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# Hadron resonance gas with repulsive interactions

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**Abstract.** We study QCD equation of state and fluctuations of baryon number using the hadron resonance gas model with repulsive mean field. We show that the inclusion of repulsive mean field into the resonance gas model leads to better description of the lattice results. The repulsive mean field is particularly important for the higher order baryon number fluctuations.

## 1. Introduction

QCD thermodynamics at net zero baryon density below the chiral crossover can be described by hadron resonance gas (HRG) model. The idea behind this model is that interactions between hadrons can be accounted for by including resonances as additional free particles. Typically resonances are treated as zero width states. Equation of State obtained in lattice QCD agree well with HRG predictions (see e.g. Refs. [1, 2, 3, 4] for some recent works). The fluctuations and correlations of conserved charges, defined as derivatives of the pressure with respect to chemical potentials can also be studied in HRG model (see e.g. Ref. [5]). The HRG model can be justified using the relativistic virial expansion based on the S-matrix approach [6]. In most cases after summation over spin and isospin channels, the second virial coefficient is dominated by the resonance contribution, and thus the interacting gas of hadrons can be approximated as a gas of non-interacting hadrons and hadronic resonances [7]. The cancellation of non-resonant contributions is somewhat accidental [8] and does not happen for all thermodynamic quantities [9]. Furthermore, not all hadronic interactions are dominated by resonances. One example is the nucleon-nucleon interactions, which are non-resonant. Finally, close to the chiral transition one may worry about in-medium modifications of hadrons [10]. Lattice calculation indicate that negative parity baryon masses experience shift compared to their zero temperature values <sup>1</sup>. Therefore, it is important to check the HRG model by comparing it to recent lattice QCD calculations of different thermodynamic quantities.

For higher order fluctuations of conserved charges the agreement between the lattice and HRG is no longer good for temperatures close to the chiral transition. It has been argued that this is due to repulsive baryon-baryon interactions [14, 15, 16]. As the baryon density increases

<sup>1</sup> Here we note that extraction of hadron properties at non-zero temperature is a very difficult task, see e.g. discussions in Refs. [11, 12, 13].



the role of repulsive interactions will increase. Therefore in order to extend HRG to non-zero baryon density repulsive baryon-baryon interactions have to be included in the description. As a step toward this goal we will study in this contribution the trace anomaly and fluctuations of baryon number using HRG with repulsive mean field.

## 2. Nucleon gas with repulsive mean field and virial expansion

Using the virial expansion the pressure of the nucleon gas can be written as [17]

$$p(T, \mu) = p_0(T) \cosh(\beta\mu) + 2b_2(T)T \cosh(2\beta\mu), \quad \beta = 1/T. \quad (1)$$

Here

$$p_0(T) = \frac{4M^2T^2}{\pi^2} K_2(\beta M) \quad (2)$$

is the pressure of free nucleon gas at zero chemical potential, and the second virial coefficient can be written as

$$b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE \left( \frac{ME}{2} + M^2 \right) K_2 \left( 2\beta \sqrt{\frac{ME}{2} + M^2} \right) \frac{1}{4i} \text{Tr} \left[ S^\dagger \frac{dS}{dE} - \frac{dS^\dagger}{dE} S \right], \quad (3)$$

with  $S$  being the scattering S-matrix and  $E$  is the kinetic energy in the lab frame. Furthermore,  $M$  is the nucleon mass and  $K_2(x)$  is the Bessel function of second kind. Using the parameterization of the  $S$ -matrix in terms of the phase shifts and their numerical values the virial coefficient  $b_2(T)$  can be calculated, and it turns out that  $b_2(T)$  is negative [17]. For the comparison with the mean-field approach discussed below it is convenient to write the above expression for the pressure in the following form

$$p(T, \mu) = p_0(T) (\cosh(\beta\mu) + \bar{b}_2(T) K_2(\beta M) \cosh(2\beta\mu)), \quad (4)$$

with

$$\bar{b}_2(T) = \frac{2Tb_2(T)}{p_0(T)K_2(\beta M)}. \quad (5)$$

being the reduced virial coefficient.

In the mean-field approach the pressure can be written as [17]

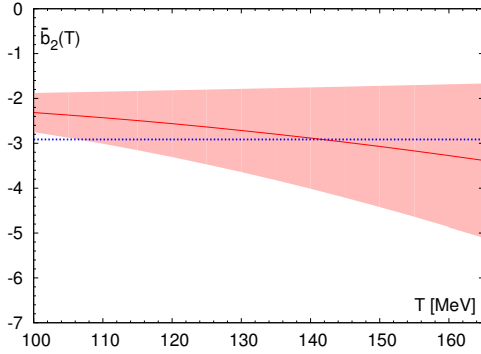
$$p(T, \mu) = T(n_b + \bar{n}_b) + \frac{K}{2}(n_b^2 + \bar{n}_b^2), \quad (6)$$

where  $n_b$  and  $\bar{n}_b$  are the densities of nucleons and anti-nucleons, respectively, defined by the following self-consistent relations:

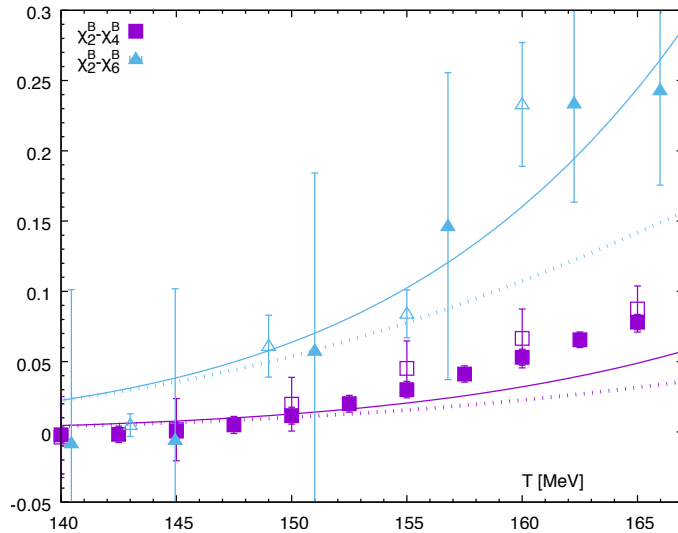
$$n_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}, \quad \bar{n}_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p + \mu + \bar{U})}, \quad E_p^2 = p^2 + M^2. \quad (7)$$

Here  $U = Kn_b$  and  $\bar{U} = K\bar{n}_b$  are the mean-field potentials for nucleons and anti-nucleons. The chemical potential corresponding to the net nucleon density is denoted by  $\mu$ . The above form of the pressure ensures thermodynamic consistency, i.e.  $\partial p / \partial \mu = n_b - \bar{n}_b$ . Since we are mostly interested in the region of not too high baryon densities one can expand the above expressions in  $\beta U = \beta Kn_b$  and  $\bar{U} = K\bar{n}_b$  and keep only the leading order terms in  $K$ . Then we get

$$p(T, \mu) = T(n_b^0 + \bar{n}_b^0) - \frac{K}{2} \left( (n_b^0)^2 + (\bar{n}_b^0)^2 \right), \quad (8)$$



**Figure 1.** The reduced second virial coefficient  $\bar{b}_2$  as function of temperature, and the mean-field parameter  $K$ .



**Figure 2.** The differences between fourth order and second order baryon number fluctuations (squares) as well as the the sixth and second order baryon number fluctuations (triangles). The dashed lines correspond to the exact mean-field calculations, while the solid line to the expanded mean field (see text). The filled symbols correspond to lattice results of Ref. [18]. The open symbols are the lattice data for  $\chi_4^B$  from Ref. [19] and  $\chi_6^B$  from Ref. [20], respectively.

with  $n_b^0$  ( $\bar{n}_b^0$ ) being the free nucleon(anti-nucleon) densities. The above equation can also be written as

$$p(T, \mu) = \frac{4T^2 M^2}{\pi^2} K_2(\beta M) \cosh(\beta \mu) - 4K \frac{T^2 M^4}{\pi^4} K_2^2(\beta M) \cosh(2\beta \mu). \quad (9)$$

Now we see that the virial expansion and the expanded mean-field approach give very similar results. From the comparison of these two results one can determine the value of  $K$  in a limited temperature range. Since  $\bar{b}_2(T)$  turns out to be negative,  $K > 0$ , i.e. the mean field is repulsive. The temperature dependence of  $\bar{b}_2$  is shown in Fig. 1 and it turns out to be relatively mild except maybe at the highest temperature. The value of  $\bar{b}_2$  is consistent with  $K = 250 \text{ MeV/fm}^3$ . The largest value allowed for  $K$  by the virial expansion is around  $K = 450 \text{ MeV/fm}^3$ . It is straightforward to generalize the mean-field approach described above to a multicomponent

system if one assumes that the repulsive mean-field is the same for all ground state baryons [17] and the baryon resonances are not affected by the mean field. Within this approach we calculate baryon number fluctuations defined as derivatives of the pressure with respect to baryon chemical potential [17]

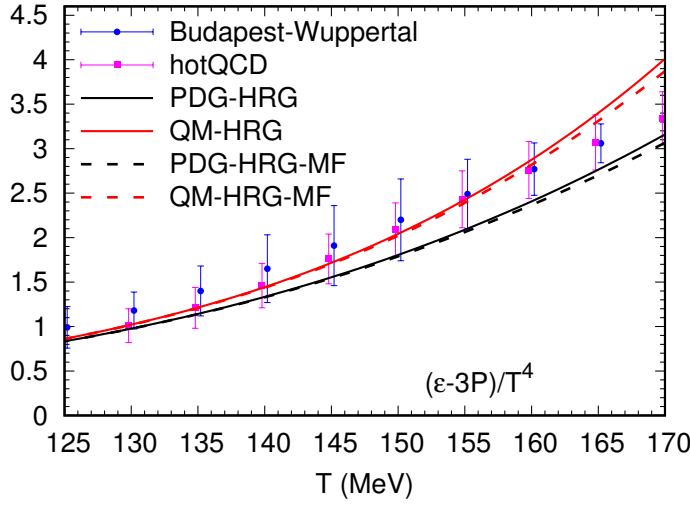
$$\chi_n^B = T^n \frac{\partial^n p / T^4}{\partial \mu_B^n}. \quad (10)$$

In Fig. 2 we show the differences of the baryon number fluctuations,  $\chi_4^B - \chi_2^B$  and  $\chi_6^B - \chi_2^B$  obtained in HRG model with the mean field and the value of  $K = 450 \text{ MeV}/\text{fm}^3$ . The model can qualitatively explain these differences as one can see from Fig. 2, however, at quantitative level it underpredicts the lattice data even for the largest allowed value of  $K$ . We note that at temperatures above 150 MeV the expanded mean-field expression (8) is no longer accurate and one should use the exact mean-field expressions with particle densities determined self-consistently [17]. The exact mean-field results are shown as dashed lines in the figure.

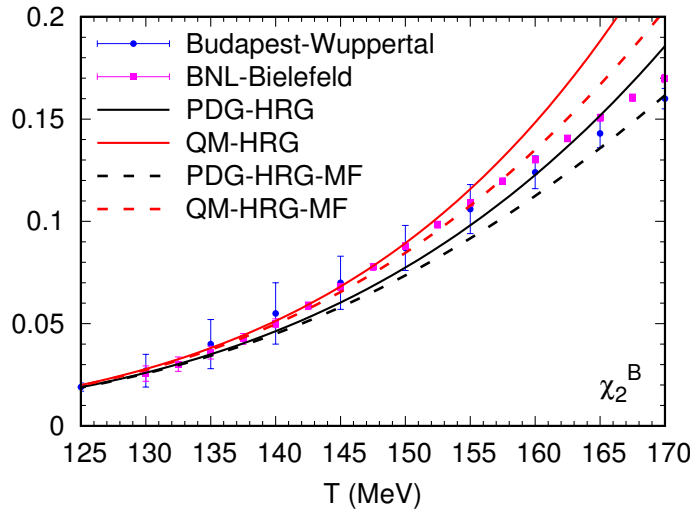
### 3. Comparison with lattice QCD

In this section we compare the calculations of HRG with repulsive mean field with lattice QCD results on the trace anomaly and fluctuations of baryon number up to sixth order. First we note that there are many baryon resonances not listed in Particle Data Group (PDG) but are predicted by quark models and lattice QCD. These are the so-called missing states. It was found that it is important to include these missing states in HRG description in order to describe the strangeness fluctuations and baryon-strangeness correlation [21]. Therefore, in our analysis we will include the missing baryons from the quark model calculations of Ref. [22, 23] and label the corresponding results as QM-HRG, whereas the HRG based on the PDG resonance list [24] is labeled PDG-HRG. In the previous analysis it was assumed that baryon resonances are not affected by the mean field [17]. This is not realistic because if the density of ground state baryons is reduced there will be also fewer baryon resonances in the system. Therefore, in the present analysis we included the effect of the mean field on the baryon resonances as well. Furthermore, we use a more realistic value for the mean-field parameter,  $K = 250 \text{ MeV}/\text{fm}^3$ . In Fig. 3 we show our calculations for the trace anomaly,  $\epsilon - 3P$  compared to the lattice results [2, 3] at zero baryon chemical potential,  $\mu_B = 0$ . We clearly see the effects of the missing states on the trace anomaly. These states lead to significant increase of the trace anomaly around temperatures of 150 MeV and higher compared to the HRG calculations with only PDG states (PDG-HRG). The HRG calculations with repulsive mean field are shown as dashed lines. The effects of the repulsive interactions turns out relatively small, because the contributions of baryons is small too. The situation will be different at sufficiently large  $\mu_B$ , when the contribution of mesons and baryons to the equation of state is comparable.

In Fig. 4 we show our results for second order baryon number fluctuations. Again we show results for QM-HRG and PDG-HRG with and without the effect of the repulsive mean field. First, we see the the inclusion of missing baryons is important also here. If all the missing baryons are included the HRG overshoots the lattice results. The repulsive mean field has an opposite effect and is comparable in size. Therefore, it restores the agreement with the lattice data. Higher order baryon number fluctuations are more sensitive to the effects of the repulsive interactions as shown in Fig. 5. The HRG-QM model again overpredicts the lattice results and lies significantly above the HRG-PDG result. The repulsive interactions reduce the HRG prediction and lead to reasonably good agreement with the lattice data. Note that the effects of the repulsive mean field are now larger than in the previous analysis shown in Fig. 2 even though we use smaller value of  $K$ . This is because the resonances are also affected by the repulsive mean field. We note that at temperatures above 155 MeV the HRG model does not do a good job in describing the lattice results even when repulsive meanfield is included.



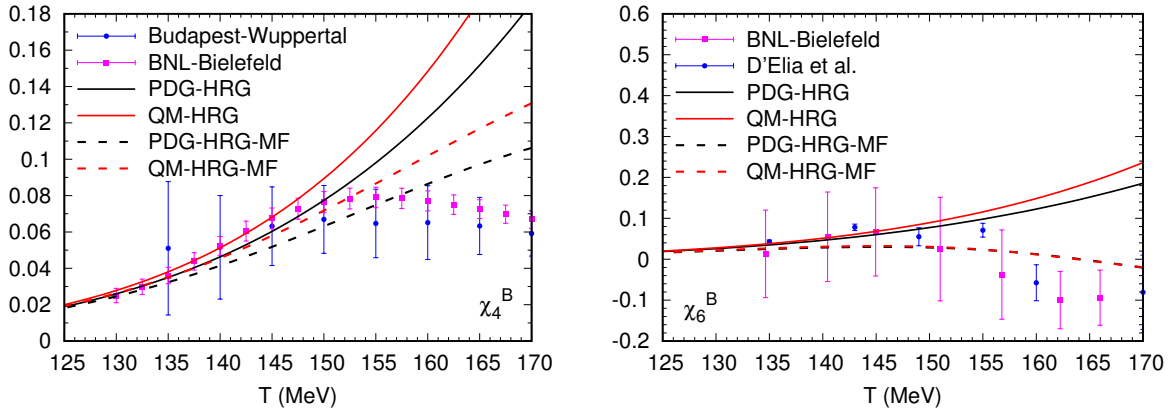
**Figure 3.** The trace anomaly in HRG model with the Particle Data Group (PDG-HRG) and Quark Model (QM-HRG) lists of resonances compared to the lattice results [2, 3]. The dashed lines correspond to HRG model with repulsive mean field.



**Figure 4.** The second order baryon number fluctuations in HRG model with the Particle Data Group (PDG-HRG) and Quark Model (QM-HRG) lists of resonances compared to the lattice results [18, 19]. The dashed lines correspond to HRG model with repulsive mean field.

#### 4. Conclusion

We studied the equation of state and fluctuations of baryon number within the HRG framework, where the effect of the repulsive baryon-baryon interactions are included using the mean-field approach. We also studied the effects of missing baryon resonances on these quantities. We have found that the missing states lead to significant increase of thermodynamic quantities. In the case of baryon number fluctuations missing states cause the HRG model to overshoot the lattice results for  $T \geq 150$  MeV. The repulsive mean field has the opposite effect. The effects of repulsive interactions are small for the trace anomaly but are significant for baryon number fluctuations, where they are needed to bring the HRG calculations in agreement with the lattice



**Figure 5.** The fourth order (left) and sixth order (right) baryon number fluctuations in HRG-QM and HRG-PDG models. The lattice results for  $\chi_4^B$  are from Refs. [18, 19], while the lattice results for  $\chi_6^B$  are from [18, 20]. The dashed lines correspond to HRG model with repulsive mean field.

results for temperatures  $T < 155$  MeV. This implies that when extending the HRG model to calculate equation of state at non-zero baryon density the repulsive interactions have to be taken into account.

### Acknowledgment

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