APPLICATIONS OF DUAL RESONANCE MODELS TO INCLUSIVE REACTIONS

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Before beginning, let me apologize to those authors whose papers should have been included under this title but which were omitted because of lack of time, space, or understanding on my part. I have attempted to be more understandable than comprehensive; inevitably a review reflects its author's predilections and biases.

I Some General Considerations

According to the generalized optical theorem,[1] the single particle inclusive cross section is related to a particular discontinuity of the three-body forward scattering amplitude (Fig. 1):

$$\sqrt{s(s-4m^2)}\, E_c \frac{d\sigma}{d^3p_c} = \text{Disc}_{ab\bar{c}}\ F \tag{1}$$

Assuming the three body amplitude has Regge asymptotic behavior, the fragmentation (a:c|b) of particle a into particle c on b will be controlled by the dominant singularities in the $b\bar{b}$ channel[2] (Fig. 2):

$$E_c \frac{d\sigma}{d^3p_c} = \underbrace{f^P(x,p_\perp)}_{\text{Pomeron in } b\bar{b}} + \sum_R \underbrace{f^R(x,p_\perp)\left(\frac{s}{s_o}\right)^{\alpha_R(0)-1}}_{\text{Reggeons in } b\bar{b}} \tag{2}$$

$$\left[\ x = \frac{2p_{\parallel}}{\sqrt{s}} \quad \text{where } p_c = (E_c, p_\perp^x, p_\perp^y, p_\parallel) \text{ in the ab center of mass frame.}\right]$$

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Here we have assumed for phenomenological purposes that the leading singularity is an effective trajectory of intercept one. Similarly, the secondary trajectories are assumed to have intercept $\alpha_R(0) \approx 1/2.$ (3) So far, nothing has been said about duality, but having obtained this asymptotic form, one may ask, in analogy with two-body scattering, whether these effective trajectories in the $b\bar{b}$ channel are related to specific dynamical mechanisms in the crossed channels. For example, is the pomeron in $b\bar{b}$ associated with non-resonant background and in which channels? Similarly, are the Reggeons in $b\bar{b}$ related to resonances in certain crossed channels. From the outset, these questions are more complicated than in two-body scattering since, in the fragmentation $(a{:}c \mid b)$, three channel invariants $s = (a + b)^2$, $u = (b - c)^2$, and $M^2 = (a + b - c)^2$ all tend to infinity proportionately to each other. [$u \approx -xs$, $M^2 \approx (1 - x)s$ as $s \to \infty$.]

These questions have been kicked around already for a couple of years, and good reviews already exist[4] concerning the attempts to answer them. Consequently, I feel relieved from performing a comprehensive review and will emphasize what I believe to be the central issue.

The main problem is that two-component duality[5] (or the Harari-Freund hypothesis) cannot be generalized to multibody amplitudes in a straightforward, model independent way. One must consider a particular framework for this hypothesis which is both compatible with unitarity and generalizable to many-body reactions. Within that framework, one can discuss the generalization of two-component duality. With these preconditions, the ambiguity involved in the generalization is largely reduced. Two approaches that more or less share this philosophy have been pursued during the past year. One by Green, and Virasoro and myself[6] and another by Tye and Veneziano,[7] based on earlier work by Veneziano[8] and by Gordon and Veneziano.[9] [For earlier proposals, not based on generalizations of the two-component theory, see the reviews of Ref. 4.]

At the two body level, both groups assume the hypothesis of Freund and Rivers[10] for the pomeron P, viz, it is assumed that the twisted loop amplitude (Fig. 3), having no resonances in the direct channel (s-channel), has a pomeron in the crossed channel (t-channel).[11] However, this duality diagram also has the P′ or f^0 trajectory in the crossed channel. Thus the duality diagrams suggest that in general resonances are dual to Reggeons (including the P′) but background is dual to the pomeron and to the P′ Regge trajectory. This form of duality has been called the "weak Harari-Freund hypothesis" by Tye and Veneziano. This is in contrast to the usual two-component theory which has resonances dual to Reggeons and background dual to the pomeron but not to the P′. This is called the "strong Harari-Freund hypothesis" by Tye and Veneziano. Thus, under the weak H-F hypothesis, one maintains exchange degeneracy among the ρ-ω-A_2 trajectories, but the P′ is no longer constrained to be degenerate with them. The set of predictions based on the weak H-F hypothesis is a naturally subset of consequences of the strong HF hypothesis. Thus, e.g., weak H-F implies $\sigma_{tot}(K^- p) > \sigma_{tot}(K^+ p)$, but the $K^+ p$ total cross section could well be energy dependent. The strong H-F hypothesis implies $\sigma_{tot}(K^+ p)$ is energy independent, as is the case experimentally.

Now, the first named group above[6] attempts to generalize the usual two-component theory. Tye and Veneziano, on the other hand, generalize the weak Harari-Freund hypothesis, regarding the success of the strong H-F hypothesis at the two-body level as something yet to be explained. It is no wonder then that these two groups, starting from different hypotheses at the level of two-body scattering, reach different conclusions for inclusive cross sections. What I am trying to suggest is that the rules of the game are far less arbitrary than it might seem at first sight, and, although the generalization from total cross sections to inclusive cross sections is not deductive and requires further assumptions, the basic disagreements between the two results can be traced back to basically different initial hypotheses. Tye and Veneziano's weak H-F hypothesis is not without motivation, for the predictions obtained by the first group from the strong HF hypothesis are indeed very strong and, it is argued by Tye and Veneziano, perhaps even too strong to satisfy the requirements of inclusive sum rules (See Section III below).

I have neither time nor space to describe these schemes in greater detail; however, I have indicated something of the spirit of the two approaches.[12] Tye and Veneziano arrive at a great many predictions which are neatly summarized in a tabular form convenient for reference by experimentalists. They stress that many of them are " a very stringent test of planar duality" for multiparticle amplitudes. Some of them are in the form of equalities of differences of cross sections, such as

$$d\sigma(pp \to K^- X) - d\sigma(pn \to K^- X) = 0$$

$$d\sigma(\pi^+ p \to K^- X) - d\sigma(\pi^+ n \to K^- X) = d\sigma(\pi^- n \to K^- X) - d\sigma(\pi^- p \to K^- X).$$

[These relations are valid for all values of x and $p_\perp$ and are true of leading energy dependent effects as well as of the asymptotic cross sections.] These are essentially a consequence of the fact that exchange degeneracy is maintained in non-vacuum channels and are important relations to test. As these authors stress, "the importance of doing inclusive experiments on deuterium targets appears obvious."

A second class of predictions are in the form of inequalities, such as

$$d\sigma(K^- p \to K^- X) > d\sigma(K^+ p \to K^- X)$$

$$d\sigma(K^+ p \to p X) > d\sigma(K^+ p \to \bar{p} X) .$$

These I do not regard as peculiar to planar dual models and would be shared by any model assuming the absence of interference terms (e.g., multi-peripheral model). Loosely speaking, if one reaction is less exotic than another, more channels will be open for it; if there is no interference, each channel will contribute positively to the cross section and lead to inequalities like the ones above. Thus, these are not sensitive tests of the particular model of Tye and Veneziano.

If, on the other hand, one tries to generalize the hypothesis that all Reggeons (secondaries) are dual to resonances in crossed channels (strong HF hypothesis), one inevitably concludes that for a reaction $ab \to cX$ to be energy independent, it would certainly be sufficient to have all channels (ab, $a\bar{c}$, $b\bar{c}$, and $ab\bar{c}$) exotic,[13] such as $\pi^+\pi^+ \to \pi^- X$, $K^+p \to K^- X$, $pp \to K^- X$, $pp \to \bar{p} X$. (One hardly needs a theory to arrive at this conclusion!) However, the last two reactions have been measured at the ISR[14] and show a large increase (in the case of the $\bar{p}$, by nearly an order of magnitude from $p_{lab} \sim 24$ GeV/c to $p_{lab} \sim 450$ GeV/c (for $0.1 \lesssim x \lesssim 0.25$.)) Thus it would appear that for inclusive reactions, there is no anologue of exoticity for total cross sections. (See however the discussion of scale below.)

Before giving up the strong HF hypothesis, it is worthwhile to discuss in detail some of the other theoretical assumptions which could be wrong.

(1) Exoticity criteria are normally derived in a world of only mesons and then simply assumed to hold even when baryons are present. This is highly suspect, for it is equivalent to assuming a planar model for baryons which implies a rather different baryon spectrum.[15] If one begins with a symmetric model for baryons, such as the non-planar model of Mandelstam,[16] then the strong HF hypothesis seems incorrect even at

the two-body level. New diagrams[17] appear for meson-baryon scattering (e.g., Fig. 4) which are exotic in the s-channel but which have ordinary Reggeons in the t-channel. [Unlike the breaking of exchange degeneracy for the P′ trajectory alone, these diagrams involve net quark exchange in the t-channel and break exchange degeneracy in channels having non-vacuum quantum numbers as well.] Notice that the difficulty appears even with just one baryon present.

(2) The results depend crucially on the particular model for the pomeron, universally assumed to be the twisted loop. While this is the only candidate around for a dual pomeron, it is rather diseased in dual perturbation theory.

(3) Regge poles may be a poor approximation. Even though absorptive corrections are known to be important for two-body scattering,[18] they are not so strong as to disturb the basic predictions of the two-component theory. It may well be that absorption is even more important for three-body scattering, especially at large t (small x or large $p_\perp$).[19]

(4) Finally, it is difficult to interpret experimental results in terms of a Mueller-Regge picture without knowing more about the energy scale s_o. For two-body scattering, we have learned that a useful scale is 1 GeV.^{2} The energy dependence above about $p_{lab} \sim 2.5$ GeV/c can be parameterized with only a few terms and the couplings of the pomeron and of secondary Reggeons to particles are of the same order in these units. Exoticity and exchange degeneracy work very well. Exotic total cross sections (K^+p and pp total cross sections) show less than a 5% variation with energy

from $p_{lab} \sim 1.5$ GeV/c to the highest energies measured.[20]

How rapidly do the components of inclusive cross sections approach their asymptotic expansions? And for what choice of scale are the relative magnitudes of the pomeron and Reggeon terms comparable, i.e., for what choice of scale s_o is the ratio

$$R(x, p_\perp) = \frac{f^R(x, p_\perp)}{f^P(x, p_\perp)}$$

of order unity? We must allow for the possibility that the answer is dependent on x and $p_\perp$, i.e., the scale s_o may be a function of x and $p_\perp$.

One might guess that all the invariants which are not held fixed must be large compared to the external masses or other fixed invariants (such as t). Thus we guess that, in the fragmentation $(a:c|b)$, one must have

$$s, |u|, M^2 >> m^2, \ |t|, \frac{1}{\alpha'} \sim 1 \text{ GeV}^2.$$

In general, then, this would mean that for $\frac{2m_\perp}{\sqrt{s}} \lesssim x \lesssim 0.5$. The u-channel sets the scale so that,

$$R(x, p_\perp) \sim x^{\alpha_R(0)-1} \left(\frac{2m_\perp}{\sqrt{s}} \lesssim x \lesssim 0.5 \right).$$

Such a conjecture is consistant with the pionization limit being approached more slowly and this scale goes continuously over to the pionization result $s^{\frac{\alpha_R(0)-1}{2}}$. Similarly, we would expect that, for $\frac{1}{2} < x < 1$, the missing mass would set the scale so that $R(x, p_\perp) \sim (1-x)^{\alpha_R(0)-1}$ $\frac{1}{2} \lesssim x < 1$.

In summary, then, the effective scale for fragmentation is expected to be

$$\frac{s_o}{x} \quad \text{for} \quad \frac{2m_\perp}{\sqrt{s}} \leq x \lesssim 0.5,$$

$$\frac{s_o}{1-x} \quad \text{for} \quad 0.5 \lesssim x \lesssim 1.$$

This conjecture should be investigated experimentally and, in models, numerically.

Of course when $s, |u|$ or M^2 are too near threshold, we expect large energy variations due to threshold effects. (Even exotic total cross sections show significant energy variation when there is less than 500 - 1000 MeV kinetic energy in the center of mass.[21]) It has been suggested that the large energy variation seen in $pp \to \bar{p} X$, for example, is due to threshold effects at the lower energies (p_{lab} = 24 GeV/c). At this energy for $x \approx 0.25$, we find the following values for s, $|u|$ and M^2, well above their channel thresholds.

$$s = 47 \text{ GeV}^2 >> 4m_N^2$$

$$|u| \approx 12 \text{ GeV}^2 >> 4\,m_N^2$$

$$M^2 \approx 35 \text{ GeV}^2 >> 9\,m_N^2$$

(m_N = nucleon mass). Consequently, from the point of view of three-body scattering we expect there to be a sufficient amount of phase space and a sufficient number of intermediate states to smooth out kinematical "threshold effects". If, indeed the energy variation is due to "threshold effects", it is probably a different mechanism than in total cross sections.[22]

Perhaps it is related to the fact that we are not dealing with the three-body total cross section ($ab\bar{c} \to X$) but rather the production cross section ($ab \to \mathbf{c}X$), but no convincing argument has been offered which suggests that the scale of Regge behavior will change as a result of this analytic continuation.

To summarize the problems discussed above in generalizing duality to inclusive cross sections, I would say that progress depends on finding a reasonable dual phenomenological model for the pomeron, consistant with the Harari-Freund hypothesis for meson-meson, meson-baryon, and baryon-baryon total cross sections.

II. More Detailed Dynamical Questions

Let us now turn to more detailed questions such as the form of the fragmentation residues $f(x, p_\perp)$. It is difficult to see on general grounds what form these functions should take; even for the planar six-point dual amplitude, B_6, their form is analytically complicated. Accordingly, there have been a number of numerical evaluations of these residues among the papers submitted to this conference.[23] From these papers, we may gleen the following general features. (1) The approach to asymptopia is slower for small x ($x \lesssim 0.1$) than for larger values ($0.1 < x < 0.9$). This can be easily understood since the non-leading trajectories in this case correspond to daughters. (2) Variations in x and $p_\perp$ due to signature factors persist outside the triple Regge region. The effect of daughter trajectories is to slowly vary the modulus of the amplitude. These oscillations will be most pronounced in those fragmentations having t-channel exchanges of one signature only. For this reason, the pion fragmentation ($\pi^- : \pi^0 | p$) is an excellent place to look for them.[24] [Notice that the doubly differential cross section is required since an integration over x or $p_\perp$ will wash out this behavior.] (3) Examples of numerical calculations are shown in Figs. 5 and 6, taken from Biebel, et al., Ref. 23. Note, in Figs. 5(b) and 6(b), the dips due to the signature zeros discussed above. Although we will pass over the point here, we would like to remark that the shapes of the distributions are quite sensitive to the masses of the particle and its fragment.[25]

The strong damping of the $p_\perp$ distribution is generally considered to be one of the great successes of this model and was already emphasized in the first applications of the model.[26] For large $p_\perp$, the distribution is given by[27]

$$f(x, p_\perp) \approx \left(1 - x^2\right)^3 \left[\frac{4x}{(1+x)^2}\right]^{1-\alpha_R} \frac{1}{p_\perp^5} \, e^{-\frac{2p_\perp^2}{x} \ln\left(\frac{1+x}{1-x}\right)} \qquad (4)$$

which, for $x \to 0$, gives

$$f(x, p_\perp) \sim x^{1-\alpha_R} \frac{1}{p_\perp^5} \, e^{-4p_\perp^2} \qquad (x \to 0) \qquad (5)$$

This transverse momentum distribution falls rapidly with $p_\perp$ and so is qualitatively correct. However, experimental distributions[28] tend to fall less rapidly than indicated by the preceding formula. This should not be considered a failure of the model, since the pomeron has not been properly described. It is possible that the twisted loop diagram might lead to less rapidly falling distributions. A first step in this direction has been taken by Alessandrini and Amati,[27] who calculate the effect of a single twisted loop in the $b\bar{b}$ channel (Fig. 7a). [For phenomenological purposes, these authors simply adjust the intercept of the pomeron singularity to one.] They find

$$f(x, p_+) \sim (1+x)^2 \, x^{1-\alpha_R} (1-x)^{1+2\alpha_R} \frac{1}{p_\perp^4} \, e^{\frac{-2p_\perp^2}{x} \log\left(\frac{1+x}{1-x}\right)} \qquad (6)$$

so that, for $x \to 0$

$$f(x, p_\perp) \sim x^{1-\alpha_R} \frac{1}{p_\perp^4} \, e^{-4 p_\perp^2} \qquad (7)$$

Thus, although the power falloff is less severe, the exponential is unchanged. Of course, the diagram leading to pionization will have pomerons in both $a\bar{a}$ and $b\bar{b}$ (Fig. 7(b)), and these authors are working on this. There is some reason to suspect that the exponential falloff will be $e^{-2p_\perp^2}$, but let me pass over this speculation.

III Constraints on Duality Schemes from Inclusive Sum Rules.

Although inclusive sum rules will be discussed in other sessions, I would like to mention in passing one application of them to dual models. From the conservation of energy-momentum and conservation of probability, it follows that[29]

$$(p_a + p_b)^\mu \, \sigma_{tot}^{ab} = \sum_c \int E_c \frac{d\sigma}{d^3 p_c} \, p_c^\mu \, \frac{d^3 p_c}{E_c} \quad . \tag{8}$$

This basically says that the total energy-momentum carried away by a fragment times the probability of producing that fragment (summed over all fragments) is equal to the total energy-momentum entering the reaction times the total probability of interaction. It is useful to evaluate this in the center of mass frame. The sum of energy and longitudinal momentum components leads to

$$\sigma_{tot}(s) = 1/2 \sum \int f(s, x, p_\perp) \left[1 + \frac{x}{\sqrt{x^2 + 4m_\perp^2/s}} \right] d^2 p_\perp \, dx \tag{9}$$

where we've defined $f \equiv E_c \dfrac{d\sigma}{d^3 p_c}$, a quantity finite in the limit $s \to \infty$ and also finite at $x = 0$. [However, in studying the energy dependence of the right hand side, one must be very careful to treat properly the regions $x \approx 0$ and $x \approx 1$.] Besides being a detailed constraint on any duality scheme for inclusive cross sections, it was pointed out[30, 7] that this had some simple general consequences. If, for example, one chooses ab exotic, the left hand side will be energy independent. This means that not all inclusive cross sections can fall to their asymptotic values at least not for all values of x and p_+. Since some inclusive cross sections, such as $pp \to pX$, seem to fall to their asymptotic values, some others must rise to compensate. Therefore, unlike total cross sections, some inclusive cross sections must rise to their asymptotic values (at least for a range of x and $p_\perp$). This feature and other experimental data are presumably what induced Tye and Veneziano to introduce their weak version of the Harari–Freund conjecture. However, one should notice that the sum rules are not compatible with pure Regge pole behavior, and that in fact, starting from Regge poles in the inclusive cross section, one will obtain in general Regge cuts of the AFS type in the total cross section.[31]

Other similar constraints arise from other conservation laws such as charge, hypercharge, and isospin, but I will omit their discussion.

IV Reggeon - Particle Scattering

Consider the exclusive process for $a + b \to c + N$, where N is some definite multiparticle state. Define $s = (a + b)^2$, $t = (a - c)^2$, $u = (b - c)^2$, $M^2 = (a + b - c)^2 = s + t + u - 3m^2$. For small t and large energy $[s, |u| >> M^2, m^2, s_o]$, we expect the amplitude for this process to have Regge behavior:

$$\sum_i \beta^i_{a\bar{c}}(t)\, \beta^i_{b\bar{N}}(t, N)\, \xi_i\, \eta^{\alpha_i(t)} , \tag{10}$$

where we have denoted the signature factor by $\xi_i = -\tau - e^{-i\pi\alpha_i}$ and defined the crossing energy $\eta = \frac{s-u}{2} = b\cdot(a+c)$. (All other variables required by N have been suppressed.) Square this amplitude and sum over all states N (consistant with energy-momentum conservation) to get

$$\sum \beta^i_{a\bar{c}}(t)\xi_i \left(\beta^j_{a\bar{c}}(t)\, \xi_j\right)^* \eta^{\alpha_i(t)+\alpha_j(t)}\, A^{ij}_{b\bar{b}}(t; M^2) \tag{11}$$

where (see Fig. 8(a))

$$\begin{aligned} A^{ij}_{b\bar{b}} &= \sum_N (ib \to N)(jb \to N)^* \,\delta(a+b-c-N) \\ &= \operatorname{Disc}_{M^2} F^{ij}_{b\bar{b}}(t; M^2). \end{aligned} \tag{12}$$

The quantity $A^{ij}_{b\bar{b}}$ may be regarded as the absorptive part of the Reggeon-particle forward scattering amplitude $F^{ij}_{b\bar{b}}$. This Reggeon-particle amplitude is assumed to have Regge asymptotic behavior as $M^2 \to \infty$ (see Fig. 8(b)).

$$F^{ij}_{b\bar{b}}(t;\nu) \sim -\sum \Bigg\{ \beta^{k}_{b\bar{b}}(0)\, g^{k}_{ij}(t;0) \quad \times \tag{13}$$

$$\times \left[\frac{\tau + e^{-i\pi(\alpha_k - \alpha_i - \alpha_j)}}{\sin \pi (\alpha_k - \alpha_i - \alpha_j)} \right] \nu^{\alpha_k(0) - \alpha_i(t) - \alpha_j(t)} \Bigg\} + \tilde{F}^{ij}_{b\bar{b}}(t;\nu)$$

where $\tau = \tau_i \tau_j \tau_k$ is the triple Reggeon signature[32] and $\nu = M^2 - t - m^2$. $\tilde{F}$ represents possible fixed integral powers in ν. If one supposes that the Reggeon-particle amplitude F will have analyticity in ν similar to particle-particle amplitudes, then $A^{ij}_{b\bar{b}}(t;\nu)$ will satisfy finite energy sum rules:

$$\int_0^N d\nu \left[A^{ij}_{b\bar{b}}(t;\nu) + (-)^{n+1} A^{ij}_{b\bar{b}}(t;-\nu) \right] \nu^n =$$

$$= \sum \frac{\beta_{b\bar{b}}\, g^{k}_{ij}\, N^{\alpha_k - \alpha_i - \alpha_j + 1}}{\alpha_k - \alpha_i - \alpha_j + 1} \tag{14}$$

Inclusive cross sections in the limit in which Regge exchanges in the t-channel dominate will therefore satisfy finite missing mass sum rules (FMSR)[32, 33]

$$\int_0^N d\nu \left[E_c \frac{d\sigma}{d^3 p_c} (ab \to c X) + (-)^{n+1} E_a \frac{d\sigma}{d^3 p_c} (cb \to a X) \right] \nu^n =$$

(15)

$$\sum_{\tau = (-)^n} \frac{\beta^i_{a\bar{c}}(t)\, \xi_i\, (\beta^j_{a\bar{c}}\, \xi_j)^* \, \beta^k_{b\bar{b}}(0)\, g^k_{ij}(t)}{\alpha_k + n + 1 - \alpha_i(t) - \alpha_j(t)} \left(\frac{\eta}{N} \right)^{\alpha_i - \alpha_j - 1} N^{\alpha_k + n} .$$

Let me discuss a number of potential uses of such sum rules.

(1) The triple-Regge couplings g^k_{ij} are of considerable theoretical interest, especially when one or more of the Regge poles is the pomeron pole. However, at energies below 30 GeV/c, it is difficult to satisfy the conditions necessary for the triple- Regge limit, via, $M^2 >> s_o$, $\frac{s}{M^2} >> 1$. It is especially difficult to isolate couplings which are thought to be suppressed, such as the triple pomeron or pomeron-pomeron-Reggeon couplings. The FMSR enables one to use data throughout the low missing mass region to predict the magnitude of triple-Reggeon couplings. Thereby predictions can be made for experiments at ISR and NAL and the large missing mass data correlated with the low missing mass spectrum. To uniquely identify the triple-Reggeon couplings, data on the missing mass spectrum are required at several energies with reliable normalization and good resolution. Although such data is not yet available, fits[34]

have been performed to existing data under a variety of assumptions as to the important couplings, and predictions have been made for experiments currently underway at NAL and ISR energies. (2) The Reggeon-particle absorptive parts $A^{ij}_{b\bar{b}}(t;M^2)$ are interesting in themselves, for one can begin to explore the duality properties of Reggeon-particle scattering. For example, Tye and Veneziano[7] have suggested that the duality properties will change as t becomes larger and that the P′ trajectory will become dual to both resonances and background. As a second example, when one of the exchanged Reggeons is a pomeron, we have suggested[6] that the duality properties will be surprisingly different from particle scattering. (3) Analogues of superconvergence relations can be sought for Reggeon-particle amplitudes. A recent discussion by Finkelstein leads to some interesting relations.[35] (4) If one writes FMSR s for amplitudes of definite signature, one obtains[32] sum rules similar to Eq. (15) except for the replacement of $(-)^{n+1}$ by $(-)^n$ and the addition on the right hand side of residues $R^{(n)}_{ij}(t)$ at possible nonsense wrong signature fixed poles. The lowest order of these, $R^{(0)}_{ij}(t)$, is the fixed pole residue through which the i-j Reggeon-Reggeon cut couples to the forward ab elastic amplitude.[36] Thus measurements of single particle inclusive spectra determine in principle the scale of cut contributions, so the Regge pole approximation might be checked for self-consistancy.[37]

One of the many problems involved in actually evaluating such sum rules from data is the problem of large cancellations between terms.

The point can be made already for particle-particle scattering. The simplest sum rule for the nonsense, wrong-signature fixed pole residue in the scattering of equal mass scalar particles reads

$$R^{(0)} = \int_0^N d\nu \, \mathrm{Im}\, F(\nu, 0) - \sum \beta_k^2(0) \frac{N^{\alpha_k(0)+1}}{\alpha_k(0)+1} . \qquad (16)$$

The first term on the right hand side is to be evaluated using data on the total cross section $\sigma_{tot}(\nu)$ and the optical theorem $\mathrm{Im}\, F(\nu, 0) = \sqrt{\nu^2 - 4m^2}\ \sigma_{tot}(\nu)$. The second term involves a sum over even signature Regge trajectories whose intercepts $\alpha_k(0)$ are presumed known and whose coupling to particles $\beta_k(0)$ are presumably known from evaluation of "right-signature" finite energy sum rules. In general, there will be significant cancellations between the two terms. Any planar model, such as the Veneziano model, illustrates the magnitude of the cancellations, for in such a model, the amplitude is the sum of three terms:

$$F = V(s,t) + V(u,t) + V(s,u). \qquad (17)$$

The first two terms have Regge asymptotic behavior and no wrong signature fixed pole; the third term vanishes faster than any power but contains the wrong signature fixed pole R^0. The first and third terms contribute to the discontinuity in the s-channel needed in the sum rule.

$$\mathrm{Im}\ F = \frac{1}{2i}\left[\mathrm{Disc}_s\ V(s,t) + \mathrm{Disc}_s\ V(s,u)\right]$$

The contribution of $\mathrm{Disc}_s\ V(s,t)$ to the first term of (16) is precisely and completely cancelled by the Reggeons from the second term of (16) so that, one obtains the expected result

$$R^{(0)} = \int_0^N d\nu\ \mathrm{Im}\ V_{su}\ . \tag{18}$$

To put it another way, the difference appearing in (16) is precisely what is needed to isolate the contribution which, in the language of the Mandelstam representation, comes from the third double spectral function. (Eq. 18).

This discussion leads me to believe that the accurate evaluation of analogous sum rules for Reggeon-particle amplitudes must be extremely difficult. In order to get this cancellation correct, one must have sufficiently good data over a wide enough energy range so that one has confidence that he has properly isolated a given Reggeon-particle cross section and the triple Reggeon couplings. (5) There is one worrisome point about the derivation of these FMSR which must influence their reliability when actually applied to data. We know there are Regge-Regge cuts, and we suspect that the pomeron should not be assumed to be a pole, even as a first phenomenological approximation. One can understand how ordinary FESR for particle-particle scattering will work even if the poles assumed are effective trajectories, which include absorptive cuts and need not factorize. However, the derivation of the FMSR depended on an assumption on the analyticity of Reggeon-particle amplitudes which may well not hold for an effective trajectory-particle amplitude. It would therefore be useful to have a derivation which depended less on the assumption of Regge poles in the t-channel and more on the analyticity in M^2 at fixed t and s of the full three-to-three amplitude and on the generalized optical theorem.[38] In the meantime, an iterative scheme is probably worth trying, in which one assumes only poles at the outset and then, via fixed pole residues, one estimates the magnitude of cut corrections. At least, it provides us with the first real hope for a fairly model independent calculation of Regge cuts.

In conclusion, let me say that, in retrospect, I see that my review emphasizes more the problems we face in applying duality concepts to data and less the successes achieved. While this probably accurately reflects the work over the past year, only as firm a believer in duality as I could have been so unfair as to concentrate on its failings. Perhaps, then, these notes will stimulate a quick resolution of its difficulties, for I am certain that duality, in some form or other, will provide as useful a framework for understanding the qualitative features of multiparticle reactions as it has for two-body phenomenology.

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[3] If these singularities are Regge poles, then the distribution functions $f^P(x, p_\perp)$ and $f^R(x, p_\perp)$ will have factorization properties. The effective singularities α_p and α_R may be thought of as the poles plus its absorptive cuts.

[4] H-M. Chan, Review presented at the IVth International Conference on High Energy Collisions (Stony Brook Series) Oxford, 1972, Rutherford Laboratory Preprint RPP/T/17; P.H. Frampton, "Duality Considerations in Inclusive Reactions (Review)", Preprint Ref. TH.1497-CERN, 18 May 1972; M. Jacob, Review presented at the Argonne Symposium on some applications of duality, April, 1972, National Accelerator Laboratory Preprint NAL-THY-47, June 1972; M. B. Einhorn, in Proceedings of the International Conference on Inclusive Reactions (R.L. Lander, ed.), University of California, Davis, February 4-5, 1972, Particle Physics (Irvine Conference - 1971), (M. Bander, et al., eds), AIP Conference Proceedings Number 6, AIP, New York, 1972.

[5]H. Harari, Phys. Rev. Letters 20, 1395 (1968); P. G. O. Freund, ibid 20, 235 (1968).

[6]M. B. Einhorn, M. B. Green, and M. A. Virasoro, Phys. Letters, 37B, 292 (1971), Phys. Rev. D6, 1675 (1972), LBL preprint No 768, to be published in Phys. Rev. January 1973.

[7]S-H. H. Tye and G. Veneziano, Phys. Letters 38B, 30 (1972), MIT Preprint No. CTP 292, (1972) submitted to this conference.

[8]G. Veneziano, Letter Nuovo Cimento 1, 681 (1971), in the Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, (E. Goteman, ed.), Jerusalem, Weizman Science Press, 1971.

[9]D. Gordon and G. Veneziano, Phys. Rev. D3, 2116 (1971).

[10]P. G. O. Freund and R. J. Rivers, Phys. Letters 29B, 510 (1969), P. G. O. Freund, Nuovo Cimento Letters 4, 147 (1970).

[11]The existence of a strange singularity fondly called the pomeron was verified by D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwarz, Phys. Rev. D2, 697 (1970); C. S. Hsue, B. Sakita, and M. A. Virasoro, ibid 2, 2857 (1970); G. Frye and L. Susskind, Phys. Letters 31B, 589 (1970).

[12]In Ref. (6), conclusions about inclusive cross sections are reached via the generalized optical theorem, whereas Ref. (7) analyzes the classes of distinct production amplitudes contributing to the inclusive

cross section. To the extent that both references construct unitary theories, the differences between the approaches are more superficial than basic. For example, the arguments of Ref. (6) can easily be replaced in terms of production amplitudes by examining the discontinuities explicitly. (This procedure is illustrated in Section IV of the second paper cited in Ref. (6).)

[13] This is true of every proposal (See Ref. 4). Disagreements exist concerning necessary conditions. It seems also true that, if one gives up the strong HF hypothesis, he finds no criterion sufficient for energy independence. One can, of course, hypothesize an intermediate situation such as the assumption of Tye and Veneziano (Ref. 7) that, in their seventh component, the P′ decouples.

[14] A. Bertin, et al., paper No. 747 contributed to the XVI International Conference on High Energy Physics, Sept 6-13, 1972, Chicago-Batavia. See also the review by M. Jacob, "Production Processes at High Energies," these proceedings.

[15] S. Mandelstam, Phys. Rev. D1, 1734 and 1745 (1970); P.H. Frampton and P.G.O. Freund, Nucl. Phys. B24, 453 (1970); P.H. Frampton, Bielefeld University preprint No. Bi-72/07, August 1972.

[16] S. Mandelstam, Phys. Rev. D1, 1720 (1970).

[17] The importance of such diagrams were pointed out to me by M.B. Green (private communication) and is why we were unable to extend the

considerations of Ref.(6) to non-planar baryons. Frampton, Ref. (15), found more complicated diagrams with similar difficulties but apparently overlooked these complications already at the one baryon level.

[18] See e.g., H. Harari, in Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, (E. Goteman, ed.), Weizmann Science Press, Jerusalem, 1971,

[19] This possibility has been pointed out, for example, in Ref. (6). A recent preprint seems to argue for such an interpretation of the data. See H-M. Chan, et al., Rutherford Laboratory preprint No. RPP/T/15.

[20] See, e.g., the rapporteur's talk by G. Giacomelli, "Strong Interactions at High Energies," these proceedings.

[21] See Table I of the second paper of Ref. (6).

[22] For different viewpoints, see M. Jacob, Ref. (4) and Tye and Veneziano, Ref. (7).

[23] G. H. Thomas, Argonne National Laboratory preprint ANL/HEP 7144, November 1971, paper No. 582, this conference; D. Bebel, et al., Numerical Results for the B_6- Model of Single Particle Distributions, paper No. 645, this conference; K. Kang and P. Shen, Brown University preprint No. C00-3130TA-267, paper No. 229, this conference.

[24]We should remark that, in order to apply the planar model phenomenologically, one is obliged to simulate unitary corrections by averaging over rapid oscillations, leading to smooth behavior. Consequently, we should not expect that the rapid oscillations, depicted for example in Fig. 5(b), will actually be seen in nature. However, we do believe that the first oscillation or two should be seen and that a study of $(\pi^-:\pi^0 \mid p)$ as a function of x and $p_\perp$ might teach us something about how rapidly these oscillations become smoothed out.

[25]See, especially, the discussion by Thomas, Ref. (23), and by Ph. Salin and G.H. Thomas, Nuclear Phys. B38, 375 (1972), paper No. 580, this conference.

[26]M.A. Virasoro, Phys. Rev. D3, 2384 (1971); C.E. DeTar, etal., Phys. Rev. D4, 425 (1971).

[27]This expression has been taken from the paper by V. Alessandrini and D. Amati, CERN preprint CERN-TH-1534, 1972. The scale has been chosen so that $\alpha' = 1$, and the intercept $\alpha_p(0)$ of one of the trajectories has been set equal to one. Of course, one obtains a finite pionization limit only if the other trajectory also has intercept α_R equal to one.

[28]See the review of ISR data in the summary talk by E. Lillethun, High Energy Collisions III, these proceedings and also Scientific/Technical Report No. 47, U. of Bergen, November 1972. A simple exponential $e^{-6.2p_\perp}$ provides an excellent fit to pion distributions at 90° for

$0.2 < p_\perp < 1.2$ GeV/c.

Data at even larger values of $p_\perp$ seem to flatten out. See M. Bander, et al., paper No. 478, these proceedings. The slope between 3.5 and 5 GeV/c transverse momentum is 2.5 ± 0.4 $(\mathrm{GeV/c})^{-1}$ compared to the value of 6.2 quoted above.

[29] T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1972 (1970); C. E. DeTar, D. Z. Freedman, and G. Veneziano, Phys. Rev. D4, 906 (1971); see the review by Z. Koba, Niels Bohr Institute, Preprint NBI-HE-72-9, July, 1972 (to be published in Acta Phys. Polon.)

[30] L. Caneschi, Phys. Letters 37B, 288 (1971).

[31] This fact was first called to my attention by M. A. Virasoro (private communication, Jan, 1972). This has been discussed in detail in Appendix A of Ref. 7(b).

[32] M. B. Einhorn, J. Ellis, J. Finkelstein, Phys. Rev. D5, 2063 (1972).

[33] J. Kwiecinski, Nuovo Cimento Letters 3, 619 (1972); A. I. Sanda, Phys. Rev. D6, 280 (1972).

[34] S. D. Ellis and A. I. Sanda, Phys. Letters 41B, 87 (1972); J. Dias de Deus and W. S. Lam, Phys. Letters 38B, 220 (1972); F. E. Paige and L-L. Wang, Brookhaven National Lab Preprint No. BNL-16782, April 1972 .

[35] J. Finkelstein, Stanford Univ., Institute for Theoretical Physics preprint ITP-407, April 1972.

[36] A discussion of this relation can be found in A. B. Kaidalov, Yad Fix. 13, 401 (1971), [Sov. J. Nucl. Phys. 13, 226 (1971)]; I. J. Muzinich, F. E. Paige, T. L. Trueman, L-L. Wang, Phys. Rev. Letters 28, 850 (1972). A controversy currently rages over the correct formula for the cut. [See, e. g., G. F. Chew, "Arguments Supporting a Positive 2-Reggeon Discontinuity," Paper No. 250 contributed to this conference.] However, all suggestions involve this fixed pole residue, so it will be worthwhile to know it.

[37] A first attempt to evaluate such a residue and a discussion of the problems involved may be found in D. P. Roy and R. G. Roberts, Phys. Letters 40B, 555 (1972.

[38] Presumably, a formula like Equation (15) does have a stronger basis since, on the left-hand side, no Reggeons appear and, on the right-hand side, all t dependent factors can be gathered together into one grand complicated, effective coupling.

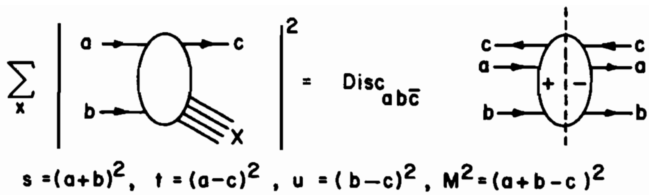


Fig. 1. The generalized optical theorem.

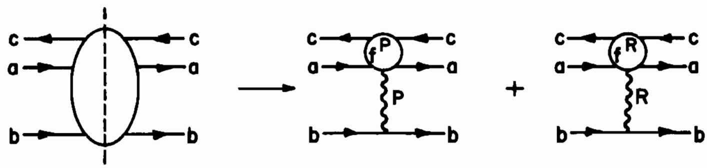


Fig. 2. High-energy behavior of the fragmentation $(a:c|b)$ is determined by singularities in $b\bar{b}$ channel.

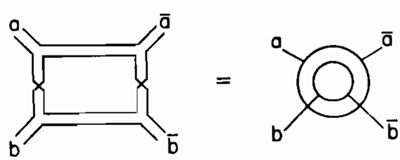


Fig. 3. The twisted loop model for the pomeron.

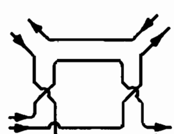
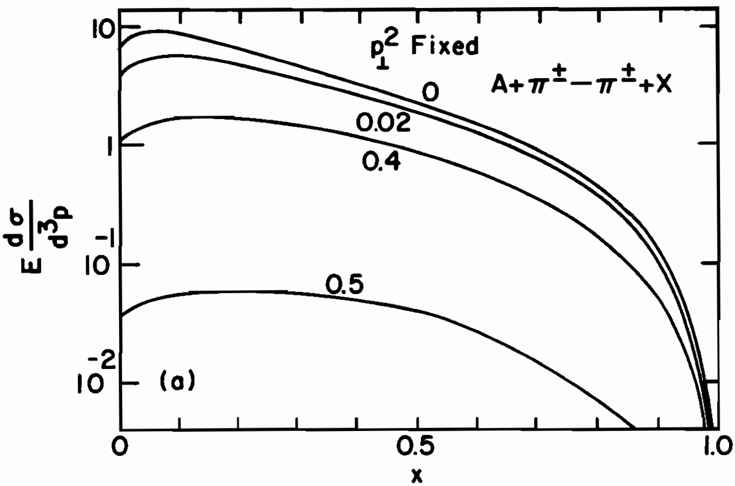
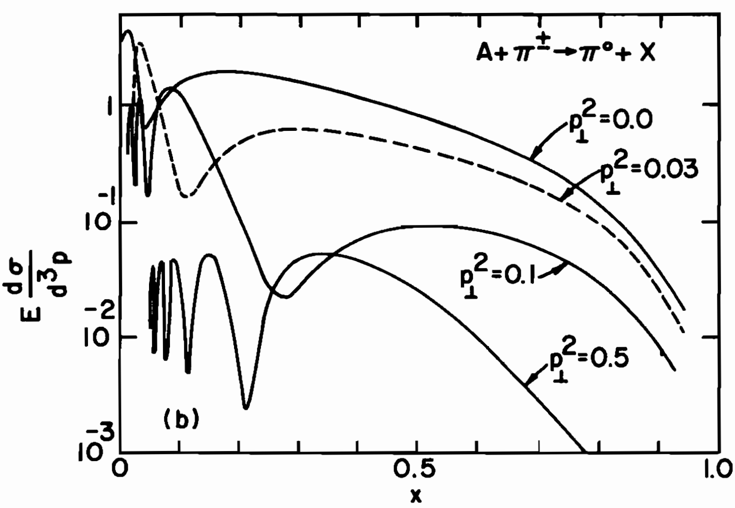


Fig. 4. A diagram contributing to meson baryon total cross sections (such as πp) which has only exotic states in the direct channel but a non-exotic Reggeon in the crossed channel.

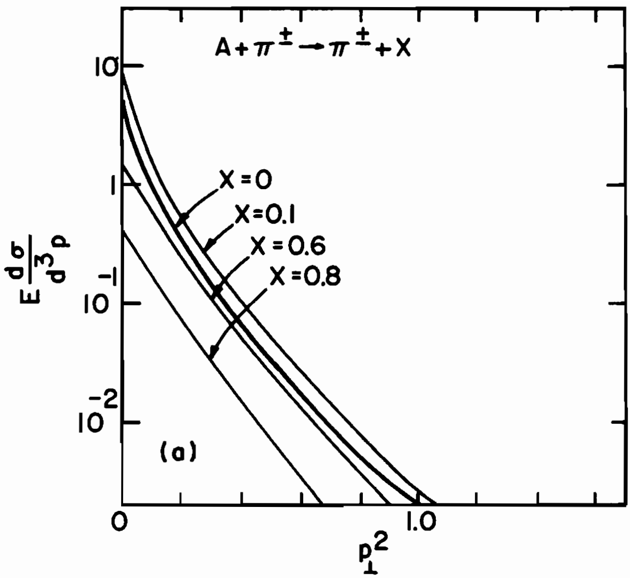


(a)

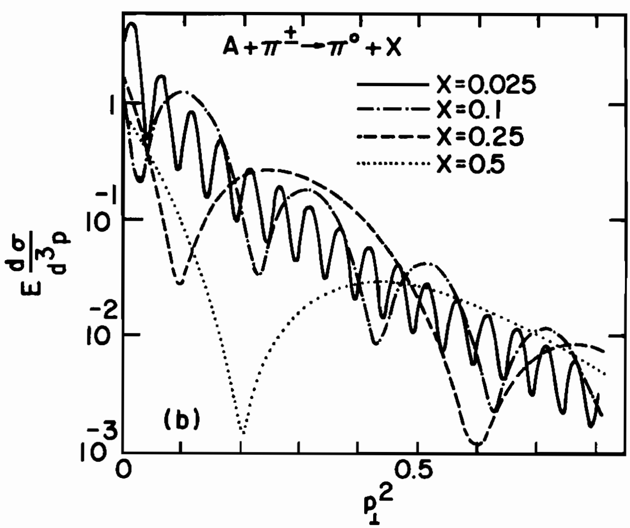


(b)

Fig. 5. Typical x distributions taken from Ref. 23. (a) $A + \pi^\pm \rightarrow \pi^\pm + X$; (b) $A + \pi^\pm \rightarrow \pi^0 + X$.



(a)



(b)

Fig. 6. Typical $p_\perp$ distributions taken from Ref. 23. (a) $A + \pi^\pm \to \pi^\pm + X$; (b) $A + \pi^\pm \to \pi^0 + X$.

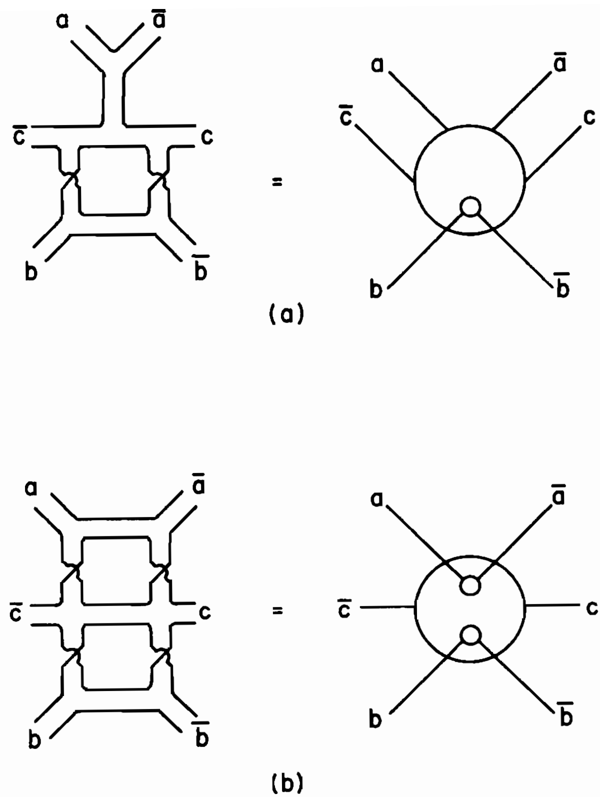


Fig. 7. Pomeron contributions to inclusive cross sections. (a) A Pomeron in $b\bar{b}$ only; (b) Pomerons in both $a\bar{a}$ and $b\bar{b}$.

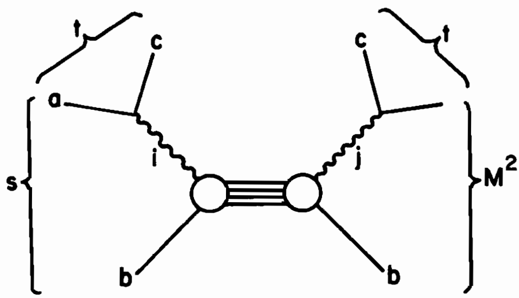


Fig. 8(a). Inclusive reactions determine Reggeon-particle total cross sections.

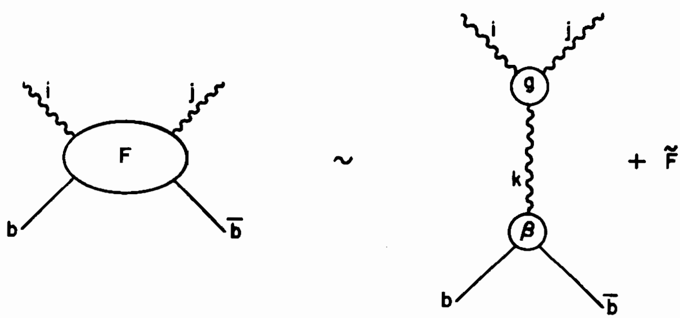


Fig. 8(b). Asymptotic behavior of the Reggeon-particle amplitude.