FINITE ENERGY SUM RULES - RECENT DEVELOPMENTS

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In the past two years FESR evaluations have increased both in number and in accuracy. This progress has been made possible by the steady accumulation of experimental data leading to better low energy phase shift analysis. Therefore it is possible for the first time to look for general features in a large number of FESRs, rather than try to guess general trends from one or two sumrules.
We shall see that several general features do emerge - some expected, some not.

Table I lists the processes which I shall consider, with their recent phase shift analyses[1] in column 2, and the resulting FESR evaluations[2-15] in column 3. Column 4 gives the FESR cutoffs in terms of the variable $\nu = \frac{1}{2}(s-u)$.

An FESR is an integral at fixed momentum transfer (t) of a crossing-odd amplitude $\nu^M T(\nu,t) = -[(-\nu)^M T(-\nu,t)]$ around the contour shown in figure 1. The FESR states simply that the integral is zero, so that

$$- \int_B^C T(\nu)\nu^M d\nu = \int_A^B T(\nu)\nu^M d\nu$$
$$= \int_{\nu_0}^{\nu_c} \nu^M \, \mathrm{Im}(T(\nu)) \, d\nu \qquad (1)$$

The right-hand side of the equation (which I shall refer to as the FESR) is evaluated from the low energy phase shifts, or in a resonance saturation approximation. The left-hand side is evaluated from a high energy model or is assumed to be similar to the imaginary part of the high energy amplitude. Sincethe cutoff N is fairly low, duality is a vital ingredient, either when testing a model, to assert[16] that the model amplitudes can be continued down to the cutoff ν_c to evaluate the LHS, or in a model-independent approach, to assert that the low-energy integral is somehow related to the imaginary part of the high-energy Regge amplitude.

I shall not consider CMSRs or optimised FESRs, but shall only consider the lowest moment FESRs. These generally have M=0 (odd signature) or M=±1 (even signature) in equation (1), where the amplitude $T(\nu)$ is defined so as to have Regge behaviour $T(\nu) \sim \nu^\alpha$. I shall look for general features as a function of t-channel quantum numbers (i.e. allowed Regge poles) and of s-channel helicity structure (motivated by absorption). Other sumrules for different amplitudes can also be interesting, but

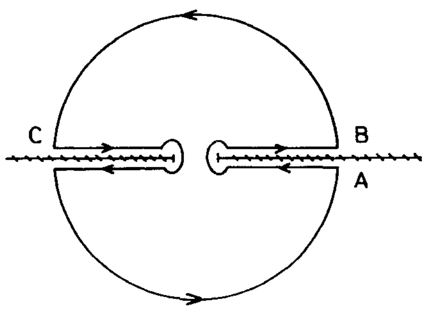

Fig. 1 The contour of integration for a finite energy sumrule.

TABLE I

Reaction	Phase Shifts		FESRs	Cutoff $\frac{1}{2}(s-u)$
$\pi N \to \pi N$	Almehed-Lovelace	71	Elvekjaer Inami & Ringland	4.0 $\to$5.2
	Saclay	72 (73)	Phillips	
$KN \to KN$	Albrow et al.	71	Lyberg	3.0
	B.G.R.T.	73	Elvekjaer & Martin	
	Chao et al.	73		
	Langbein-Wagner	72		
	L.O.M.M.	73	Argyres et al.	
	Litchfield et al.	71		
$\pi N \to K\Lambda$, $\bar{K}N \to \pi\Lambda$	Devenish et al.	74	Devenish Froggatt & Martin	3.4
	Lovelace-Wagner	71		
	Van Horn	72	Vanryckeghem	
$\pi N \to K\Sigma$, $\bar{K}N \to \pi\Sigma$	Kalmus et al.	71	Vanryckeghem	3.0
	Langbein-Wagner	73		
$\gamma N \to \pi N$	K.M.O.R.	73 (74)	Barker et al.	2.3 $\to$3.2
	Metcalfe-Walker	74	Worden	
	Devenish et al.	73-74		
$\pi N \to \pi\Delta$	LBL-SLAC	72-74	Froggatt & Parsons	2.7
	Kernan et al.	73		
$\pi\pi \to \pi\pi$	CERN-Munich	73-74	Shaw & Martin	1.9 $\to$3.5
	Estabrooks Martin	74	Pennington	
			Froggatt & Petersen	

I believe the above selections will convey the maximum of useful information in the available time.

It is widely accepted as a working hypothesis[17] that s-channel helicity flip amplitudes show only weak absorption effects, and are dominated by exchange degenerate Regge poles. If this is so, the spinflip FESR integrals, which I shall denote by $F^{\pm}$ ($\pm$ for signature) have the following properties:- The integral F^{+} vanishes where the Regge trajectory α goes through zero (implied by pole dominance alone). F^{-} vanishes when $\alpha=0$ (implied by pole dominance and EXD). EXD also relates the signs and magnitudes of the FESRs $F^{\pm}$ for a given process:-

$$\frac{F^{+}}{F^{-}} = \frac{\alpha+1}{\alpha+2}\,\nu_c \qquad (2)$$

We shall now examine the FESR integrals to test these properties, and so test the hypotheses of pole-dominance and EXD for the spinflip amplitudes.

Spinflip ρ exchange FESRs (figure 2) all show [3,5,10,11] the predicted EXD zero at $t\simeq-0.5$, in the processes $\pi N\to\pi N$, $KN\to KN$, $\pi N\to\pi\Delta$ and $\gamma N\to\pi N$. Furthermore the relative magnitude and sign of πN and KN FESRs agree with SU(3), and the two $\pi N\to\pi\Delta$ FESRs have relative sizes consistent with the Stodolsky-Sakurai distribution*. Therefore these FESRs are remarkably consistent with pole dominance and EXD for ρ exchange, as are the high energy data[18] (although they do not rule out a strong absorption model[19] in which the ρ has no EXD zero). However, the well-determined spinflip ω exchange FESR[9,10] in $\gamma p\to\pi^{o}p$ (figure 3) has no EXD zero near $t=-0.5$ (the position of the $\frac{d\sigma}{dt}$ dip at high energy), indicating that this amplitude is probably not

*However, there may be a problem with their overall magnitudes, (reference 11).

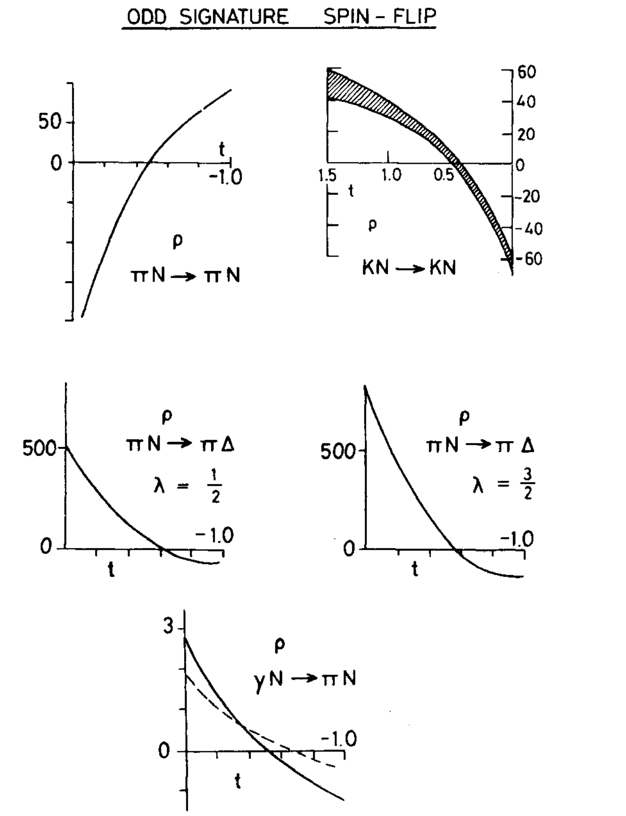

Fig. 2 Spinflip ρ exchange FESRs as a function of momentum transfer t, from refs 3, 5, 10, 11. In the photoproduction FESR, the solid curve is an evaluation from the multipoles of Metcalfe and Walker, and the dashed curve is evaluated from those of Devenish et al.

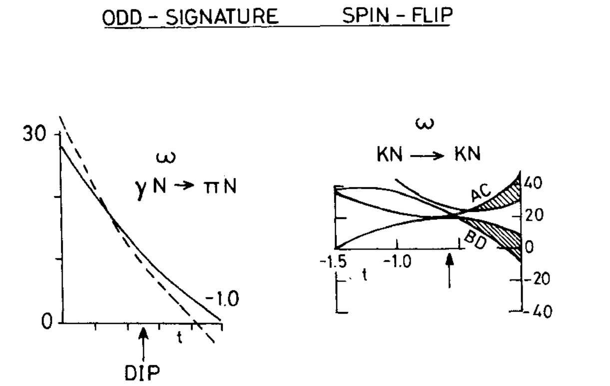

Fig. 3 Spinflip ω exchanges FESRs from refs 5, 10. The two curves for the photoproduction FESR are as in fig. 2. The arrow marks the position of the high energy dip in $\gamma p \to \pi^0 p$, where pole dominance would imply a zero in the FESR.

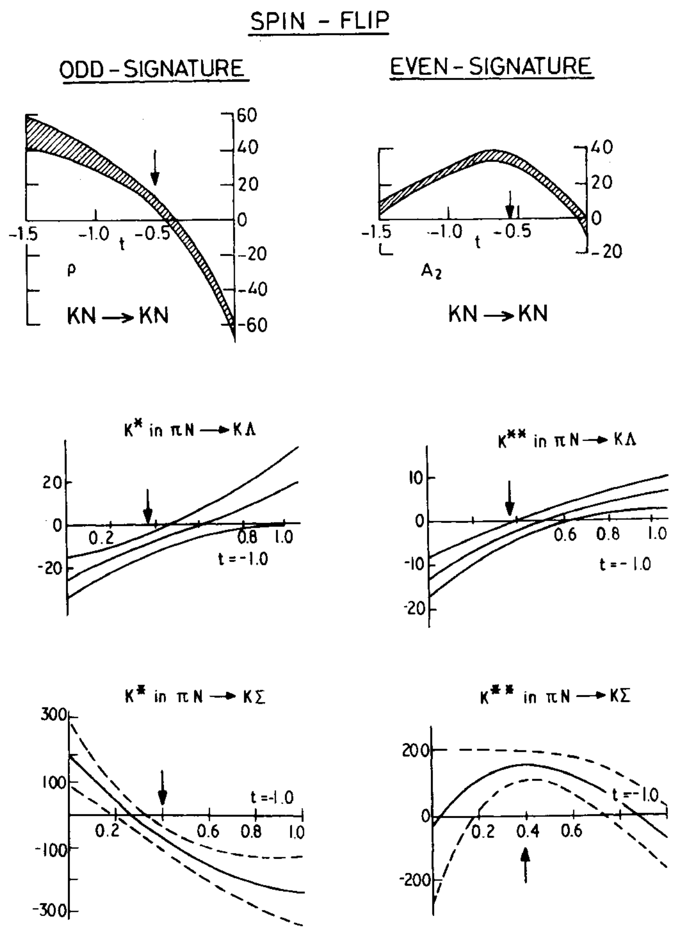

Fig. 4 Even and odd signature spinflip FESRs in KN→KN and hypercharge exchange reactions from refs 5, 7, 8. Exchange degeneracy implies approximate equality of the ρ and A_2 exchanges FESRs, and requires opposite signs K^* and K^{**} FESRs. Pole dominance and EXD require FESR zeros near t values marked by the arrows.

pole-dominated. This suspicion is confirmed[20] by high energy $\gamma p \to \pi^0 p$ data; in particular by their lack of Regge pole shrinkage, by the movement of the dip with energy, and by the large polarisation in the dip region. The spinflip ω FESR in KN→KN (figure 3) also appears[5] not to have a zero at t=−0.5; but the large errors prevent any definite conclusions.

Figure 4 shows three even signature spinflip FESRs opposite the corresponding odd-signature FESRs, to test the EXD relation of equation (2). The A_2 FESR in KN→KN has no zero[5] at t=−0.5 (so cannot be pole dominated) and is not EXD with the ρ.

The K* and K** FESRs should have opposite signs to satisfy EXD; this is plainly not the case[7,8] and in fact there is almost maximal violation of EXD. Furthermore in $\pi N \to K\Sigma$ the K** FESR shows[8] no Regge pole zero.

These results can be interpreted in three different ways:-

(a) The low energy integrals are inaccurate, through inaccuracies of the phase shift amplitudes or inadequacy of the resonance saturation approximation.

(b) FESRs are not simply related to high energy amplitudes, because nonleading Regge singularities[16], unimportant at high energies, are important in the FESRs.

(c) FESRs are not misleading, and high energy spin-flip amplitudes really are not pole-dominated (eg, have strong absorptive corrections) and are not EXD.

(a) can only be resolved by better phase shift analyses, which presumably will come in time. However, it would be surprising if the large EXD violations of figure 4 all disappeared. (b) simply implies that duality does not work; this would be very interesting if it were true. However, one cannot really decide between (a), (b) and (c) until we have direct analyses of high energy amplitudes for tensor exchange, which do not rely on strong theoretical input assumptions.

We can make some indirect tests between possibilities (a), (b) (that FESRs are somehow misleading) and (c) (that FESR are correct, and high energy amplitudes are complicated). For instance, if the FESR are simply related to high energy amplitudes, they should obey the same SU(3) relations as the high energy Regge poles (which are unaffected by SU(3) singlet absorption effects). Vanryckeghem[8] has compared $\pi N \to K\Lambda$ and $\pi N \to K\Sigma$ FESRs in this way, and one can also compare $\pi N \to \pi N$ with $KN \to KN$ – the results generally support the view* that FESR are simply related to high energy amplitudes

High energy data show important violations of line-reversal symmetry, which cannot be wholly attributed to nonflip amplitudes[21]. This tends to support possibility (c). However, in some reactions (eg, KN charge exchange) the line reversal violations[22] occur mainly in the $p_{lab} \lesssim 5$ GeV/c region; this suggests that they are associated with a nonleading Regge singularity, which could also cause the violation of ρ-A_2 EXD in the FESR of figure 4. This interpretation favours possibility (b).

Nonflip amplitudes are known to show important absorption effects, which could in principle have a complicated energy dependence. Therefore the nonflip FESRs are a priori more difficult to interpret, and it is paradoxical that in fact they show a more simple and consistent structure than the spinflip FESRs. All odd-signature nonflip FESRs have a crossover zero at some small t-value; some examples are shown in figure 5. Therefore they closely resemble the imaginary parts of high energy

*They do not prove it, since there could be strong daughter poles with the same SU(3) properties as the parents.

♠The zero in the $\pi\pi$ FESR, at $t \simeq -0.4$, may appear to be merely an EXD zero, slightly displaced by weak absorption. However, the small $\pi\pi$ total cross-section may imply a small interaction radius, which would give an absorptive zero at fairly large t.

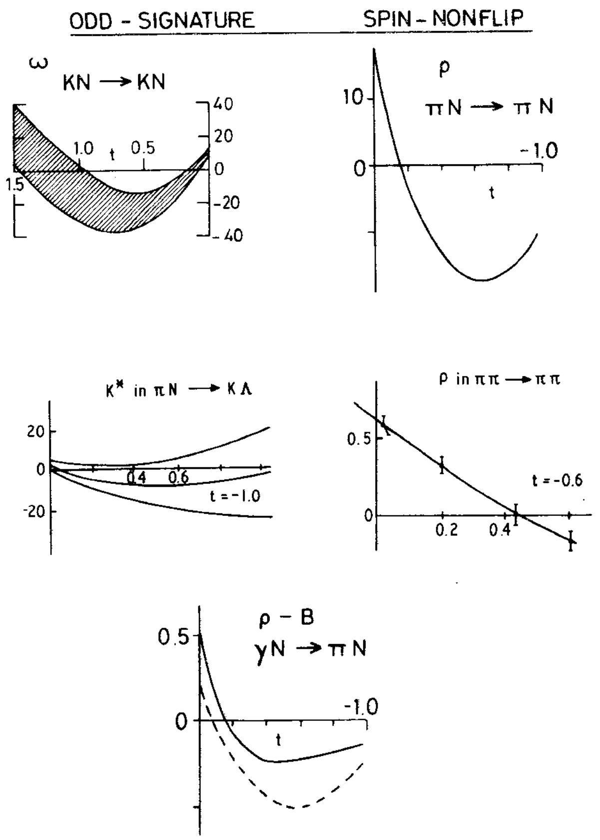

Fig. 5 Odd signature nonflip FESRs from refs 3, 5, 7, 10, 13. The solid and dashed curves for the photoproduction FESR are evaluated as in fig. 2.

nonflip odd-signature amplitudes, as deduced from amplitude analyses. This resemblance is nontrivial (in fact it does not occur in many models[16]) and tells us something important about the energy dependence of the absorptive corrections; they are very closely matched to the energy dependence of the Regge poles, and have a shrinkage which is not predicted by conventional absorption models.[2,25] This energy dependence has been incorporated in the Hartley-Kane model[19,23].

In even signature nonflip FESRs (figure 6), the "crossover" zero occurs only at larger t-values, if at all. This also agrees with imaginary parts deduced from high energy amplitude analyses[21]. There is therefore an obvious breaking of EXD in

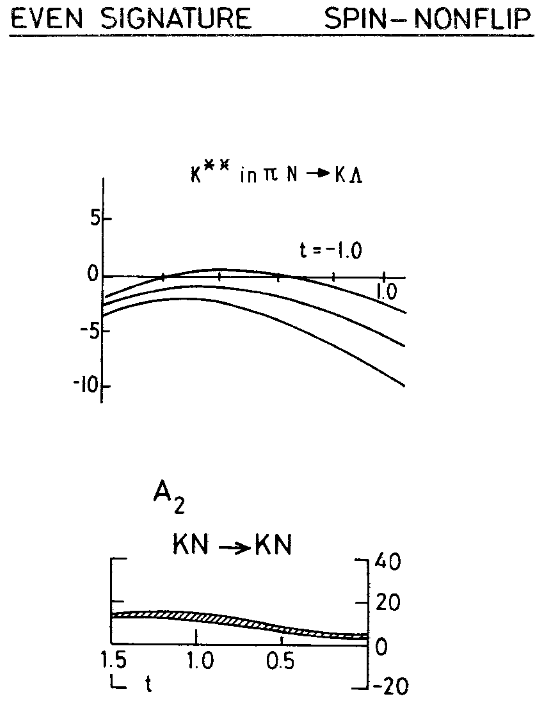

Fig. 6 Even signature nonflip FESRs, from refs 5, 7.

the nonflip FESRs and amplitudes. However, the derivatives (with respect to t) of nonflip FESRs at t=0, which emphasise the peripheral high partial waves of the amplitude, apparently do have exchange degeneracy of sign and approximate magnitude. The breaking of EXD occurs mainly in the low partial waves. This would agree with a model in which Regge poles are exchange degenerate, and the absorption effects (which break EXD) are mainly in the low partial waves[24].

The main conclusions of this survey are listed in Table II. This table poses many questions:-
Are high energy spinflip tensor exchange amplitudes really un-pole like, and not EXD with vector exchange amplitudes, as the FESRs seem to indicate? Or are the FESRs misleading in this case, perhaps dominated by low-lying j-plane singularities, which are unimportant in the high energy amplitude? Why

TABLE II

FESR SYSTEMATICS

	Nonflip	Flip
Odd Signature	Crossover zero at small t.	ρ is pole-like.
		ω is not.
	Like high energy Amplitude	K*?
Even Signature	Crossover not at small t.	Not EXD.
		Not pole-like.
	Like high energy Amplitude?	

are spinflip ρ exchange amplitudes and FESRs so simple and pole-like, while other exchanges (such as ω) apparently are not? Do the simple systematics of nonflip FESRs reflect a simple structure of all high energy nonflip amplitudes? Should we be looking for systematics in terms of s-channel helicity amplitudes or in terms of some other amplitudes? Answers to these questions will come from further low-energy phase shift analysis (allowing more and better FESR evaluations), but above all from model-independent amplitude analyses at high energy.

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