore, the Bekenstein–Hawking area-entropy relation holds in the vacuum quantum-corrected Schwarzschild space–time [54]. Now, consider the matter-part in the Lagrangian for the presence of the scalar field of quintessence matter, independently of the quantum correction. For simplicity, the quintessence field is taken to be conformally invariant. The Noether current $(n - 1)$ -form for the scalar field in four dimensions is [64]

$$J_{\mu_2\mu_3\mu_4} = \frac{1}{8\pi G_N} \nabla_{\lambda} \left[\nabla_{\sigma} \left(g^{\sigma[\lambda} \xi^{\alpha]} \right) \right] \epsilon_{\alpha\mu_2\mu_3\mu_4},$$

where ϵ is the volume form associated with the metric. The Noether charge $(n - 2)$ -form is

$$Q_{\mu_3\mu_4} = -\frac{1}{16\pi G_N} \left[g^{\alpha\sigma} \left(g^{\beta\lambda} \nabla_{\sigma} \xi_{\lambda} + \nabla_{\alpha} \xi^{\beta} \right) \right] \epsilon_{\alpha\beta\mu_3\mu_4}. \tag{A.2}$$

On the bifurcation surface, $\xi^{\alpha} = 0$, and Eq. (A.2) reduces to

$$Q_{\mu_3\mu_4} = -\frac{1}{16\pi G_N} (\nabla^\alpha \xi^\beta) \epsilon_{\alpha\beta\mu_3\mu_4},$$

leading to the black hole entropy surrounded by quintessence as

$$S = \frac{A}{4G_N}.$$

Therefore, neither the quantum correction nor the quintessence matter does modify the Bekenstein–Hawking relation. Although it is expected that the entropy of the quantum-corrected Schwarzschild black hole surrounded by quintessence is

$$S = \frac{A}{4G_R},$$

where G_R is the quantum-deformed gravitational coupling constant related to the Newton's gravitational constant by $G_R = G_N \ln\left(\frac{\mu}{\mu_0}\right)$. $G_R = 1$ when $G_N = 1$ and $\mu = \epsilon\mu_0$.

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