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RPC Simulations

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Abstract

This note discusses simulation results of several important RPC performance characteristics. We discuss single gap RPCs with 2 mm gap that are used in ATLAS and LHCb. Signal formation as well as the dependence of the time resolution on amplifier characteristics and noise are discussed. The signal propagation along the RPC strips, ideal termination networks and crosstalk are analyzed in detail. Primary ionization was calculated with HEED [1], the electrical RPC parameters and fields were calculated with MAXWELL [2]. The signal propagation was simulated with PSPICE [3] and MATHEMATICA [4].

Contents

Chapter 1

Introduction

RPCs are heavily used in LHC experiments. The hostile rate environment puts severe demands on all LHC detectors. Therefore a detailed understanding of the detectors is very useful. The approximate RPC geometry used in ATLAS and LHCb is shown in Figure 1.1 [5] [6]. A simple avalanche model was used to study signal characteristics and time resolution. Directly induced crosstalk was studied by calculating the weighting fields of the strips with Maxwell. This note discusses the RPC as a inhomogeneous multi-conductor transmission line which was shown to be a good model [7]. We discuss ideal termination networks and electrical crosstalk by using a very powerful matrix formalism for transmission lines.

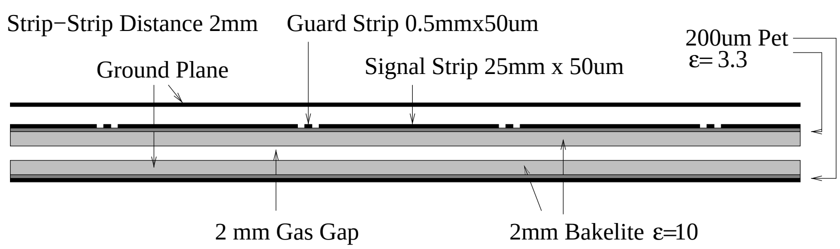

Figure 1.1 *: RPC geometry used for this study. A gas gap of 2 mm is formed by two 2 mm Bakelite plates with a volume resistivity of* $10^9 - 10^{11}\,\Omega cm$*. Two graphite layers with a resistance of about 1 M*Ω*/square are used to apply the voltage of about 10 kV supplying the electric field of 50 kV/cm in the gas gap. A polyethylene foil isolates the graphite plates. On top of the foil there are readout strips that transport the induced signal to the amplifiers at the end of the lines. A guard strip is used in order to reduce the strip-strip capacitance. The* ϵ *of the Bakelite is around 10 and losses in the Bakelite can be neglected up to 150 MHz [7].*

Chapter 2

Signal Characteristics

In this section we study the signal shapes for a 2 mm RPC filled with an $C_2H_2F_4$/i-C_4H_{10}/SF_6 mixture operated at a voltage of 10 kV, i.e. an electric field of 50 kV/cm [9].

2.1 Gas Properties

Figure 2.1 shows a HEED [1] simulation of the number of electrons deposited by a 10 GeV muon in the 2 mm gap for the gas mixture $C_2H_2F_4$/i-C_4H_10/SF_6 95/4/1. We find a most probable value of 30 electrons.

A drift-velocity of 100 $\mu m/ns$ was assumed. This number is not very crucial for the simulation procedure since the time resolution is strictly proportional to the driftvelocity around this value, so if the driftvelocity turns out to be different we just scale the result. With the assumption of 100 μm/ns the maximum electron drift time (signal duration) is 20 ns.

The calculation of the Townsend coefficient is usually not very reliable, so we have to somehow extract it from measurements. If the above numbers were correct we could naively just tune the Townsend coefficient until the measured charge distribution agrees with the simulated one. By this estimation we arrive at a Townsend coefficient of 90-100 cm^{-1}. In reality we have however two additional effects entering the game, attachment and space charge.

The attachment coefficient is also not known very well. However the effect for our study will only be a smaller effective Townsend coefficient, so it doesn't play a role.

Space charge effects are more important but difficult to simulate since the space charge dynamically changes during the avalanche development. In this simulation we neglect the space charge effect which leads to a exponential charge distribution which is observed for RPCs in avalanche mode. At the working point of 10 kV however the RPCs show a

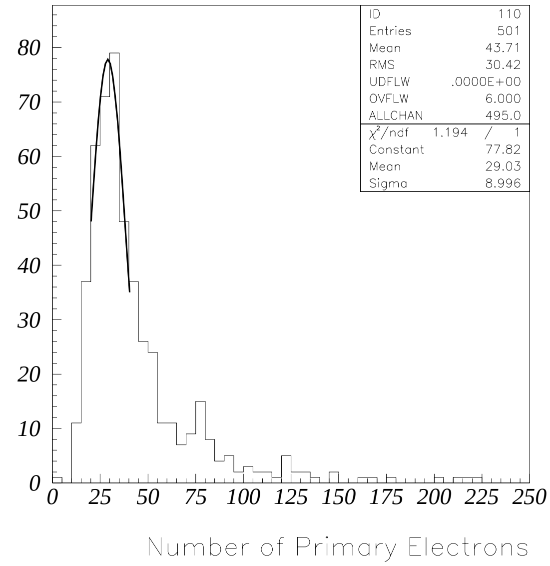

Figure 2.1 *: Primary electrons in a gas gap of 2 mm for* $C_2H_2F_4$/i-C_4H_10/SF_6 *95/4/1 calculated by HEED. The most probable value is around 30.*

more 'Landau' or 'Gaussian' like distribution [8] which indicates that the RPCs work in a saturated avalanche regime i.e. the avalanches are space charge limited. This simulation will therefore not completely describe the real situation. The details will be discussed later.

2.2 Induced Signal

Using the primary charge deposit from HEED, the driftvelocity v and the Townsend coefficient α we can calculate the signal time development. In a wire chamber the avalanches happen very close to the wire, so the signal has a very fast, short electron component and a long tail from the ion movement that induces most of the total charge. In an RPC the situation is entirely different. The avalanche region is equal to the entire gas gap and the ions move quite slowly, so for our studies we completely neglect the ion part of the signal. To find the induced signal we first have to calculate the weighting field of the strip.

Fig. 2.2 shows the weighting field (E_w) for signal and a guard strip with the geometry described in the previous chapter. Since the weighting field varies only weakly across the gas gap we can assume a constant weighting field in the gas gap. Depending on the polarity of the High Voltage the avalanche happens either close or far from the strip so the weighting field is either given by the solid or the dotted line in Fig. 2.2.

First we consider a particle crossing the chamber in the center of the strip, the weighting field corresponds to 3.74 V/cm, there is no signal induced on the neighboring strip. If a

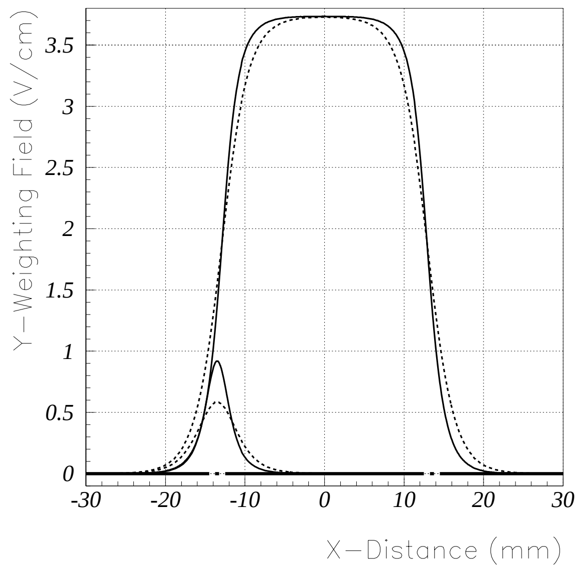

Figure 2.2 *: Weighting field for the signal strip and the guard strip for the geometry described in the previous chapter. The solid line shows the weighting field in the gas gap close to the strips, the dotted line shows the weighting field far from the strips. In the center of the strip we find* E_w*=3.75* cm^{-1}*. The signal induced on the guard strip can also be significant.*

single electron is moving in the gas gap we find an induced current of

$$I = E_w v e_0 = 6\,pA \tag{2.1}$$

If we simulate the time development of the entire avalanche we find an induced current of

$$I(t) = E_w v e_0 N(t) \tag{2.2}$$

where $N(t)$ is the number of electrons present at time t. To estimate the order of magnitude we assume $N_0 = 30$ electrons deposited in the gas gap at equal distances. Every electron will now start an avalanche according to $N(t) = e^{\alpha v t}$ until all the electrons hit the end of the gap. So on average we find

$$N(t) = \sum_{n=0}^{N_0} e^{\alpha v t}\Theta[\frac{d}{v}(1 - \frac{n}{N_0}) - t] \qquad I(t) = E_w e_0 v N(t) \tag{2.3}$$

The total induced charge i.e. the integrated signal evaluates to

$$Q = \int I(t)dt = \frac{E_w e_0}{\alpha}(N_{tot} - N_0) \qquad N_{tot} = \frac{1 - e^{\alpha d}}{1 - e^{\frac{\alpha d}{N_0}}} \tag{2.4}$$

where N_{tot} is the total number of electrons produced in the gap. For 30 primary electrons and $\alpha = 90$, 100, 110 this gives $8 \times 10^7, 5.1 \times 10^8, 3.3 \times 10^9$ electrons and an induced charge

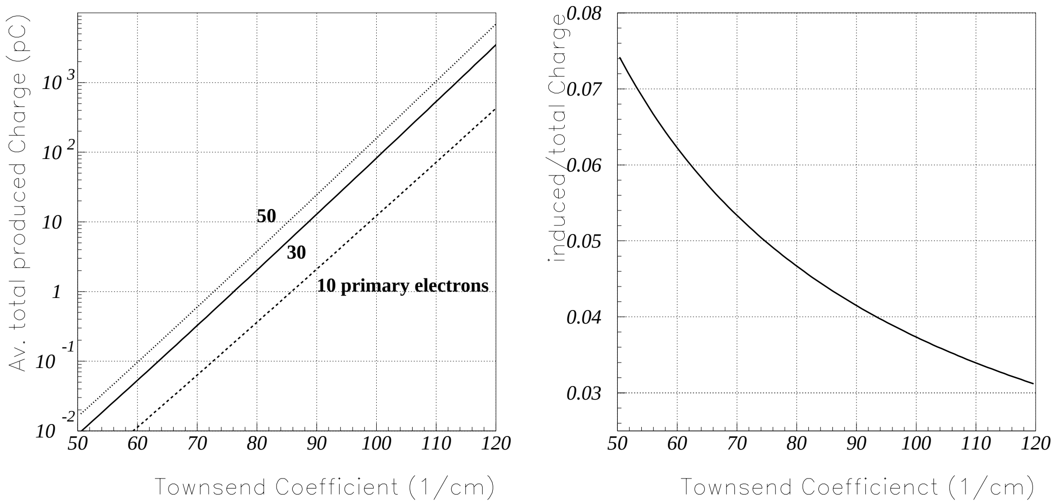

***Figure 2.3** : The left plot shows the total avalanche charge for different primary ionizations and Townsend coefficients. The right plot shows the ratio between induced and produced charge. E.g. for a Townsend coefficient of 96 we find a total avalanche charge of 40 pC and a total induced charge of 1.52 pC.*

of 0.53, 3.1, 18.1 pC. Figure 2.3 shows the total produced and induced charge for different primary ionization numbers.

From the above formula we see that the ratio between total *produced* and total *induced* charge depends on the Townsend coefficient which seems not intuitive. To discuss this point let's assume an avalanche in the center of a strip. There is no induced signal on the other strips and we can consider the RPC as a two electrode system (strip, ground). In a two electrode system it holds that the induced signals on the two electrodes are equal size and opposite polarity, and that the total *induced* charge on each electrode is equal to the total *produced* charge. The signal shown above is however only the electron component, we neglected the ion component since the current is very small. The rule about the total induced charge holds only for the signals induced by ions+electrons. If the Townsend coefficient is higher there are more electrons produced close to the anode plane and therefore the total charge induced by the electrons is reduced relative to the charge induced by the ions. This is the reason for the dependence on the Townsend coefficient.

The local average of the above signal is given by

$$I(t) = E_w e_0 v N_0 (1 + \frac{1}{2N_0} - \frac{td}{v}) e^{\alpha v t} \tag{2.5}$$

Figure 2.4 shows this current signal calculated with Equation 2.3 for the two different Townsend coefficients and the average of 500 Monte Carlo events. Figure 2.5 finally shows the examples of four Monte Carlo events for a Townsend coefficient of 110.

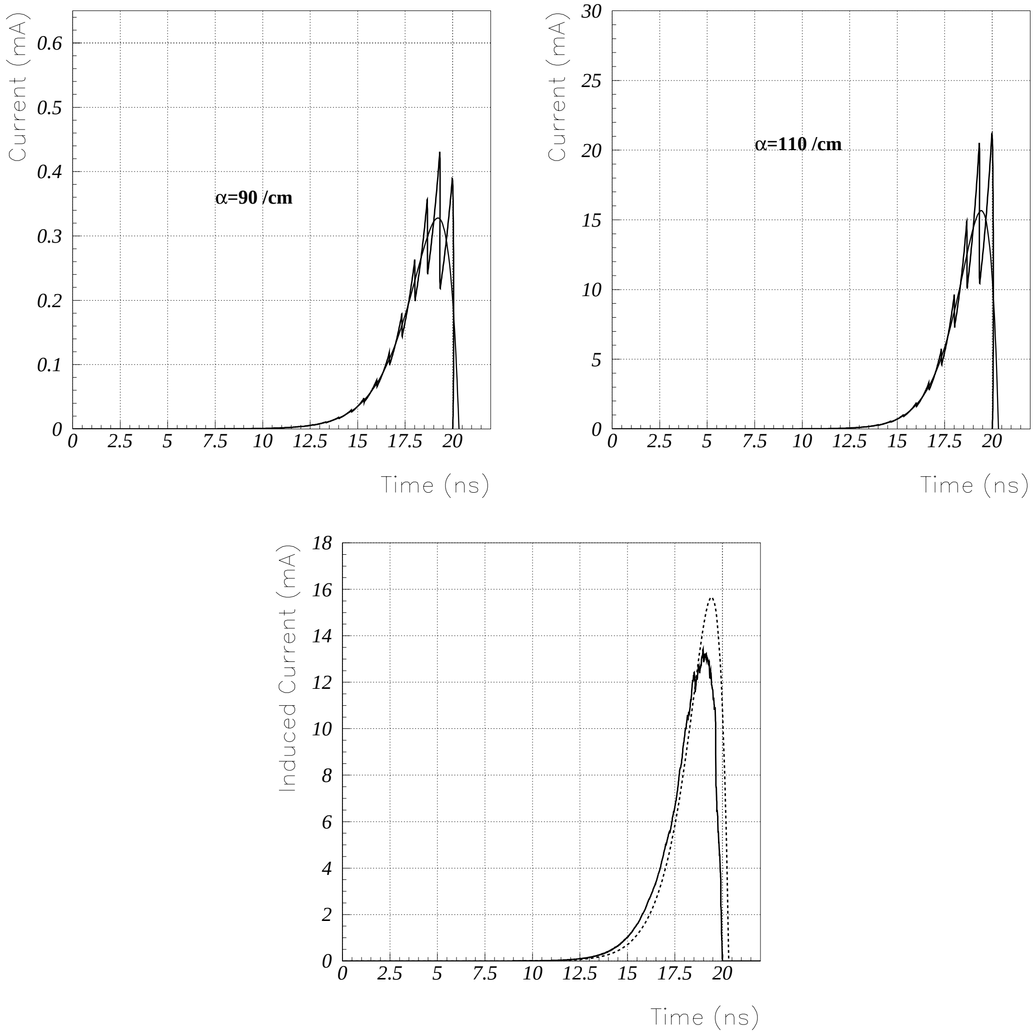

Figure 2.4 *: The top figures show signals calculated from Eq. 2.3 for two different Townsend Coefficients. The solid line shows the averaged signal from Equation 2.5. The bottom figure shows the average of 500 MC simulated signals together with the function from Equation 2.5 for* $\alpha = 110\,\mathrm{cm}^{-1}$*, v= 100* $\mu m/ns$*, 2 mm gap and Weighting Field of 3.75 V/cm. We see that the function is a good approximation of the average RPC signal.*

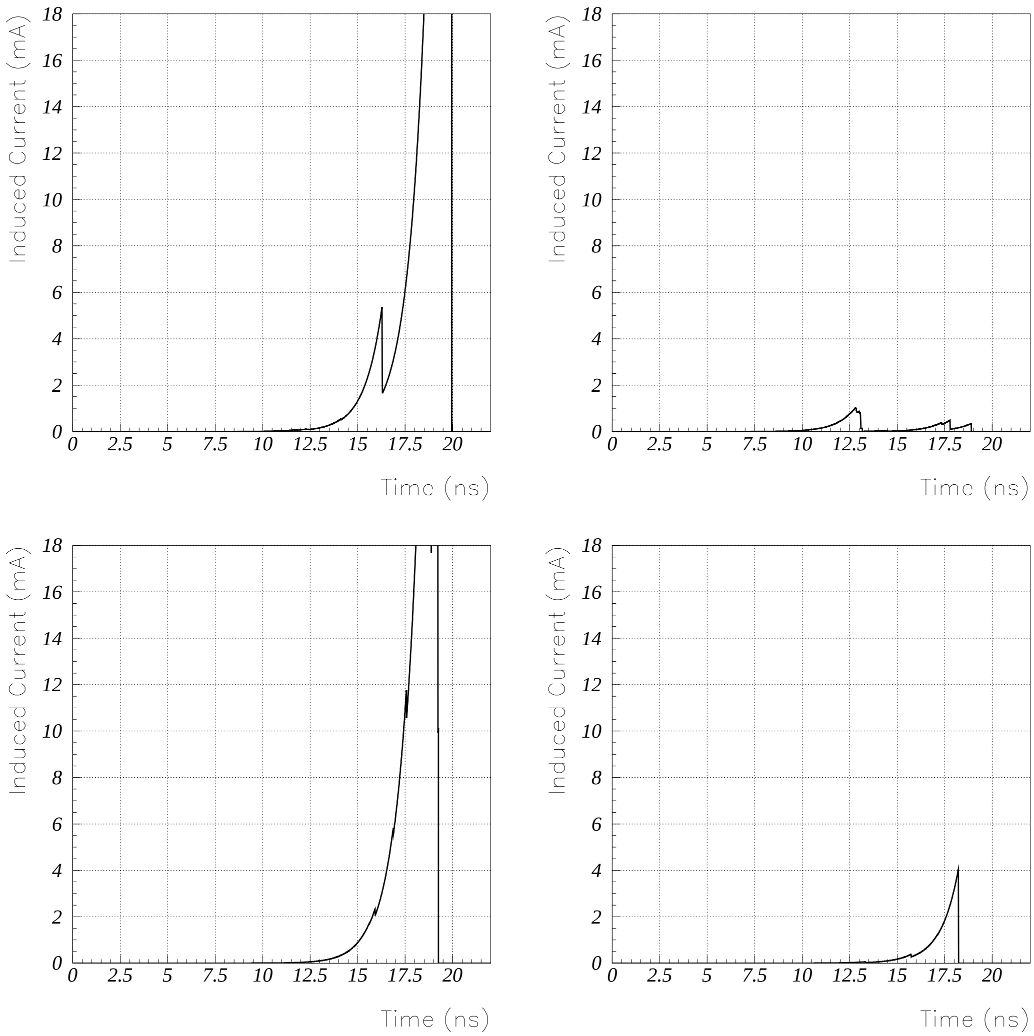

Figure 2.5 : *Four MC events for* $\alpha = 110\,cm^{-1}$. *We see that the pulseheight fluctuations are very large.*

Chapter 3

Frontend Electronics

We characterize the front-end electronics by it's delta response $f(t)$, i.e. the front end output signal for a delta function input. For our discussion we assume a delta response of the form

$$h(s) = \frac{n^{-n}e^n n!\tau}{(1+s\tau)^{n+1}} \quad \rightarrow \quad f(t) = n^{-n}e^n\left(\frac{t}{\tau}\right)^n e^{-\frac{t}{\tau}} \tag{3.1}$$

where $t_p = n\tau$ is the peaking time and n corresponds to the number of integration stages. The delta response is normalized to one. Figure 3.1 shows the two functions for different peaking times. The output $g(t)$ of the front-end for a general current input $i(t)$ can be computed by convoluting with the delta response

$$g(t) = \int_0^t f(t-t')i(t')dt' \tag{3.2}$$

The equivalent noise charge due to serial and parallel noise is given by [10]

$$ENC^2 = \frac{1}{2}e_n^2C^2\int_{-\infty}^{\infty} f'(t)^2dt + \frac{1}{2}i_n^2\int_{-\infty}^{\infty} f(t)^2dt \tag{3.3}$$

where $f(t)$ is the front-end electronics delta response normalized to give and output voltage peak of unity for an input current delta function of charge unity. C is the total capacitance at the front-end input (detector capacitance+ trace capacitance + amplifier input capacitance). e_n and i_n are the spectral series and parallel noise densities. These values depend on the front-end chip technology and design. If we would be free to chose the delta response $f(t)$ the ENC becomes a minimum for the function

$$f(t)_{opt} = e^{-\frac{|t|}{\tau_c}} \qquad \tau_c = \frac{e_n}{i_n}C \qquad ENC_{min} = \sqrt{e_n i_n C} \tag{3.4}$$

For optimum ENC it is therefore desired to have a delta-response close to this one. For any given delta-response with a given peaking time t_p the above integral will evaluate to

$$ENC^2 = \frac{1}{2}e_n^2C^2\left(at_p\frac{i_n^2}{e_n^2C^2} + \frac{b}{t_p}\right) \tag{3.5}$$

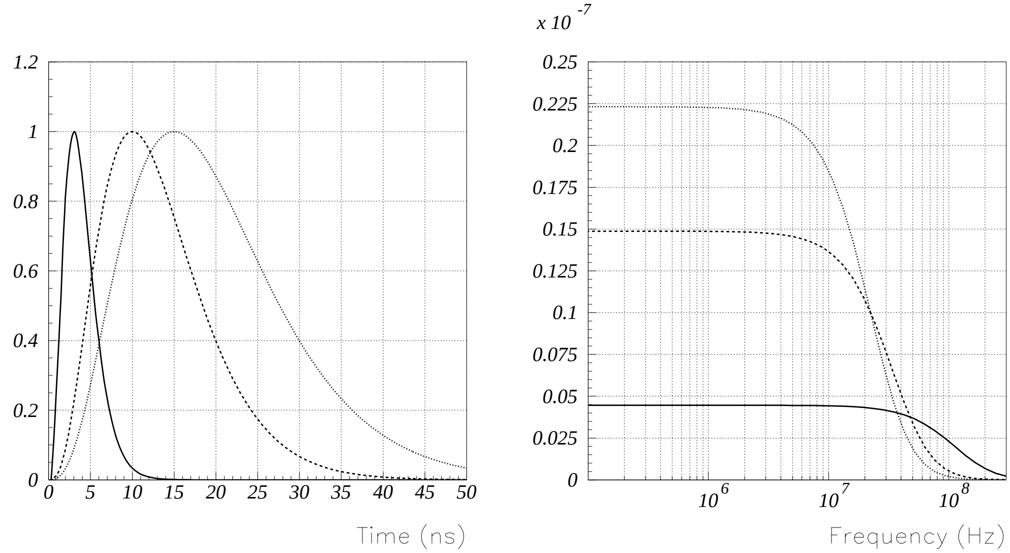

Figure 3.1 *: The left figure shows the front-end delta response for different peaking times. The right figure shows the transfer function. In order to achieve the same sensitivity for the slower amplifier the gain has to be higher.*

where a and b depend on the shape of the delta response. The peaking time for minimum noise and the ENC minimum are then given by

$$t_p = \tau_c \sqrt{\frac{b}{a}} \qquad ENC_{min} = \sqrt[4]{ab}\sqrt{e_n i_n C} \tag{3.6}$$

so in order to minimize the noise, the delta response shape should be such that the product ab is close to 1, which is the case for symmetric delta responses. (The choice of the optimum peaking time will however be determined by time resolution requirements). For the delta response 3.1 the integral evaluates to

$$a = (\frac{e}{2n})^{2n}(2n-1)! \qquad b = (\frac{e}{2n})^{2n}(n^2(2n-1)! + 2n^3(2n-2)! - n(2n)!) \tag{3.7}$$

the optimum peaking time is given by

$$t_p^{ENC} = \tau_c \sqrt{\frac{b}{a}} = 1,\ 1.15,\ 1.34,\ 1.51,\ 1.67\tau_c \quad \text{for} \quad n = 1, 2, 3, 4, 5 \tag{3.8}$$

and the noise at the minimum is given by

$$\sqrt[4]{ab} = 1.35, 1.21, 1.17, 1.16, 1.15 \qquad \text{for} \qquad n = 1, 2, 3, 4, 5 \tag{3.9}$$

We see that for a delta function consisting of 5 integrations the ENC is only 15% higher that the optimum case.

Finally we want to plug in numbers for the RPC. In case we terminate the strips properly the strip capacitance has no effect. Neglecting the amplifier input capacitance and trace capacitances we have

$$i_n^2 = \frac{4kT}{R_p} \tag{3.10}$$

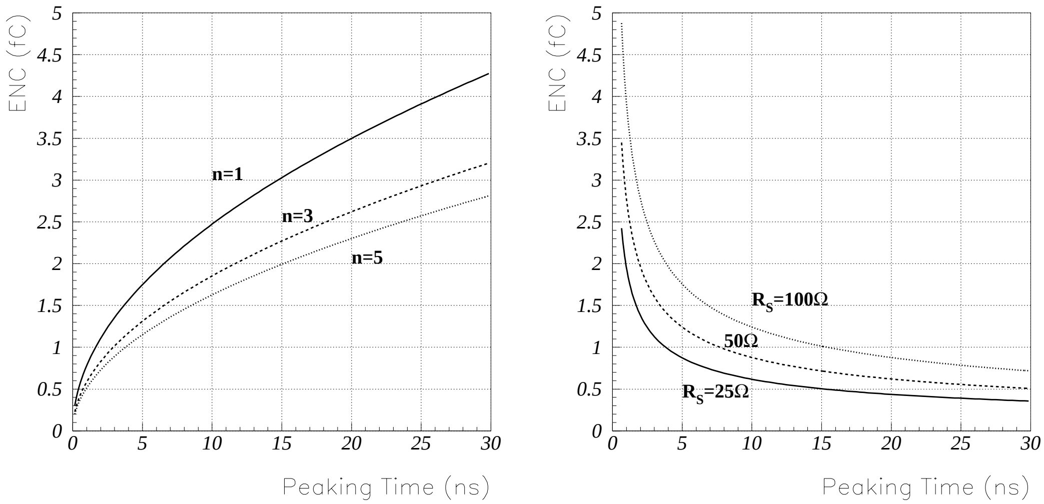

***Figure 3.2** : The left plot shows the parallel noise ENC for $R_p = 25\,\Omega$ for different peaking times and integration numbers which applies for an RPC with perfect termination. The right plot shows the series ENC for $C_{strip} = 100\,pF$ for different serial noise resistances which applies for an unterminated RPC strip. In the unterminated scheme the ENC scales with the capacitance.*

where R_p is the impedance of the RPC strip and we find an ENC of

$$ENC_p = \sqrt{\frac{1}{2}\left(\frac{e}{2n}\right)^{2n}(2n-1)!\frac{4kT}{R_p}t_p} \tag{3.11}$$

If we assume a very short RPC ($< 40cm$) we don't terminate the strips at all. In that case we can neglect the parallel noise and we only have serial noise which depends on the strip capacitance. The serial spectral noise density is given by

$$e_n = 4kTR_S \tag{3.12}$$

where R_S is the series noise resistance which depends on the amplifier design and we find

$$ENC_s = C\sqrt{\frac{1}{2}(\frac{e}{2n})^{2n}(n^2(2n-1)! + 2n^3(2n-2)! - n(2n)!)\frac{4kTR_S}{t_p}} \tag{3.13}$$

Figure 3.2 shows these numbers for different amplifier parameters. We see that in terms of noise the terminated RPC favours short peaking time while the unterminated RPC favours long peaking time.

Chapter 4

Time Resolution

The RPC time resolution for different peaking times was studied by simulating MC signals as described in chapter 2 and convoluting them with the front-end delta response. For this study we chose a preamp with $n = 3$, a sensitivity of $1\,\mathrm{mV/fC}$, and a parallel noise resistance of $25\,\Omega$. We chose Townsend coefficients of 90, 95 and $100\,\mathrm{cm}^{-1}$ which gives a total charge that should be close to our working point. Figure 4.1 shows an example of a pulse-height distribution and induced charge distribution. As discussed earlier the distributions have exponential shape because space charge effects where neglected while measurements at the working point show a more 'Landau' like distribution i.e. in reality space charge effect play a role. We should however expect that the time resolution dependence on the peaking time should be reduced in that case since the fluctuations are smaller. The numbers here should therefore be pessimistic. This however has to be checked experimentally.

We chose three different peaking times: 3, 10 and 15 ns. Figure 4.2 shows the time resolution and efficiency for the different Townsend coefficients. We find three important points:

- Plotting the time resolution versus threshold (in units of % of average pulseheight) we see no difference for $\alpha = 90, 95, 100\,\mathrm{cm}^{-1}$.
- For a given threshold (in units of % of average pulseheight) the 'hit-efficiency' i.e. the probability that a signal crosses the threshold independent of the peaking time.
- The only difference for the three different Townsend coefficients is the signal to noise ratio i.e. the minimum possible threshold. However, due to crosstalk effects we have to use a threshold around 5% of the average signal.
- At a threshold of 5% of the average signal we find a time resolution of 1.2, 1.5, 1.8 ns for peaking times of 3,10,15 ns.

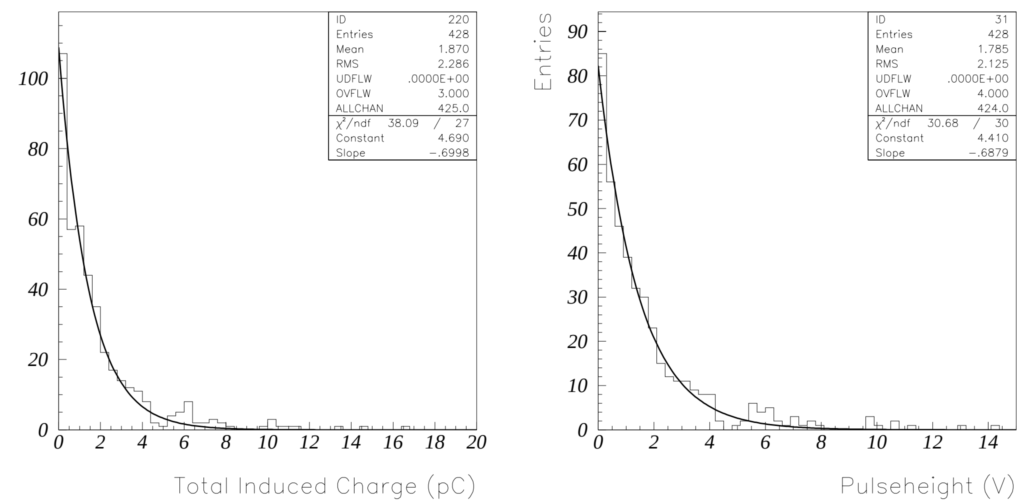

Figure 4.1 *: The left plot shows the total induced charge for* $\alpha = 95$ *which gives an average of 1.43* pC*. The right plot shows the corresponding pulseheight distribution for* $t_p = 10\,ns$ *and a preamp sensitivity* $1\,mV/fC$*. We find an average pulseheight of 1.45* V*.*

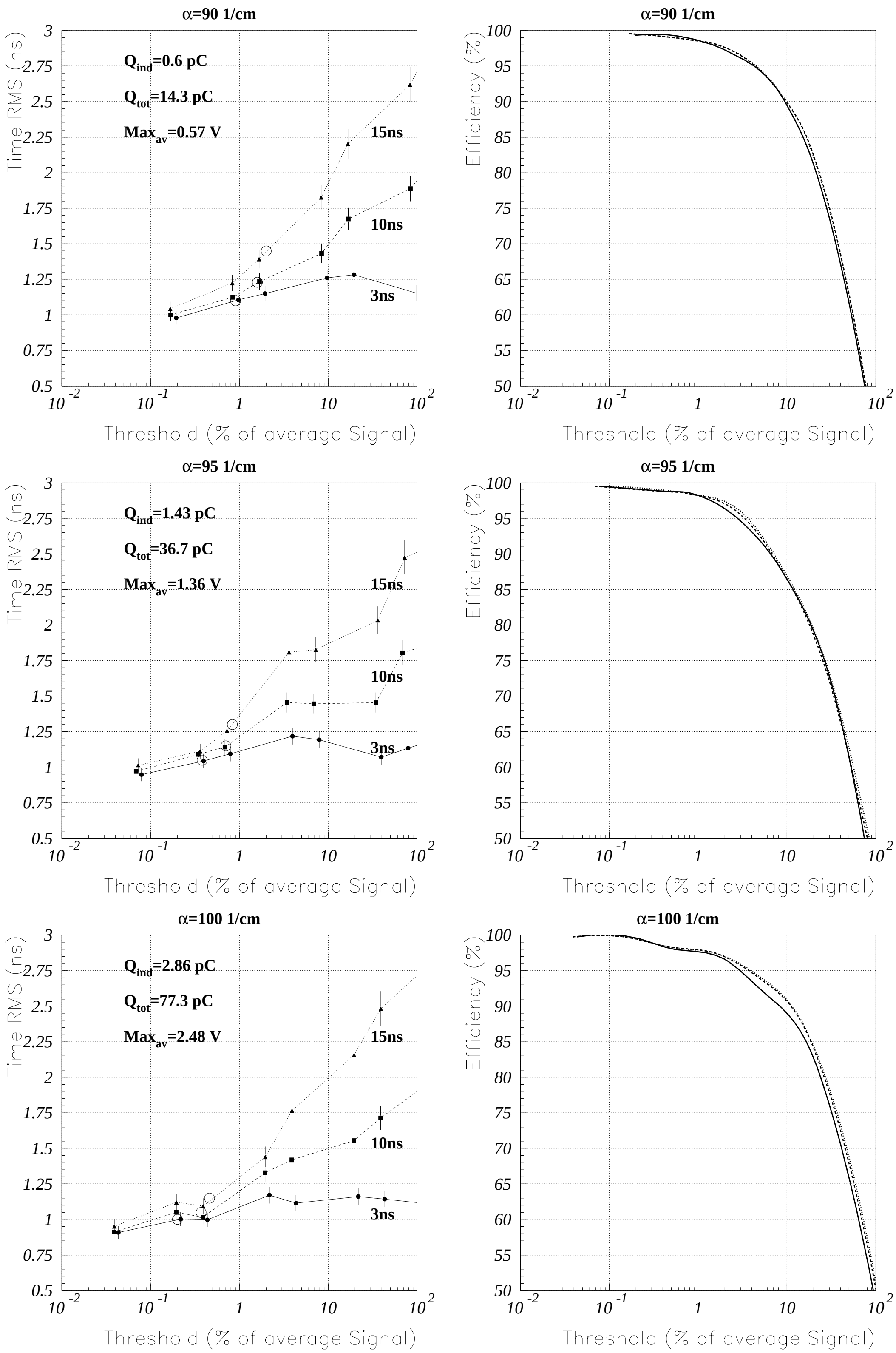

Figure 4.2 *: Time resolution and efficiency for different peaking times and Townsend coefficients. The open circles show the* $5\times$ *noise level, i.e. the minimum possible threshold.*

Chapter 5

Directly induced Crosstalk

In order to find the directly induced crosstalk i.e. the signal induced on the neighbouring strip if a particle crosses at a strip position X we normalize the weighting field of Fig. 2.2 and move the origin of the X-axis to the point between the strips (5.1). We see that for a particle passing the RPC in between two strips, the signal induced on both strips is 40% of the signal induced on a strip when the particle is passing through the center of a strip.

The second plot in Fig. 5.1 shows the crosstalk for a given threshold. If we set the threshold to 5% of the average signal we find a hit of the strip even if a particle crosses 3(4.5) mm from the strip.

From these curves we can calculate the average cluster size due to direct signal induction for different thresholds. For uniform illumination of the RPC we find

$$\text{clustersize} = 1 + \frac{2\Delta x(thr)}{w} \qquad (5.1)$$

were Δx is the distance from the edge of the strip at which both strips still fire and w is the width of the strip which for this formula has to be $\geq 2.5\,cm$. Figure 5.2 finally shows these numbers for different thresholds. If we set the threshold to 1% of the average signal we find a cluster size of 1.46(1.624). For a threshold of 5% we find a cluster size of 1.25(1.35).

We see that in order to keep the cluster size below 1.2 we have to keep the threshold above 5% of the average signal.

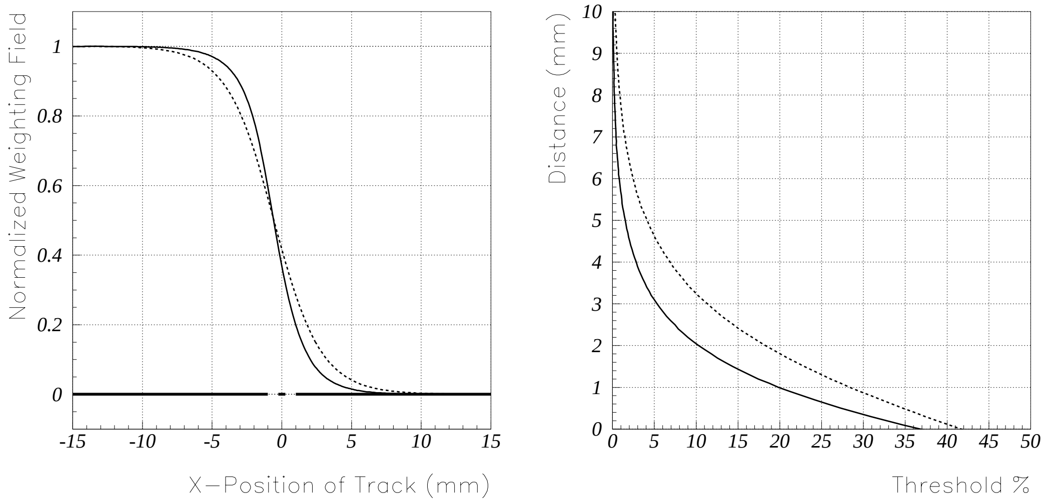

Figure 5.1 *: Normalized weighting fields. The left plot is centred in between the signal strips. The right plot shows the maximal position of the track that causes a hit on the signal strip for a given threshold. If the threshold is set to 5% of the average signal the crosstalk signal fires the neighbouring strip even if it is 3(4.5) mm from the edge.*

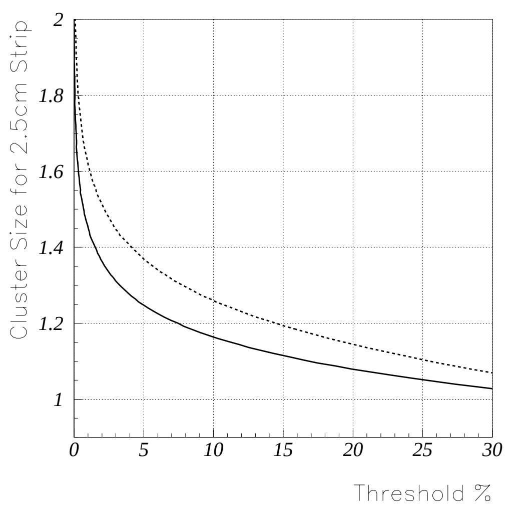

Figure 5.2 *: Cluster size for different thresholds for a 2.5 cm strip. If we set the threshold to 5% of the average signal we find a cluster size of 1.25(1.35).*

Chapter 6

General Discussion of Transmission Lines

There are several programs for simulation of transmission lines. In order to discuss the general features it is however essential to analyze the analytic solutions of the transmission line problem. In this context we also have to carefully define all the parameters that we use to describe the RPC. An excellent overview of the theory of Multi Conductor Transmission Lines is given in [12]. We will first write down the formal solutions of the transmission line problem and define all the necessary parameters. Then we will apply this formalism to a RPC with a single strip in order to discuss signal reflections, attenuation and distortion. After that we will study a RPC with two strips to discuss crosstalk and ideal termination networks. After that we apply the formalism to the N-conductor transmission line, representing the entire RPC. Although the general solutions are very mathematical the formalism very powerful for the discussion of general features and results of transmission lines as we will see later.

Our RPC is a two dimensional transmission line consisting of N signal lines and one reference line (ground line). The equations describing the most general 2-dimensional $(N+1)$ conductor transmission line in the TEM approximation [12] are

$$\frac{\partial}{\partial z}\mathbf{V}(z,t) = -\hat{\mathbf{R}}\mathbf{I}(z,t) - \hat{\mathbf{L}}\frac{\partial}{\partial t}\mathbf{I}(z,t) \tag{6.1}$$

$$\frac{\partial}{\partial z}\mathbf{I}(z,t) = -\hat{\mathbf{G}}\mathbf{V}(z,t) - \hat{\mathbf{C}}\frac{\partial}{\partial t}\mathbf{V}(z,t) \tag{6.2}$$

where

$$\mathbf{V}(z,t) = \begin{pmatrix} V_1(z,t) \\ . \\ . \\ . \\ V_N(z,t) \end{pmatrix} \qquad \mathbf{I}(z,t) = \begin{pmatrix} I_1(z,t) \\ . \\ . \\ . \\ I_N(z,t) \end{pmatrix} \tag{6.3}$$

are the currents and voltages at position z of the N conductors. The $N \times N$ matrices $\hat{\mathbf{C}}, \hat{\mathbf{L}}, \hat{\mathbf{R}}, \hat{\mathbf{G}}$ are the 'per unit length parameters' of Capacitance, Inductance, Resistance and Conductance. They are defined by the geometry and materials of the transmission line and completely determine the line characteristics. For the RPC case we calculate them with Maxwell [2].

6.1 The Matrices $\hat{\mathbf{R}}, \hat{\mathbf{L}}, \hat{\mathbf{G}}, \hat{\mathbf{C}}$

A transmission line is completely determined by four matrices that contain the 'per unit length' parameters. Here we will discuss their properties and frequency dependencies.

6.1.1 Resistance Matrix $\hat{\mathbf{R}}$

If the signal lines are perfect conductors, the resistance matrix is zero. If the signal lines have resistances of $r_i\,\Omega$/m and the reference conductor has a resistance of $r_0\,\Omega$/m the resistance matrix for low frequencies is given by

$$\hat{\mathbf{R}} = \begin{pmatrix} r_1 + r_0 & r_0 & . & r_0 \\ r_0 & r_2 + r_0 & . & r_0 \\ . & . & . & . \\ r_0 & r_0 & . & r_n + r_0 \end{pmatrix} \tag{6.4}$$

The RPC strips are made of aluminium ($\sigma = 3.8 \times 10^8$ S/m). The ground resistance r_0 is negligible and for low frequencies the the resistances for the signal strip and the guard strip are $0.002\,\Omega$/m and $0.1\,\Omega$/m which can be neglected as well.

For higher frequencies the line resistance increases as $\sqrt{f}$ due to the well known skin effect i.e. the fact that the current flows close to the conductor surface. The skin depth (of a planar conductor) is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{6.5}$$

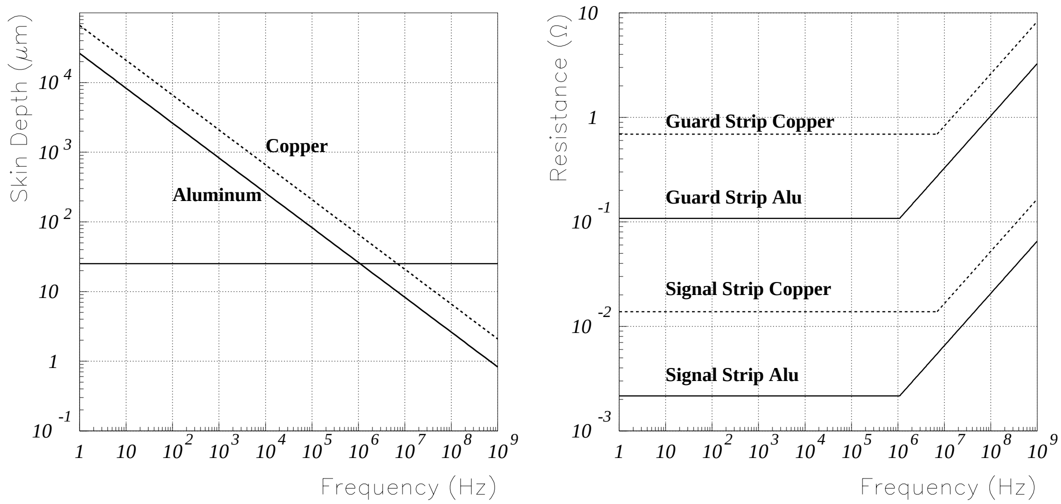

Figure 6.1 *: The left plot shows the skin depth versus frequency for Copper and Aluminium. The right plot shows the approximate resistance versus frequency for signal and guard strip.*

where f is the frequency, μ the magnetic permeability of the conductor and σ is the conductivity of the conductor. The current density in the conductor decreases as $e^{-x/\delta}$ where x is the distance from the conductor surface. We expect that the skin effect starts to play a role when the skin depth becomes equal to the smallest dimension of the conductor which in our case is the thickness of 50 μm. We approximate the resistance as

$$R = \frac{1}{\sigma w t} \quad \delta < \frac{t}{2} \qquad R = \frac{1}{2\delta\sigma(w+t)} \quad \delta > \frac{t}{2} \tag{6.6}$$

where w is the width and t is the thickness of the strip. The results are shown in Figure 6.1.

6.1.2 Capacitance Matrix $\hat{\mathbf{C}}$

The capacitance matrix $\hat{\mathbf{C}}$ is defined as

$$\hat{\mathbf{C}} = \begin{pmatrix} \sum_{k=1}^{N} C_{1k} & -C_{12} & . & -C_{1N} \\ -C_{21} & \sum_{k=1}^{N} C_{2k} & . & -C_{2N} \\ . & . & . & . \\ -C_{N1} & -C_{N2} & . & \sum_{k=1}^{N} C_{Nk} \end{pmatrix} \tag{6.7}$$

where $C_{ij} \quad i \neq j$ is the mutual capacitance between conductor i and j and C_{ii} is the capacitance of conductor i to the reference conductor. The capacitances are given by the conductor geometry and the dielectrics surrounding the conductors. Since the dielectric constants are in general not frequency dependent below $\approx 1\,\text{GHz}$ the capacitance matrix is constant for our application.

6.1.3 Inductance Matrix $\hat{\mathbf{L}}$

The Inductance Matrix $\hat{\mathbf{L}}$ contains the self inductances L_{ii} of the signal lines and the mutual inductances L_{ij} between the conductors.

$$\hat{\mathbf{L}} = \begin{pmatrix} L_{11} & L_{12} & . & L_{1N} \\ L_{21} & L_{22} & . & L_{2N} \\ . & . & . & . \\ L_{N1} & L_{N2} & . & L_{NN} \end{pmatrix} \tag{6.8}$$

The values depend on the conductor geometry and the magnetic permeability of the surrounding materials. The inductances are constant for low frequencies and they decrease as $1/\sqrt{f}$ for higher frequencies due to an effect that is strongly linked to the skin effect. It approximately holds that for high frequencies the inductance is given by $r(f)/f$ where r is the resistance.

6.1.4 Conductance Matrix $\hat{\mathbf{G}}$

The Conductance Matrix $\hat{\mathbf{G}}$ relates the transverse conduction currents passing between the conductors to their voltages. It is defined by

$$\hat{\mathbf{G}} = \begin{pmatrix} \sum_{k=1}^{N} G_{1k} & -G_{12} & . & -G_{1N} \\ -G_{21} & \sum_{k=1}^{N} G_{2k} & . & -G_{2N} \\ . & . & . & . \\ -G_{N1} & -G_{N2} & . & \sum_{k=1}^{N} G_{Nk} \end{pmatrix} \tag{6.9}$$

where $G_{ij} \quad i \neq j$ relate the currents between conductors i and j to their voltage difference and G_{ii} relate the currents to ground to their voltage differences. The matrix is constant if the surrounding medium has a constant conductivity σ. A frequency dependence is introduced by an imaginary part of the electric permittivity.

6.1.5 Properties of $\hat{\mathbf{R}}, \hat{\mathbf{L}}, \hat{\mathbf{G}}, \hat{\mathbf{C}}$

The matrices $\hat{\mathbf{L}}, \hat{\mathbf{G}}, \hat{\mathbf{C}}$ are symmetric and positive definite. If the medium surrounding the conductors is homogeneous with parameters σ, ϵ, μ and the conductors are lossless we have the relations

$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \hat{\mathbf{C}}\hat{\mathbf{L}} = \mu\epsilon\hat{\mathbf{1}} \qquad \hat{\mathbf{L}}\hat{\mathbf{G}} = \hat{\mathbf{G}}\hat{\mathbf{L}} = \mu\sigma\hat{\mathbf{1}} \tag{6.10}$$

where $1/\sqrt{\mu\epsilon}$ is the propagation velocity of the waves.

6.2 Formal General Frequency Domain Solution

Substituting

$$\mathbf{V}(z,t) = \mathbf{V}(z)e^{i\omega t} \qquad \mathbf{I}(z,t) = \mathbf{I}(z)e^{i\omega t} \tag{6.11}$$

into 6.1 6.2 and decoupling the equations they simplify to

$$\frac{d^2}{dz^2}\mathbf{V}(z) = \hat{\mathbf{Z}}\hat{\mathbf{Y}}\mathbf{V}(z) \quad \frac{d^2}{dz^2}\mathbf{I}(z) = \hat{\mathbf{Y}}\hat{\mathbf{Z}}\mathbf{I}(z) \quad \text{with} \quad \hat{\mathbf{Z}} = \hat{\mathbf{R}} + i\omega\hat{\mathbf{L}} \quad \hat{\mathbf{Y}} = \hat{\mathbf{G}} + i\omega\hat{\mathbf{C}}$$

the $\hat{\mathbf{Z}}$ the 'per unit length' impedance matrix and $\hat{\mathbf{Y}}$ the 'per unit length' admittance matrix (not to be confused with the *characteristic* impedance and admittance matrix). The equations are second order linear coupled differential equations that can be solved by diagonalizing the matrix $\hat{\mathbf{Y}}\hat{\mathbf{Z}}$ as

$$\hat{\mathbf{T}}^{-1}(\hat{\mathbf{Y}}\hat{\mathbf{Z}})\mathbf{T} = \hat{\gamma}^2 \qquad \hat{\gamma}^2 = \begin{pmatrix} \gamma_1^2 & 0 & . & 0 \\ 0 & \gamma_2^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \gamma_N^2 \end{pmatrix} \tag{6.12}$$

where γ_n^2 are the eigenvalues of $(\hat{\mathbf{Y}}\hat{\mathbf{Z}})$ and $\hat{\mathbf{T}}$ is the matrix containing the corresponding normalized eigenvectors. The general solution of the transmission line equations is then given by

$$\mathbf{I}(z) = \hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\mathbf{I}^+ + \mathbf{e}^{\hat{\gamma}z}\mathbf{I}^-) \qquad \mathbf{V}(z) = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}(\mathbf{e}^{-\hat{\gamma}z}\mathbf{I}^+ - \mathbf{e}^{\hat{\gamma}z}\mathbf{I}^-) \tag{6.13}$$

where

$$\mathbf{e}^{\pm\hat{\gamma}z} = \begin{pmatrix} e^{\pm\gamma_1 z} & 0 & . & 0 \\ 0 & e^{\pm\gamma_2 z} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & e^{\pm\gamma_N z} \end{pmatrix} \tag{6.14}$$

and we define

$$\hat{\mathbf{Z}}_C = \sqrt{\left(\hat{\mathbf{Z}}\hat{\mathbf{Y}}^{-1}\right)} = \hat{\mathbf{Z}}\hat{\mathbf{T}}\hat{\gamma}^{-1}\hat{\mathbf{T}}^{-1} \qquad \hat{\mathbf{Y}}_C = \hat{\mathbf{Z}}_C^{-1} \tag{6.15}$$

with $\hat{\mathbf{Z}}_C$ the *characteristic* impedance matrix and $\hat{\mathbf{Y}}_C$ the *characteristic* admittance matrix. The two vectors $\mathbf{I}^+$ and $\mathbf{I}^-$ contain $2N$ constants that will be determined by boundary conditions on the two ends of each of the N lines and the signal sources on the line. Note that the solution is still completely general!

6.3 Formal Lossless Time Domain Solution

If the transmission line is lossless i.e. $\hat{\mathbf{R}}$ and $\hat{\mathbf{G}}$ are negligible, the solutions are independent of the frequency and we can write down the explicit time domain solution. The differential equations to be solved are

$$\frac{d^2}{dz^2}\mathbf{V}(z,t) = \hat{\mathbf{L}}\hat{\mathbf{C}}\frac{d^2}{dt^2}\mathbf{V}(z,t) \qquad \frac{d^2}{dz^2}\mathbf{I}(z,t) = \hat{\mathbf{C}}\hat{\mathbf{L}}\frac{d^2}{dt^2}\mathbf{I}(z,t) \tag{6.16}$$

with the general solution

$$\mathbf{I}(z,t) = \begin{pmatrix} I_1(z,t) \\ . \\ . \\ . \\ I_N(z,t) \end{pmatrix} = \hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+(t-\frac{z}{v_1}) \\ . \\ . \\ . \\ I_N^+(t-\frac{z}{v_N}) \end{pmatrix} + \begin{pmatrix} I_1^-(t+\frac{z}{v_1}) \\ . \\ . \\ . \\ I_N^-(t+\frac{z}{v_N}) \end{pmatrix} \right) \tag{6.17}$$

$$\mathbf{V}(z,t) = \begin{pmatrix} V_1(z,t) \\ . \\ . \\ . \\ V_n(z,t) \end{pmatrix} = \hat{\mathbf{Z}}_C\hat{\mathbf{T}}\left(\begin{pmatrix} I_1^+(t-\frac{z}{v_1}) \\ . \\ . \\ . \\ I_N^+(t-\frac{z}{v_N}) \end{pmatrix} - \begin{pmatrix} I_1^-(t+\frac{z}{v_1}) \\ . \\ . \\ . \\ I_N^-(t+\frac{z}{v_N}) \end{pmatrix} \right) \tag{6.18}$$

where the $I_n^+(x)$ and $I_n^-(x)$ are $2N$ arbitrary functions and

$$\hat{\mathbf{T}}^{-1}(\hat{\mathbf{C}}\hat{\mathbf{L}})\hat{\mathbf{T}} = \hat{\mathbf{v}}^{-2} \qquad \hat{\mathbf{Z}}_C = \hat{\mathbf{L}}\hat{\mathbf{T}}\hat{\mathbf{v}}\hat{\mathbf{T}}^{-1} \tag{6.19}$$

with

$$\hat{\mathbf{v}}^{-2} = \begin{pmatrix} \frac{1}{v_1^2} & 0 & . & 0 \\ 0 & \frac{1}{v_2^2} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \frac{1}{v_N^2} \end{pmatrix} \qquad \hat{\mathbf{v}} = \begin{pmatrix} v_1 & 0 & . & 0 \\ 0 & v_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & v_N \end{pmatrix} \tag{6.20}$$

The individual functions represent pulses that are running along the lines in positive and negative direction without changing their shape. Note that the signal propagation happens with N different velocities. Only if the line is homogeneous i.e. if $\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2}\hat{\mathbf{1}}$ there is only one propagation velocity v. The form of these functions will be determined by the line excitation mechanism and the line boundaries.

6.4 Boundary Conditions, Reflection Matrix

The $2N$ constants in the solution 6.13 or the $2N$ functions in solution 6.17 and 6.18 are defined by the networks at the end of the lines at $z = 0$ and $z = L$ and the signal sources. Let's look at the left line end $z = 0$ (Fig. 6.2). We interconnect ('load') all the N conductors with resistors, capacitors, inductors i.e. we assume the most general linear network at $z = 0$. If $z_{nm} \quad n \neq m$ is the impedance of the network between line n and line m and z_{nn} is the impedance of the network between line n and the reference conductor (ground), the voltages and currents at the line end are related by

$$\mathbf{V}(0,t) = \hat{\mathbf{Z}}_L \mathbf{I}(0,t) \qquad \hat{\mathbf{Z}}_L = \hat{\mathbf{Y}}_L^{-1} \qquad Y_{nm}^L = -\frac{1}{z_{nm}} \quad n \neq m \qquad Y_{nn}^L = \sum_{m=1}^{N} \frac{1}{z_{nm}} \tag{6.21}$$

where we define $\hat{\mathbf{Z}}_L$ as the load impedance matrix. If we e.g. connect all the conductors with a resistor R to ground and don't interconnect them, the load impedance matrix is a diagonal matrix with the R as diagonal elements. If we interconnect lines the matrix has off-diagonal elements that have to be calculated from the above equations. The same relations are of course true for the other line end at $z = L$.

It can be shown that the forward and backward running *voltage* waves at a line end loaded by Z_L are connected by the reflection coefficient matrix $\hat{\mathbf{\Gamma}}_L$ according to

$$\mathbf{V}^- = \hat{\mathbf{\Gamma}}_L \mathbf{V}^+ \qquad \hat{\mathbf{\Gamma}}_L = (\hat{\mathbf{Z}}_L - \hat{\mathbf{Z}}_C)(\hat{\mathbf{Z}}_L + \hat{\mathbf{Z}}_C)^{-1} \tag{6.22}$$

The voltage $\mathbf{V}$ at the line end is then given by

$$\mathbf{V} = \mathbf{V}^+ + \mathbf{V}^- = (\hat{\mathbf{1}} + \hat{\mathbf{\Gamma}}_L)\mathbf{V}^+ \tag{6.23}$$

In the next section we will discuss the current pulse excitation at some position $z = z_0$ of a single strip of a lossless transmission line with $N + 1$ conductors. This represents the RPC case.

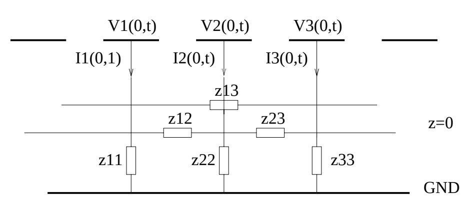

Figure 6.2 : *Line end loaded with a general impedance network.*

6.5 Current Pulse Excitation of a Lossless Line

This scenario is most important for the real RPC application where the avalanches in the gas gap induce a current signal on a strip. We assume an induced signal $I^0(t)$ on strip n at position z_0 along the RPC strip. Substituting into 6.17 we find the equation

$$\begin{pmatrix} 0 \\ . \\ I^0(t) \\ . \\ 0 \end{pmatrix} = \hat{\mathbf{T}}(\begin{pmatrix} I_1^+(t-\frac{z_0}{v_1}) \\ . \\ . \\ . \\ I_N^+(t-\frac{z_0}{v_N}) \end{pmatrix} + \begin{pmatrix} I_1^-(t+\frac{z_0}{v_1}) \\ . \\ . \\ . \\ I_N^-(t+\frac{z_0}{v_N}) \end{pmatrix}) \tag{6.24}$$

which defines the $2N$ functions and we find the *general* solution

$$\mathbf{I}(z,t) = \frac{1}{2}\hat{\mathbf{T}}(\begin{pmatrix} t_{1n}^{-1} I^0(t-\frac{z-z_0}{v_1}) \\ t_{2n}^{-1} I^0(t-\frac{z-z_0}{v_2}) \\ . \\ . \\ t_{Nn}^{-1} I^0(t-\frac{z-z_0}{v_N}) \end{pmatrix} + \begin{pmatrix} t_{1n}^{-1} I^0(t+\frac{z-z_0}{v_1}) \\ t_{2n}^{-1} I^0(t+\frac{z-z_0}{v_2}) \\ . \\ . \\ t_{Nn}^{-1} I^0(t+\frac{z-z_0}{v_N}) \end{pmatrix}) \tag{6.25}$$

where the t_{nm}^{-1} are the elements of the matrix $\hat{\mathbf{T}}^{-1}$. This solution shows that there are two pulses running symmetrically to the left and the right from the point z_0. The current pulse running along one conductor is a superposition of N times the same pulse-shape running with different velocities v_n. Therefore we have signal dispersion even for the lossless case.

One side of the RPC we want to terminate, the other side we want to read with an amplifier. Equation 6.22 shows that in order to avoid reflections on one line end we have to load the line such that $\hat{\mathbf{Z}}_L = \hat{\mathbf{Z}}_C$. Using 6.21 we find the termination resistors as

$$\hat{\mathbf{Y}}_C = \hat{\mathbf{Z}}_C^{-1} \qquad R_{nm} = -\frac{1}{Y_{nm}^C} \quad n \neq m \qquad \frac{1}{R_{nn}} = \sum_{m=0}^{m=N} Y_{mn}^C \tag{6.26}$$

where R_{nn} are the resistors between conductor n and ground and R_{nm} are the resistors between conductor n and m. We see that we eliminate reflections only of we interconnect **all** conductors with resistors. We'll discuss examples later.

For now we assume the far end of the RPC to be terminated. The other end of the RPC is read out by amplifiers connected to each line. To keep the discussion general we assume that the lines are also interconnected with resistors. From the preamp input resistance and the interconnection resistors we can calculate the load impedance matrix $\hat{\mathbf{Z}}_P$ on the amplifier side with Eq. 6.21. The voltage and current at the line end are then given by

$$\mathbf{V}(L,t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C (\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C)^{-1} \hat{\mathbf{T}} \begin{pmatrix} t_{1n}^{-1} I^0(t - \frac{L-z_0}{v_1}) \\ t_{2n}^{-1} I^0(t - \frac{L-z_0}{v_2}) \\ . \\ . \\ t_{Nn}^{-1} I^0(t - \frac{L-z_0}{v_N}) \end{pmatrix} \qquad \mathbf{I}(L,t) = \hat{\mathbf{Z}}_P^{-1} \mathbf{V}(L,t) \qquad (6.27)$$

$\mathbf{V}(L,t)$ is the voltage at the preamp inputs, the current through the preamps is simply given by $\mathbf{I}_{amp} = \frac{1}{R_{in}} \mathbf{V}(L,t)$ where R_{in} is the amplifier input resistance. (The current $\mathbf{I}(L,t)$ in the above expression is not the current through the amplifier but the current at the line end which is only equal to $\mathbf{I}_{amp}$ if the lines are not interconnected.) This is the most general result for a lossless transmission line. The result is true for every line length. The rest of this note will discuss the consequences of the above formula. Here we will examine a few limiting cases to draw general conclusions.

6.5.1 Homogeneous or Short Terminated RPC

In a homogeneous RPC all the propagation velocities are the same and the results simplifies to

$$\mathbf{V}(L,t) = \hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C (\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C)^{-1} \begin{pmatrix} 0 \\ . \\ 0 \\ I^0(t - \frac{L-z_0}{v}) \\ 0 \\ . \\ 0 \end{pmatrix} \qquad \mathbf{I}(L,t) = \hat{\mathbf{Z}}_P^{-1} \mathbf{V}(L,t) \qquad (6.28)$$

This result is also valid for a short unterminated RPC since there is not enough time for the pulses to disperse. From this we learn that the signal and crosstalk pulses will have exactly the same shape as the original induced signal. The relative pulse-heights of the strips are given by the n^{th} column of the matrix $\hat{\mathbf{Z}}_P \hat{\mathbf{Z}}_C (\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C)^{-1}$. The crosstalk is independent of the amplifier peaking time. If the transmission line is homogeneous (which is *not* the case for the RPC) the crosstalk is also independent of the distance of the induced signal from the amplifier.

If we adjust the preamp input resistance and interconnecting resistors such that $\hat{\mathbf{Z}}_P = \hat{\mathbf{Z}}_C$

i.e. if we also ideally terminate the preamp side, the solution is given by

$$\mathbf{V}(L,t) = \frac{1}{2}\hat{\mathbf{Z}}_C \begin{pmatrix} 0 \\ . \\ 0 \\ I^0(t - \frac{L-z_0}{v}) \\ 0 \\ . \\ 0 \end{pmatrix} \qquad \mathbf{I}(L,t) = \hat{\mathbf{Z}}_P^{-1}\mathbf{V}(L,t) \tag{6.29}$$

The crosstalk is then given by the ratio of the cross impedances $Z^C_{nj}\, n \neq j$ to the impedance of the signal line Z^C_{nn}. This is however *not* the scenario giving the smallest crosstalk.

If we don't interconnect the lines on the amplifier side but just connect each line to the preamp it holds that $\hat{\mathbf{Z}}_P = \text{Diag}(R_{in}, R_{in}, ..., R_{in})$ where R_{in} is the preamp input resistance, or $\hat{\mathbf{Z}}_P = \text{Diag}(R_{in}, R, R_{in}, R, ..., R, R_{in})$ if we use a guard strip with a resistance R to ground. It can be shown that off-diagonal elements in the matrix $\hat{\mathbf{Z}}_P$ will always increase the crosstalk i.e. we don't want to interconnect the lines on the preamp side.

In case the amplifier input resistance is $R_{in} = 0$ we have $\hat{\mathbf{Z}}_P = 0$, $\mathbf{V}(L,t) = 0$ and

$$I(L,t) = (0, 0, ..., I^0(t - \tfrac{z-z_0}{v}), ..., 0)^T \tag{6.30}$$

i.e. we measure exactly the pulse induced on line n and zero on all the other lines. The whole scenario looks the following: If a current is induced at point $z = z_0$, half of it runs to the left and half of it to the right. The pulse running to the left is terminated on the left end, the pulse running to the right is totally negatively reflected and we measure the difference i.e. the entire current signal. The reflection again runs to the left where it is terminated. This way we measure the maximum signal with minimum crosstalk – it is the desired scenario.

6.5.2 Inhomogeneous or Long Terminated RPC

For a long RPC the signals will disperse as they run along the strips and the pulse-shapes will change. The crosstalk will always increase as a function of distance from the amplifier and will only be equal to the numbers from above in the limit of very long preamp peaking times. To discuss this fact let's look again at amplifiers with input resistance $R_{in} = 0$. The general solution is given by

$$\mathbf{I}(L,t) = \hat{\mathbf{T}} \begin{pmatrix} t^{-1}_{1n} I^0(t - \frac{L-z_0}{v_1}) \\ t^{-1}_{2n} I^0(t - \frac{L-z_0}{v_2}) \\ . \\ . \\ t^{-1}_{Nn} I^0(t - \frac{L-z_0}{v_N}) \end{pmatrix} \tag{6.31}$$

The crosstalk will not be zero as above, but since all the individual pulses have the same shape is holds that

$$\int I(L,t)dt = (0,0,...,\int I^0(t-\tfrac{z-z_0}{v})dt,...,0)^T \tag{6.32}$$

i.e in case $R_{in} = 0$ the crosstalk signals are **perfectly bipolar**. The total charge on all neighbours is zero. All the induced charge is measured on conductor n. From this we learn that the crosstalk will be less if the peaking time is longer. (This is of course only true for long RPCs. For short strips there is not enough time for dispersion and the crosstalk will be independent of the peaking time.) Therefore we will try to chose the longest possible preamp peaking time that is still compatible with our timing requirements. We also see that the crosstalk will be position dependent since for a pulse far from the preamp side the individual modes will disperse by a larger amount.

6.5.3 Short Unterminated RPC

In case the RPC strips are short i.e. the pulse-width is longer than the propagation time, we can omit the termination and leave the 'far end' open. If we interconnect the lines on the preamp side such that it represents ideal termination the measured signals will again have the same shape as the original signal and the result is independent of the amplifier peaking time. The crosstalk is given by the relative magnitude of the impedance matrix elements. If we however omit the interconnections of the lines and just connect amplifiers with lowest possible input resistance we can reduce the crosstalk. However now we will have multiple reflections since both line ends are not ideally terminated, the pulse-shape will be different and the crosstalk becomes again peaking time dependent. If we imagine that we use amplifiers with input resistance of $R_{in} = 0\,\Omega$ (or very high) the signal will be reflected back and forth forever and the system becomes unstable. In the unterminated RPC we therefore want the amplifier impedance as close as possible to the impedance of the signal strip. Since we don't interconnect the lines we have multiple reflections and the signal shapes will be different from the induced signal.

6.6 Geometry independent Conclusion on RPC Signal Propagation

- The RPC signal acts as an ideal current source on an RPC strip.
- The RPC is modelled as a lossless inhomogeneous multi-conductor transmission line. The signal propagation happens in N modes where N is the number of RPC strips. This means that the signal is propagating along the strips as a superposition of N pulses that have the same shape as the original signal and travel with N different velocities. The signal experiences 'modal dispersion'.
- In order to avoid reflections for long RPC strips the RPC has to be terminated at the characteristic impedance of the transmission line which is theoretically only possible if one interconnects all the strips with resistors i.e. one needs $N \times (N-1)$ resistors.
- On the preamp side we want the lowest possible impedance since in that case the reflection coefficient matrix becomes $\hat{\mathbf{\Gamma}} = \mathrm{Diag}(-1, -1, ..., -1)$ and the measured signal becomes maximal. The reflection is then absorbed at the terminated end.
- If the amplifier input impedance is zero the crosstalk signals on the neighbouring strips are strictly bipolar i.e the total measured charge is zero. Therefore the crosstalk will be smaller for long peaking times and low input impedances (this is only true for long strips).
- For a signal that is induced far from the preamp, the different modes have more time to disperse i.e. we expect that the crosstalk increases with the distance from the amplifier.
- It can be shown that interconnecting the strips on the preamp side will always increase the crosstalk.
- For a 'short' terminated RPC (where short means that the modes don't run apart too far) the measured signal on the 'hit' strip and the crosstalk signals have exactly the same shape as the original induced signal.
- In terms of noise, an ideally terminated RPC is equal to a set of resistors equal to the characteristic impedance, i.e. the RPC represents a parallel noise source.
- Since for a short RPC reflections are not as issue we can leave the termination out. If the lines are not interconnected at the preamp side (which is desired for crosstalk) the the measured signal does not have the same shape as the induced signal because of the multiple reflections. It will be shown later that in that case the amplifier input impedance should be close to the impedance of the strip in order to avoid instabilities.
- In terms of noise the unterminated RPC represents a serial noise source.

Chapter 7

Electrical Crosstalk in RPCs

In order to discuss all the the different effects in RPC detectors we first analyze a RPC with a single strip to discuss attenuation and reflections. Then we'll discuss a homogeneous and inhomogeneous RPC with two strips to examine modes and termination networks. Then we finally discuss a N-conductor transmission line representing the RPC. A modal analysis of RPC crosstalk is presented in [11].

7.1 Single Strip RPC

An example of a transmission line representing an RPC with a single strip is shown in Figure 7.1. The 'per unit length' matrices are scalars R, G, L, C. We will keep the discussion still general to see the effects of the different parameters. We will insert the values only at the end.

It holds that

$$LC = \mu\epsilon_r^{eff} \qquad G = \frac{\mu(\sigma^{eff} + \omega\epsilon_i^{eff})}{L} \tag{7.1}$$

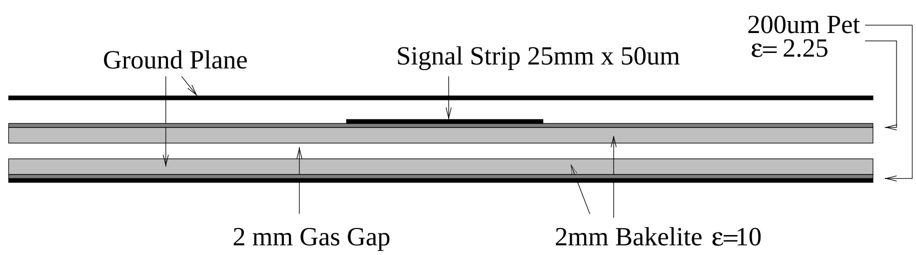

***Figure 7.1** : RPC with a single signal strip.*

We find the characteristic parameters

$$Z_C = \sqrt{\frac{R + i\omega L}{G + i\omega C}} \qquad \Gamma_1 = \frac{Z_C - R_1}{Z_C + R_1} \qquad \Gamma_2 = \frac{Z_C - R_2}{Z_C + R_2} \tag{7.2}$$

As we see the reflection factor is in general frequency dependent, so it is not possible to terminate all waves with a passive resistor. Only in the high frequency limit Z_L reaches a constant real value. The 'eigenvalue' is

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)} = \alpha + i\beta \tag{7.3}$$

with

$$\alpha = \sqrt{\frac{1}{2}(\sqrt{(G^2 + \omega^2 C^2)(R^2 + \omega^2 L^2)} + RG - \omega^2 LC)} \tag{7.4}$$

$$\beta = \sqrt{\frac{1}{2}(\sqrt{(G^2 + \omega^2 C^2)(R^2 + \omega^2 L^2)} - RG + \omega^2 LC)} \tag{7.5}$$

The general solution is then

$$I(z,t) = (e^{-(\alpha+i\beta)} I^+ + e^{(\alpha+i\beta)} I^-) e^{i\omega t} \qquad V(z,t) = Z_C (e^{-(\alpha+i\beta)} I^+ - e^{(\alpha+i\beta)} I^-) e^{i\omega t} \tag{7.6}$$

The two constants I^+ and I^- will be determined by boundary conditions. We see that α leads to frequency dependent exponential attenuation (even if L, C, G, R themselves are independent of frequency). The factor β defines the propagation velocity i.e. $v = \omega/\beta$ which also depends on the frequency. This leads to dispersion of pulses. Only if R and G are zero we don't have attenuation. R is zero if the signal line is a perfect conductor. From (7.1) we see that G is zero if the conductivity of the medium is zero and the imaginary part of the permittivity is zero.

In case $R << \omega L$ and $G = 0$ (which is a good approximation for the RPC) the factors α and β can be approximated by

$$\alpha = \frac{1}{2}\frac{R}{Z_C} \qquad \beta = \frac{\omega}{v} \tag{7.7}$$

so in this approximation the propagation velocity is the same for all frequencies and the the attenuation length is given by the ratio of 'per unit length' resistance and characteristic impedance. Since the typical strip impedance is around $20\,\Omega$ and since in the frequency range $< 1\,GHz$ the strip resistance R is $< 1\,\Omega$ the attenuation length is $> 50\,m$, so we can completely neglect the losses. In that case we have

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC} = \frac{\omega}{v} \tag{7.8}$$

where v is the phase velocity of the waves which is equal for all frequencies. The characteristic impedance becomes constant and is equal to $Z_C = \sqrt{\frac{L}{C}}$.

From here we continue with the lossless case. The parameters for the transmission line in Fig. 7.1 are

$$C = 205\,\mathrm{pF/m} \quad L = 89.3\,\mathrm{nH/m} \quad \rightarrow \quad Z_C = 20.87\,\Omega \quad v = 2.34 \times 10^8\,\mathrm{m/s} \tag{7.9}$$

The RPC pulse acts as a current source $I^0(t)$ somewhere along the strip. The general solution for the lossless case is well known

$$I(z,t) = \frac{1}{2}(I^0(t - \tfrac{z-z_0}{v}) + I^0(t + \tfrac{z-z_0}{v})) \qquad V(z,t) = \frac{1}{2}Z_C(I^0(t - \tfrac{z-z_0}{v}) - I^0(t + \tfrac{z-z_0}{v}))$$

so half the current pulse is running to the left and half to the right. At the line ends the voltage pulses are reflected according to the reflection coefficients determined by the loads. For a long RPC we want to terminate the strip on one side in order to avoid reflections, i.e. we want $R_2 = Z_C$. On the preamp side the measured current pulse is

$$I(t) = \frac{Z_C}{Z_C + R_1} I^0(t - \tfrac{z-z_0}{v}) \tag{7.10}$$

The measured pulse-shape is exactly equal to the original induced current. We see that for input resistance of $0\,\Omega$ the measured current is equal to the induced current, the reflection coefficient is -1 and the reflection is absorbed again on the terminated side. This result is true for every line length.

For short strips i.e. if the preamp peaking time is longer than the propagation time, we don't have to terminate the strip at the far end and we can look at a different scenario. We can use a quasi-static approximation where we concentrate all the 'parameters' C and L in on element (Fig. 7.2). This represents the circuit for the damped harmonic oscillator. For a strip of length l we can express the total capacitance and inductance as

$$C = \frac{l}{vZ_C} \qquad L = \frac{lZ_C}{v} \qquad Z_C = \sqrt{\frac{L}{C}}$$

and the approximate delta responses become

$$g(t) = \frac{1}{R_1 C} e^{-\frac{t}{R_1 C}} \quad R_1 >> 2Z_C \qquad g(t) = \frac{1}{\sqrt{LC}} e^{-\frac{R_1}{2L}t} \sin\frac{t}{\sqrt{LC}} \quad R_1 << 2Z_C \tag{7.11}$$

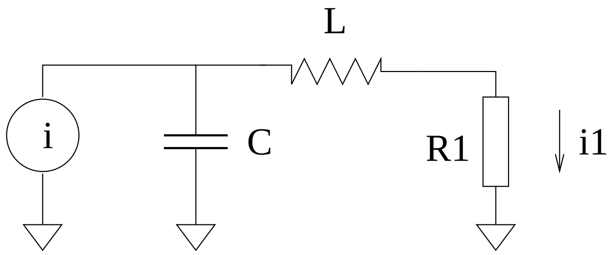

***Figure 7.2** : Equivalent circuit for a short lossless RPC strip.*

For $R_1 \geq 2Z_C$ the circuit shows critical damping and there is no oscillation but only exponential attenuation. The signal is 'integrated' and the pulse-width is increased. For $R_1 \leq 2Z_C$ there is an oscillation that is damped with a time constant of $\tau = \frac{2L}{R_1} = \frac{2Z}{R_1}T$ where T is the propagation delay along the strip length. Also this is an unwanted effect. Therefore we want the preamp input impedance to be close to the characteristic strip impedance in case we don't terminate the strip on the 'far side' i.e. we want to terminate the strip with the preamp.

It is interesting to see how these expressions from Eq. 7.11 come out of the exact transmission line solution. If we assume a current signal in the center of the strip we can calculate the exact solution by multiply reflecting the two current pulses that run to the right and the left. The signal is the given by the sum

$$I(t) = \frac{1}{2}(1+\Gamma_1)(1+\Gamma_2)\frac{Z_C}{R_1}\sum_{n=0}^{N}(\Gamma_1\Gamma_2)^n I^0(t-2nT) \tag{7.12}$$

with $T = l/v$ and $N = t/2T$. For a short strip we can now replace the sum by an integral

$$\sum_{n=0}^{N}(\Gamma_1\Gamma_2)^n I^0(t-2nT) \approx \frac{1}{2T}\int_0^t e^{\ln(\Gamma_1\Gamma_2)\frac{t-x}{T}} I^0(x)dx \tag{7.13}$$

If we set $\Gamma_2 = 1$ (unterminated end) and if $\Gamma_1 > 0$ which is the case for $R_1 > Z_C$

$$\ln(\Gamma_1\Gamma_2) \approx -2\frac{Z_C}{R_1} \qquad \text{and} \qquad \frac{1}{2}(1+\Gamma_1)(1+\Gamma_2)\frac{Z_C}{R_1} \approx \frac{2Z}{R_1} \tag{7.14}$$

and 7.11 becomes equal to 7.12. If $Z_C < R$ the reflection coefficient Γ_1 is negative and we replace $(-1)^n$ by $sin(n\pi + \frac{\pi}{2})$

$$I(t) = \frac{1}{2}(1+\Gamma_1)(1+\Gamma_2)\frac{Z_C}{R_1}\sum_{n=0}^{N}\sin(n\pi+\frac{\pi}{2})(\Gamma_1\Gamma_2)^n I^0(t-2nT) \tag{7.15}$$

which again becomes equal to Eq. 7.11 after replacing the sum by an integral.

7.2 Homogeneous Double Strip Line

Figure 7.3 shows an example of a transmission line with two strips. The dielectric properties are equal in the entire area where the waves propagate. We call this type of transmission line **homogeneous**. For this type of transmission line it holds that

$$\hat{\mathbf{L}}\hat{\mathbf{C}} = \frac{1}{v^2} \qquad \hat{\mathbf{Z}}_C = v\hat{\mathbf{L}} \tag{7.16}$$

i.e. there is only one propagation velocity. The transmission line is fully described by the characteristic impedance matrix and the propagation velocity v which are calculated from from Maxwell. We find

$$\hat{\mathbf{C}} = \begin{pmatrix} 126 & 6.4 \\ 6.4 & 126 \end{pmatrix} \mathrm{pF/m} \qquad L = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \mathrm{nH/m} \rightarrow \tag{7.17}$$

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} 26.5 & 1.34 \\ 1.34 & 26.5 \end{pmatrix} \Omega \qquad v = 3 \times 10^8 \,\mathrm{m/s} \tag{7.18}$$

Since the matrix $\hat{\mathbf{L}}\hat{\mathbf{C}}$ is already diagonal the matrix $\hat{\mathbf{T}}$ is undefined and any set of two orthonormal vectors will do for it. The general solution for a current pulse $I^0(t)$ at position $z = z_0$ on conductor 1 following Eq. 6.25

$$\begin{pmatrix} I_1(z,t) \\ I_2(z,t) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} I^0(t - \frac{z-z_0}{c}) + I^0(t + \frac{z-z_0}{c}) \\ 0 \end{pmatrix} \tag{7.19}$$

and for the voltage pulses

$$\begin{pmatrix} V_1(z,t) \\ V_2(z,t) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} Z_{11} \\ Z_{12} \end{pmatrix} I^0(t - \tfrac{z-z_0}{c}) + \frac{1}{2} \begin{pmatrix} Z_{11} \\ Z_{12} \end{pmatrix} I^0(t + \tfrac{z-z_0}{c}) \tag{7.20}$$

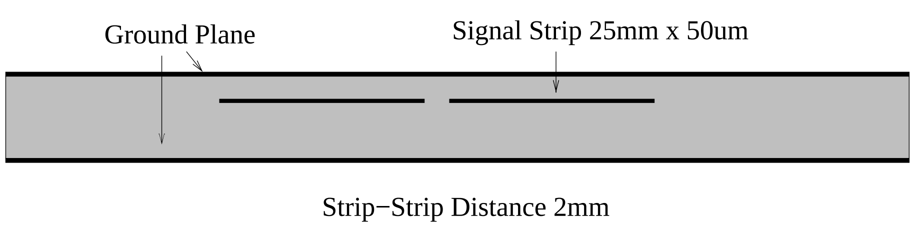

Figure 7.3 *: Homogeneous transmission line with two strips*

These two voltage waves are now running with velocity v to the left and to the right. At the line ends they will be reflected according to the networks connected to the line end.

First we want to terminate one end such that there is no reflection. The reflection coefficient at the line end is given by

$$\hat{\mathbf{\Gamma}}_L = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} = \frac{\hat{\mathbf{Z}}_L - \hat{\mathbf{Z}}_C}{\hat{\mathbf{Z}}_L + \hat{\mathbf{Z}}_C} \tag{7.21}$$

The reflection matrix is zero if the load impedance is equal to the characteristic impedance. Let's now calculate the resistor values that give $\hat{\mathbf{\Gamma}}_L = 0$. According to Equation 6.26 we find

$$R_{11} = R_{22} = 27.8\,\Omega \qquad R_{12} = 522.7\,\Omega \tag{7.22}$$

Only if we interconnect *all* the lines we avoid reflections (Figure 7.4). The other side of the strips we finally want to read out with an amplifier of input resistance R. Since the lines are not interconnected the load impedance matrix is a diagonal matrix with R as diagonal elements. The measured current is given by Eq. 6.27 and evaluates to

$$\begin{pmatrix} I_1(t) \\ I_2(t) \end{pmatrix} = \frac{\hat{\mathbf{Z}}_C}{\hat{\mathbf{Z}}_P + \hat{\mathbf{Z}}_C} \begin{pmatrix} I^0(t - \frac{z-z_0}{c}) \\ 0 \end{pmatrix} \tag{7.23}$$

$$= \frac{1}{R^2 + 2RZ_{11} + Z_{11}^2 - Z_{12}^2} \begin{pmatrix} RZ_{11} + Z_{11}^2 - Z_{12}^2 \\ RZ_{12} \end{pmatrix} I^0(t - \tfrac{z-z_0}{c}) \tag{7.24}$$

so we find a crosstalk of

$$\frac{I_2(t)}{I_1(t)} = \frac{RZ_{12}}{RZ_{11} + Z_{11}^2 - Z_{12}^2} \tag{7.25}$$

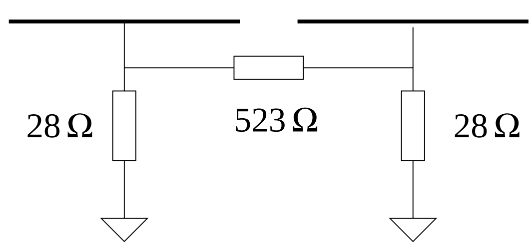

Figure 7.4 *: In order terminate the strips properly we have to interconnect all the lines.*

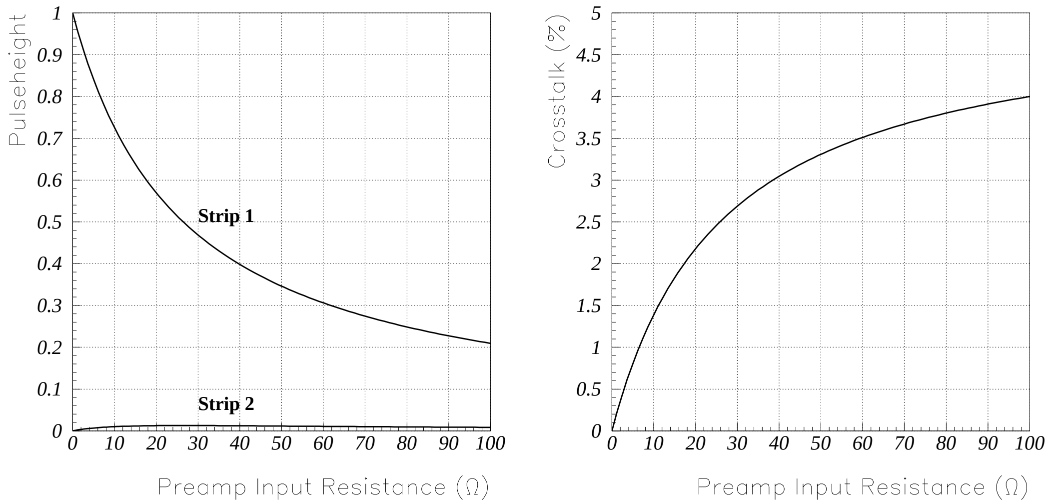

Figure 7.5 *: The left figure shows the pulse-height on both strips. The right figure shows the crosstalk. We see that crosstalk is zero if the input resistance is zero.*

which is illustrated in Figure 7.5. The measured signals on both strips have exactly the same shape as the original induced signal. The crosstalk is zero if the preamp input resistance is zero. In that case we measure exactly the induced current signal. In order to keep the crosstalk small we want the ratio Z_{12}/Z_{11} to be small. It can be easily shown that by interconnecting the strips with a resistor the crosstalk is always increasing.

7.3 Double Strip RPC

An RPC with two strips is shown in Figure 7.6. The transmission line is *inhomogeneous* and the parameters calculated from Maxwell are

$$\hat{\mathbf{C}} = \begin{pmatrix} 216 & -30 \\ -30 & 216 \end{pmatrix} \quad \text{pF/cm} \qquad \hat{\mathbf{L}} = \begin{pmatrix} 88.4 & 4.47 \\ 4.47 & 88.4 \end{pmatrix} \quad \text{nH/m} \qquad \Rightarrow \tag{7.26}$$

$$\hat{\mathbf{Z}} = \begin{pmatrix} 20.4 & 1.93 \\ 1.93 & 20.4 \end{pmatrix} \quad \Omega/\text{m} \qquad \hat{\mathbf{T}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \tag{7.27}$$

$$v_1 = 2.2 \times 10^8 \quad \text{m/s} \qquad v_2 = 2.4 \times 10^8 \quad \text{m/s} \tag{7.28}$$

The ideal termination for one side of the RPC gives the values

$$R_{11} = R_{22} = 22.3 \quad \Omega \qquad R_{12} = 213.7 \quad \Omega \tag{7.29}$$

The effect of the two modes with different propagation velocities is illustrated in Figure 7.7. In general the signal is a superposition of the two modes and we therefore find dispersion. We will see that this dispersion plays a significant role for the crosstalk of long strips.

Exciting the RPC as before by putting a current $I^0(t)$ on strip one at position $z = z_0$ we get the general solution

$$\begin{pmatrix} I_1(z,t) \\ I_2(z,t) \end{pmatrix} = \frac{1}{4} \begin{pmatrix} I^0(t - \frac{z-z_0}{v_1}) + I^0(t - \frac{z-z_0}{v_2}) \\ -I^0(t - \frac{z-z_0}{v_1}) + I^0(t - \frac{z-z_0}{v_2}) \end{pmatrix} + \frac{1}{4} \begin{pmatrix} I^0(t + \frac{z-z_0}{v_1}) + I^0(t + \frac{z-z_0}{v_2}) \\ -I^0(t + \frac{z-z_0}{v_1}) + I^0(t + \frac{z-z_0}{v_2}) \end{pmatrix}$$

We see that this time the wave propagation happens in two different waves with two different velocities i.e. in different 'modes'. The longer the pulse is travelling the more the waves will run apart. This effect is called 'modal dispersion' and is illustrated in Figure 7.8.

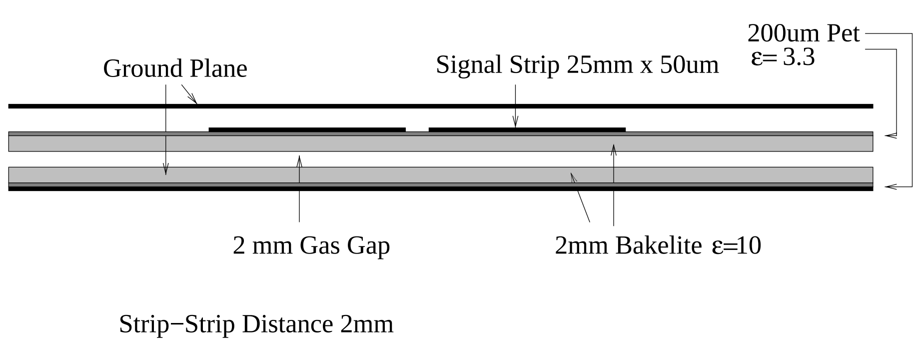

Figure 7.6 *: RPC with two strips.*

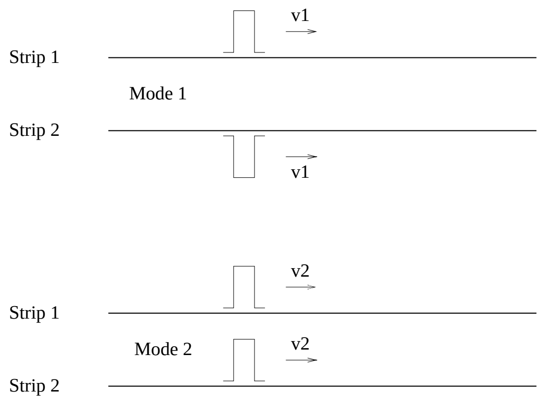

Figure 7.7 *: This Figure shows the two modes of the symmetric RPC with two strips. The relative pulse-heights for the given mode are determined by the eigenvectors. In our example they have the same pulse-height. In general a signal is the superposition of the modes which leads to dispersion.*

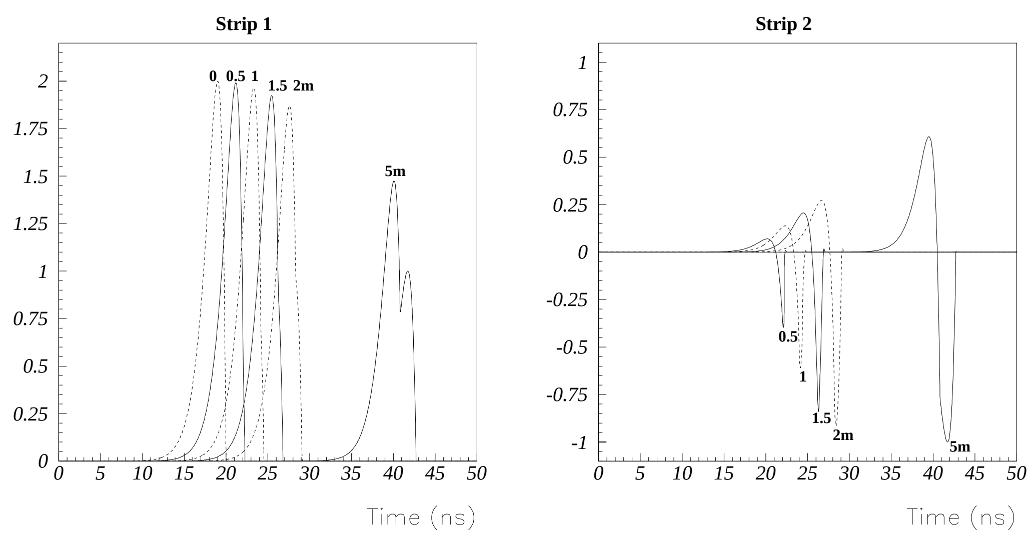

Figure 7.8 *: This Figure shows the effect of a current pulse on one strip at position 0. The left plot shows the signal strip. One can see that the two 'modes' are running apart after some distance. The right plot shows the signal travelling on the neighbouring strip. At the position where the signal is induced the crosstalk signal is zero, as the modes are running apart the crosstalk from the two becomes visible. The integral over the crosstalk current wave is zero.*

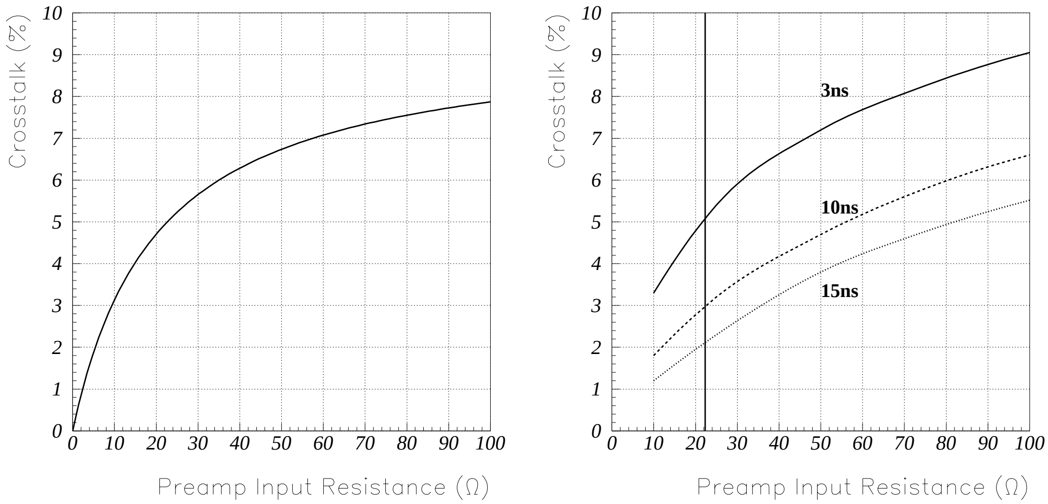

Figure 7.9 *: The left plot shows the crosstalk for a short terminated RPC. The crosstalk signal has the same shape as the induced signal and is independent of the peaking time. Comparing with the homogeneous RPC we see that the Bakelite increases the crosstalk by a factor 2. The right plot shows the unterminated short RPC. The crosstalk signal has a different shape from the original signal. The crosstalk decreases for longer peaking times. The vertical line shows the strip impedance which is the desired value for the preamp input resistance in the unterminated scenario.*

The current measured on the readout side for preamps with input resistance R is given by

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \frac{1}{2} c_1 \begin{pmatrix} RZ_{11} + Z_{11}^2 - Z_{12}^2 & RZ_{12} \\ RZ_{12} & RZ_{11} + Z_{11}^2 - Z_{12}^2 \end{pmatrix} \begin{pmatrix} I^0(t + \frac{z-z_0}{v_1}) + I^0(t + \frac{z-z_0}{v_2}) \\ -I^0(t + \frac{z-z_0}{v_1}) + I^0(t + \frac{z-z_0}{v_2}) \end{pmatrix}$$

$$c1 = \frac{1}{R^2 + 2RZ_{11} + Z_{11}^2 - Z_{12}^2}$$

If the strip is short we can neglect the different propagation times, the crosstalk is again given by Eq. 7.25 and is illustrated in Figure 7.9. In that case the crosstalk is independent of the peaking time and the crosstalk signal has the same shape as the actual induced signal. If we omit the termination the the crosstalk becomes again peaking time dependent.

The result for long RPC strips is illustrated in Figure 7.10. We find a very strong dependence of the crosstalk on the peaking time and the distance.

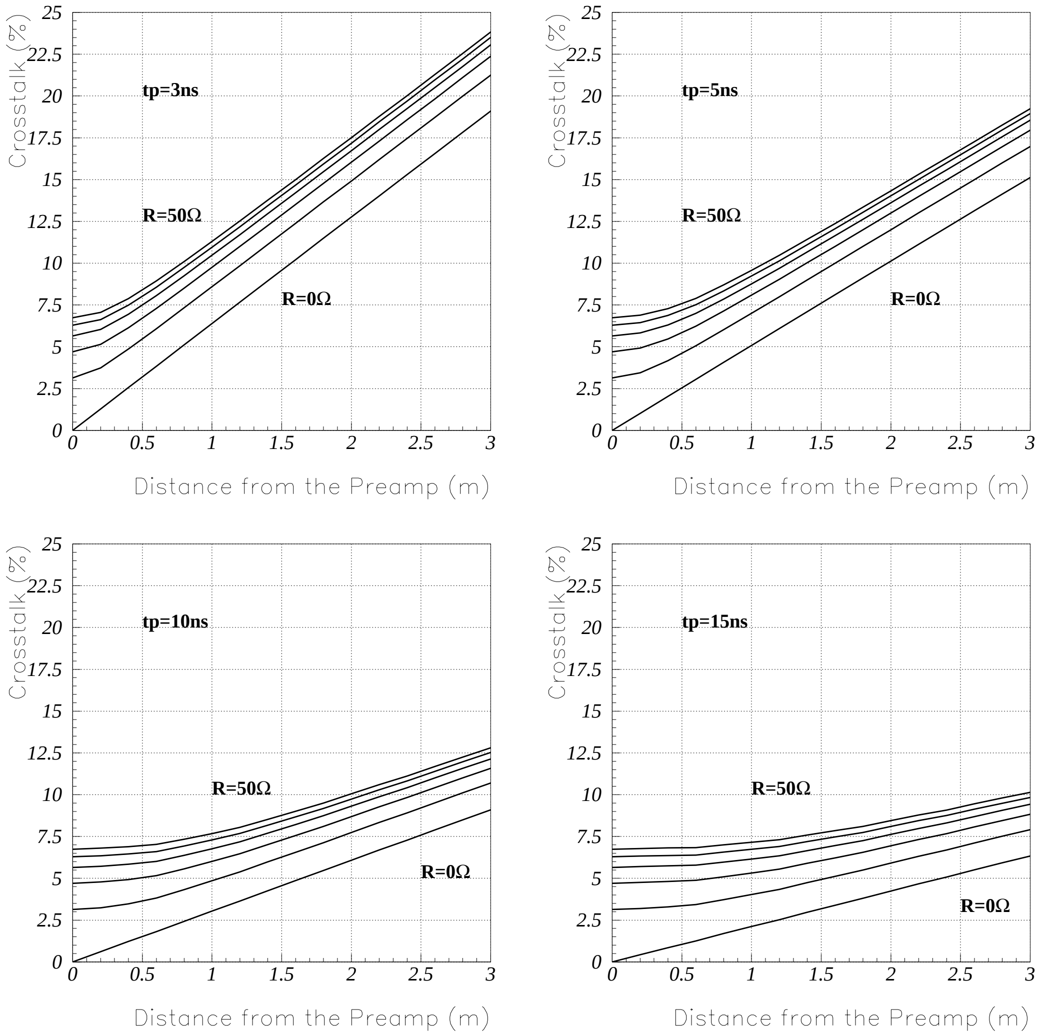

Figure 7.10 *: Crosstalk for different peaking times, distances from the preamp and input resistances 0,10,20,30,40,50 Ω. The further the pulse is from the preamp side the more the modes disperse which increases the crosstalk.*

7.4 Double Strip RPC with Guard Strip

In order to reduce the crosstalk we can try to put a guard strip between the two signal strips as shown in Fig. 7.11. The characteristic parameters are now given by

$$\hat{\mathbf{C}} = \begin{pmatrix} 225 & -20 & -21.6 \\ -20 & 46.5 & -20 \\ -21.6 & -20 & 225 \end{pmatrix} \quad \mathrm{pF/cm} \qquad \hat{\mathbf{L}} = \begin{pmatrix} 88.3 & 4.47 & 4.6 \\ 4.47 & 526 & 36.3 \\ 4.6 & 36.3 & 88.3 \end{pmatrix} \quad \mathrm{nH/m} \qquad \Rightarrow \tag{7.30}$$

$$\hat{\mathbf{Z}} = \begin{pmatrix} 20.4 & 9.2 & 2 \\ 9.2 & 111 & 9.2 \\ 2 & 9.2 & 20.4 \end{pmatrix} \quad \Omega \qquad \hat{\mathbf{T}} = \begin{pmatrix} -0.43 & 0.707 & 0.706 \\ 0.79 & 0 & 0.0041 \\ -0.43 & 0.707 & 0.706 \end{pmatrix} \tag{7.31}$$

$$v_1 = 2.01 \times 10^8 \quad \mathrm{m/s} \quad v_2 = 2.2 \times 10^8 \quad \mathrm{m/s} \quad v_3 = 2.4 \times 10^8 \quad \mathrm{m/s} \tag{7.32}$$

For the ideal termination we find

$$R_{11} = R_{33} = 22.7 \quad R_{22} = 582 \quad R_{12} = 252 \quad R_{13} = 319\,\Omega \tag{7.33}$$

which is illustrated in Figure 7.12. On the preamp side we ground the guard strip. Figure 7.13 shows the crosstalk versus distance of the hit from the preamp for different peaking times and a input resistance of $25\,\Omega$. For comparison the result without guard strip is plotted. Although the crosstalk is reduced by about 30% for short distances the relative difference becomes smaller since the slopes of the curves are the same. Finally the crosstalk for the short terminated and unterminated RPC is shown in Fig. 7.14.

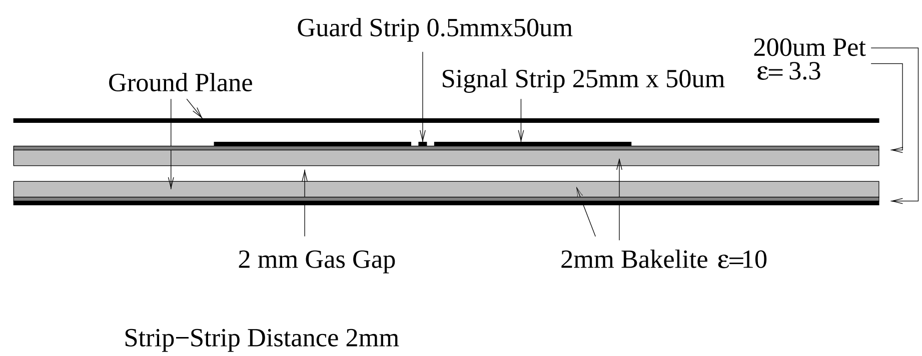

Figure 7.11 *: RPC with two strips and one guard strip.*

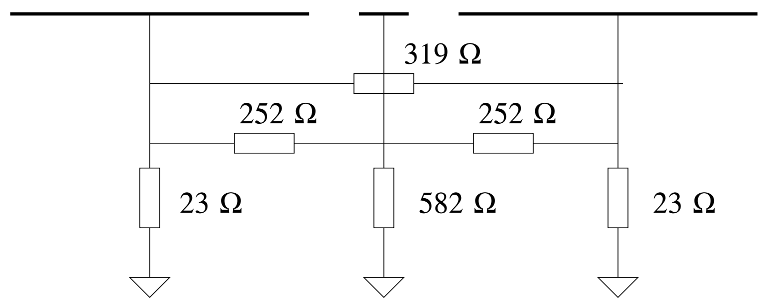

Figure 7.12 *: In order terminate the strips properly we have to interconnect all the lines.*

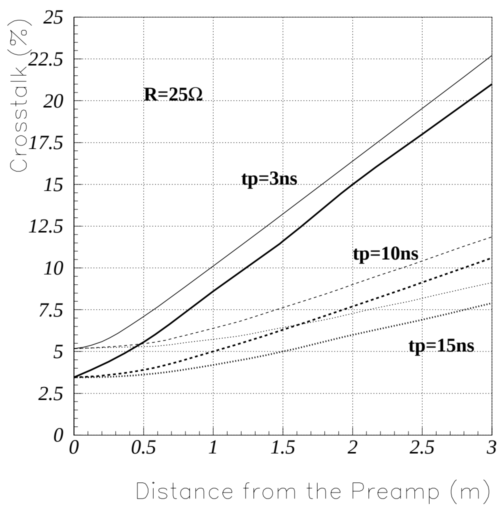

Figure 7.13 *: Crosstalk versus distance from the preamp for the RPC with guard strip. The thin lines show the results from the RPC without guard strip for comparison. The relative difference becomes marginal for long strips.*

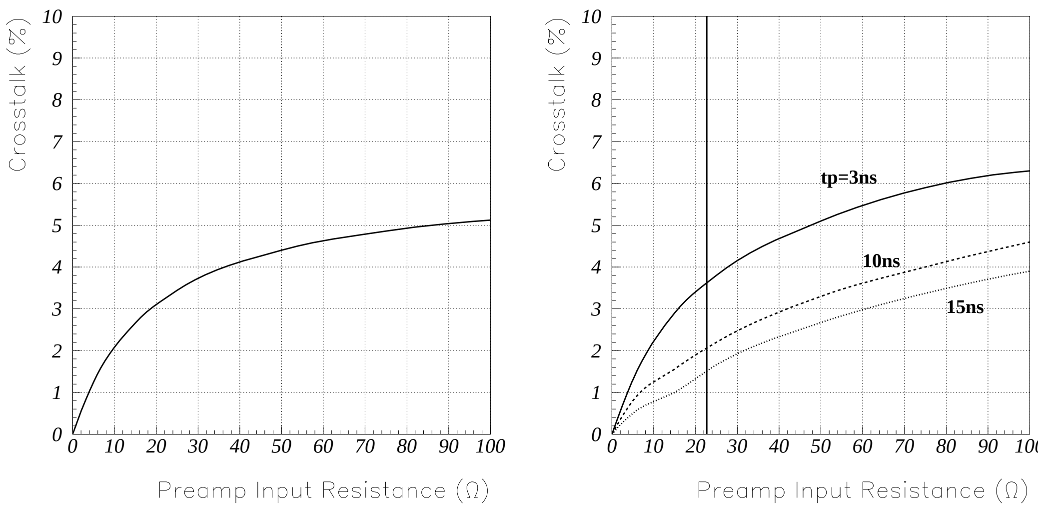

Figure 7.14 *: Crosstalk for a short terminated (left) and short unterminated RPC (right). For explanation see Fig. 7.9.*

7.5 RPC with Many Strips

Finally we can investigate the crosstalk for an RPC with many strips (Fig. 1.1). Capacitances $< 0.1\,\mathrm{pF/m}$, inductances $< 0.1\,\mathrm{nH/m}$ and resistances $< 0.1\,\Omega$ are set to zero. Resistances $> 1\,\mathrm{M\Omega}$ are set to ∞. The underlined numbers are the diagonal values. $\hat{\mathbf{R}}$ is the matrix containing the resistors for ideal termination.

$$\hat{\mathbf{C}} = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & 0 & 0 & . & . & . \\ . & -20 & -21.7 & . & . & . \\ . & \underline{46.4} & -20 & 0 & 0 & . \\ . & -20 & \underline{245.1} & -20 & -21.7 & . \\ . & 0 & -20 & \underline{46.4} & -20 & . \\ . & 0 & -21.7 & -20 & \underline{245.1} & . \\ . & 0 & 0 & 0 & -20 & . \\ . & 0 & 0 & 0 & -21.7 & . \\ . & 0 & 0 & 0 & 0 & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix} \mathrm{pF/m} \qquad \hat{\mathbf{L}} = \begin{pmatrix} . & . & . & . & . & . \\ . & 0.1 & 0 & 0 & 0 & . \\ . & 0.76 & 0.1 & 0 & 0 & . \\ . & 1.86 & 0.23 & 0.1 & 0 & . \\ . & 14.7 & 1.86 & 0.76 & 0.1 & . \\ . & 35.8 & 4.52 & 1.86 & 0.23 & . \\ . & \underline{525.5} & 35.8 & 14.7 & 1.86 & . \\ . & 35.8 & \underline{87.2} & 35.8 & 4.52 & . \\ . & 14.7 & 35.8 & \underline{525.5} & 35.8 & . \\ . & 1.86 & 4.52 & 35.8 & \underline{87.2} & . \\ . & 0.76 & 1.86 & 14.7 & 35.8 & . \\ . & 0.1 & 0.23 & 1.86 & 4.52 & . \\ . & 0 & 0.1 & 0.76 & 1.86 & . \\ . & 0 & 0 & 0.1 & 0.23 & . \\ . & 0 & 0 & 0 & 0.1 & . \\ . & 0 & 0 & 0 & 0 & . \\ . & . & . & . & . & . \end{pmatrix} \mathrm{nH/m}$$

$$\hat{\mathbf{Z}}_C = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & 0.1 & 0 & 0 & 0 & . \\ . & 0.39 & 0.1 & 0 & 0 & . \\ . & 0.85 & 0.19 & 0.1 & 0 & . \\ . & 4.03 & 0.85 & 0.39 & 0.1 & . \\ . & 8.94 & 1.86 & 0.85 & 0.19 & . \\ . & \underline{110.9} & 8.93 & 4.03 & 0.85 & . \\ . & 8.94 & \underline{19.9} & 8.94 & 1.86 & . \\ . & 4.03 & 8.93 & \underline{110.9} & 8.93 & . \\ . & 0.85 & 1.86 & 8.94 & \underline{19.9} & . \\ . & 0.39 & 0.85 & 4.03 & 8.93 & . \\ . & 0.1 & 0.19 & 0.85 & 1.86 & . \\ . & 0 & 0.1 & 0.39 & 0.85 & . \\ . & 0 & 0 & 0.1 & 0.19 & . \\ . & 0 & 0 & 0 & 0.1 & . \\ . & 0 & 0 & 0 & 0 & . \\ . & . & . & . & . & . \end{pmatrix} \Omega \qquad \hat{\mathbf{R}} = \begin{pmatrix} . & . & . & . & . & . \\ . & \infty & \infty & \infty & \infty & . \\ . & \infty & 269966 & \infty & \infty & . \\ . & \infty & \infty & \infty & \infty & . \\ . & 245851 & 25218 & \infty & 269966 & . \\ . & \infty & 246005 & \infty & \infty & . \\ . & 251.8 & 322.9 & 245851 & 25218 & . \\ . & \underline{584.3} & 251.8 & \infty & 246005 & . \\ . & 251.8 & \underline{24.9} & 251.8 & 322.9 & . \\ . & \infty & 251.8 & \underline{584.3} & 251.8 & . \\ . & 245851 & 322.9 & 251.8 & \underline{24.9} & . \\ . & \infty & 246005 & \infty & 251.8 & . \\ . & \infty & 25218 & 245851 & 322.9 & . \\ . & \infty & \infty & \infty & 246005 & . \\ . & \infty & 26966 & \infty & 25218 & . \\ . & \infty & \infty & \infty & \infty & . \\ . & . & . & . & 26966 & . \\ . & . & . & . & \infty & . \end{pmatrix} \Omega$$

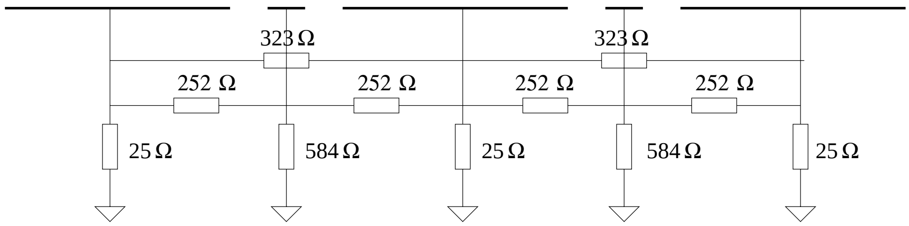

***Figure 7.15** : Ideal termination for the RPC with many strips. All other interconnections are $> 25\,k\Omega$ and have negligible effect.*

The velocities of the modes range from 2.0 to 2.5×10^8 m/s. The capacitive coupling between signal strips that are not direct neighbours is negligible. The capacitance of signal strip and guard strip to ground is given by the sum of the columns in the capacitance matrix (Eq. 6.7) and evaluates to 161.8 and 6.7 pF/m. The resistances for ideal termination are given by the matrix $\hat{\mathbf{R}}$ and are represented in Figure 7.15.

Figure 7.16 shows pulse-shapes for different distances from the preamp side. As discussed before the crosstalk increases and changes shape as a function of distance from the preamp side. The figure also shows the pulses for the scenario where the guard strips are grounded on both ends and there are no interconnections on the 'far end'. In that case we find reflections as expected.

The crosstalk to the first and the second neighbour is shown in Fig. 7.17. We find a very strong dependence on the peaking time. The numbers are very similar to the RPC model with only two strips and a guard strip. This is expected because the coupling between strips that are not direct neighbours is negligible.

For preamp input resistances $> 20\,\Omega$ the crosstalk dependence on the resistance is very small. The dependence on the peaking time for long strips is however significant.

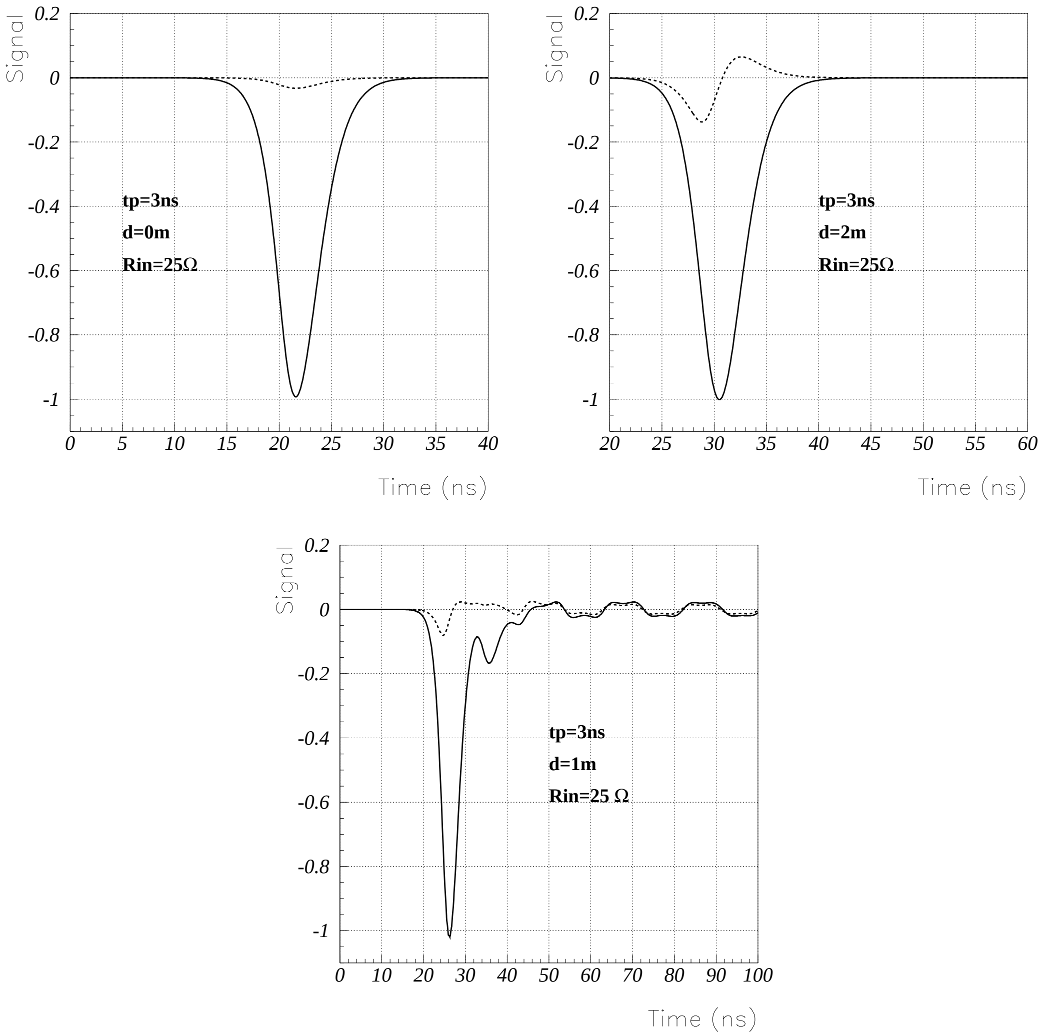

Figure 7.16 *: The top two figures show the RPC pulse together with the crosstalk to the first neighbour at two different distances from the preamp. We see that the shape changes. If the particle crosses the RPC close to the preamp side the pulse-shape is the same as the original one. For events further from the preamp the crosstalk increases and becomes bipolar. The bottom figure shows an event where the RPC strips are terminated at* $25\,\Omega$*, the intermediate strips are grounded on both sides and the strips are not interconnected. As expected we find reflections.*

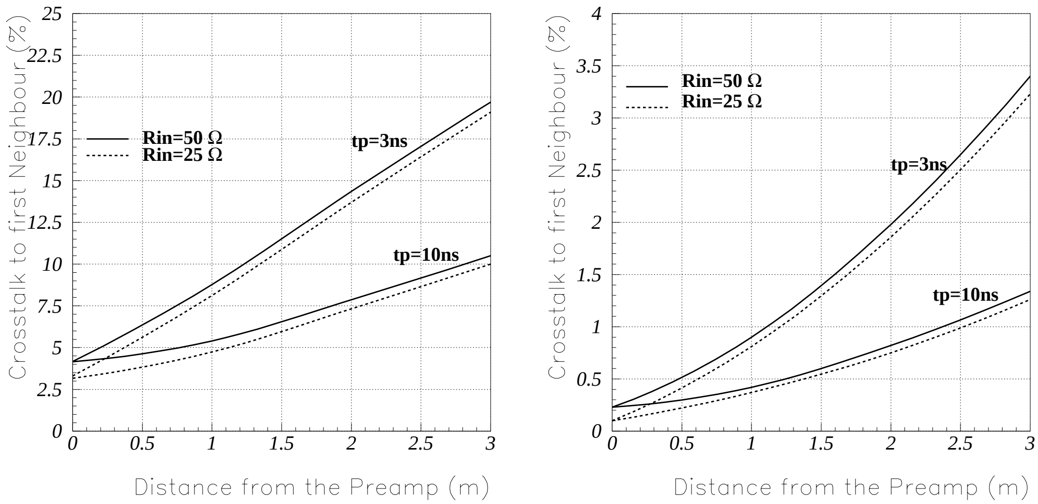

Figure 7.17 *: Crosstalk to the first and second neighbour for different peaking times and preamp input resistances. In the limit of very long peaking times the dependence on the distance would vanish and the curves would be flat. The crosstalk to the second neighbour happens entirely through the first neighbour and not through direct coupling.*

7.6 Short RPC with Many Strips

Here we want to study the short unterminated RPC since it is very important for the application in LHCb. In this approximation we have to know only the capacitances which we find from MAXWELL as

$$C_S = 161.8\,\mathrm{pF/m} \quad C_G = 6.7\,\mathrm{pF/m} \quad C_{SG} = 19.9\,\mathrm{pF/m} \quad C_{SS} = 21.7\,\mathrm{pF/m} \tag{7.34}$$

where C_S is the capacitance of strip to ground, C_G is the capacitance of the guard to ground, C_{SG} is the capacitance between strip and guard and C_{SS} is the strip-strip capacitance. If the guard strip is grounded the equivalent circuit describing the RPC is very simple as shown in Fig. 7.18. The effective capacitance to ground is $C = C_S + 2C_{SG} = 201.6\,\mathrm{pF/m}$. The capacitance determining the serial noise is given by $C_{noise} = C_S{+}2(C_{SG}{+}C_{SS}) = 245\,pF/m$.

For a RPC of 30 cm length the numbers are $C = 60.5\,\mathrm{pF}$, $C_{SS} = 6.5\,\mathrm{pF}$. The crosstalk for this RPC is shown in Figure 7.19.

We see that the crosstalk is smaller for long peaking times. Since the unterminated RPC is a serial noise source the noise will also be smaller for long peaking times. The crosstalk is approximately proportional to C_{SS}. This capacitance can be reduced by a more optimized geometry.

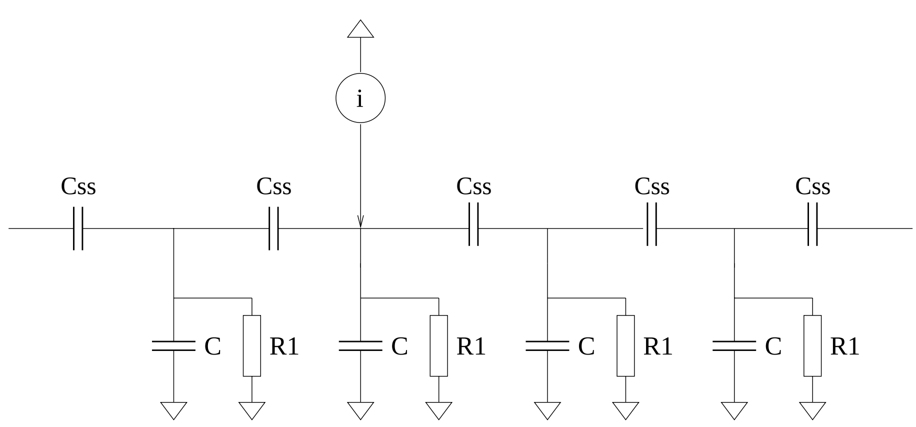

Figure 7.18 *: Equivalent circuit for the short unterminated RPC.* $C = C_S + 2C_{SG}$.

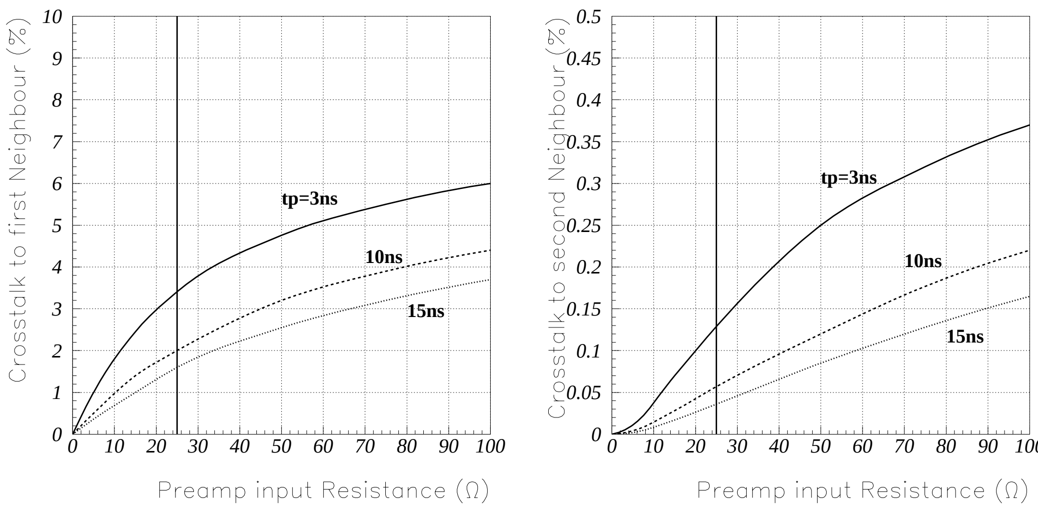

Figure 7.19 *: Crosstalk for an unterminated 30 cm RPC. The left plot shows the crosstalk to the first neighbour, the right plot shows the crosstalk to the second neighbour.*

7.7 Conclusions

- RPCs with two Bakelite plates forming a 2 mm gas gap where studied in detail.
- An avalanche model neglecting space charge effects indicates that for a total induced signal charge of about 1.5 pC an frontend electronics peaking time of 10 ns would still give a time resolution < 2 ns.
- For 2.5 cm readout strips and a threshold of 5-10% of the average signal we find a cluster size of about 1.2 from direct signal induction.
- For crosstalk studies the RPC was modelled as a lossless inhomogeneous multi-conductor transmission line.
- To terminate the RPC properly on one end one has to interconnect the signal lines.
- In case one side is ideally terminated the measured signal shape is equal to the original induced signal shape.
- For RPC signals induced close to the preamp side the crosstalk signal has the same shape as the original signal and the amount of crosstalk depends only on the preamp input resistance (not on the peaking time). In the limit of $R_{in} = 0$ the crosstalk is zero.
- The crosstalk is a strong function of the distance from the amplifier due to modal dispersion. Therefore the crosstalk signal shape depends on the distance of the induced signal from the preamp and the amount of crosstalk also depends on the preamp peaking time. In the limit of very long peaking time the crosstalk becomes independent of the distance.
- If the RPC is short the 'far end' termination can be omitted and the preamp input resistance should be close to the impedance of the signal strip. The amount of crosstalk decreases for long peaking times.
- The RPC model discussed in this note gives the following crosstalk numbers: For a preamp input resistance of 25 Ω and a peaking time of 3 ns the crosstalk increases from 4% to 14% at a distance of 0 to 2m from the amplifier. For a peaking time of 10 ns the crosstalk at 0 m is the same and reduces to 8% at 2 m.

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