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Review of impedance issues for B-factory*

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Abstract

Review of recent analytic and experimental results on impedance relevant for the performance of the B-factories are overviewed.

1 Introduction

Impedance budget is one of the main characteristics of the high-current storage rings such as B-factories affecting the beam stability and the thermal conditions of the vacuum chamber. PEP-II luminosity exceeds now the design by a factor of three. That became possible, in particular, due to careful design of the vacuum chamber. Nevertheless, several weak components were identified and cured. Today, however, there are plans to upgrade the machine to luminosity $10^{36}\ cm^{-2}s^{-1}$ with beam currents up to 6×18 A. That is good time to summarize what we learned from the operational experience and recent progress in theory and simulations since the first impedance budget was estimated [1].

The main contributors to the impedance responsible for the multi-bunch instabilities, the RF cavities and the resistive wall impedance, are well understood and can be reliably defined. Here, we concentrate on small vacuum components in the ring such as bellows, tapers, collimators, masks, flanges, BPMs, valves, screens, pumping slots, etc. They define the broad-band impedance driving the single-bunch instabilities and are responsible for heating of the vacuum chamber. The power deposited to the higher order modes (HOMs) is

$$P = (\frac{eN_bc}{s_b})^2 \sum_p Z_l(pn_b\omega_0)e^{-p^2(\frac{2\pi\sigma_b}{s_b})^2}, \qquad (1)$$

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where ω_0 is the revolution frequency, n_b is number of bunches, N_b, s_b, and σ_b are the bunch population, the bunch spacing and the rms bunch length, respectively. Approximately, the summation can be replaced by integration over frequency giving result in terms the loss factor κ,

$$P = \frac{Z_0 I_{beam}^2 s_b}{4\pi} \kappa, \tag{2}$$

where $Z_0 = 120\pi$ Ohm. Here the bunch spacing s_b is taken in cm, and κ in $1/cm$, $V/pC = 1.11\,1/cm$.For 3 A beam current and the total $\kappa \simeq 15$ V/pC, the power is 0.5 MW for PEP-II $s_b = 2\lambda_{RF} = 126$ cm. This number should be compared with the power limit $P_{bell} \simeq 100$ W (i.e. 0.5 W/cm^2) accepted for a bellows.

Although the loss factor of each vacuum component is small, the number of such components can be as large as several hundreds. The electrical stability of such components may be even more important than heating or beam stability. It has to be emphasized that the performance of the whole machine may depend on a single small component. The luminosity of the PEP-II B-factory was limited for a while by excessive heating of a single bellows near the interaction region (IR).

In this review we summarized results of recent studies of small vacuum components. We concentrate on issues which are important in the design of the vacuum system as the operational experience of the PEP-II indicates and, still, sparely described in literature. Hopefully, that will be useful for the planning upgrades of the machines such as PEP-II B-factory at SLAC and BEPC in Beijing.

2 General comments

Although some simple estimates can be done sometimes quite easily, accurate calculations of impedance is difficult and time consuming. Additional problem arises due to possible cross-talk between different components separated sometimes by tens of meters.

Three basic ideas are used in theoretical modelling of the impedance.

At low frequencies, where the HOM wave length is large compared with the dimension of the obstacle (radius of a hole, width of a groove, etc) impedance is always inductive. The inductance can be defined from Maxwell equations in the limit $\omega -> 0$ and components are characterized by the polarizabilities which define the induced electric and magnetic dipoles.

At high frequencies, where the rms bunch length is small compared to the dimension of the obstacle, the generation of the HOMs is essentially the diffraction of the field of the beam.

For smooth variations of the beam pipe radius (tapers), the constructive approach is perturbation theory, where tangential component of the HOM on the perfectly conductive wall is taken equal (with the opposite sign) to the tangential component of the field of the beam. That, basically, defines the surface impedance of the tilted wall.

2.1 Symmetry of the beam-pipe

In the cylindrically symmetric structures there are separate TE and TM modes and only the latter is coupled to the beam. The transverse dipole force in this case is proportional to the offset of the leading bunch and is independent of the offset of the test particle. For an arbitrary structure, the longitudinal impedance depends both on the offset of the leading (x_l) and the trailing (x_t) particles. For small offsets $x << b$, the longitudinal impedance can be expanded in Taylor series:

$$Z_l(\omega, x_l, x_t) = Z_1(\omega) + (x_l + x_t)Z_2(\omega) + (x_l^2 + x_t^2)Z_3(\omega) + x_l x_t Z_4(\omega) + ... \qquad (3)$$

Here we used the symmetry with respect to exchange in x_t and x_l and have not written similar terms depending on the offsets in y-direction. For structures with additional symmetry, some terms may vanish. For example, there are no linear terms in x if the structure is mirror symmetric with respect $x- > -x$.

The Panofsky-Wenzel theorem gives the Fourier transform of the transverse wake $\tilde{W}_t$

$$\begin{aligned} \tilde{W}_x(\omega, x_l, x_t) &= \frac{1}{k}\frac{\partial Z_l}{\partial x_t}, \\ W_x(\omega, x_l, x_t) &= \frac{1}{k}\,[Z_2 + 2x_t Z_3 + x_l Z_4 + ..]. \end{aligned} \qquad (4)$$

The terms proportional to Z_2 generate an orbit distortion, Z_3 give the tune shift, and Z_4 define the beam stability.

To define coefficients, we represent the impedance as a sum of resonance terms,

$$Z_l(\omega, x_l, x_t) = \sum_n R_n(\omega) V_n^*(\omega, x_l) V_n(\omega, x_t), \qquad (5)$$

where the voltage V_n of a mode is, usually, calculated numerically integrating the field excited in a structure along the trajectory with the same offset for both particles,

$$V_n(\omega, x) = \int dz E_n^{\omega}(x, z) e^{i\omega z/c}. \qquad (6)$$

Expanding $V_n(x) = a + bx + cx^2$, it is easy to see that four parameters

$$Z_1 = R_n a, \quad Z_2 = R_n ab, \quad Z_3 = R_n ac, \quad Z_4 = R_n b \qquad (7)$$

are given by three coefficients a, b, c which can be defined from $V_n(x)$ calculated for three different offsets. That allows the full description of the asymmetric structures including dipole and quadrupole modes.

Several issues have to be addressed for an asymmetric structures. First, the quadrupole wakes affect focusing and change the tune dependence on current dQ/dI, see Figs. (1),

(2). This issue is discussed in more details later. Second, the modes in asymmetric structures are usually the hybrid HOMs and have both TM and TE components. Third, such structures may lead to mode conversion of the wakes generated somewhere else generating TE modes which can penetrate screens and cause local heating problems.

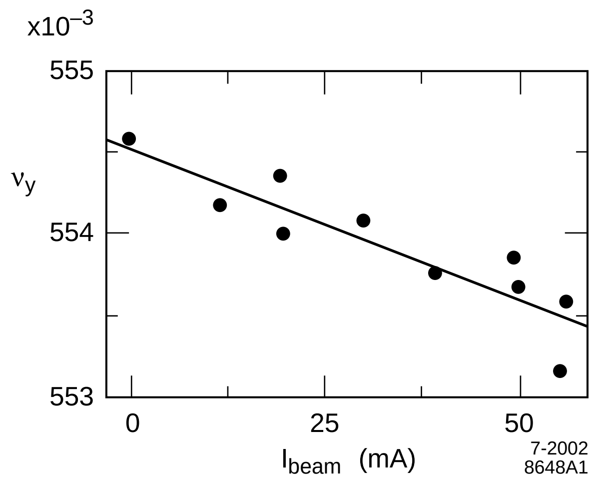

Figure 1: Variation of the tune Q_y with the beam current due to asymmetry of the beam pipe.

It is worth noting that the mode conversion is roughly proportional to the number of asymmetric components within the absorbtion length of a mode (of the order of 200 m) and can become large even if the conversion coefficient of each component is only few percent.

2.2 Cross-talk of components

An HOM with the frequency ω above the beam pipe cut off propagates along the beam pipe and can produce heating far away from the component where it was generated. Absorbtion length of the mode depends on the type of the mode but is of the order of $l \simeq 2Q\lambda$, where $\lambda = 2\pi c/\omega$ and Q-factor is $Q \simeq b/(2\delta)$. At typical frequencies $\omega/c \simeq 1/\sigma_B$ for the bunch with the rms length $\sigma_B = 1$ cm and the beam pipe radius in the straight sections $b = 4.5$ cm, the skin depth $\delta \simeq 1$ μm, and $l \simeq 250$ m.

The wake field is, usually, generated at the wall and starts interacting with the parent bunch some distance downstream. The catch-up distance $l \simeq (b-a)^2/\sigma$. For small σ, that distance can be longer than the distance between impedance generating components. An example of that gives the periodic array of cavities as in a linac structure. The real

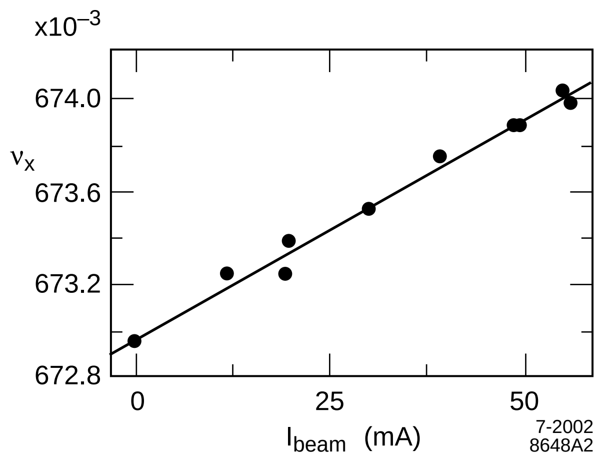

Figure 2: Variation of the tune Q_x with the beam current due to asymmetry of the beam pipe. The sign of the slope is opposite to the slope in Fig. 1

part of the impedance of such structure may roll off as $ReZ_l \propto (1/\omega)^{3/2}$ and $ImZ_l \propto 1/\omega$ while the impedance of a single pill-box cavity rolls off as $Z_l \propto (1+i)/\sqrt{\omega}$.

The numerical analysis of the impedance of a long mask made as two tapers (up and down) separated by a flat section shows that the cross-talk between tapers is negligible for the flat sections sufficiently long compared to the catch-up distance defined by the bunch length. In this case, the inductive parts of both tapers are additive. The real part though is defined by the cavity formed by two tapers (not the case for collimators).

The cross-talk of well separated vacuum components was observed in PEP-II and is discussed below.

2.3 Trapped modes

Trapped modes closed to the cut-off of the beam pipe can cause the local heating. Such modes can be expected where the pipe bulges from the radius $r = a$ to $r = b > a$, within $|z| < l$. The localization of the mode $E \propto e^{-q|z|}$ depends on the volume V_b of the bulge

$$q = \zeta \frac{k_c^2 V_b}{2S}, \tag{8}$$

where $k_c = \omega_c/c_0$, ω_c is the cut-off frequency, S is the beam pipe cross-section, $\zeta = |H(r = b)|^2 / < |H|^2 >$. For a round pipe, $\zeta = 1$.

The frequency shift $\delta\omega$ is given by the difference $V_b[H^2 - E^2]$ of the energy stored in magnetic and electric fields within the additional volume V_b of the bulge. To generate a trapped mode, the frequency shift

$$\delta\omega = \frac{q^2 c_0^2}{2\omega_c} \tag{9}$$

has to be larger than the width of the resonance $\omega_c/Q \simeq \omega_c\delta/b$ where δ is the skin depth at frequency ω_c.

The shunt impedance R_s of the trapped mode is given by the resistivity of the wall. For a round beam with radius b,

$$R_s = \frac{Z_0}{2\pi}\frac{\nu}{J_1^2(\nu)}\frac{b}{\delta}(\frac{\alpha_m}{\pi b^3})^3. \tag{10}$$

The estimate for 40 slots with $l = 9$ cm and $w = 3.26$ mm in the beam pipe $b = 4.5$ cm gives the shift $\Delta\omega/\omega_c = 3.5\,10^{-4}$ below the cutoff frequency $f_c = 2.58$ GHz. That is larger than the resistive wall width $\Delta\omega/\omega_c \simeq \delta/2b = 0.36\,10^{-4}$. The latter is of the order of the revolution frequency $\omega_{rev}/\omega_s \simeq 0.52\,10^{-4}$, the frequency separation of the of couple bunch modes. Hence, one of the coupled bunch mode can be affected by such a trapped mode.

The trapped modes potentially dangerous and should be avoided. As an example we consider the recess of the BPM button. Such a recess is desirable to shield the button from the SR but may be limited by the possibility to have a trapped mode. Other examples are discussed below.

The bulging beam pipe is not the only way to generate the trapped mode. The trapped modes were found also both in gasket and momentum collimators [2]. We describe this result below considering trapped modes in collimators.

2.4 Bunch spacing resonances

The resonance can occur if the HOM frequency is close to the resonance frequency ω_r, $\omega_r s_b/c_0 = 2\pi n$, where n is integer and s_b is bunch spacing. In this case, the wake fields generated by individual bunches within the coherence length $L \simeq 2Q_L(c/\omega_r)$ in the train are build up.

The energy loss in this case is

$$\Delta U = N_b^2 e^2 \kappa \frac{1}{2\pi n Q_L}\frac{1}{(\Delta\omega/\omega_r)^2 + 1/(2Q_L)^2}, \tag{11}$$

where Q_L is loaded Q-factor of the HOM, and $\Delta\omega = \omega_{HOM} - \omega_r$ is the detuning of the mode from the resonance. The width of the resonance $\Delta\omega/\omega_r = 1/(2Q_L)$. The enhancement takes place provided $\Delta\omega << c/(Q_L s_B)$. The enhancement factor

$$D_n = \frac{4Q_L}{\omega_r s_b}. \tag{12}$$

The resonances are separated by $\Delta f = c/s_B$ (250 MHz for by-two fill). The width of the resonance usually is larger than the revolution frequency, but additional heating due to coupled-bunch (CB) motion depends on the amplitude of the CB oscillations.

The paper [4] maybe gives an example of such a resonance. The mode $f = 2.384$ GHz has the largest amplitude in the spectrum. The HOM excited in the 5.5 m long pumping section penetrates to the LER ante-chamber and then to the TSP. It was measured with a short wire antenna at the TSP feedthrough outside the pumping section. With the bunch spacing $s_b = 126$ cm, the mode corresponds to the bunch spacing resonance $n = 10$.

It is worth noting that heating may produce the detuning mechanism from the resonance. The detuning is due to the change of the dimensions of the structure with temperature T. Typically,

$$\frac{\Delta f}{f} \simeq \frac{\Delta L}{l}, \quad \frac{\Delta L}{L} \simeq 10^{-5} \Delta T. \tag{13}$$

For the beam pipe with the length $L \simeq 5$ m attached to a bellows with the length $l = 10$ cm, the heating by $10\,K^o$ provides detuning for the modes with $Q_L > 200$.

The bunch spacing resonances enhance heating of the vacuum components. They affect the beam stability only if the width of the resonances is large enough to overlap with the sidebands frequencies.

2.5 Excitation of the cavity behind the slots and fingers

In many cases there is a cavity-type structure linked with the beam pipe by a slots or holes. That is true for bellows, screens separating distributed ion pumps (DIP) from the beam, ante-chamber in LER, screens of the vacuum ports, etc, see an example in Fig. (3).

Radiation through the slots at low frequencies can be described as radiation of an electric $d = \alpha_e E_r$ and magnetic $m = \alpha_m H_\phi$ dipoles induced by the field E, H of a bunch [5]. The energy U radiated by a Gaussian bunch through a longitudinal slot with the length l and width w in a beam pipe with radius b is [6]

$$U = \frac{N_b^2 e^2}{3\pi (2\pi b)^2} (\alpha_e^2 + \alpha_m^2) \int \frac{d\omega}{c} (\frac{\omega}{c})^4 e^{-(\omega \sigma_b / c)^2}. \tag{14}$$

For a slot in a wall with the thickness $d > w$, the radiated energy is reduced by a factor $e^{-2\pi d/w}$. Polarizabilities $\alpha_{e,m}$ for a long slot are given in Appendix.

For the TE modes, the ratio of the radiated power P to the incident power of the mode is [7]

$$\frac{P}{P_{in}} = \frac{2}{3\pi} (\frac{\omega}{c})^3 (\frac{\omega_c^2 \alpha_m^2}{qabc^2}), \tag{15}$$

where ω_c is the cut-off frequency in a rectangular beam pipe $a \times b$, and q is propagating constant.

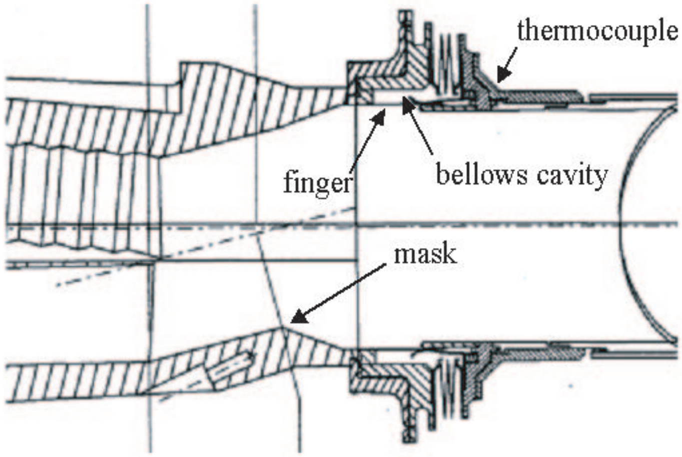

Figure 3: Shielded vertex bellows cavity. It can be heated by the TE modes penetrating through the fingers.

For TE modes, $\alpha_m \propto l^3$. The main contribution is given by the modes with frequencies $\omega/c \simeq 1/\sigma_b$. For $b^2 << a^2$, we get

$$\frac{P}{P_{in}} = \frac{2}{3\pi}(\frac{\pi}{24})^2 \frac{\sigma_b}{a \ln^2(4l/w)} (\frac{l^2}{\sigma_b b})^3. \tag{16}$$

The ratio is $P/P_{in} \simeq 1$ with $l = 10$ cm, $a = 10$ cm, $b = 5$ cm, $w = 3$ mm, and $\sigma_b = 1$ cm.

In the case of the high-frequency TE HOM, $kb >> 1$, $kl >> 1$ where $k = \omega/c$, and polarization perpendicular to the slot, the fraction of the power flowing through the slot is proportional to the surface area of the slot wl and the ratio $k_\perp/k$. The fraction of the power flow through the slot in a cylindrical beam pipe is given by Stupakov [8]. For example, for the TE_{11} mode, the fraction of the power penetrating through n_{sl} slots between bellows fingers with the width and length of a slot w and l, respectively, is

$$\frac{P}{P_{in}} = n_{sl} \frac{wl\nu}{\pi q b^3} \tau. \tag{17}$$

Here $\nu \simeq 1.84$ is the root of the Bessel function $dJ_1(\nu)/d\nu = 0$, q is propagating constant of the mode, $k = \omega/c = \sqrt{q^2 + (\nu/b)^2}$, and

$$\tau = \frac{\pi^2}{2k}[(\ln \frac{kw \sin\alpha}{8} + 0.577)^2 + \frac{\pi^2}{4}]^{-1}, \quad \sin\alpha = \frac{\nu}{kb}. \tag{18}$$

The ratio is about 4.5% for $n_{sl} = 50$ slots with $l = 1.25$ cm, $b = 4.5$ cm, $w = 0.075$ cm, and $k = 2.0\ cm^{-1}$.

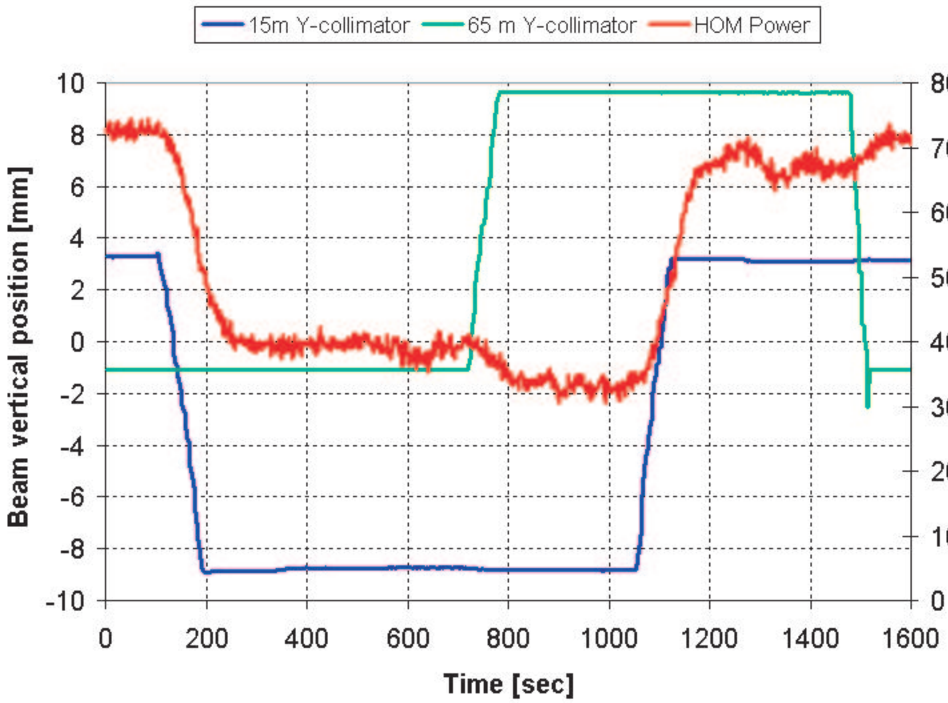

Figure 4: Correlation between distance of the beam to momentum collimator with the heating of the pumping section.

Generally, radiation back to the beam pipe has to be taken into account to find the equilibrium stored energy in the cavity. The energy U stored in a cavity with the surface area S_{cav} and volume V_{cav} is related to the power $P = \eta_{in} P_{TE}$ penetrating through the slot,

$$U = \frac{Q_L}{\omega} P, \tag{19}$$

where the loaded Q-factor $1/Q_L = 1/Q_0 + 1/Q_{ext}$ and Q_0 defines the wall power, $P_w = \omega U / Q_0$,

$$P_w = \frac{Q_L}{Q_0} P = \frac{P}{1 + Q_0/Q_{ext}}. \tag{20}$$

The field in the cavity can be decomposed to the waves to the slot and in the opposite direction. The power of the wave going toward the slot is $P_+ = (c/4\pi)|E_+|^2 S_{cav}$ can be written in terms of the stored energy

$$P_+ = (c/2V_{cav}) U S_{cav}. \tag{21}$$

If the radiated power to the beam pipe $P_{ext} = \eta_{out} P_+ S_{slot}/S_{cav}$, then the external Q-factor

$$\frac{1}{Q_{ext}} = P_{ext}\omega U = \eta_{out}\frac{c}{2\omega}\frac{S_{slot}}{V_{cav}}. \tag{22}$$

Hence, the wall power is

$$\frac{P_w}{P_{TE}} = \frac{\eta_{in}}{1 + \eta_{out} Q_0 (c/\omega)(S_{slot}/2V_{cav})}. \tag{23}$$

Excitation of the HOMs in a gap between the flange and the RF gasket of a vacuum valve was observed in experiment [9].

The excessive heating of the shielded vertex bellows 20 cm from IP was a limiting factor for PEP-II [10]. The $m = 0, 1, 2$ modes were found in 2D simulations of the cavity behind the fingers of the shielded vertex bellows, Fig. (5). Calculated frequencies are in the range 4.75 to 9.8 GHz. These modes can be coupled with the HOMs in the beam pipe and were detected in the BPM 50 cm away from the bellows. The coupling is inversely proportional to the polarizibility of the slot $[\ln(4L/w) - 1]^{-1}$. ($w = 0.81$ mm, $L = 13$ mm).

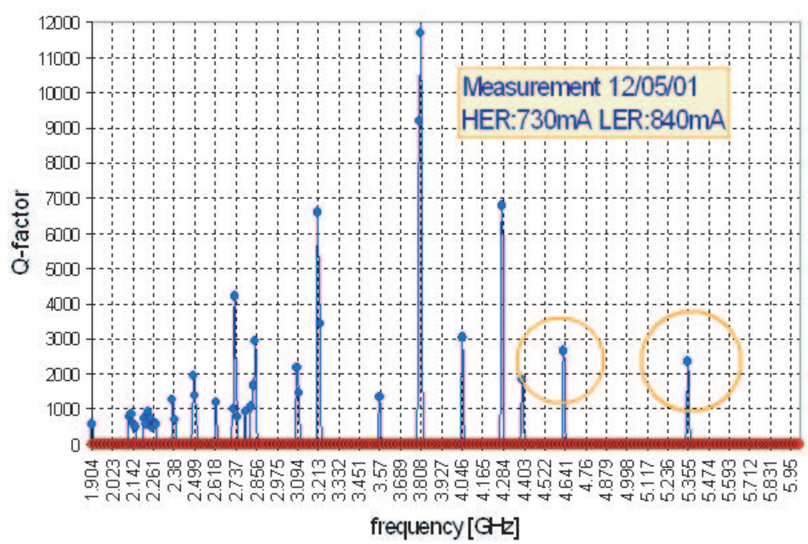

Figure 5: Q-factors of the modes in the spectrum of the BPM signal taken in the gap between trains. The BPM is 50 cm away from the the IR bellows. The circles show modes with amplitudes correlated with the thermocouple temperature.

Another example is given by the pumping section where heating was detected at the absorber installed in the chamber. The heating is strongly correlated with the position of the collimator 15 m upstream from the pumping chamber, see Fig.(4). That indicates the collimator as the source of the HOMs. That is supported by the linear dependence of the HOM power with the RF voltage, (i.e. $1/\sigma_b^2$ dependence), Fig.(6). Such dependence corresponds to real part of the impedance approximately linear with ω within the bunch spectrum contrary to the impedances of resistive wall or cavities.

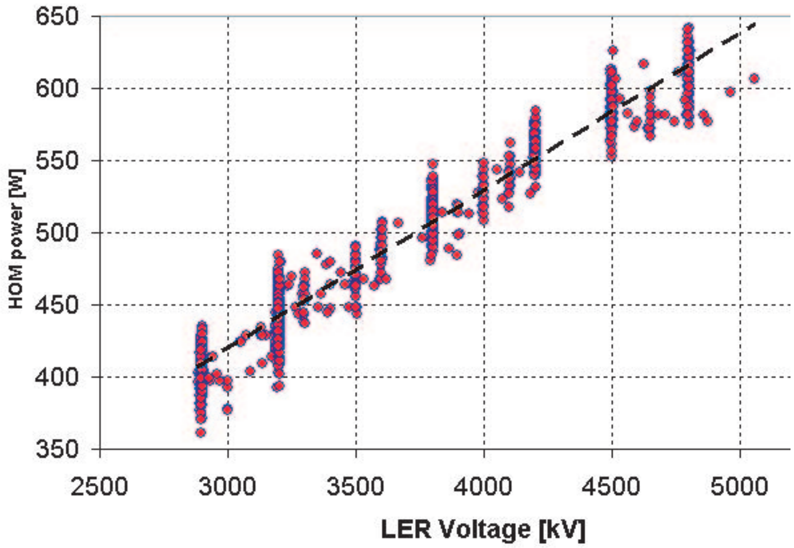

Figure 6: Linear dependence of the power deposited in the pumping section with the RF voltage. Beam current 1400 mA.

3 Impedance budget

3.1 Resistive wall

Resistive wall is an old problem which is under constant development. The classic formula for the impedance of the ring gives

$$\frac{Z_l(n)}{n} = Z_0 \frac{1-i}{2} \frac{\delta_w}{b},$$
$$W_l(s) = -\frac{c_0}{2\pi b} \sqrt{\frac{Z_0}{\pi \sigma_w(\omega) s}},$$
$$k_l = \frac{\Gamma(3/4)}{2\pi \sigma_b} \frac{\delta_\sigma}{\sigma_b} \frac{2\pi R}{b}, \quad \Gamma(\frac{3}{4}) = 1.225. \tag{24}$$

Here the skin depth $\delta_w(\omega) = c/\sqrt{2\pi\sigma_w\omega}$, $\delta_\sigma = \delta_w(c/\sigma_b)$, and $n = \omega R/c_0$ is the revolution harmonic number.

The form-factor was found for a non-circular beam pipe and formulas obtained for multi-layer walls (Burov, Fermilab).

The transverse kick can be written as

$$c\Delta p_x = -N_b e^2 (a' x_l + a x_t) W_\perp, \quad c\Delta p_y = -N_b e^2 (b' y_l + b y_t) W_\perp, \tag{25}$$

where (x, y) and (x', y') are transverse offsets of the test and leading particles, respectively,

$$W_\perp(s) = -\frac{4\delta_0}{b^3}\sqrt{\frac{2\pi R}{s}}, \tag{26}$$

and δ_0 is the skin depth at the revolution frequency. The coefficients are $a' = 1$, $a = 0$, $b' = 1$, $b = 0$ for a circular beam pipe, and $a' = \pi^2/24$, $a = -\pi^2/24$, $b' = \pi^2/12$, $b = \pi^2/24$ for two parallel with the vertical separation of $2b$.

The classic formula is also has to be corrected at small frequencies where the skin depth is larger than the wall thickness due to the leak of the field through the wall. For the B-factories the classic formula with proper form factors should work pretty well because the bunch length will remain about 1 cm (with bunch lengthening) even for the upgraded design.

The resistive wall impedance gives major contribution to the tune shift with current. However, the tune shift measured in a train of bunches in the PEP-II HER gave unexpected result: the slope dQ/dI_b in the train of bunches had opposite signs in x and y-planes, Fig. (1), (2). The effect [11] is explained by the asymmetry of the beam pipe which leads to quadrupolar wake fields excited in the beam pipe. Contrary to the wake in a cylindrically symmetric pipes, the transverse wake in this case depends on the offset of both leading and trailing particles. The wake proportional to the offset of the trailing particle changes the tune. Such a wake builds up for the number of turns defined by the diffusion of the fields through the beam pipe wall of the finite thickness.

It is worth noting that the classic formula has to be corrected at small distances [12]

$$s \simeq s_0 = (\frac{b^2 \delta_\sigma^2}{\sigma_b})^{1/3}. \tag{27}$$

3.2 Inductive impedance

The broad-band impedance of small vacuum components is traditionally described as a single HOM mode impedance with $Q = 1$ and the mode frequency equal to the cut-off of the beam pipe $\omega/c = \pi/a$. At PEP-II we tried to build the broad-band impedance as the sum of the impedances of all vacuum components. Inductance and loss factor of each of them were estimated or calculated numerically. The total broad-band impedance obtained in that way is mostly inductive but the total loss factor is not zero and has to be taken into account. We do that using a simple model of inductive-like impedance [13]

$$Z(\omega) = -\frac{i\omega L}{(1 - i\omega T)^3/2}. \tag{28}$$

That gives the loss factor

$$\kappa_l = \frac{3}{8\sqrt{\pi}}(\frac{La}{\sigma_b^3}). \tag{29}$$

Such impedance with parameters $L = 80$ nH and $\kappa = 3$ V/pC gives reasonable prediction for the bunch lengthening, $d\sigma/dI_{bunch} \simeq 1\ mm/mA$.

It is worth noting that the same vacuum component may change from mostly inductive to resistive with shorter bunch length. It is also important to remember that for the single bunch instabilities such as microwave instability the figure of merit may be the length several times smaller than the rms bunch length σ_b if the instability is due to quadrupole or sextupole modes.

The pure inductive impedance stabilizes the microwave instability. On the other hand, the inductive impedance leads to the bunch lengthening and affects luminosity through the hour-glass effect.

The inductive impedance $Z_L = -i\omega L/c^2$ may lead to the energy-position correlation at IP

$$\delta \simeq \frac{N_b r_e L}{\gamma_b \sigma_b^3 \sqrt{2\pi}} \tag{30}$$

although it does not change the uncorrelated energy spread in the storage ring below the threshold of the microwave instability.

The typical example of a small vacuum discontinuity is the misalignment of two sections of the beam pipes with radius b. Impedance of a small misalignment δ is inductive with inductance

$$L = \frac{4}{3}\frac{\delta^2}{b}. \tag{31}$$

300 hundred of $\delta = 2$ mm misalignments of the beam pipes with radius $b = 2.5$ cm give $L = 7$ nH, about 10% of the total inductance of the machine. That set the acceptable limit on $\delta < 2$ mm.

3.3 Ceramic coating and rough surface

Absorbers were introduced to reduce the heating, Fig. (7). Ceramics of the absorber has very have high $\epsilon \simeq 30$. For thick dialectric layers, $b - a >> a/(2\epsilon)$ and $\epsilon > 1$, the wake potential per unit length is equal to the filed on the beam pipe axis is [14]

$$E(z) = -\frac{4}{a^2}[e^{-z/s_0} - \frac{1}{4\epsilon(1 + z^2/(2a^2(\epsilon - 1))}], \quad s = \frac{a}{2\epsilon}\sqrt{\epsilon - 1}. \tag{32}$$

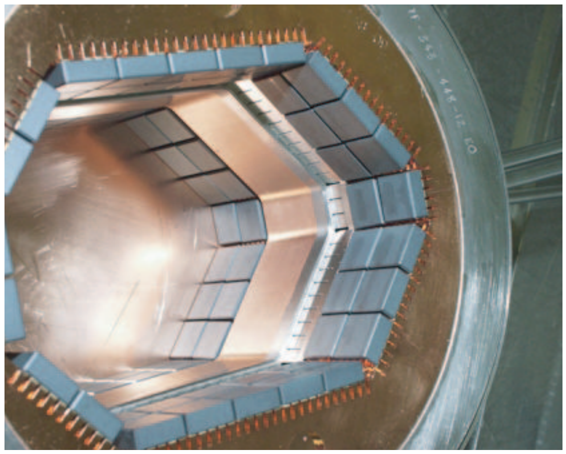

Figure 7: Absorber installed in Q2 bellows.

The energy loss by bunch per unit length is

$$\Delta E = \frac{2N_b^2 e^2}{a^2}, \quad \sigma_b << s$$
$$\Delta E = \frac{2N_b^2 e^2}{a^2}\frac{s}{\sigma_b\sqrt{\pi}}, \quad s << \sigma_b << a\sqrt{\epsilon - 1}. \tag{33}$$

At very large distances, $z > z_m = (b-a)\sqrt{\epsilon}$, the field is, approximately, periodic with the wave length equal to $4z_m$.

Transverse force can be found in [14].

A rough surface of the beam pipe can be considered as a thin dialectric layer with $\epsilon \simeq 2$ [15]. The main effect of the rough surface is generation of the wave which can be synchronous with the beam. The frequency of the mode $\omega_0/c = k_0$ can be estimated from the wave equation

$$k_0^2 = \frac{2\epsilon}{a\delta(\epsilon - 1)}, \quad \epsilon\frac{\delta}{b} << 1. \tag{34}$$

The longitudinal wake [16]

$$w_l(s) = W_c\cos(k_0 s), \quad W_c = \frac{Z_0 c}{\pi a^2} \tag{35}$$

For B-factories, $k_0\sigma >> 1$. In this case, the wake potential

$$W(s) = \int ds'\rho(s')w_l(s - s') = -\frac{W_c}{k_0^2}\frac{d\rho(s)}{ds} \tag{36}$$

corresponding to small inductive impedance.

The thin coating of the ceramic such as in kickers can affect the wake [17]. Results depend on the parameter

$$V = \frac{\sigma_b}{Z_0 \sigma_c w t}, \tag{37}$$

where σ_c is conductivity of the coating ($1/\sigma_c = 43\mu\, Ohm\, cm$ for Ti coating), w is thickness of the ceramic, and t is the thickness of the coating. For $V << 1$ the wake is mostly resistive and follows the bunch density $\rho_b(s)$,

$$W(s) = \frac{2l}{Z_0 \sigma_c b t}\, \rho_b(s). \tag{38}$$

That gives the loss factor

$$\kappa_l = \frac{2R\sqrt{\pi}}{Z_0 \sigma_b}, \quad \text{where } R = \frac{l}{2\pi \sigma_c b t}. \tag{39}$$

4 Individual components

4.1 IR

The heating is a serious problem at IR because the cooling here is extremely difficult. Several issues has to be address in modelling IR. First, it is the narrow-band impedance due to HOMs in the Be beam pipe (±20 cm from IP) and modes trapped between crotches ("Y-shapes"). Simulations [1] show about 12 pill-box like HOMs within the frequency range from 4.6 to 5.92 GHz. Both TM and TE modes were found [18]. The impedance was analyzed using MAFIA in time domain. The Q-factor of the modes in the Be pipe is given by the coupling to the propagating modes due to tunnelling of the modes through the masks. It was estimated that only 10% of the power loss goes to the Be pipe walls [1]. The power deposited by HOMs in Be pipe was estimated as 2.5 W, less than 12 W resistive wall heating.

The broad-band loss factor 0.12 V/pC of the total structure was found with MAFIA. The main source of it is caused by the broad band HOMs generated at the crotches. Such a loss factor corresponds to 1.5 kW of the generated power within the bunch spectrum with $I_b = 1.5$ A beam current. Most of this power goes outside of the IR to the walls, in particular, to the IR bellows. The TE component of the generated HOMs goes through the fingers and causes excessive heating in the IR bellows. The heating corresponding to several kW of the generated power, indeed, was observed and it was a serious limitation on the maximum stored current.

To reduce heating, additional cooling and ceramic absorbers was used. However, the absorbers have to be carefully placed not to introduce additional impedance.

In PEP-II IR, there is is a cut in the crotch from one side of the IR in PEP-II, see Fig. (8), Fig. (9). Such a cut may work as a taper and reduce the generated power. The length of the cut is limited to avoid additional parasitic crossing of two beams. It would be important to carry out more simulations of such a design.

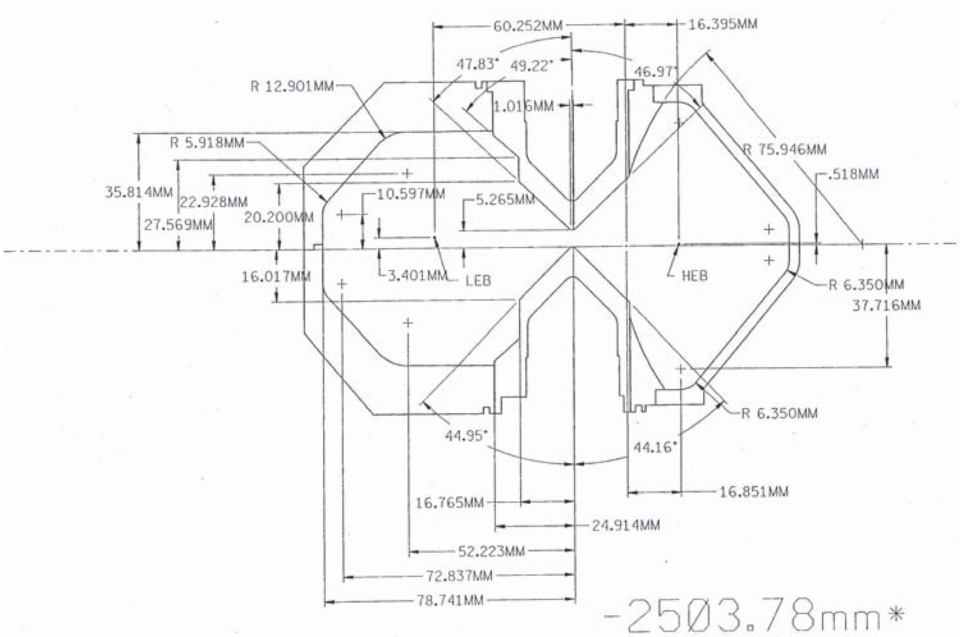

Figure 8: Cross-section of the crotch at IP at the distance 2503 mm from IP.

Another issue is the interference of the wakes generated by two beams simultaneously in the Be pipe. If the IP is at the distances l_1 and l_2 from the end of the pipe, $l_1 + l_2 = 40$ cm, then the amplitudes of the even and odd modes is proportional to the factor

$$A_{\pm} \propto (N_+ + N_-)[\sin(kl_1) \pm \sin(kl_2)] + (N_+ - N_-)[\cos(kl_1) \pm \cos(kl_2)], \qquad (40)$$

where $N_{\pm}$ is the population of two colliding bunches. The power in the even modes scales as $(N_+ + N_-)^2$. For the asymmetric collision point, $l_1 \neq l_2$, the odd modes can be excited even for equal number of particles in both beams.

The fields excited in IR region by one bunch were experimentally detected in the beam pipe of another bunch (A. Novokhatski).

Some ports in the beam tube are introduced for diagnostics and have structure similar to the crotches. For example, the SR monitor in PEP-II uses a long (1.8m, diameter $2a = 3.5$ cm) round pipe to deliver the SR from the beam to the monitor. The pipe goes at a shallow angle $\alpha = 0.1$ to the beam tube (the radius $b = 4.5$ cm). At the joint there is a long elliptical opening in the beam tube wall with the length $14a$. The issue here is to limit the power propagating to the monitor which can not take a lot of the heating. We estimate the loss factor using the code ABCI for a cylindrically symmetric structure scaling result down with azimuthal factor proportional to the ratio a/b and taking into account only part of the bunch spectrum above the cut-off frequency of the pipe.

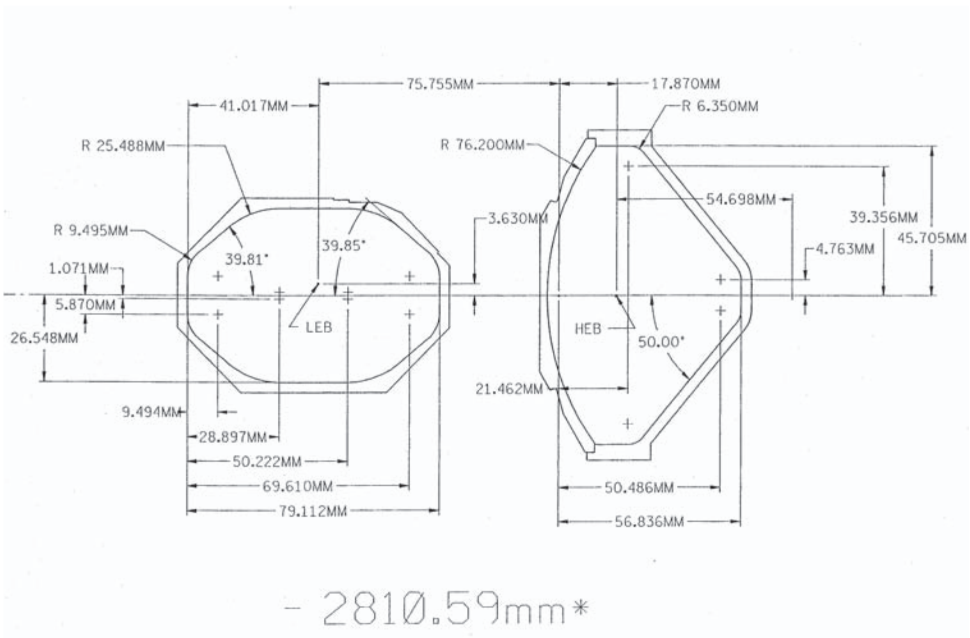

Figure 9: Cross-section of the crotch at IP at the distance 2810 mm from IP.

4.2 Screens

Here there are several issues. First, penetration of the fields into a hole produces an effective bulge. In a thin screen with holes of the radius a, the field penetrates to the distances $\simeq a$ down to the hole (for a slot, the field leaks into a slot by the distance $w/2$). The mesh with the surface area $S = 2L \times b$ of $n_h = S/(8a^2)$ holes placed in a checkered fashion, can be considered as the additional volume $V_b \simeq n_h \pi a^3$ generating a trapped mode. The localization length l of the mode in the rectangular beam pipe $a \times b$, $a > b$ due to a randomly placed slot is

$$\frac{1}{l} = \alpha_m \frac{(2\pi)^2}{4ab^3}. \tag{41}$$

$l < L$ provided $L > (b/\pi)\sqrt{24b/a}$. In the case of a slot in a thin wall, radiation from a slot can give additional to the resistivity of the wall width of the resonance.

At PEP-II, there are two types of screens which differ by the screen thickness. The thin screens, indeed, give problems with excessive heating.

That can be avoided if the the grid with the mesh is bowing into the beam pipe generating a negative bulge or introducing a shallow tapered bump on the inner surface of the screen. Another way is to cover the mesh with bars with the hight d along the beam pipe axes. That makes channels d cm deep. If the bar spacing w is small, the TM HOMs are attenuated by a factor $e^{-2\pi d/w}$ [1].

Secondly, the system of induced dipoles at each hole can be considered as a thin dialectric coating. Such a coating give for long bunches inductive impedance if $\sigma_b > d/\pi$, where d is hole separation. For shorter bunches, the dialectric coating may produce a mode which can be synchronous with the beam and give a substantial loss factor [19].

The third issue is penetration of the HOMs excited by the beam through the screen. While bars attenuate TM modes, they do not help with the TE modes. The latter may require additional bars or a thick mesh on the outer side of the mesh in transverse to the beam direction. Such bars do not produce beam impedance but prevent penetration of the TE modes and excitation of the cavities which may exist behind the screen.

Finally, the holes (or slots) placed periodically produced reflected waves propagating in the beam pipe. The amplitude of the waves can be enhanced coherently at frequencies equal multiple of the hole spacing a, $\omega a/c = 2\pi n$.

4.3 Tapers and collimators

Tapers are used to reduce the impedance generated by the variation of the beam pipe radius.

Shallow tapers $a < r < b$ with the length l are inductive $L = (b-a)^2/l$. The real part is non-zero above cut-off $\omega/c > \pi/a$.

For a shallow taper the inductive part is given by Yokoya [20] and (applicability was corrected later [22])

$$\begin{aligned} Z_l &= -i\frac{\omega L}{c_0^2} \quad L = \int dz[b'(z)]^2, \quad (provided\ kb^2/l << 1) \\ Z_t(\omega) &= -i\frac{Z_0}{2\pi}\int dz(\frac{b'(z)}{b(z)})^2. \end{aligned} \tag{42}$$

Yokoya's analytic formula is valid for small angles.

The real part [21]

$$ReZ_l(\omega) = \frac{Z_0 k}{4\pi b^2}\sum_m \frac{1}{k_{0m}}[|b'(k-k_{0,m})|^2 + |b'(k+k_{0,m})|^2]. \tag{43}$$

Here, $k = \omega/c$, $k_{0,m} = \sqrt{k^2 - k_{0,m}^2}$, $J_0[k_{0,m}b] = 0$, $b'(k)$ is Fourier transform of $b'(z)$. The sum is taken over $k_{0,m} < k$.

Results is generalized to smooth but not necessarily shallow tapers [22].

The kick factor

$$\begin{aligned} \kappa_\perp &= -\frac{1}{\pi}\int_0^\infty d\omega|\rho(k)|^2 ImZ_t(\omega), \\ k_t &= -\frac{c}{2\sigma_b\sqrt{\pi}}ImZ_t. \end{aligned} \tag{44}$$

Note $Z_t(\omega) = Z_t(0)$.

Tapers in PEP-II used to match the large aperture of the RF cavities to the adjacent beam pipe radius. The total loss factor can, actually, larger than of the cavities on the beam pipe with radius equal to the aperture of the cavities, see [19].

Tapers are also used as transition sections between octagonal beam pipes of the arcs and the round beam pipes of straight sections and to reduce the impedance generated in some collimators.

Two type of collimators were studied for PEP-II. The purpose of the gasket collimator was to reduce background in HER. It was a thin iris, Figs. (10), (11). Such a collimator would have the length of 1 cm, the edge of the jaw is about 1 cm from the beam axis, and collimators are about 6 m apart. The LER momentum collimators, Fig. (12), (13), are long two sided tapers with the 10 cm long flat region in the center, the the taper length 32 cm, and the thickness of the collimator 1 inch. The flat region is 12 mm from the beam axis.

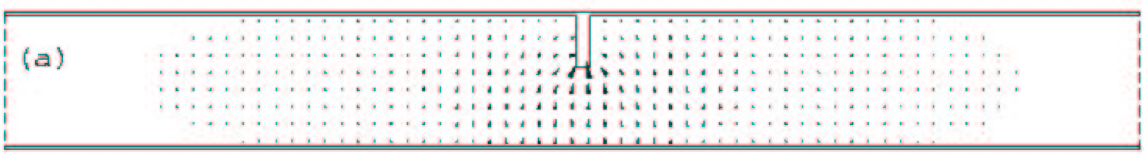

Figure 10: TE-like mode at the gasket collimator: electric field pattern.

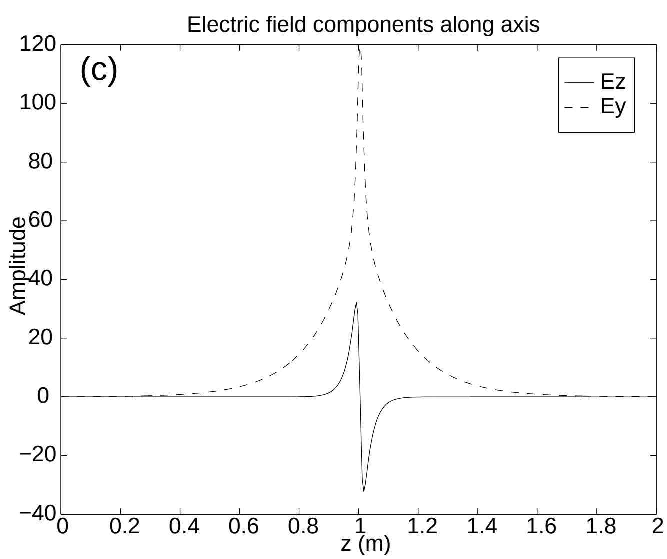

Figure 11: TE-like mode at the gasket collimator: electric field along the beam pipe axis.

Figure 12: TE-like mode at the LER momentum collimator: electric field pattern.

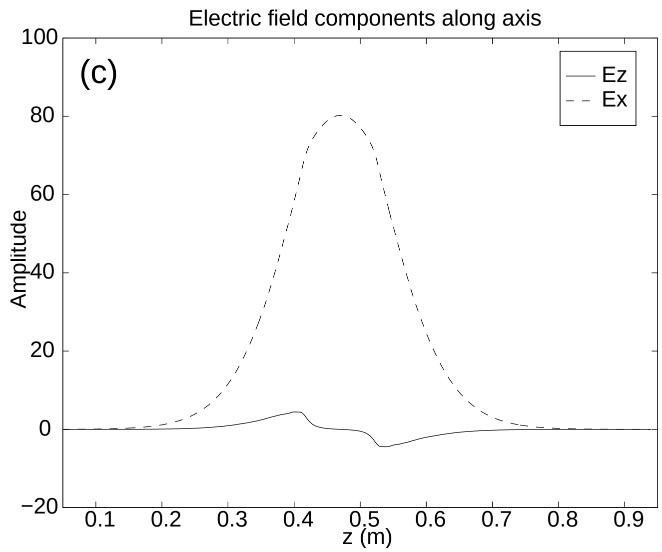

Figure 13: TE-like mode at the LER momentum collimator: electric field along the beam pipe axis.

The trapped modes in collimators are hybrid modes. They are mostly TE modes and were found in both cases [2] and were confirmed in measurements [3]. Similar modes exist in movable collimators. For a gasket collimator, the mode with frequency 1.94 GHz and the transverse shunt impedance $R_s = 26.7$ KOhm is located at position of a jaw. At the momentum collimator the 1.512 GHz mode and $R_s = 0.23$ KOhm is located in the flat region and decays to the ends of the tapers. The beam impedance in the last case is due to the longitudinal TM components of the mode at the ends of the tapers and strongly depends on the transient time along the collimator. The transverse impedance is affected by the thickness of the collimator: increasing the thickness from 1 inch to 2 inches the transverse impedance can be reduced by more than an order of magnitude. The field pattern near the beam axis remains essentially the same but the frequency shift changes the transient time and the impedance substantially for a long collimator. The absorbers placed sidewise on the collimator may substantially reduce the heating [3].

Additional to the trapped modes, the TM-like modes were found between two adjacent gasket collimators. They are slightly above the TM_{01} cutoff frequency of the beam pipe and have the frequency separation about 30-40 MHz. Most of them are strongly coupled to the propagating modes and have low Q_{ext} factor. The highest $Q_{ext} = 1500$ but the shunt impedance is low and such modes are not dangerous for the beam stability.

4.4 Flanges

The joint of two flanges leave, on average, a $100\,\mu m$ wide slot which is terminated by a vacuum seal at the radius $b + l$ from the beam line where b is the beam pipe radius. Here we estimate the power which goes into a slot of flanges [2]). The method we give for illustration and can be used for similar problems. We assume $b = 3.5$ cm and $l = 1$ cm. Actually, there is an RF gasket but we ignore it considering a slot as a terminated quarter wave waveguide. A mode with the frequency $f = 4l/c$ can be excited in the slot depositing P_d power to the wall of flanges. We assume that the amplitude of the azimuthal component of the magnetic field of the mode is H_0 and $E_z = H_0$. The tangential to the flanges electric field is given by the surface impedance $E_t = (1 - i)(k\delta/2)H_0$ where δ is the skin depth of the stainless steel flanges at the frequency f and $k = \omega/c$.

The dissipated power is given by the Pointing vector

$$P_d = \frac{S}{Z_0}\frac{k\delta}{2} < |H_0|^2 > \tag{45}$$

where $S = 2 \times (2\pi bl)$ is the surface area of the flanges and angular brackets mean time average.

The stored energy in the mode in the volume of the slot $2\pi blw$ is

$$< U >= \frac{2\pi blw}{4\pi} < |H_0|^2 > . \tag{46}$$

The voltage across the slot $V = wE_0$, where $E_0 = H_0$ (in CGS units). That defines the shunt impedance $R_s = |V|^2/(2P_d)$ and the Q_0-factor of the mode $Q_0 = \omega U/P_d$,

$$R_s = \frac{Z_0}{2\pi^2}\frac{w^2}{b\delta}, \quad Q = \frac{w}{\delta}. \tag{47}$$

Here we neglected radiation from the slot back to the beam pipe assuming that coupling is small. Otherwise, we have to take into account the external Q_{ext} factor, and use the loaded Q_L-factor, $1/Q_L = 1/Q_0 + 1/Q_{ext}$.

The loss factor is $\kappa = (\pi f)(R_s/Q_0)$.

The power loss of the beam due to excitation of the mode is

$$P = \frac{Z_0 I_{beam}^2 s_b}{4\pi}\kappa. \tag{48}$$

That can be enhanced by factor $D = 4Q/(ks_b)$ if frequency is on the bunch spacing resonance $fs_b/c = integer$.

It is instructive to consider a flange as LCR contour. The image current equal to the beam current I_b splits into current I_C through the parasitic capacitor C and the current I_r which goes through the resistor r and inductance L, $I_b = I_C + I_r$. The eigenfrequency of the contour $(\omega_0/c_0)^2 = 1/(LC)$ and the Q-factor $1/Q = \omega_0 rC$. The induced voltages on the capacitor and $L - r$ chain are the same, $V_C = I_C/(i\omega C) = I_r Z_L$, where $Z_L = r + i\omega L/c_0^2$. Hence, $I_C = i\omega CV$, $I_r = (V/r)(1 + iQ\omega/\omega_0)^1$, and for $Q >> 1$,

$$V = -i\frac{I_b rQ}{\omega/\omega_0 - \omega_0/\omega - i/Q}. \tag{49}$$

The shunt impedance R_s is given by the maximum voltage $R_s = V_{max}/I_b = rQ^2$ at the resonance $\omega = \omega_0$.

The maximum field induced in the gap

$$E_0 = (\frac{V}{w})_{max} = \frac{I_b R_s}{w}. \tag{50}$$

Estimating

$$C = \frac{S}{4\pi w} = \frac{bl}{2w}, \quad r = \frac{2l}{2\pi b\delta\sigma_{wall}} = \frac{2l}{b}\,(\frac{\omega_0}{c_0^2})\,\delta, \tag{51}$$

we get again $R_s = Z_0 w^2/(b\delta)$, $1/Q \simeq \delta/w$, and

$$E_0 \simeq Z_0 I_b \frac{w}{b\delta}. \tag{52}$$

With parameters given above, $\delta \simeq 1\,\mu m$. For the beam current $I_b = 3A$, $E_0 = 45$ kV/cm and can exceed the electric stability.

The reciprocity theorem $Re(V_b I_b^*) = Re(V I_r^*)$, gives that the maximum beam impedance is also equal to R_s.

4.5 BPMs

The PEP-II BPMs are 4-button BPMs with the 1.5 cm diameter of a button. The MAFIA calculations show that there is an HOM mode along the annular gap. One of the questions here is the voltage across the annular gap. Estimate can be done using reciprocity theorem [23].

The voltage across the gap with the width w around the button with radius a is $V_g = I_g Z_g$. The gap impedance

$$\frac{1}{Z_g} = \frac{1}{Z_C} + \frac{1}{R + Z_L}. \tag{53}$$

Here impedance of the parasitic capacitor $Z_C = 1/(i\omega C)$, $C = S/(4\pi w)$, and the surface area $S = 2\pi ad$, where d is the depth of penetration of the field in the gap. For small w the mode with frequencies within the bunch spectrum $\omega/c < 1/\sigma_B$, the field decays exponentially, $d \simeq w/\pi$. Hence, $Z_C = -(i/\omega)(4\pi w)/(2aw) = -iZ_0/(2\omega a/c)$. The resistance of the path across the gap $R = d/(\sigma_W S_\omega)$, where σ_W is conductivity of the wall, and $S_\omega = 2\pi a\delta_\omega$ depends on the skin depth δ_ω. Finally, $Z_L = -i\omega l/c^2$, where $l \simeq w/\pi$ gives the main contribution,

$$Z_g \simeq i\frac{Z_0}{4\pi^2}\frac{w\omega}{c}. \tag{54}$$

The voltage across the gap

$$V_g \simeq \frac{Z_0 I_g}{4\pi^2}\frac{w}{\sigma_B}. \tag{55}$$

For the gap current $I_g \simeq I_b$, with the beam current $I_b = 3$ A, $V_g \simeq w/\sigma_B$ volt, and the field $E_g = V_g/w \simeq 30$ volt. The beam coupling impedance $Z_{cp,g}$ generated by the current across the gap can be found from the reciprocity theorem: $V_b I_b = V_g I_g$, and the definition $V_b = Z_{cp,g} I_b$. For $I_g \simeq I_b$ that gives $Z_{cp,g} \simeq Z_g$, much smaller than the impedance of the annular gap in the beam pipe with radius b. Tha latter is inductive with the inductance [24]

$$L = \frac{\pi a^3}{2b^2[\ln(32a/w) - 2]} \tag{56}$$

and is generated mostly by the current around the gap. More accurate estimate of the current across the gap can be found in the note [23].

4.6 CSR and impedance of the undulator

A bunch in free space radiates wave at low frequencies $\omega\sigma_b/c < 1$ coherently. The coherent synchrotron radiation (CSR) introduces the energy spread in a bunch and may lead to instabilities.

The steady-state CSR impedance per unit length for a dipole is [25]

$$Z(\omega) = -i\frac{Z_0 A}{2}(\frac{\omega}{cR^2})^{1/3}, \quad A = 3^{-1/3}\Gamma(\frac{2}{3})(i\sqrt{3} - 1). \tag{57}$$

The longitudinal wake function per unit length in a week undulator $K << 1$ due to CSR (additional to Coulomb force) is [26]

$$W_l(s) = K^2(\frac{\omega}{c})^2(\frac{\sin 2s}{s} + \frac{3\cos(2s)}{2s^2}), \tag{58}$$

where $s = \omega z/c$, z is distance from the source particle to the test particle, and $K = eB/(mc^2 k_u)$ is the undulator parameter.

The wake of a strong undulator can be found in [27].

Fortunately, radiation of the modes with frequencies $\omega/c = \sqrt{q^2 + k_\perp^2}$ is suppressed in the beam pipe with radius b if $k_\perp < \pi/b$. The synchrotron radiation from the bean with the bend radius ρ at the harmonic $n = \omega\rho/c$ goes in the angle $\theta \simeq n^{-1/3}$. In the beam pipe radiation of such modes is possible only if $k_\perp = (\omega/c)\theta > \pi/b$. That gives the limit

$$\frac{\omega}{c} > (\frac{\pi}{b})^{3/2}\rho^{1/2}. \quad (59)$$

For a bunch with the rms length σ_b that condition is consistent with $\omega\sigma_b/c < 1$ only for short bunches,

$$\frac{1}{\sigma_b} > \frac{\pi}{b}\sqrt{\frac{\pi R}{b}}. \quad (60)$$

However, if the bunch profile has a high-frequency modulation with the wave length small compared with σ_b, the modulation can radiates coherently and the instability can take place. Such instability was observed experimentally. The stabilizing mechanism is given by the energy spread in a bunch.

5 Numerical methods

There are several method used to calculate the impedance of the vacuum components.

The Kroll-Yu method [28] gives Q-factor of a HOM in a cavity connected to a damping waveguide which is assumed to be terminated by a matched load. The method uses as input the frequency of the mode calculated with MAFIA with lossless walls and assumes that there is a single propagating waveguide mode. The Q-factor is determined from calculations with 4 different length of the wave guide (4-point method). Fitting of the spectrum with the Breit-Wigner formula can be useful. When there are several close modes, the identification of the modes in the runs with different waveguide lengths is difficult.

Time domain calculations define the field excited by a bunch going through the structure. The wake is obtained by integration the longitudinal electric field over the pass of the bunch (direct method, preferable for collimators) or along the ports (indirect method for couplers and up-and-down tapers). The calculations for collimators can be simplified closing the structure with the beam pipes and calculating the wakes twice, with and without collimators. The difference gives the result provided the artificial beam pipes are sufficiently separated.

The Fourier analysis of the wake gives impedance. For a narrow-band impedance, the width of the peaks of the impedance gives, in principal, the Q-factor.

The transmission coefficient as function of frequency can be calculated with MAFIA. Deviation of the transmission coefficient from one indicates the losses within the structure and frequencies of the trapped modes. In this way the radiation through the screen slots was detected [18].

The further study of the HOM in the structure can be done with an antenna (a dipole) within the bandwidth of the mode.

The time domain method requires a long downstream beam pipe and a fine mesh with the step $1/5-1/6$ of σ_B. To determine the wake function $W(s)$ within the range $s < s_{max}$, the downstream beam pipe has to be longer than the catch-up distance $\simeq s_{max}/(1-v_g/c)$, where v_g is the group velocity of the scattered waves. Limitations on these method comes from discreteness of the mesh, final resolution of the Fourier transform, and large number of mesh points. The very long downstream pipes are required to determine the long-range wakefields and computation may become not feasible. Analysis of individual propagating modes may give better results in this case.

Much better results can be obtained in the time domain calculations with the moving frame. That allows effective analysis of long structures and short bunches [29]. At the present time, the method is limited to 2D structures.

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6 Appendix 1. Polarizabilities

Impedances of small holes and slots are mostly inductive. Generally, the longitudinal impedance is given [30] by the surface integral over the hole of the harmonics of the azimuthal current $I_\phi(\omega)$ excited by the charge q

$$Z(\omega) = -\frac{1}{2\pi bq}\int dSe^{-ikz}I_\phi(\omega). \tag{61}$$

At low frequencies,

$$Z_l = -i\frac{L_l\omega}{c_0^2}, \quad Z_t = -i\frac{L_t}{b^2}. \tag{62}$$

The inductances are [24]

$$\begin{aligned} L_l &= \frac{\alpha_m + \alpha_e}{\pi b^2}, \\ L_t &= \frac{4(\alpha_m + \alpha_e)}{\pi b^2}\cos(\phi_h - \phi_b). \end{aligned} \tag{63}$$

Transverse kick is directed to the hole in the cross-section containing the hole, ϕ are azimuthal angles to the beam and the hole in this cross-section. For a circular hole

$$\alpha_m = \frac{4a^3}{3}, \quad \alpha_e = -\frac{2a^3}{3}. \tag{64}$$

For thick walls $t > a$ polarizabilities have to be multiplied by 0.56. For an elliptic hole with the angle θ of the ellipse major axes and the beam pipe axes,

$$\alpha_m = \alpha_{m,l} \sin^2(\theta) + \alpha_{m,t} \cos^2 \theta, \tag{65}$$

where

$$\begin{aligned} \alpha_{m,l} &= \frac{\pi l_1^3 e^2}{3[K(e) - E(e)]}, \\ \alpha_{m,t} &= \frac{\pi l_1^3 e^2 (1-e^2)}{3[K(e) - (1-e^2)E(e)]}, \\ \alpha_e &= -\frac{\pi l_1^3 (1-e^2)}{3E(e)}. \end{aligned} \tag{66}$$

The eccentricity $e = \sqrt{1-(l_2/l_1)^2}$, $l_1 > l_2$ are ellipse semi-axes, $K(e)$ and $E(e)$ are complete elliptical integrals of the first and second kind.

For the longitudinal slot

$$\begin{aligned} \alpha_m &\simeq -\alpha_l = -\frac{\pi w^2 l}{16} \\ \alpha_m + \alpha_e &= const w^3 (1 + o(w/l)), \end{aligned} \tag{67}$$

$const = 0.184$ for a rectangular slot and 0.133 for a rounded slot.

For transverse rectangular slots ($w/l < 0.2$, magnetic field is along the slot)

$$\alpha_m = \frac{\pi l^3}{24[\ln(4l/w) - 1]}, \tag{68}$$

and α_e is negligibly small. In the thick wall

$$\alpha_m = \frac{\pi l^3}{24 \ln(8l/w) + \frac{\pi t}{2w} - \frac{7}{3}}, \quad (thick\ wall). \tag{69}$$

7 Appendix 2. Transition from a cavity to a step regime

The high-frequency tail of a cavity with the longitudinal dimension g and beam pipe radius a in the case $g << (\omega/c)a^2$, $g << a^2/\sigma$ can be described [31]

$$
\begin{aligned}
Z_l(\omega) &= \frac{Z_0(1+i)}{2\pi}\sqrt{\frac{cg}{\pi\omega a^2}} \quad , Z_l(-\omega) = Z_l^*(\omega), \\
k_l &= \frac{\Gamma(1/4)}{\pi a}\sqrt{\frac{g}{\pi\sigma}}, \quad \frac{\Gamma(1/4)}{\pi} = 1.15, \\
k_{bot} &= \frac{1}{a^3}\sqrt{\pi g\sigma}.
\end{aligned} \tag{70}
$$

In this case, the diffracted wave can not reach the outer wall of the cavity at $r = b$ and the results are independent of b. Note that the contribution of the high-frequency tail to the transverse kick is small and the kick is dominated by the low frequencies.

In the opposite case of a long step, $g >> (\omega/c)a^2$, $g >> a^2/\sigma$,

$$
\begin{aligned}
Z_l(\omega) &= \frac{Z_0}{\pi}\ln\frac{b}{a}, \\
k_l &= \frac{2}{\sqrt{\pi}\sigma}\ln\frac{b}{a}, \\
k_{bot} &= \frac{2}{a^2\sqrt{\pi}}\ln\frac{b}{a}\, ln\frac{b}{\sigma}.
\end{aligned} \tag{71}
$$

Transition form the cavity-regime to a step-regime takes place at $\sigma g \simeq a(b-a)$.

We can try to get a rough estimate the loss factor of a crotch considering the crotch as a step. If the radius of the beam pipe at IP is the same as radii of the pipe joining at the crotch, the simple consideration of the geometry shows that the maximum variation of the distance of the beam from the wall is $2/\sqrt{3}$. Taking that as the ratio b/a and using the formula for the loss factor of a step, see Appendix 3, we can expect the loss factor

$$
\kappa_l = \frac{2}{\sqrt{\pi}}\ln(\frac{2}{\sqrt{3}}). \tag{72}
$$

That gives $\kappa_l = 0.16$ V/pC for a bunch with the rms length $\sigma = 1.0$ cm (0.11 V/pC for $\sigma_b = 1.5$ cm).