The BES equation and its strong coupling limit

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1 Introduction

The maximally supersymmetric Yang-Mills theory in four dimensions (" $\mathcal{N} = 4$ SYM") has been conjectured to be dual to IIB super string theory on AdS₅× S⁵ [1]. This "AdS/CFT correspondence" is a weak/strong-coupling duality, i.e. the strong coupling regime of the gauge theory is described by the string theory. The $\mathcal{N} = 4$ model is not realistic, but it is capable of furnishing parts of QCD expressions.

The $\mathcal{N} = 4$ theory is perturbatively finite in the sense that no renormalisation of its action is required. On the other hand, most gauge invariant composite operators are infinite, and renormalisation leads to corrections to their naive scaling dimensions ("anomalous dimensions"). This corresponds to the energy of the dual string states.

In the planar limit, the computation of the operator spectrum may be dealt with by spin chain methods [2]. Here we discuss derivative operators (or "twist operators")

$$\{s_1, s_2, s_3, \ldots\} = Tr((\mathcal{D}_z^{s_1}X)(\mathcal{D}_z^{s_2}X)(\mathcal{D}_z^{s_3}X)\ldots)$$

where X is a complex scalar field of the $\mathcal{N} = 4$ SYM theory with SU(N) gauge group and $\mathcal{D}_{\mu} = \partial_{\mu} + i g_{YM} A_{\mu}$. The operators are assumed to carry traceless symmetric Lorentz representations of spin $s = s_1 + s_2 + s_3 + \ldots$

The spin chain picture emerges most clearly in the study of two-point functions: In the large N limit only nearest neighbour interactions survive. One-loop graphs connect two sites in the first composite operator to two sites in the other, and (at least for a certain tensor component) the total spin is conserved. The one-loop anomalous dimension is related to the leading singularity in the Feynman diagrams.

If we view the derivatives as excitations or "magnons" moving on the chain of scalar fields, the coefficient of the one-loop divergence defines an "amplitude" or Hamiltonian for the transfer of derivatives between two sites [3]:

$$\mathcal{H}_{i}^{(0)}(\{s_{1}, s_{2}\} \to \{s_{1}, s_{2}\}) = h(s_{1}) + h(s_{2}), \qquad (1)$$
$$\mathcal{H}_{i}^{(0)}(\{s_{1}, s_{2}\} \to \{s_{1} - d, s_{2} + d\}) = -\frac{1}{|d|},$$

where h(s) is a harmonic number. One has to sum over all positions *i* in the chain. We recognize the Hamiltonian of the Heisenberg XXX chain with spin $-\frac{1}{2}$. The dynamics of the system is captured by the Bethe ansatz

$$\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right)^L = \prod_{j \neq k} \left(\frac{u_k - u_j - i}{u_k - u_j + i}\right), \quad j, k \in \{1, \dots, s\}, \qquad \prod_{k=1}^s \left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right) = 1.$$
(2)

Here s is the spin of the operator and L is the number of scalar fields, i.e. the length of the spin chain. There is one "Bethe root" u_k per magnon. For each s, L there will in general be several solutions, corresponding to various operators of the same spin and length. The energy of any given solution is

$$E = \sum_{k=1}^{s} \left(\frac{i}{u_k + \frac{i}{2}} - \frac{i}{u_k - \frac{i}{2}} \right) \,. \tag{3}$$

This reproduces the one-loop anomalous dimensions obtained by direct methods, e.g. diagonalisation and renormalisation of two-point functions.

Higher loop diagrams define a perturbation of the Hamiltonian, which in turn requires a deformation of the Bethe ansatz [4]:

$$u \pm \frac{i}{2} = x^{\pm} + \frac{g^2}{2x^{\pm}}, \qquad g = \frac{\sqrt{\lambda}}{4\pi}, \qquad \lambda = g_{YM}^2 N,$$
 (4)

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{j \neq k} \frac{x_k^- - x_j^+}{x_k^+ - x_j^-} \frac{1 - g^2/2x_k^+ x_j^-}{1 - g^2/2x_k^- x_j^+},$$
(5)

$$\prod_{k=1}^{s} \left(\frac{x_{k}^{+}}{x_{k}^{-}}\right) = 1, \qquad E(g) = \sum_{k=1}^{s} \left(\frac{i}{x_{k}^{+}} - \frac{i}{x_{k}^{-}}\right).$$
(6)

2 Large spin limit for length two operators

We present the discussion from [5] and [6].

At one-loop order, the length two case is numerically solvable to very high precision¹ because the Bethe roots u_k are the zeroes of certain Hahn polynomials. The roots are real and symmetrically distributed around zero. The outermost roots grow as $\max\{|u_k|\} \rightarrow s/2$. As the spin increases we can follow how the distribution of the roots along the interval [-s/2, s/2] converges to a smooth density function.

For an analytic derivation we start by taking the logarithm of the one-loop Bethe equations (2):

$$-i L \log\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right) = 2\pi n_k - i \sum_{j \neq k} \log\frac{u_k - u_j - i}{u_k - u_j + i}$$
(7)

At length two the mode numbers turn out to be $n_k = \pm 1$ for negative/positive roots. For L > 2 there is more than one state. However, the lowest state is "universal" in that the root distribution is again real and symmetric with $n = \epsilon(u)$. For large spin, we rescale $u \to s \bar{u}$, expand in 1/s, and take a continuum limit:

$$0 = 2\pi \epsilon(\bar{u}) - 2 \int_{-1/2}^{1/2} d\bar{u}' \, \frac{\bar{\rho}_0(\bar{u}')}{\bar{u} - \bar{u}'} \tag{8}$$

One may solve by an inverse Hilbert transform:

$$\bar{\rho}_{0}(\bar{u}) = \frac{1}{\pi} \log \frac{1 + \sqrt{1 - 4\,\bar{u}^{2}}}{1 - \sqrt{1 - 4\,\bar{u}^{2}}} = \frac{2}{\pi} \operatorname{arctanh}\left(\sqrt{1 - 4\,\bar{u}^{2}}\right) \tag{9}$$

The one-loop energy is

$$E_0 = \frac{1}{s} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\bar{u} \, \frac{\bar{\rho}_0(\bar{u})}{\bar{u}^2 + \frac{1}{4s^2}} = 4 \, \log(s) + \mathcal{O}(s^0) \,. \tag{10}$$

It is problematic to discuss the higher loop corrections to the density by means of the Hilbert transform because Taylor expanding the square root functions x^{\pm} leads to non-integrable singularities. In [5] the following approach was devised: We write the logarithm of the all-loops Bethe equations (5) as

$$2L \arctan(2u_k) + iL \log\left(\frac{1+g^2/2(x_k^-)^2}{1+g^2/2(x_k^+)^2}\right) = 2\pi \tilde{n}_k - \frac{1}{2}\sum_{j\neq k} \arctan\left(u_k - u_j\right) + 2i\sum_{j\neq k} \log\left(\frac{1-g^2/2x_k^+ x_j^-}{1-g^2/2x_k^- x_j^+}\right).$$
(11)

In this formulation the lowest state has mode numbers $\tilde{n}_k = k + \epsilon(k) (L-2)/2$. As $s \to \infty$ we introduce a smooth continuum variable $x = \frac{k}{s}$. The excitation density is $\rho(u) = \frac{dx}{du}$. We divide by s, replace the sums by integrals, and differentiate w.r.t. u. Note that there is no rescaling of u by 1/s. Last, we split

$$\rho(u) = \rho_0(u) - g^2 \frac{E_0}{s} \sigma(u) .$$
(12)

The final integral equation for the higher-loop density $\sigma(u)$ is

$$0 = 2\pi\sigma(u) - 2\int_{-\infty}^{\infty} du' \frac{\sigma(u')}{(u-u')^2 + 1} - \left(\frac{1}{2}\frac{d}{du}\right) \left[\frac{1}{x^+(u)} + \frac{1}{x^-(u)}\right] + 2i\int_{-\infty}^{\infty} du'\sigma(u')\frac{d}{du}\log\left(\frac{1 - g^2/2x^+(u)x^-(u')}{1 - g^2/2x^-(u)x^+(u')}\right)$$
(13)

at leading order in s. The potential arises from integrating out the one-loop density.

Note that the Bethe equations (5) were derived for the asymptotic regime of infinite spin chain length. For operators of finite length L the ansatz is expected to break down at g^{2L} . We are interested in length two, thus in the shortest possible chain. In order to justify applying the Bethe ansatz beyond one loop we make use of the aforementioned "universality" of the lowest lying state: At each order in g we tacitly take the lowest state of sufficient length instead of the length two state. Consistency can be seen from the L independence of the final equation.

Upon taking the Fourier transform equation (13) becomes

$$\hat{\sigma}(t) = \frac{t}{e^t - 1} \left[\frac{J_1(2gt)}{2gt} - 4g^2 \int_0^\infty dt' \, \hat{K}(2gt, 2gt') \, \hat{\sigma}(t') \right] \tag{14}$$

with the non-singular kernel

$$\hat{K}(t, t') = \frac{J_1(t) J_0(t') - J_0(t) J_1(t')}{t - t'}.$$

At small g the equation can be solved iteratively. On substituting the resulting density into the Fourier transformed energy formula we find

$$f(g) = \frac{E(g)}{\log(s)} = 8 g^2 - 16 \zeta(2) g^4 + \left(4 \zeta(2)^2 + 12 \zeta(4)\right) 8 g^6 - \left(4 \zeta(2)^3 + 24 \zeta(2) \zeta(4) - 4 \zeta(3)^2 + 50 \zeta(6)\right) 16 g^8 + \dots$$
(15)

or alternatively:

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - \left(\frac{73}{630}\pi^6 - 4\zeta(3)^2\right) 16g^8 + \dots$$
(16)

The scaling function f(g) obeys a principle of uniform transcendentality: In each term in (15) the arguments of the ζ -functions add up to the power of the coupling constant minus two. Further, odd ζ -values only occur in pairs.

Up to two loops the anomalous dimensions of the length two operators have been calculated by direct means. They are given by harmonic sums organised by the very same transcendentality principle [7]. It was observed at two loops that the $\mathcal{N} = 4$ result is identical to the highest transcendentality part of the corresponding anomalous dimension in QCD. On postulating this pattern at the next order [8] the three-loop values in $\mathcal{N} = 4$ became available from an impressive QCD calculation [9]. The large spin limit of these results agrees with (16) through three-loop order.

The higher-loop Bethe equations (5) may in fact receive corrections:

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{j=1, j \neq k}^S \frac{x_k^- - x_j^+}{x_k^+ - x_j^-} \frac{1 - g^2/x_k^+ x_j^-}{1 - g^2/x_k^- x_j^+} \exp\left(2i\theta(u_k, u_j)\right),\tag{17}$$

where we included a dressing phase [10-12]

$$\theta(u_k, u_j) = \sum_{r \ge 2, \nu \ge 0} \beta_{r, r+1+2\nu}(g) \left(q_r(u_k) q_{r+1+2\nu}(u_j) - q_r(u_j) q_{r+1+2\nu}(u_k) \right) \,. \tag{18}$$

The q_r are the higher conserved charges of the spin chain model. In Fourier space they are given by higher Bessel functions so that the dressing phase can be incorporated into our iteration scheme at very little expense. At weak coupling

$$\beta_{r,r+1+2\nu}(g) = \sum_{\mu=\nu}^{\infty} g^{2r+2\nu+2\mu} \beta_{r,r+1+2\nu}^{(r+\nu+\mu)} .$$
(19)

The dressing phase thus contains infinitely many unknown constants that have to be fixed by explicit higher loop calculations, e.g. at three loops consistency with [8] yields the requirement $\beta_{2,3}^{(2)} = 0$.

To make progress the AdS/CFT duality may be invoked: The same Bethe ansatz correctly describes the energy spectra of certain rotating strings if the dressing phase is assumed to have an expansion in descending powers of g [11, 13]. In this situation the phase is subject to a crossing equation [14] which was solved in [15]. Crossing fixes only one half of the coefficients of the Taylor expansion of the phase in 1/g, but the authors of [15] observed that there is a natural guess for the remaining constants.

In [6] we derived the weak coupling expansion of this "natural" dressing phase and inserted it into the calculation of the scaling function. We found

$$\begin{split} &\beta_{2,3}^{(3)} = +4\,\zeta(3), \\ &\beta_{2,3}^{(4)} = -40\,\zeta(5), \\ &\beta_{2,3}^{(5)} = +240\,\zeta(7), \quad \beta_{3,4}^{(5)} = +24\,\zeta(5), \quad \beta_{2,5}^{(5)} = -8\,\zeta(5), \\ &\beta_{2,3}^{(6)} = -4704\,\zeta(9), \quad \beta_{3,4}^{(6)} = -420\,\zeta(7), \quad \beta_{2,5}^{(6)} = +168\,\zeta(7), \ \ldots \end{split}$$

We could show that the new scaling function $f_+(g)$ is obtained from f(g) (trivial dressing phase) by multiplying all odd zeta values by the imaginary unit *i*. In particular, the term $64 \zeta(3)^2 g^8$ in (16) picks up a negative sign.

In a parallel effort the g^8 term in the scaling function was computed from the four-loop four-point MHV gluon scattering amplitude [16]. The result is

$$f_{MHV}(g) = \dots - (3.0192 \pm 0.0054) \times 10^{-6} \lambda^4 + \dots$$
 (20)

Our prediction

$$f_{+}(g) = \dots - 16 \left(\frac{73}{630} \pi^{6} + 4\zeta(3)^{2}\right) g^{8} + \dots \approx \dots - 3.01502 \times 10^{-6} \lambda^{4} + \dots$$
(21)

lies within the error bar. Subsequently, the error margin of the MHV calculation has been significantly improved so that full agreement is almost certain, and the value of the lowest non vanishing coefficient of the dressing factor has been proved by constructing the four-loop Hamiltonian in a related sector [17].

3 Strong Coupling

Let us define

$$\hat{K}_{0}(t,t') = \frac{t J_{1}(t) J_{0}(t') - t' J_{0}(t) J_{1}(t')}{(t+t')(t-t')} = 2 \sum_{1}^{\infty} (2n-1) \frac{J_{2n-1}(t)}{t} \frac{J_{2n-1}(t')}{t'}, \quad (22)$$

$$\hat{K}_{1}(t,t') = \frac{t' J_{1}(t) J_{0}(t') - t J_{0}(t) J_{1}(t')}{(t+t')(t-t')} = 2 \sum_{1}^{\infty} (2n) \frac{J_{2n}(t)}{t} \frac{J_{2n}(t')}{t'}, \quad (23)$$

i.e. the even and odd parts of the kernel (15) under sign reversal of its arguments. In [6] it was shown that the dressing phase can be written as an additional kernel

$$\hat{K}_d(t,t') = 4g^2 \int_0^\infty dt'' \,\hat{K}_1(t,2gt'') \frac{t''}{e^{t''}-1} \,\hat{K}_0(2gt'',t') \,, \tag{24}$$

which is to be added to $\hat{K}(t, t')$ in (14).

Therefore both integration kernels come as expansions in the test functions $f_n(t) = J_n(t)/t$. This observation was used in [18] to devise a numerical approach powerful enough to determine the scaling function with very good precision up to $g \approx 20$. It monotonously increases with g; there is a transition to a linear regime at $g \approx 1$ so that one may extrapolate to strong coupling. The $g \to \infty$ limit of the scaling function could be shown to agree with excellent precision with the energy of the folded spinning string [19], as required by the AdS/CFT duality.

To be more precise, the authors of [18] write the root density as an expansion in the test functions $f_n(t)$, which yields an (infinite) matrix equation for a set of coefficients s_n . Numerical analysis becomes possible upon truncation of the rank. Following these ideas the leading root density at strong coupling was first analytically obtained in [20]. In the latter paper the matrix equation was separated into the parts with odd and even open index, respectively. The even part can be simplified employing the other equation.

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Here we point out that the two matrix equations can be re-formulated as integral equations by using (22), (23) from the right to the left. Let $\hat{\sigma} = \hat{\sigma}_o + \hat{\sigma}_e$ with

$$\hat{\sigma}_{o}(t) = \frac{t}{e^{t} - 1} \sum_{1}^{\infty} s_{2n-1} \frac{J_{2n-1}(2gt)}{2gt}, \qquad \hat{\sigma}_{e}(t) = \frac{t}{e^{t} - 1} \sum_{1}^{\infty} s_{2n} \frac{J_{2n}(2gt)}{2gt}.$$
 (25)

In terms of the split root density we find the two coupled integral equations

$$\hat{\sigma}_{o}(t) = \frac{t}{e^{t} - 1} \left[\frac{J_{1}(2gt)}{2gt} - 4g^{2} \int_{0}^{\infty} dt' \, \hat{K}_{0}(2gt, 2gt') \, (\hat{\sigma}_{e} + \hat{\sigma}_{o})(t') \right] \,, \tag{26}$$

$$\hat{\sigma}_{c}(t) = \frac{t}{e^{t} - 1} \left[-4g^{2} \int_{0}^{\infty} dt' \, \hat{K}_{1}(2gt, 2gt') \, (\hat{\sigma}_{e} - \hat{\sigma}_{o})(t') \right] \,. \tag{27}$$

To address the strong coupling problem we use the inverse Fourier transform to replace the rapidly oscillating Bessel functions by square roots x^{\pm} (c.f. [5]). It is convenient to rescale $u \rightarrow u/\epsilon$, $\epsilon = 1/(2g)$, and further to rescale $\sigma_{o,e}$ in order to avoid an explicit ϵ^2 on the potential term in the first equation. The branch cut in the functions x^{\pm} makes it necessary to distinguish the regions |u| < 1 and |u| > 1, and it invalidates a straightforward Taylor expansion in ϵ in a region around |u| = 1. Nevertheless, a leading order analysis remains valid. Equation (26) implies

$$(\sigma_e^{\ell} + \sigma_o^{\ell})(u) = \frac{1}{\pi}, \qquad |u| < 1,$$
 (28)

but unfortunately no other constraint. Next, to leading order the inverse Fourier transform of the test functions $f_{2n-1}(t) = J_{2n-1}(t)/t$ vanishes outside the unit interval. Consequently,

$$\sigma_o^{\ell}(u) = 0, \qquad |u| > 1.$$
 (29)

These two facts can be used to simplify the Fourier transform of (27). We find

$$\left(\int_{-\infty}^{-1} + \int_{1}^{\infty}\right) du' \left(u\sqrt{1 - \frac{1}{u^2}} + u'\sqrt{1 - \frac{1}{(u')^2}}\right) \frac{1}{u - u'} \sigma_e^{\ell}(u')$$
(30)
$$-\frac{2}{\pi} u\sqrt{1 - \frac{1}{u^2}} \operatorname{arctanh}\left(\frac{1}{u}\right) - \int_{-\infty}^{\infty} du' \left(\sigma_e^{\ell} - \sigma_o^{\ell}\right)(u'), \quad |u| > 1.$$

(Note that the second term on the r.h.s. is a constant.) On substituting
$$u = \operatorname{coth}(x)$$
, $u' = \operatorname{coth}(y)$ this equation may be solved by one last Fourier transform in the new variables.
Transforming back we obtain

$$\sigma_e^{\ell}(u) = \frac{1}{\pi} \left(1 - \frac{1}{2} \left(\frac{u+1}{u-1} \right)^{\frac{1}{4}} - \frac{1}{2} \left(\frac{u-1}{u+1} \right)^{\frac{1}{4}} \right), \qquad |u| > 1.$$
(31)

Hence we reproduce the strong coupling solution of [20], which yields the correct leading asymptotics for the scaling function.

Very recently, our system of two coupled equations (26), (27) has been used to derive the strong coupling expansion of $f_+(g)$ to arbitrary order in 1/g [21].

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4 Conclusions

The anomalous dimension of the length two twist operators scales logarithmically with the total spin s as the number of derivatives becomes large. The coefficient of $\log(s)$ is the "scaling function" f(g).

At strong-coupling (string theory) the dressing phase in the Bethe ansatz had been conjectured on grounds of calculational data paired with crossing symmetry. We have presented the weak coupling expansion of this phase factor and discussed its effect on the scaling function. The four-loop term of the final function $f_+(g)$ agrees with a field theory calculation based on unitarity methods. Our result explains the string theory/field theory discrepancies within the AdS_5/CFT_4 duality as an order of limits problem, which is resolved by correctly incorporating the dressing phase.

The scaling function $f_+(g)$ is a first quantity in four-dimensional QFT that can be controlled at any value of g. Although it was computed in the $\mathcal{N} = 4$ SYM model it gives the highest transcendentality part of the corresponding expression in QCD.

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