Nuclear Forces. I

The importance of nucleon-nucleon forces for nuclear physics as well as particle physics is self evident. Nuclei are made of nucleons so that knowledge of their interaction plays a fundamental role in the understanding of nuclear structure. On the other hand, among the hierarchy of elementary particles, the nucleons occupy an important position since they are the ground states of the baryon system. The forces between nucleons, their relation to the forces between other baryons as well as between mesons and baryons are significant for the understanding of elementary-particle properties.

Because the two-nucleon system possesses only one bound state, the deuteron, most of our information with regard to nuclear forces comes from scattering experiments. Neutron-proton scattering at very low energy was observed soon after the discovery of the neutron. The early (p–p) scattering experiments of M. White and of Tuve, Heydenburg, and Hafstad demonstrated the existence of specifically non-Coulombic forces acting between protons. It was, however, realized very early that experiments were needed over a wide range in energies. Nearly all of the accelerators in the one to several hundred MeV range were built at least in part in response to the need for higher energy nucleon-nucleon scattering data. What have we learned about nuclear forces in these last two decades, which uncertainties remain and how can these be resolved are the questions we shall touch upon in this note and one to follow.

The particular feature which makes scattering experiments so useful for the determination of nuclear forces is the very strong influence of the centrifugal barrier potential. For a given orbital angular momentum $l\hbar$, the radial dependence of this potential is

$$V(\text{centrifugal}) = \frac{\hbar^2}{M}\frac{l(l+1)}{r^2}, \tag{1}$$

where M is the nucleon mass and r is the inter-nucleon distance. In units of the meson Compton wavelength, Eq. (1) may be rewritten

$$V(\text{centrifugal}) = \frac{\mu}{M}(\mu c^2)\frac{l(l+1)}{x^2}, \tag{2}$$

$$r \equiv x\hbar/\mu c, \quad \hbar/\mu c \sim 1.4 \times 10^{-13}\text{cm}.$$

In Fig. 1 this potential is compared to an attractive potential of the order of magnitude which prevails in the nucleon-nucleon interaction. The total potential, nucleon-nucleon + centrifugal, is given by the solid lines. The broken-line curves give the centrifugal potential. The nucleon-nucleon potential can be neglected for x greater than the value at which the broken-line and solid-line curves join. Each unit along the ordinate corresponds to about 20 MeV. Note that these energies are to be compared with the energy in the center-of-mass system, which for nucleon-nucleon scattering is $\frac{1}{2}$ the energy of the incident nucleon beam in the laboratory.

As we can see from the figure, the effect at a given energy of the centrifugal barrier is to limit the effects of the nuclear potential to just a few l-values. It is this feature which enables one to disentangle the contribution of the various partial waves and obtain a phase-shift analysis. At low energies only $l = 0$ waves enter. By approximately 20 MeV, $l = 1$ partial waves will be above the barrier. In the range $>\frac{1}{2}$ pion Compton wavelength and for $l > 1$ the centrifugal barrier dominates, this dominance becoming more complete as l increases. For nucleon-nucleon total energies of 150-MeV center-of-mass (300-MeV lab) energy the effect of the nucleon-nucleon potential on partial waves with $l > 4$ can be treated as a small perturbation.

Once the phase shifts* are known what do they tell us about the nucleon-nucleon potential? Suppose that at a given energy a particular l partial wave is well below the barrier. Although the barrier will prevent this partial wave from penetrating completely into the region where the nuclear force is important, the incident wave will have a finite amplitude there. This will reflect itself in a small but measurable phase shift. For example, the $l = 1$ phase shift at say 10-MeV center-of-mass energy is of the order of a few degrees, although the classical closest distance of approach is, for the case illustrated in Fig. 1, of the order of 2 meson Compton wavelengths. An effect occurs because quantum mechanically the incident wave extends beyond this point to somewhat smaller values of x. The phase shift for $l = 1$ will thus give a measure of the nuclear force for x somewhat less than and of the order of $x = 2$. As the energy increases so does the penetration into the internal-force region. At the same time higher partial waves, $l = 2$ for example, will start to penetrate appreciably. It should be clear that by a careful analysis of the behavior of the partial waves with energy it should become possible to extract the radial dependence of the nuclear

* A summary of the nucleon-nucleon phase shifts and other scattering parameters is presented in the Conference Summary of the Conference on the N–N Interaction, held at the University of Florida, Gainesville, Florida, in March 1967 (to be published in Reviews of Modern Physics).

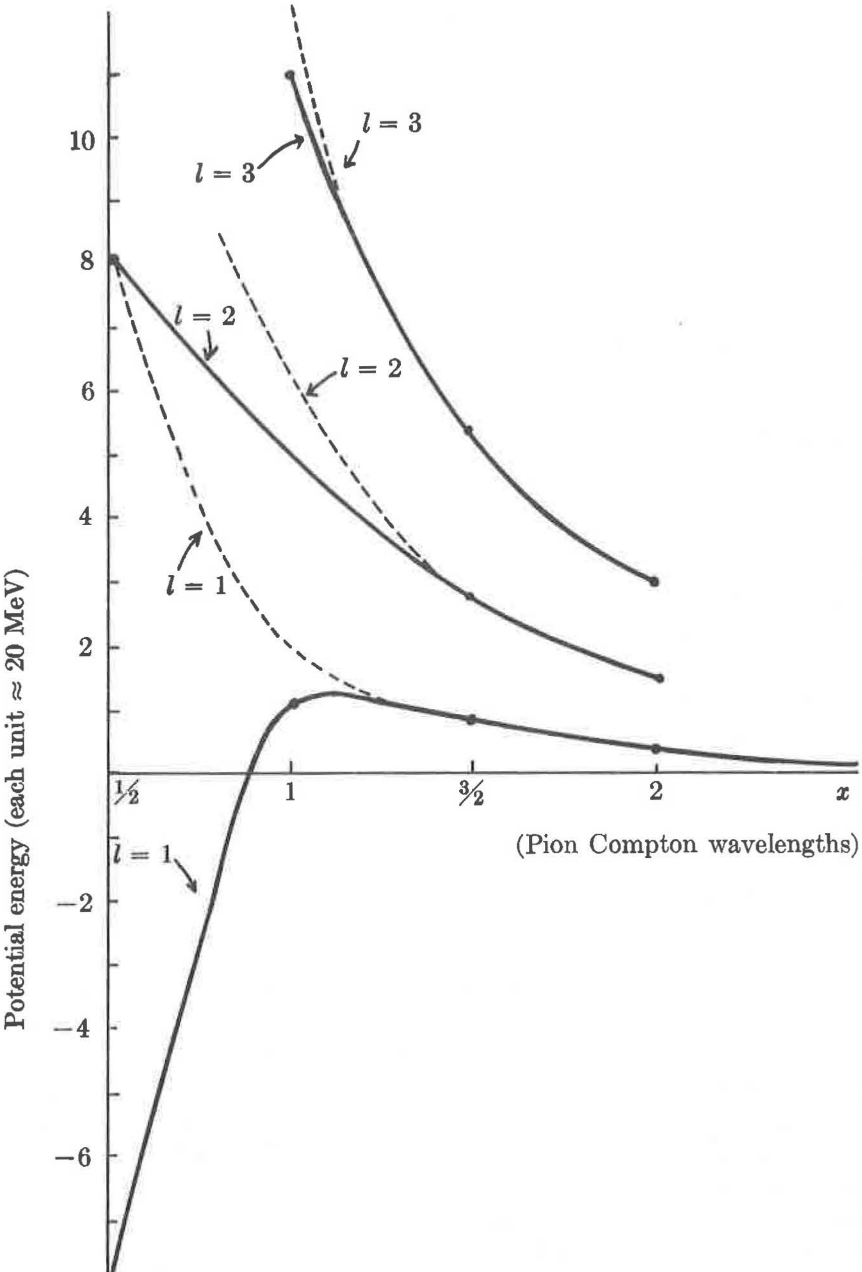
force and its dependence upon angular momentum, the radius of ignorance shrinking as the energy increases. This in essence is the plan explicitly or implicitly followed by the various attempts at the phenomenological analysis of nucleon-nucleon scattering data.

These arguments are not applicable to the $l = 0$ state. In this case the particle can penetrate to small distances. Indeed most of our information on the small-distance behavior of the potential comes from the $l = 0$ phase shifts.

We now shall outline the results which have been obtained using incident nucleons whose energies ranged up to approximately 350-MeV lab energy. In general the potential V between the two nucleons, denoted 1 and 2, has the functional dependence $V(\mathbf{r}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2, \mathbf{L})$, where $\mathbf{r}$ is the interparticle distance, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ the Pauli spin operators, and $\mathbf{L}$ the orbital angular-momentum operator. It is by now clear that for the nucleon-nucleon case dependence of V on $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$, and $\mathbf{L}$ is as complicated as possible within the restriction implied by rotational, reflection, and time-reversal invariance. There is general agreement for sufficiently large nucleon-nucleon separations that the potential is that generated by the exchange of a single pion. (For more detail see Table I below). This potential is called the OPEP (an acronym for one-pion-exchange potential). It is also generally agreed that there is a strong spin-orbit force at small distances. Similarly it is found (this is from the $l = 0$ phase shift) that the two-nucleon wave function goes to zero rapidly at small distances indicating the presence of an effective repulsion there.

Because of the complexity of the potential, it would be too time consuming to record here the various potentials which have been employed. We refer the reader to Refs. 1 and 2 for more complete discussions. Here we shall limit ourselves to those terms which can be compared with the OPEP potential.

Since the two nucleons have an isotopic and ordinary spin of $\frac{1}{2}$ it is possible to classify the states of the system according to their value of T, the total isotopic spin, and S, the total ordinary spin. They can take on values of unity (triplet) or zero (singlet) as indicated in the first two columns of Table I. Central potentials, i.e. $V(r)$, are the only ones possible for states with $S = 0$ while OPEP has both a central and a tensor term for $S = 1$ states. This is indicated in the third column of Table I. In the fourth column we place the distance x, i.e. r in units of the pion Compton wavelength, beyond which the internucleon potential is OPEP. In the fifth and last column we give the distance beyond which there is substantial agreement between the various potentials which have been employed to interpret the scattering data. The numbers in columns 4 and 5 are not to be



Potential energy (each unit ≈ 20 MeV)
10
8
6
4
2
−2
−4
−6
−8
$l = 3$
$l = 3$
$l = 2$
$l = 2$
$l = 1$
$l = 1$
½
1
3⁄2
2
x
(Pion Compton wavelengths)

FIGURE 1

TABLE I

Spin (S)	Isospin (T)	Potential type	OPEP	Region of agreement
0	1	Central	$>$ 1.7	$> \sim 0.5$
0	0	Central	$> \sim 2$	$> \sim 2$
1	0	Central	$> \sim 1.4$	$> \sim 1.1$
		Tensor	$> \sim 1.4$	$> \sim 0.9$
1	1	Central	$> \sim 1.4$	$> \sim 1.0$
		Tensor	$> \sim 1.4$	$> \sim 0.8$

taken too literally. They are intended to show what the area of qualitative agreement is. The precise values will depend on the potential models being compared and upon the criterion for "agreement" employed.

Roughly speaking, except for the $S = 0$, $T = 0$ force, the nucleon-nucleon potential can be considered to be well determined beyond one pion Compton wavelength. The $S = 0$, $T = 1$ potential is better known, down to $\frac{1}{2}$ pion Compton wavelength because of the very accurate data which can be obtained by proton-proton scattering. An example of the disagreement between two extreme models, which can occur below $x = 1$, is sketched in Fig. 2. This is the $S = 1$, $T = 1$ central potential. The bottom curve fits on to an infinitely repulsive core at $x = 0.343$. In spite of the marked difference between the two potentials they both give rise to scattering amplitudes which agree within experimental error. It is clear that very little is known about the potential below $x = 1$. Apparently a strong but short-range repulsion has the same effect to within experimental error as the much more spread out and softer repulsion of the upper curve on Fig. 2.

In the region in which agreement exists (last column of Table I) but with x less than the values at which OPEP is correct, one might ask to what extent does the potential differ from OPEP? For most of the potentials ($S = 0$, $T = 0$ is an obvious exception) the deviation is considerable; at the smallest value of x listed in the last column of Table I the potential is generally twice as large as the OPEP potential except for the $S = 0$, $T = 1$ case for which it is 16 times as large. These factors become considerably larger for even smaller x. The conclusion to be drawn is that OPEP cannot be considered the dominant term in the nucleon-nucleon potential except for x considerably larger than unity; i.e., except for r considerably larger than one Compton wavelength. Conclusions drawn in the past based on the dominance of the OPEP must, therefore, be discarded.

How can a choice be made on an empirical basis between the differing nucleon-nucleon interactions for $x < 1$? A considerable increase in the accuracy of the experimental data is one way. Recent improvements[3] in the precision p–p phase-shift fits suggest that in the near future a choice for the

p–p potential, i.e., $T = 1$ for $x < 1$ may be possible. An order of magnitude improvement in the quality of the (n–p) data is required before the $T = 0$ potential will become as accurately known as the $T = 1$ potential. Increasing the energy may not prove very useful because pion production becomes increasingly important.

Another way to proceed is to probe the two-nucleon system with a third particle. To obtain information in this way it is necessary to be able to detect the interaction of the third particle with the two nucleons when the

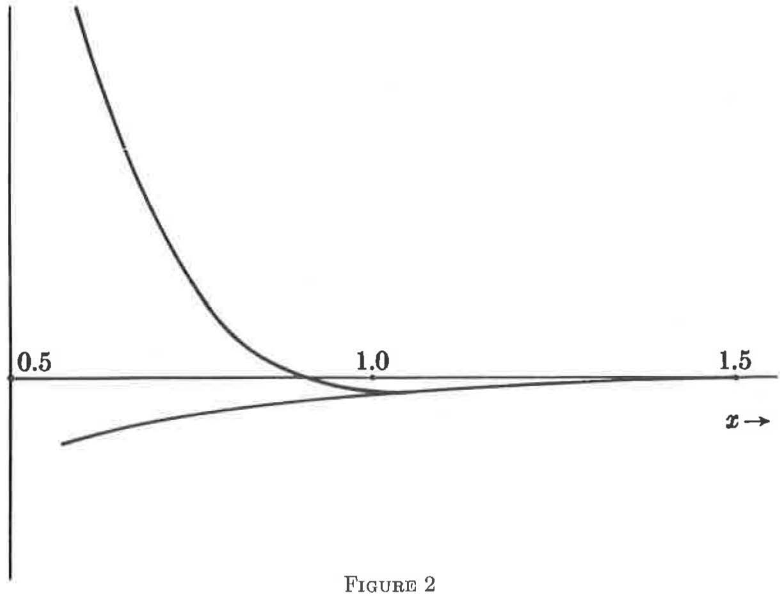


FIGURE 2

latter are close together. Although a meson or a nucleon could in principle be the third particle, the interpretation of the experimental results is much simpler when the interaction is the weaker electromagnetic interaction present when the third particle is an electron or a photon. Experiments involving the photodisintegration of the deuteron have so far been not sufficiently accurate to provide more than a verification that a good phase-shift analysis has been made. Recently measurements have been made of the radiation of photons when protons collide. It has been verified that the cross sections are of the correct magnitude, but these experiments are too recent for it to be possible to quote any deduction with regard to nucleon-nucleon forces.

The electron-deuteron interaction has yielded two results. One that the deuteron wave function is very small at small r, verifying the effective repulsion in this region as seen in nucleon-nucleon scattering. A second though more controversial result is that the e–d data are more consistent with electron-nucleon scattering data when the D-state percentage of the deuteron wave function is of the order of 4–5%.

To conclude, although the experiments of the last two decades have done much to delineate the nature of nuclear forces, precise information is not available for many of the components of the nucleon-nucleon potential for distances less than one pion Compton wavelength (see Table I for more detail). Much more experimental and theoretical work is required in order to extend our knowledge into this domain. It is somewhat ironic to realize that information in this domain is important for adequate nuclear-structure calculations.

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